SIMULATIONS OF GRANULAR FLOW: CRACKS IN A FALLING SANDPILE

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We present numerical simulations of the fall of a two dimensional (2D) model granular material in a rectangular container and study the decompaction of this granular assembly consisting of non-cohesive particles. During the fall we observe the occurrence of cracks splitting the initial pile into pieces. Cracks preferentially occur in the lower part of the array, an effect also observed experimentally. Using event driven simulations we vary dissipation and friction in order to understand the dynamics of the cracks. We classify the cracks quantitatively via an analysis using only on the coordinates of the particles.

1 Introduction

The flow behavior of granular media has received increasing interest during the last years. Flow in tubes, pipes or chutes is of crucial importance for industry; however, the fascinating features of granular flow are not yet completely understood. For a review concerning the physics of granular materials, see Ref.[1,2] and refs. therein.

Several basic phenomena like recurrent clogging[3] as a sort of 'traffic jam' problem[4] or density waves[5,6] may be observed in granular systems. Density waves have been observed in simulations of a steady state regime[6] or in experiments related to gas-particle interactions (pneumatic effects)[7]. Here we focus on the problem of a 2D pile made up of rather large beads, a convenient toy model displaying many of the features found also in real 3D granulates. We study the decompaction of such a 2D pile, initially almost at rest. Recent observations of approximately V-shaped *microcracks* in vertically vibrated granular media[8] were complemented by experiments and simulations of the discontinuous decompaction of a falling sandpile[9]. The following basic features were found: In a system with polished laterals walls cracks are unlikely to be appear during the fall, while in a system with rather poorly polished walls cracks occur frequently, displaying the following characteristics: A crack in the lower portion of the pile quite generally grows, whereas a crack in the upper part tends to disappear. These experimental findings were paralleled by numerical simulations and a continuum approach[9].

In the following we will present a method to recognize cracks in 2D granular assemblies, using only the coordinates. With this method we analyze various simulations of falling piles with different material's parameters.

2 Simulation Method

Our simulation model is an event driven (ED) method[10] based upon the following considerations: The particles undergo an undisturbed motion in the gravitational field until an event occurs. An event is either the collision of two particles or the

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collision of one particle with a wall. Particles are hard spheres and interact instantaneously; dissipation and friction are only active on contact. From the velocities just before contact, we compute the velocities after the contact, accounting for energy loss in normal direction introducing the coefficient of normal restitution, ϵ. Furthermore, we describe the roughness of surfaces and the connected energy dissipation, using the coefficient of friction, μ, and the coefficient of maximum tangential restitution β_0 [9,10]. For the interaction of a particle with a wall the parameters may be different and we mark these with an index w, e.g. μ_w. For a discussion of the problems connected to ED simulations see Ref.[9].

From the momentum conservation laws in linear and angular direction we get the change of linear momentum of particle 1: $\Delta \vec{P} = -m_{12}(1+\epsilon)\vec{v}_c^{(n)} - (2/7)m_{12}(1+\beta)\vec{v}_c^{(t)}$, with the reduced mass $m_{12} = m_1 m_2/(m_1 + m_2)$. (n) and (t) indicate the normal and tangential components of the relative velocity of the contact points, $\vec{v}_c$, relative to the surface of the particles. The factor 2/7 in the tangential part of $\Delta \vec{P}$ stems from the fact that we use solid spheres. β is the coefficient of tangential restitution limited by β_0 for sticking contacts or determined by Coulomb's law for sliding contacts. For a detailed discussion of this interaction model see Ref. [10] and refs. therein.

3 Results

Since we are interested in a situation when a rather compact array of particles begins to fall we firstly prepare a convenient initial condition. Here, we use $N = 1562$ particles of diameter $d = 1$ mm in a box of width $L = 20.2d$, initially relaxed for a time t_r under elastic and smooth conditions. The average velocity of this configuration is $\overline{v} = \sqrt{< v^2 >} \approx 0.05$ m/s. Due to the rather low kinetic energy, the array of particles is rather dense, except for a few layers at the top. At time $t = 0$ we remove the bottom, switch on dissipation and friction and let the array fall and decompact. For different initial conditions, keeping all other parameters fixed, we find cracks to occur at different positions in the pile and also at different times. In order to compare the cracks for different parameters we always use the same initial condition for the simulations presented here.

