Computer Simulations and Experiments of Dry Granular Media: Polydisperse Disks in a Vertical Pipe

Stefan Luding (1), Jaques Duran (2), Eric Clément (2), Jean Rajchenbach (2)

(1) Institute for Computer Applications 1, Pfaffenwaldring 27, 70569 Stuttgart, Germany

(2) Laboratoire d'Accoustique et d'Optique de la Matière Condensée, URA 800, Université Pierre et Marie Curie, 4 place Jussieu, 75005 Paris, France

We discuss recent experiments and simulations of monodisperse spheres in a two dimensional (2D) setup, and present new simulations of a polydisperse 2D model granular material falling in a vertical pipe. The decompaction of such a quite dense granular assembly of disks is not connected to visible cracks, contrasting the monodisperse case. From simulations with different parameters we report a rather continuous decompaction in polydisperse systems connected to a dynamic network of active particles, i.e. particles performing a great number of collisions per unit time.

Granular flow in tubes, pipes or chutes is of crucial importance for industry; however, the fascinating features of granular flow are not yet completely understood. For a review concerning the physics of granular materials, see Ref. [1,2] and refs. therein.

Density waves [3,4] may be observed in granular systems, both, in simulations of a steady state regime [4], and in experiments where gas-particle interactions (pneumatic effects) get to be important [5]. A 2D pile made of rather large particles is a convenient toy model with a comparatively weak effect of the gas on the granulate, but displaying many of the features found also in real 3D systems. In the monodisperse case, observations of approximately V-shaped *microcracks* in vertically vibrated granular media [6] were complemented by experiments and simulations of the discontinuous decompaction of a falling sandpile [7–9]. From experiments [7] one observes that in systems with smooth, polished walls cracks - separating the dense parts of the system - are unlikely to occur, whereas cracks occur frequently when the walls are rough. Cracks are stable in the lower portion of the pile and preferentially follow the order of the triangular lattice, caused by the monodispersity of the particles. From recent simulations [9], the occurrence of cracks was related to shock waves, starting at one point at the boundary and traveling through the system, towards the opposite boundary. When being reflected the momentum wave may be amplified and builds a so called dynamic arch with great pressure and thus strong friction at the walls. The consequence is that the dynamic arch is accelerated less than the free falling particles below, so that a crack opens, possibly spanning the whole system. Furthermore, in the regime of high pressure, an alternating spin order was found along the axis of great stress. Spin order means here, that neighboring granular particles (along the line of great stress) are rotating in opposite direction, in order to minimize the relative surface velocity and thus minimizing frictional energy loss.

The open questions is now, whether cracks occur in a system without long range order or not. As a supplement to the experiments and simulations of monodisperse systems we study the decompaction of a 2D pile, made of polydisperse disks.

NUMERICAL METHOD

For the dynamic simulations we use an event driven (ED) method [10–12] based upon two considerations: (i) Particles undergo an undisturbed parabolic flight in the gravitational field until an event occurs. Events are collisions of one particle with a wall or pair collisions of two particles. (ii) The particles are hard disks or cylinders, interacting instantaneously; dissipation and friction are active on contact only. The velocities after the contact are calculated accounting for energy loss in normal direction using the coefficient of normal restitution, ϵ, and accounting for the roughness of surfaces and the connected energy dissipation, using the coefficient of friction, μ, and the coefficient of maximum tangential restitution, β_0 [7–10,13,14]. Interactions with the walls need different parameters, marked with the index w, e.g. μ_w.

Momentum conservation in linear and angular direction results in a change of linear momentum of particle 1: $\Delta\vec{P} = -m_{12}(1+\epsilon)\vec{v}_c^{(n)} - (1/3)m_{12}(1+\beta)\vec{v}_c^{(t)}$, with the reduced mass $m_{12} = m_1 m_2/(m_1+m_2)$. (n) and (t) indicate the normal and tangential components of the relative velocity of the contact points, $\vec{v}_c$, relative to the surface of the particles. The factor 1/3 in the tangential part of $\Delta\vec{P}$ stems from the fact that we use solid disks or cylinders. β is the coefficient of tangential restitution limited by β_0 for sticking contacts or determined by Coulomb's law for sliding contacts. For a detailed discussion of this interaction model see Refs. [10,13,14]. Note that the ED method is dynamic, so that we first have to find a convenient initial condition as close to a static packing as possible.

