A SIMPLE METHOD TO MIX GRANULAR MATERIALS

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Abstract. We show that a mixture of two species of granular particles with equal sizes but differing densities can be either segregated or mixed by adjusting the granular temperature gradient and the magnitude of the gravitational force. In the absence of gravity, the dense, heavy particles move to the colder regions. If the temperature gradient is put into a gravitational field with the colder regions above the hotter, a uniform mixture of light and heavy particles can be attained. This situation can be realized in a container of finite height with a vibrating bottom, placed in a gravitational field. We present a relation between the height of the container, the particle properties, and the strength of gravity required to minimize segregation.

1. Introduction

Segregation and mixing of granular material is of eminent importance for industrial operations and it has been subject to research since decades. However, both effects are not yet completely understood and thus cannot be controlled under all circumstances. Traditional experimental methods and theoretical approaches are nicely complemented by numerical simulations which in the last few years have developed tremendously [1]. For a review which covers a broad practical experience of segregation see Ref. [2] and references therein.

Segregation can be driven by geometric effects, shear, percolation and also by a convective motion of the small particles in the system [3]. In vibrated systems, the segregation due to convection appears to be orders of magnitude faster than segregation due to purely geometrical effects [4, 5]. In rotating drums, another archetype of many industrial devices, several segregation processes acting in parallel are reported [6]; in three-dimensional devices, axial and longitudinal segregation are observed [7–9] simultaneously. For axial segregation, particle percolation is reported to be responsible [8], while longitudinal segregation is related to different surface flow properties in the cylinder [10, 11].

In this paper, we investigate a model segregation problem which suggests a simple way to obtain uniform mixtures of two species. We show a sketch of the system in Fig. 1.

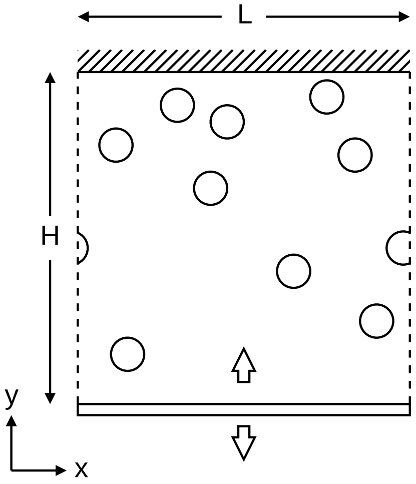

Figure 1. A sketch of the studied system

N particles are placed in a container of height H in the presence of gravity. The side walls have been replaced with periodic boundary conditions. Energy is supplied to the container by vibrating the bottom using a symmetric sawtooth wave with velocity V. The top wall is stationary. N_A of the particles have mass m_A, and the rest have mass m_B. We will take $m_A > m_B$. Though the particles have different mass, they all have the same radius a. We model the loss of energy during collisions with a restitution coefficient $r < 1$. Our detailed study of a similar system with identical particles is Ref. [12].

2. The Mixing Mechanism

We find that it is possible to obtain uniform mixtures of the two species by pitting two segregation mechanisms against each other. When the particles rarely touch the top of the container, all the dense particles are found near the bottom of the plate (see Fig. 2a). A similar effect occurs in the upper atmosphere, where different molecular species are sorted by weight [13].

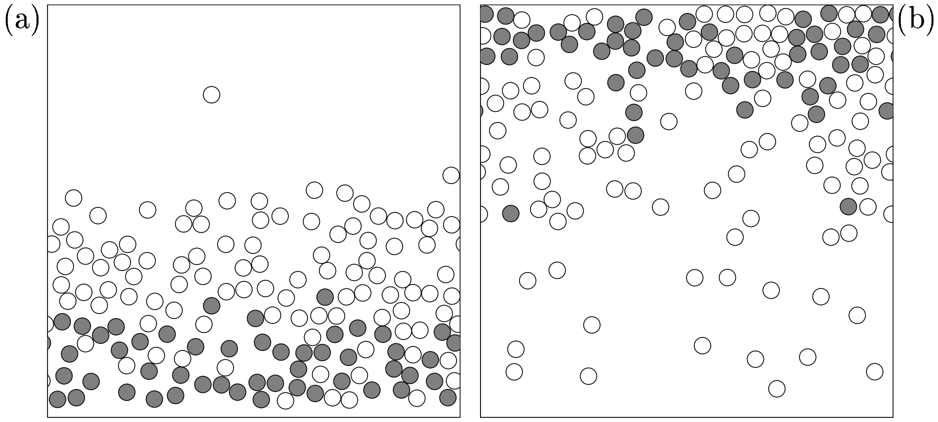

Figure 2. (a) A simulation with $N = 150$, $N_A = 50$, $L = H = 50$, $m_A/m_B = 10$, $g = 1$, $V = 1$, $r = 0.9$. The dense particles (mass m_A) are shaded. (b) same as (a), except $g = 0$.

On the other hand, when gravity is turned off, the particles are pushed against the top plate and the dense particles are found close to the upper plate (Fig. 2b). By smoothly varying between these two situations, it is possible to obtain a situation where the two species are uniformly mixed. In Fig. 3, we plot the difference between y_A, the center of mass of the heavy particles, and y_B, the center of mass of the light particles, normalized by the height of the container. Mixing is optimal when the difference between the species' centers of mass vanishes.