In Fig. 1 we plot a snapshot of one typical simulation at time $t = 0.05$ s in different ways. Initially, we mark each 8th row (beginning with the 9th row from the bottom) grey, while the other particles are black disks. With increasing time the array is accelerated in the direction of gravity. Due to friction with the walls, the particles close to the walls feel an additional acceleration in the opposite direction. Focusing on the marked particles we observe in Fig. 1(a) that the rows deform with time, i.e. the particles in the center of the array are falling faster than the particles close to the walls. This happens first at the bottom and rises up to the top of the array. We define the defects of the perfect triangular array as cracks.

For a better visualization of the cracks, found in Fig. 1(a), we plot in Fig. 1(b) the network of 'contacts'. We assume two particles i and j to be in contact when $|\vec{r}_i - \vec{r}_j| < (1 + c_0)d$. Too small values of c_0 lead to a dilute network, whereas too large values of c_0 lead to a perfect triangular lattice, even in the case of rather large separations between the particles. We found $c_0 = 0.06$ to be a reasonable

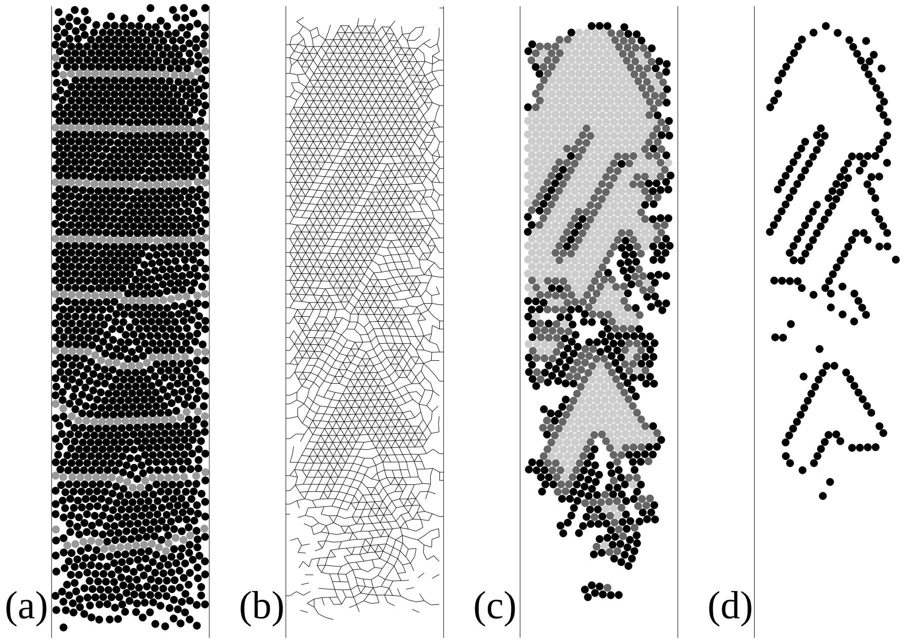

Figure 1: (a) Snapshots of a simulation with N =1562 particles, at time t = 0.05 s, in a pipe of width L/d = 20.2. We use ϵ = 0.99, ϵ_w = 0.98, μ = 0.5, μ_w = 1.0, and $\beta_0 = \beta_{0w}$ = 0.2. The grey particles were initially arranged horizontally. (b) Network of 'contacts' for the same simulation. Each line represents the connection of the centers of two particles with a distance smaller than 1.06 d. (c) Those particles with four or more neighbors being close to a dense block are plotted. (d) The edges of the cracks, i.e. the 'dots' are plotted.

compromise and will use this rather arbitrary value in the following. A particle, that is close to a wall, i.e. $|\vec{r}_i| < (1 + c_0)d/2$ is connected with the wall by a line normal to the wall. From Fig. 1(b) we get a triangular lattice, i.e. six neighbors per site, and some broken bonds i.e. less than six neigbors per site, separating the triangular blocks. Note that the cracks are visible much better in Fig. 1(b) compared to Fig. 1(a).