SIMULATION RESULTS

The initial condition for the simulations is a rather compact array of $N = 1600$ particles of diameters d, randomly chosen from the interval $[d_0 - w_0, d_0 + w_0]$, with d_0 = 1mm and w_0 = 0.2mm, in a box of width $L = 20.2$mm. The average velocity of this configuration is initially $\overline{v} = \sqrt{< v^2 >} \approx 0.03$ m/s. The volume fraction of this system is $\rho^* \approx 0.84$. The array of particles is quite dense, except for some layers at the top. At time $t = 0$ the bottom is removed, dissipation and friction are activated and the array begins to fall.

For the simulations we used various material parameters, always observing similar results, i.e. no long and stong cracks. We varied the coefficients of restitution $0.9 \leq \epsilon \leq 0.99$, $0.9 \leq \epsilon_w \leq 0.99$, the coefficients of friction $0.2 \leq \mu \leq 5.0$ and $0.2 \leq \mu_w \leq 10.0$, and the coefficients of maximum tangential restitution $-0.5 \leq \beta_0 \leq 0.5$ and

$-0.5 \leq \beta_{0w} \leq 0.5$. In addition we performed also some simulations with larger initial energy, $\overline{v} = 0.06$m/s, observing no striking difference in the behavior of the system.

In Fig. 1 we plot snapshots of a typical simulation at different times, gravity acting downwards. The parameters used for simulation are $\epsilon = 0.98$, $\epsilon_w = 0.99$, $\mu = 0.5$, and $\mu_w = 10$, i.e. an extremely frictional but flat wall. The coefficients of maximum tangential restitution are here $\beta_0 = \beta_{0w} = 0.5$. We plot particles as black or white disks when they have performed more than 300 or less than one collisions during the last millisecond respectively. The greyscale is an interpolation between these extreme values.

