GRANULAR MATTER

So much for the jamming point

The concept of an evolving jamming density explains a multitude of mechanisms in granular matter. Simulations of systems with friction now consolidate this notion and highlight that the jamming point is a variable that can move in various ways whenever the system is deformed.

Stefan Luding

Granular matter appears in various forms and contexts: as food or detergent powders in the household, as sand on the beach, as foundations under buildings, streets or railways, and as soil and stones that form our landscape — sometimes giving rise to hazards like landslides. What makes such systems so interesting? If these particulate materials would behave either as fluids or solids, we could use existing theories to understanding them; the classical limits of Newtonian fluids and static solids are well described by hydrodynamics and solid mechanics, respectively[1]. Very few theories combine both regimes from first principles[2], and many open questions remain. Soft and granular matter are special, displaying hybrid or alternating states between the classical limits, featuring peculiar mechanical properties like dilatancy (an increase in volume, like wet sand becoming dry when you put your foot on it), slow creep (under constant stress), stress relaxation (under constant volume) or ratcheting (accumulation of deformation, like roads deforming under repeated traffic).

Writing in *Nature Physics*, H. A. Vinutha and Srikanth Sastry[3] present a study of the effects of friction in granular media, highlighting the relevance of the shear strain amplitude[4,5] — next to that of the shear rate. Their simulations provide evidence of emergent, solid-like geometric features in the fluid regime, as observed earlier for frictionless systems[6], in a wide range of densities (volume fractions), well below the so-called jamming point, ϕ_J^0, at which jamming would occur under purely isotropic compression in the limits of vanishing friction, pressure, temperature and deformation rate. This fluid–solid transition under compression (often simply referred to as jamming) was believed to be governed by the system's density, ϕ, pressure, p, and the 'granular temperature' (fluctuation kinetic energy), T_g, as reflected in the (ϕ, p, T_g) phase diagram[7]. However, the phase diagram cannot explain the observed protocol dependence (different system-preparation paths lead to different states at the same point in phase space) or phenomena like shear-jamming (states that would otherwise be 'unjammed' jam upon considerable shearing) and slow compaction (like the gentle tapping of a box of powder results in an extremely slow increase in its density)[4,5,8,9].

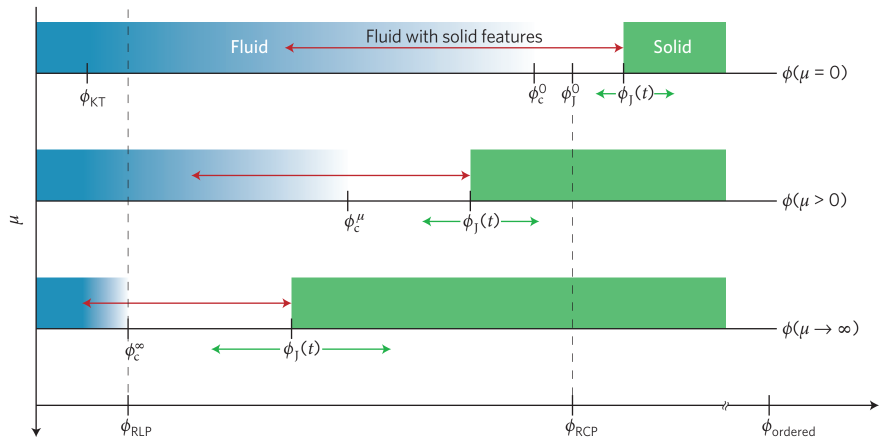

Figure 1 | Schematic of granular- and soft-matter density regimes, below and above the jamming density, for three different types of materials. Material state versus density (ϕ) for zero friction ($\mu = 0$; top), moderate friction ($\mu > 0$; middle) and large friction ($\mu \to \infty$; bottom), where μ is the coefficient of friction, blue indicates a fluid state, red arrows indicate unjammed states with solid features, and green indicates a solid state. Cases with different friction have now been investigated by means of simulations of sheared hard-sphere systems[3]. Considering one realization of a finite system, but for a different material or friction, the lowermost densities ($\phi < \phi_{KT}$) occur for random, collisional fluids that are well described by kinetic theory[11,13]. The density ϕ_{KT} above which this model fails is close to random loose packing, where $\phi_{RLP} \approx 0.54$. The intermediate regime ($\phi_{RLP} < \phi < \phi_J$) features fluids with solid features[3,4,6,11] below the variable, history-dependent jamming density ϕ_J. At higher densities ($\phi > \phi_J$), the states are jammed, 'solid-like' states[5] that are not strictly solid, but have a finite probability to flow, creep, relax, slip, yield or restructure with plastic deformations — the jamming point $\phi_J(t)$ varies with time t — and could thus be referred to as solids with fluid features. (Note that the random close packing density $\phi_{RCP} \approx 0.64$ happens to be close to the special jamming density, ϕ_J^0, but that it is by no means the upper limit for ϕ_J as both disordered and ordered structures, at and above $\phi_{ordered}$, can be present well above the fluid regime[5,12,13].) The density ϕ_c^μ, located in the fluid region, represents the well-defined, material-dependent steady-state or critical-state density[1] that is reached after applying large shear strain at vanishing pressure, with the limit values $\phi_c^0 < \phi_J^0$ and $\phi_c^\infty \approx \phi_{RLP}$ for zero and very large friction, respectively.

