

On Time Scales and Rheology of Dry and Wet Granular Materials

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ABSTRACT This paper results from an ongoing investigation of the rheology of dry and wet granular materials. We describe the different time scales involved and the related dimensionless numbers that govern dry as well as wet granular flows, with a focus to the dimensionless numbers relevant for flow in the quasistatic wet system at low confining stress. The macroscopic friction coefficient and the apparent viscosity of granular materials are the two vital parameters that can also be deduced from experiments. While the time scales are coupled to each other in the intermediate bulk of the materials and interplay among each other, they get decoupled at the extremes of the free surface and the depth of the material due to one time scale being way smaller in magnitude than the other. In the decoupled state, the smaller time scale influences the rheology in different ways, the role of the larger time scale being negligibly small. Furthermore, the capillary force related time scale results in an increase in viscosity of the materials with cohesion, especially close to the free surface.

1. INTRODUCTION

The ability to predict a material's viscosity gives manufacturers an important product property describing their flow. Knowledge on material's rheological characteristics is important in predicting their pourability, density and ease with which it may be handled, processed or used. The interrelation between rheology and other product dimensions often makes the measurement of the friction coefficient and the viscosity the most sensible and convenient way of detecting changes in flow properties. These macroscopic properties in an inhomogeneous system of granular materials are often dictated by different dimensionless numbers obtained from local time scales. Thus, a study of different local scales and the associated dimensionless numbers helps us to understand the change in flow properties e.g. with increasing cohesion.

The number of contacts of the particles and their rearrangement time are driven by the local pressure. When particles come in contact, the particles form a part of the network and open/ break after a lifetime given by the contact duration which depends on the particle stiffness. For granular system sheared at a certain shear rate, in simple shear, the contact network rotates at approximately the same rate. This makes the system eventually unstable with self-destruction and rearrangement of the particles occurring at a time scale related to the shear rate. The interaction of the particles on the other hand also depends on the presence of other external forces like gravity or attractive forces such as Van der Waals or liquid bridge capillary forces. Each of these is associated with a time scale too. While some time scales are globally invariant, others vary locally, changing the flow behavior accordingly. In general, the flow of the materials is governed by the relative importance of each time scale relative to another, expressed by several dimensionless numbers.

2. NUMERICAL SET-UP AND PARAMETERS

We use MercuryDPM [1, 2], an open-source implementation of the Discrete Particle Method, to simulate a shear cell with annular geometry and a split bottom plate, filled with dry or wet granular materials, and sheared in slow, quasistatic conditions. The details of the numerical set-up are explained in [3, 4] and thus are not discussed here. The simulations are run for different surface tensions of liquid, thereby varying the microscopic and thus the macroscopic cohesion, as discussed in [4]. The liquid bridge contact model is based on a combination of an elastic-dissipative normal repulsive force and a non-linear, irreversible, non-contact capillary force as described in [3, 4].

3. TIME SCALES

Dimensional analysis is often used to define the characteristic time scales for different physical phenomena that the system involves. Even in a homogeneously deforming granular system, the deformation of individual grains is not homogeneous. Due to geometrical and local parametric constraints at grain scale, grains are not able to displace as affine continuum mechanics dictates they should. The flow or displacement of granular materials on the grain scale depends on the timescales for the local phenomena and interactions. Each time scale can be obtained by scaling the associated parameter with a combination of particle diameter d_p and material density ρ . While some of the time scales are globally invariant, others are varying locally. The dynamics of the granular flow can be characterized based on different time scales depending on local and global variables. First, we define the time scale related to contact duration of particles which depends on the contact stiffness k as given by [5]:

$$t_k = \sqrt{\frac{\rho d_p^3}{k}} \quad (1)$$

In the special case of a linear contact model, this is invariant and thus represents a global time scale too. Two other time scales are globally invariant, the cohesive time scale t_c , i.e. the time required for a single particle to traverse unit length scale under the action of attractive capillary force and the gravitational time scale t_g , i.e. the elapsed time for a single particle to fall through half its diameter under the influence of the gravitational force. Ideally, the time scale t_c should vary locally depending on the local capillary force f_c . However, the capillary force is weakly affected by the liquid bridge volume and strongly depends on the surface tension of the liquid γ , thereby making the time scale a global parameter given by:

$$t_c = \sqrt{\frac{\rho d_p^4}{f_c}} \propto \sqrt{\frac{\rho d_p^3}{\sigma}} \quad (2)$$

with surface tension σ and capillary force $f_c \approx \pi \sigma d_p$. The surface tension of liquid is varied from 0 to 0.50 Nm⁻¹ in the simulations.