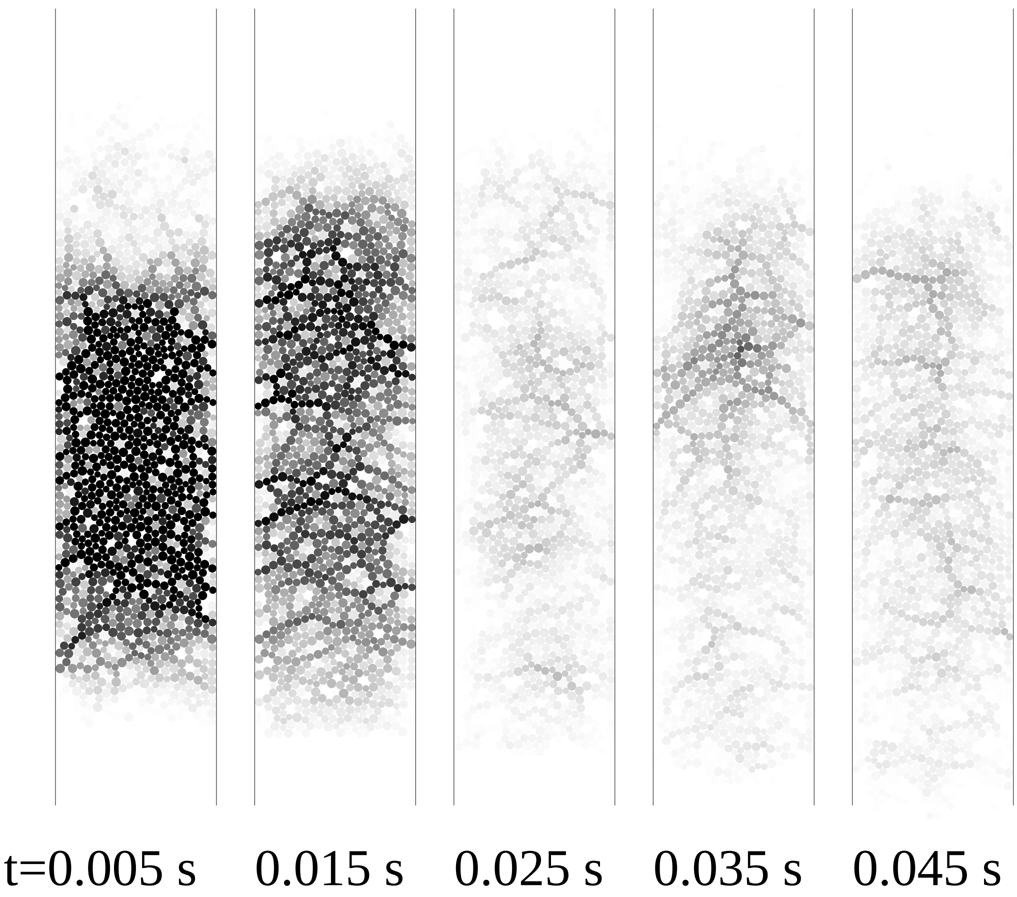

FIG. 1. Snapshots of a typical simulation with N =1600 particles, at times $t = 0.005$, 0.015, 0.025, 0.035, and 0.045 s, in a pipe of width $L/d_0 = 20.2$. We use $\epsilon = 0.98$, $\epsilon_w = 0.99$, $\mu = 0.5$, $\mu_w = 10.0$, and $\beta_0 = \beta_{0w} = 0.5$. The greyscale indicates the number of collisions performed during the last millisecond, see text for details.

In Fig. 1 the active particles are visible and the inactive ones are neglected. No long cracks are visible when the system decompacts progressively only a rather continuous dilution maybe observed. In contrast to the monodisperse case, where the dynamic arch is a block, several particle diameters high, this system shows a network of particles with a large number of collisions, i.e. active particles. The great initial activity, at $t = 0.005$s, is decreasing quickly after the quite short time of $t \approx 0.015$s. The fluctuations in activity do not lead to a strong dynamic arch in the polydisperse case, while in monodisperse systems the dynamic arch also can appear at larger times [9].

DISCUSSION

We have presented simulations of a system of polydisperse disks, falling inside a vertical, rectangular container. In contrast to the case of monodisperse particles, we observe a rather continuous decompaction as a result of the internal pressure, diluting the array. In the polydisperse case, momentum waves are propagated along selected paths and decrease in amplitude, before reaching the opposite wall. The reflected wave - connected to dynamic arches in monodisperse systems - is much weaker in a polydisperse array. Thus the dynamic arch, stable and possibly self stabilizing in the monodisperse system, is quite unstable here. One problem of the model used here may be the flat walls. The positions of particles close to the walls are less disordered as compared to particles inside the bulk, since the flat wall causes a line of particles, parallel and close to the wall. In fact, we observe, that the polydisperse system breaks preferentially along the line, one particle diameter away from the wall, i.e. we observe mass flow in the center and a much smaller downward acceleration for the particles in vicinity of the walls.

Open problems are the simulation of polydisperse systems with rough walls and of different polydispersity, different inital packing fractions, and also three dimensional simulations. Experiments of 2D polydisperse packings of disks are quite difficult to realize due to the possibility of tilting with respect to front and back wall. As an example we mention the recent experiments of a quasi static flow of cylinders in a system with rough walls [15].

ACKNOWLEDGMENTS

We gratefully acknowledge the support of the Deutsche Forschungsgemeinschaft DFG, SFB 382, A6, of the European Community (Human Capital and Mobility). The french group is part of the Groupement de Recherche sur la Matière Hétérogène et Complexe of the CNRS and of a CEE network HCM.

LITERATURE CITED

1. H. M. Jaeger and S. R. Nagel, "Physics of the Granular State", *Science* **255**, 1523 (1992).

2. H. M. Jaeger, J. B. Knight, C. h. Liu, and S. R. Nagel, "What is Shaking in the Sandbox?", MRS Bulletin May 1994 **XX**, 25 (1994).

3. G. Baxter, R. Leone, and R. Behringer, "Experimental Test of Time Scales in Flowing Sand", *Europhys. Lett.* **21**, 569 (1993).

4. J. Lee and M. Leibig, "Density waves in granular flow: a kinetic wave approach", *J. Phys. I France* **4**, 507 (1994).

5. T. Raafat, J. P. Hulin, and H. J. Herrmann, "Density waves in dry granular media falling through a vertical pipe", *Phys. Rev. E*, to appear.

6. J. Duran, T. Mazozi, E. Clément, and J. Rajchenbach, "Decompaction modes of a two-dimensional "sandpile" under vibration: model and experiments", *Phys. Rev. E* **50**, 3092 (1994).

7. J. Duran, T. Mazozi, S. Luding, E. Clément, and J. Rajchenbach, "Discontinuous Decompaction of a Falling Sandpile", *Phys. Rev. E* **53**, 1923 (1996).