Historically, many researchers have been focusing on the jamming transition itself to find the missing descriptive ingredients, and it has long been agreed that an additional state variable, for example, the coordination number (the number of contacting neighbours of a particle), is needed to explain the phenomenology of solid-like soft and granular matter. Recently, an alternative, strikingly simple idea that the jamming density, ϕ_J, itself is the missing state-variable[5] was put forward and supported by detailed studies of the evolution of ϕ_J in compressed and sheared frictionless systems, above and around ϕ_J^0. As ϕ_J is variable, one has the strain measure, $\varepsilon_J = \log(\phi/\phi_J)$, as an indicator for 'how far' the system, at density ϕ, is from (un)jamming, where ε_J changes sign. Starting from the jammed regime ($\varepsilon_J > 0$), unjamming occurs when the elastic pressure, $p = B\varepsilon_J$, with the system's bulk modulus $B \geq 0$ (refs 2,10), approaches zero. Similarly, starting from the unjammed regime ($\varepsilon_J < 0$), jamming is typified by the reverse sign change. It is crucial to realize that even for a given material, the jamming density is not a constant (even if pressure or density are held constant) and that history/protocol dependence is then a consequence of the evolution of ϕ_J.

Vinutha and Sastry[3] have now added friction to the above formalism, which was based on frictionless particle systems[5], and examined the wide open regime below the transition ($\varepsilon_J < 0$) in cohesionless granular matter. They identify the boundaries of this interesting fluid-like regime with solid features (discontinuities in the pair-correlation functions) as the random loose and the random close packing densities, ϕ_{RLP} and ϕ_{RCP}, respectively (Fig. 1). For unjammed states, friction is necessary for establishing, through finite shear strain, mechanically stable solid-like states with structural anisotropy. Only if friction is strong enough, and if the procedure is sufficiently dissipative and slow, solid, shear-jammed states[8] can be established in the full range[3]. Although it was evident in 'solid-like' states (solid with fluid features) above jamming that the jamming density changes when the system restructures (and thus the coordination number and moduli, such as B, change[5,10]), the new results support the idea of an evolving ϕ_J also below jamming.

Two questions remain: what are the mechanisms for restructuring in the absence of a mechanical-contact or force network, and what is the evolution equation for ϕ_J?

Over-compression to larger pressure[5] or tapping/tempering, as often applied in experiments[8], are (mostly) isotropic modes of perturbation that can cause irreversible (plastic) restructuring events, possibly — but not necessarily — with ongoing (local) ordering or crystallization[4,9]. Such events will, on average, lead to denser, more efficient packings that must have a higher (jamming) density after the event. Thus, such deformations are responsible for slow changes (evolution) of the jamming density[4,5,9], whereas the actual numerical values of ϕ_J (and the range available, which is narrow for frictionless materials and very wide in the presence of friction) depend on the particle-size distribution[6,11,12], the particles' shapes and the contact properties (not only friction[3], but also roughness, cohesion and so on). Both creep at fixed pressure and stress relaxation at fixed volume, particular manifestations of soft- and granular-matter behaviour as mentioned above, are then just the consequence of a slowly increasing ϕ_J that results in a decreasing volume ($1/\phi$) and pressure (p), respectively.

In contrast to mechanisms that lead to densification, that is, an increase in packing efficiency, there are fundamentally distinct modes of deformation. Shear modes result in plastic events (mostly) reducing the packing efficiency; this happens fast, with a probability increasing with the strain amplitude. Dilatancy, mentioned above, is then the consequence of a decrease in ϕ_J for general shear deformations, but also in the special cases of either constant pressure or constant volume shear. By the same token, systems that are sheared, starting from an initially unjammed state, jam at a finite shear strain[3,5,8,9], just because ε_J transits from a negative to a positive value due to a decreasing jamming density. Adding different deformation rates (not discussed here) and the consequent variation in ϕ_J (ref. 4) will complete the picture.

The work by Vinutha and Sastry[3] thus adds important insights, complementing other recent research: fundamentally different roles are played by tapping (T_g), isotropic (compression) and deviatoric (shear) deformations — both above and below jamming.

In the opinion of this author, the evolution of the microstructure due to previously applied deformations is the most essential ingredient for a meaningful model for granular (and soft) matter. The microstructure contains the information on how different deformation paths have affected the present mechanical state (structure) of the system. In other words, the structure — both isotropic (ϕ_J) and anisotropic (not discussed here, see refs 5,10) — memorizes the history of the packing. The many peculiar effects like hysteresis, ratcheting, dilatancy, creep, relaxation and so on are then a consequence of the evolution of ϕ_J — but not new mechanisms. ❐

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