The corresponding time scale due to gravity which is of significance under small stress is defined as:

$$t_g = \sqrt{\frac{d_p}{g}} \quad (3)$$

The global time scales for granular flow are complemented by locally varying time scales. Granular materials subjected to strain undergo constant rearrangement and thus the contact network re-arranges (by extension and compression and by rotation) with a time scale related to the local strain rate field:

$$t_{\dot{\gamma}} = \frac{1}{\dot{\gamma}} \quad (4)$$

Finally, the time for rearrangement of the particles under a certain pressure constraint is driven by the local pressure p . This microscopic local time scale based on pressure is:

$$t_p = d_p \sqrt{\frac{\rho}{p}} \quad (5)$$

As the shear cell has an unconstrained top surface, where the pressure is infinitely small, this time scale varies locally from very low (at the base) to very high (at the surface). Likewise, the strain rate is high in the shear band and low outside, so that also this time scale varies between low and high, respectively.

All the dimensionless numbers in our system are discussed in brief for the sake of completeness, even though not all are of equal significance.

4. DIMENSIONLESS NUMBERS

Dimensionless numbers in fluid and granular mechanics are a set of dimensionless quantities that have a dominant role in describing the flow behavior. These dimensionless numbers are often defined as the ratio of different time scales or forces, thus signifying the relative dominance of one phenomenon over another. In general, we expect five time scales (t_g , t_p , t_c , t_γ and t_k) to influence the rheology of our system. The ratio of the square of the first two time scales is given by:

$$P_g^* = \frac{f_p}{f_g} = \frac{t_g^2}{t_p^2} \quad (6)$$

where, $f_g = \rho g d_p^3$ is the gravitational force on a single particle and $f_p = p d_p^2$ is the normal force on a single particle due to the confining stress. This local dimensionless number vanishes at small pressures close to a free surface, where the effect of the confining pressure is less significant than that of gravity ($t_p > t_g$).

The other dimensionless number which is only significant for cohesive granular materials is the Bond number Bo . While the conventional way of defining the Bond number as the ratio of the forces f_c and f_g [6] is appropriate for single particles, or close to the free surface, we define the local Bond number relative to the confining force:

$$Bo = \frac{f_c}{f_p} = \frac{t_p^2}{t_c^2} \quad (7)$$

where, f_c is the capillary (attractive) force between two particles in contact. The experimentally measurable global Bond number Bo_g is defined as the minimum of the local Bond number, i.e. the Bond number corresponding to measured pressure at the base of the flow.

The other dimensionless number which is of significance in dynamic flow as shown in [7] and less relevant in quasistatic flow is the inertial number [8, 9]:

$$I = \frac{\dot{\gamma} d_p}{\sqrt{p/\rho}} = \frac{t_p}{t_\gamma} \quad (8)$$

The Inertial number is the ratio of the time scales related to pressure and strain rate. This provides an estimate of the local rapidity of the flow. In quasistatic flow, particles interact over enduring contacts and inertial effects are negligible ($t_p \ll t_\gamma$).

The fourth dimensionless number which is often relevant in granular flow but of little relevance for the relatively stiff particles studied here (t_k very small), is the local compressibility defined as:

$$P^* = \frac{p d_p}{k} = \frac{t_k^2}{t_p^2} \quad (9)$$

which is a good way to define the dimensionless pressure.

In summary, five time scales allow to define four dimensionless numbers, where $p_g^* > 5$ defines the bulk of the system, and in the bulk three unique numbers control the rheology, namely, I , Bo , p^* , which dominate in inertial, cohesive and soft (high confining stress) systems, respectively. While the effect of both inertial number and softness was reported in [4] and [10], we consider cohesion and the fourth dimensionless number p_g^* both to be predominant close to the free surface, identified by the regime $p_g^* < 5$, where compression is weak and both f_c and f_g are comparable to f_p , *i.e.* the corresponding time scales t_c , t_g and t_p are interfering. In the bulk one should consider t_p .