8. S. Luding, J. Duran, T. Mazozi, E. Clément, and J. Rajchenbach, "Simulations of Granular Flow: Cracks in a Falling Sandpile", in *Traffic and Granular Flow* edited by D. Wolf (World Scientific, Singapore, 1996).

9. S. Luding, J. Duran, E. Clément, and J. Rajchenbach, "Simulations of dense granular flow: Dynamic Arches and Spin Organization", *J. Phys. I (France)*, to appear in June 1996.

10. S. Luding, "Granular Materials under Vibration: Simulations of Rotating Spheres", *Phys. Rev. E* **52**, 4442 (1995).

11. B. D. Lubachevsky, "How to Simulate Billards and Similar Systems", *J. of Comp. Phys.* **94**, 255 (1991).

12. O. Walton and R. Braun, "Viscosity, Granular-Temperature, and Stress Calculations for Shearing Assemblies of Inelastic, Frictional Disks", *Journal of Rheology* **30**, 949 (1986).

13. S. F. Foerster, M. Louge, H. Chang, and K. Allia, "Measurements of the collision properties of small spheres", *Phys. Fluids* **6**, 1108 (1994).

14. O. R. Walton, "Numerical simulations of inelastic frictional particle-particle interactions", in *Particulate Two Phase Flow*, edited by M. C. Roco (Butterworth-Heinemann, Boston, 1992).

15. O. Pouliquen and R. Gutfraind, "Stress fluctuations and shear zones in quasi-static granular chute flows", *Phys. Rev. E* **53**, 552 (1996).

[1] H. M. Jaeger and S. R. Nagel, *Physics of the Granular State*, Science **255**, 1523 (1992).

[2] H. M. Jaeger, J. B. Knight, C. h. Liu, and S. R. Nagel, *What is Shaking in the Sandbox?*, MRS Bulletin May 1994 **XX**, 25 (1994).

[3] G. Baxter, R. Leone, and R. Behringer, *Experimental Test of Time Scales in Flowing Sand*, Europhysics Letters **21**, 569 (1993).

[4] J. Lee and M. Leibig, *Density waves in granular flow: a kinetic wave approach*, J. Phys. I France **4**, 507 (1994).

[5] T. Raafat, J. P. Hulin, and H. J. Herrmann, *Density waves in dry granular media falling through a vertical pipe*, to be published in Phys. Rev. E (unpublished).

[6] J. Duran, T. Mazozi, E. Clément, and J. Rajchenbach, *Decompaction modes of a two-dimensional "sandpile" under vibration: model and experiments*, Physical Review E **50**, 3092 (1994).

[7] J. Duran, T. Mazozi, S. Luding, E. Clément, and J. Rajchenbach, *Discontinuous Decompaction of a Falling Sandpile*, Phys. Rev. E **53**, 1923 (1996).

[8] S. Luding, J. Duran, T. Mazozi, E. Clément, and J. Rajchenbach, *Simulations of Granular Flow: Cracks in a Falling Sandpile*, in *Traffic and Granular Flow* (World Scientific, Singapore, 1996).

[9] S. Luding, J. Duran, E. Clément, and J. Rajchenbach, *Simulations of dense granular flow: Dynamic Arches and Spin Organization*, to appear in June 1996, J. Phys. I (unpublished).

[10] S. Luding, *Granular Materials under Vibration: Simulations of Rotating Spheres*, Phys. Rev. E **52**, 4442 (1995).

[11] B. D. Lubachevsky, *How to Simulate Billards and Similar Systems*, J. of Comp. Phys. **94**, 255 (1991).

[12] O. Walton and R. Braun, *Viscosity, Granular-Temperature, and Stress Calculations for Shearing Assemblies of Inelastic, Frictional Disks*, Journal of Rheology **30**, 949 (1986).

[13] S. F. Foerster, M. Louge, H. Chang, and K. Allia, *Measurements of the collision properties of small spheres*, Phys. Fluids **6**, 1108 (1994).

[14] O. R. Walton, *Numerical simulations of inelastic frictional particle-particle interactions*, in *Particulate Two Phase Flow*, edited by M. C. Roco (Butterworth-Heinemann, Boston, 1992).

[15] O. Pouliquen and R. Gutfraind, *Stress fluctuations and shear zones in quasi-static granular chute flows*, Phys. Rev. E **53**, 552 (1996).