For a quantitative description of the cracks we plot in Fig. 1(c) all particles i with four or more neighbors, being close to a rather dense part of the system. We compare for each particle the number of neighbors with less than four, n_i^-, to the number of neighbors with more than or exactly four neighbors, n_i^+. All particles with $n_i^- > n_i^+$ are neglected, such that only those particles with many neighbors close to a dense region are accepted. The particles in contact with the walls are also neglected. A black circle corresponds to four neighbors, grey and light grey correspond to five and six neighbors respectively.

Simply counting the number of particles with four or five neighbors, i.e. dark circles in Fig. 1(c), would lead to an overestimate of the number of cracks. Therefore, we account for particles with just a small number of equivalent neighbors, i.e. we accept particles with one, two or three neighbors with the same number of

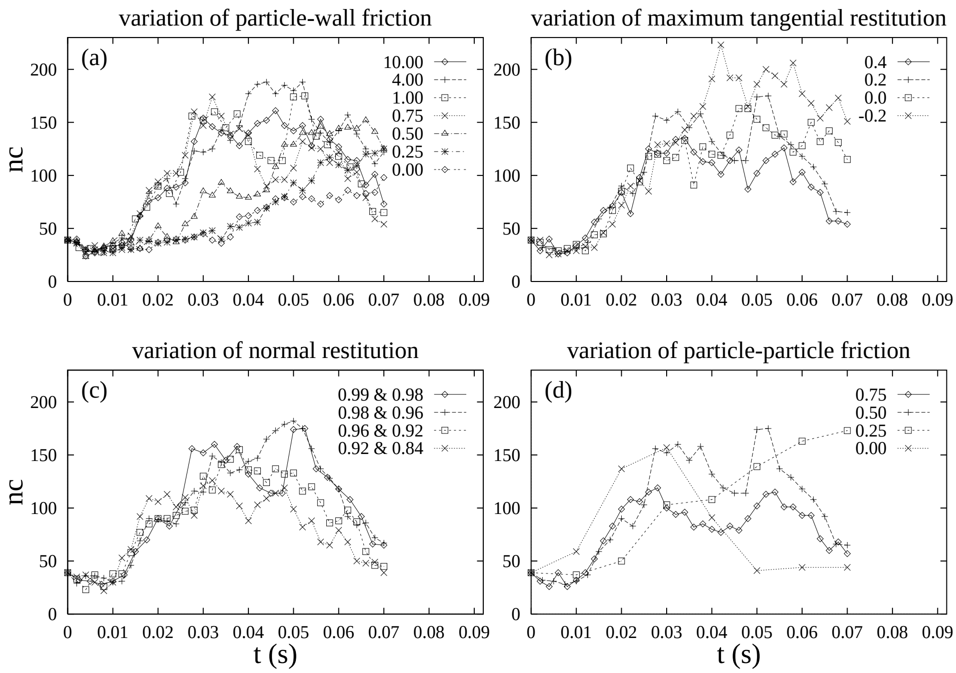

Figure 2: Plot of the typical crack density, nc, as a function of time for various simulations with N =1562 particles in a pipe of width $L/d = 20.2$. We use $\epsilon = 0.99$, $\epsilon_w = 0.98$, $\mu = 0.5$, $\mu_w = 1.0$, and $\beta_0 = \beta_{0w} = 0.2$. We vary μ_w (a), β_0 and β_{0w} (b), ϵ and ϵ_w (c), and μ (d). The values of the varied parameters are given in the inserts.

neigbors. As an example, all particles belonging to a dense block have more than three equivalent neigbors and are thus disregarded. A particle with no equivalent neighbor obviously belongs to the dilute regime and is also disregarded. Finally, we accept only particles with at least one neighbor inside a dense part of the system. All these rules are consistent with the idea that a crack is the boundary between a 'solid' block and a dilute region. We assume that the particles which follow the above rules are correlated to a crack and define them as 'dots', see Fig. 1(d). Note that the dots are not only arranged on long, straight lines, i.e. slip planes, but some dots are also scattered in the volume, indicating small defects in the triangular array.