5. LOCAL MACROSCOPIC FRICTION COEFFICIENT

The effects of p_g^* and Bo on the macroscopic friction coefficient μ are predicted as:

$$\mu(p_g^*, Bo) = \mu(p^*, I) \left[1 + aBo \left[1 - b \exp\left(-\frac{P_g^*}{p_{go}^*}\right) \right] \right] \quad (10)$$

where in the right side of the above equation, the first function denotes the dependence of μ on Bo and the second function denotes the dependence of μ on p_g^* .

The values of the fitting constants are reported in [4] and are $a \approx 0.22$, $b \approx 0.72$ and $p_{go}^* \approx 2.38$. Figure 1 shows the local macroscopic friction coefficient $\mu = \tau/p$ as a function of the scaled pressure p_g^* for different intensity of cohesion in sheared granular materials. The dash dotted lines represent the predicted friction coefficient for different surface tension of liquid (see Bo_g inset). While we show the prediction of macroscopic friction coefficient for weakly to moderately cohesive granular materials (up to $Bo_g = 0.12$), we discuss about the effects on viscosity for higher cohesion as well in the next section. A comparison of the local friction coefficient as a function of the scaled pressure shows that μ approaches exponentially to the dry, rigid, quasistatic limit value $\mu_o = 0.15$ in the high pressure limit ($t_g \gg t_p$). It is further observed that trend of the macroscopic friction coefficient changes with the global Bond number and $Bo_g = 0.06$ marks the intermediate case of the change in this trend. For higher wet cohesive materials, μ is very high at small pressure due to the contribution from high local Bond number ($t_c \ll t_p$).

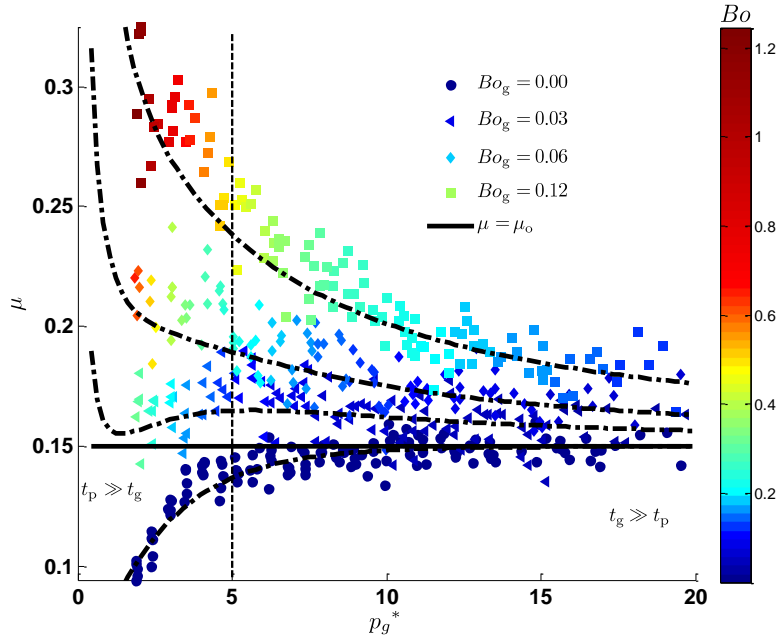


Figure 1 Predicted local macroscopic friction coefficient (μ) as compared with the simulation results as a function of pressure (p_g^*) with the representation of different timescales. Different symbols indicate different global Bond numbers (Bo_g). The dotted lines indicate μ for different global Bond numbers as predicted by Eq. (10). The vertical dashed line represents the demarcation between the free surface and the bulk of the material given by $p_g^* \approx 5$.

6. GLOBAL APPARENT VISCOSITY

In continuation of earlier discussions on the system parameters, time scales, dimensionless numbers and the locally varying macroscopic friction coefficient, now we focus our attention on bulk properties e.g. the global apparent viscosity of the granular fluid. Any gradual deformation of the material gives rise to process-changed forces in its interior, due to restructuring and friction between adjacent particles. Deformation at a finite rate gives rise to the “apparent” viscosity ($\tau/\dot{\gamma}$) i.e., shear stress per shear rate with interesting non-Newtonian flow features for dry as well as wet granular materials. For fluids, it is critical to apply rheometrical techniques for their characterization, specifically for measuring their physical flow properties like viscosity. However, for granular materials this approach is still a challenge for several reasons; for example, these materials may exhibit a strongly non-linear behavior depending on the flow conditions like strain localization as shear banding. There is added complexity for wet granular materials due to their variable cohesion due to liquid migration (neglected here), with unknown constitutive flow behavior.

The shear cell set-up constrained by pressure under gravity has the advantage of getting data at different pressures, strain rates and densities from a single simulation. One can extract quantitative data best in the shear band region by long-term averaging. We analyze the mean apparent viscosity at the center of the shear band (termed as the global viscosity $\tilde{\eta}$) as a function of the global Bond number Bo_g . Figure 2 shows that the global viscosity increases linearly with the global Bond number. The solid line represents the linear function:

$$\tilde{\eta} - \eta_{dry} = \eta' Bo_g \quad (11)$$

where $\eta' = 747 \text{ Pa}\cdot\text{s}^{-1}$ and $\eta_{dry} = 233 \text{ Pa}\cdot\text{s}^{-1}$ are the specific cohesive viscosity (per Bo_g) and the global viscosity for dry granular materials respectively. However these constants are not done validated in systems with different gravity, rotation rate and filling height. Thus, the dominance of the capillary force relative to the pressure results in an increase in global Bond number, thereby increasing the global viscosity of the material in an experimental set-up. This increasing viscosity can be related to the increasing torque and shear stress for cohesive wet granular materials as discussed in [11].

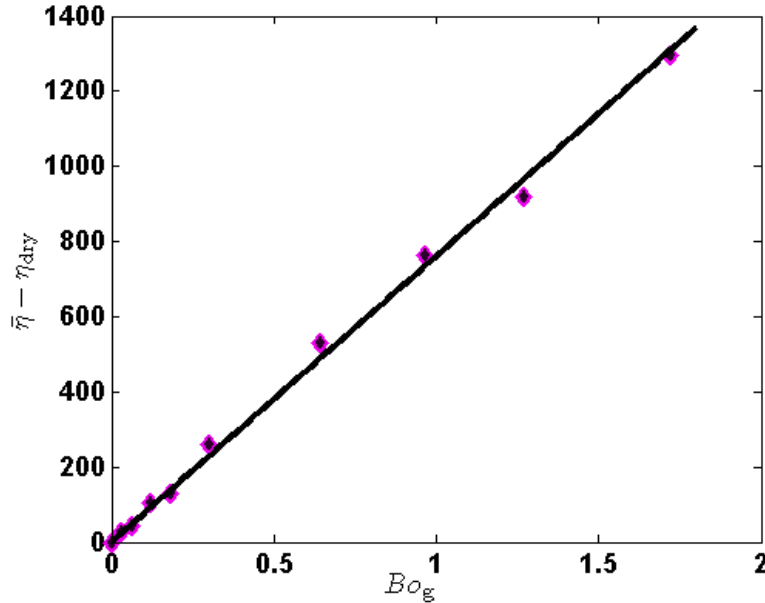


Figure 2 Global apparent viscosity due to wet cohesion $\tilde{\eta} - \eta_{dry}$ as a function of the global Bond number Bo_g . The solid line represents Eq. (11).

7. CONCLUSIONS

We studied the time scales that are significant in the bulk and close to the free surface of the granular flows, respectively. The time scales can be associated to define different dimensionless numbers relevant for a general granular rheology. More specifically, we focused on the dimensionless numbers that are significant for the quasistatic flows in our system with almost rigid particles. The significant flow properties like the macroscopic friction coefficient and the apparent shear viscosity are studied for dry and wet granular materials and are shown to be controlled by different dimensionless numbers. Thus, the flow properties are highly affected by locally dominated phenomena, which are represented by the interplay of local time scales. The presence of strong gravitational forces and the cohesive capillary forces are shown to play the dominant role for particles constrained under small pressure close to the free surface. The global apparent viscosity in the critical state increases linearly with the intensity of wet cohesion, represented by the global Bond number.

9. ACKNOWLEDGEMENTS

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10. NOMENCLATURE

d_p	: mean particle diameter [m]
ρ	: particle density [kg/m^3]
k	: particle stiffness [N/m]
σ	: surface tension of liquid [N/m]
g	: acceleration due to gravity [m/s^2]

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