In Fig. 2 we plot the number of dots, nc, versus time for various simulations with different values of μ_w (a), β_0 and β_{0w} (b), ϵ and ϵ_w (c), and μ (d). Note that the initial number of dots is $nc \approx 40$, since the initial dense block has an upper and a lower end, also being identified as a crack. We use the parameters already used for Fig. 1 and give the values of the modified parameters in the inserts of the figures.

From Fig. 2(a) we learn that cracks start to occur typically at time $t \approx 0.015$ s, if μ_w is sufficiently large, i.e. $\mu_w \geq 0.75$. For decreasing $\mu_w = 0.5$, 0.25, and 0.0 the cracks systematiclly occur later and nc decreases. From a dot representation for μ_w

= 0.0 we find no long cracks, i.e. no slip planes, like in Fig. 1, but only a random decompaction, i.e. scattered dots, beginning from the upper and lower boundaries.

In Fig.2(b) the onset of cracks depends slightly on β_0. For large $\beta_0 = 0.4$, i.e. an inversion of the tangential velocity, the cracks occur faster than for small $\beta_0 =$ -0.2, i.e. a simple reduction of the tangential velocity. The number of cracks nc seems to increase with decreasing β_0.

For Fig. 2(c) we vary the coefficients of normal restitution such that the ratio $(1-\epsilon)/(1-\epsilon_w) = 1/2$ is fixed. We use $\epsilon = 0.99, 0.98, 0.96$, and 0.92. The cracks seem to occur faster for smaller epsilon, i.e. for stronger dissipation. Furthermore, the number of cracks nc decreases with increasing dissipation, since strong dissipation is reducing the internal kinetic energy and thus the internal pressure.

In Fig. 2(d) we vary the coefficient of particle-particle friction, μ. The onset of cracks occurs independent of μ, for our values of $\mu \geq 0.50$. The number nc is quite small for large μ due to the strong dissipation connected to strong friction. Cracks occur less rapidly for small μ values, i.e. $\mu = 0.25$. Interestingly, we observe a fast increase followed by a rapid decrease of nc for $\mu = 0$, i.e. no particle-particle friction at all. We connect this to a fast random decompaction due to the internal pressure of the block which is not reduced by frictional energy loss.

4 Summary and Outlook

We presented simulations of a 2D granular system, falling inside a vertical, rectangular container. We observe discontinuous decompaction as a result of cracks breaking the array into pieces from the bottom to the top. Cracks may be identified either as randomly scattered defects of a perfect triangular array, or as slip planes. We proposed a method to recognize cracks in the array by selecting those particles which, roughly speaking, separate close packed from dilute regions. Using this method we calculated the number of 'dots', i.e. a measure for the crack density. Obviously, the number of dots does not contain any information on the height dependence of the cracks. Furthermore, our method does not distinguish between long, straight, stable cracks and short, often unstable cracks. Stable and unstable mean here long lasting and rather short lasting density fluctuations. However, we discuss the decompaction dynamics for different material combinations, i.e. different restitution and friction coefficients. The occurrence of cracks is obviously connected to the friction at the walls; for smooth walls we do not find any cracks connected to slip planes, but we only find random decompaction due to the internal pressure. Also particle-particle friction is important since smooth particles are not able to keep a crack stable. Thus we observe a random decompaction of the block without slip planes for weak friction. Cracks are also related to the normal dissipation of energy, i.e. cracks occur faster for greater normal dissipation. This is connected to the fact that a dense block dissipates its internal energy faster for greater dissipation.

Cracks in 2D systems are visible without great problems due to the optical accessibility of the system. An extension of our method possibly allows a classification of cracks in 3D. Since we did not examine the height dependence of the cracks, here further research is necessary in order to understand the propagation of cracks inside the material and also the reasons for the occurrence of cracks as long as the whole system in horizontal direction. Furthermore, it is not yet clear if cracks will occur in polydisperse systems or in three dimensions.

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