We see that the state of maximum mixing is obtained near $m_A gH/T \sim 2$ for all values of the parameters, except for almost elastic particles ($r = 0.99$). Here, T is the granular temperature, defined as the average kinetic energy per particle: $T \equiv (1/2N) \sum m_i v_i^2$. When the ratio of both energies is near unity, it means that the kinetic and potential energies of the particles are comparable. The tendency of $(y_A - y_B)/H$ to approach 0 for large gravities $m_A gH/T > 10$ is due to the initial conditions. Initially, all particles are arranged in a lattice just above the vibrating floor. Due to the large gravity, it is very difficult for particles to change places, and the mixture keeps its original configuration for a very long time. For $m_A gH/T < 10$,

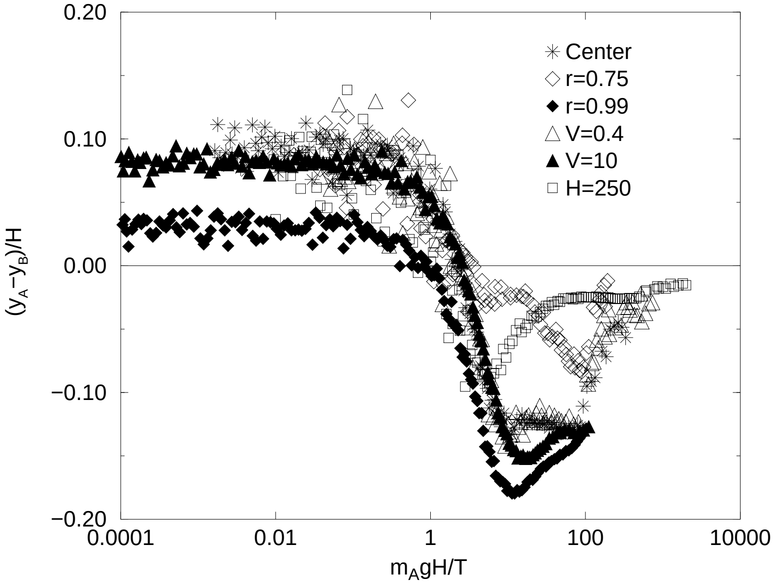

Figure 3. Segregation of the two species measured by the difference between y_A, the center of mass of species A and y_B, the center of mass of species B. This difference is plotted as a function of $m_A gH/T$, where T is the granular temperature, here defined as the average kinetic energy per particle. The points marked "Center" have $V = 2$, $m_A/m_B = 2$, $N_A = 50$, $N_B = 100$, $H = L = 50a$, and $r = 0.95$, with g swept over six orders of magnitude. The other points are the same, except for the parameter values marked on the graph. In all cases, the time unit is fixed by the wall vibration period, and the mixture is equilibrated for 300 time units, and then data is averaged for 1500 time units.

the particles change places often, and $(y_A - y_B)/H$ is independent of initial conditions.

To show more closely what happens with the densities of the different species, we show in Fig. 4 the concentrations of each as a function of height for three different simulations; one at small g, one at large g, and one where the particles are nicely mixed. In the situations with extremal g values, we obtain rather strong density gradients, while in the case of optimal mixing the density gradients are small, i.e. the density is almost constant throughout the system.

3. Discussion and Conclusion

Each of the two segregation mechanisms can be observed also with perfectly elastic (dissipationless) particles. In Fig. 5(a), we show a binary gas of elastic particles under gravity in the absence of forcing. The heavy par-

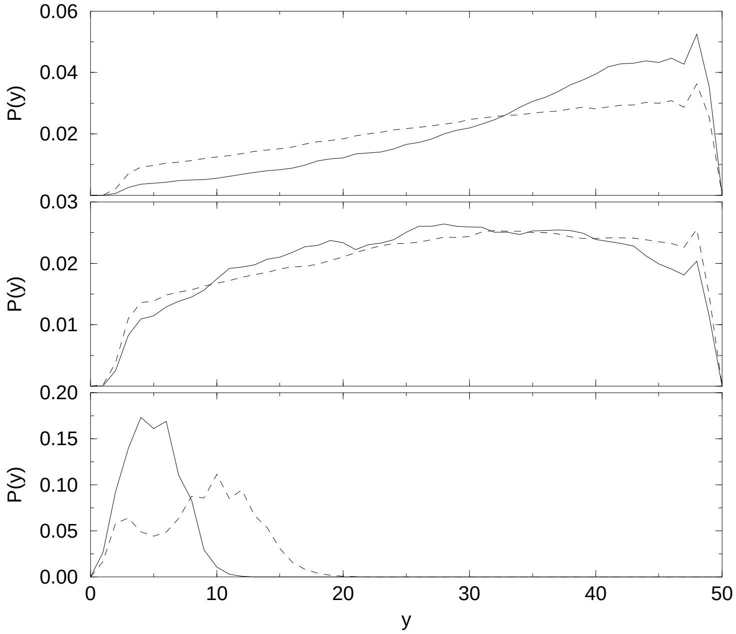

Figure 4. Profiles of the density of each species as a function of y. The solid curve represents the heavy species with mass m_A, and the dashed curve is the light species with m_B. All simulations have $m_A/m_B = 2$, $V = 2$, $H = 50$, $N_A = 50$, $N_B = 100$. The top figure has $g = 10^{-4}$, the middle has $g = 0.3169$, and the bottom has $g = 10^2$. The densities are given in terms of the probability of a single particle of each species to have a given height; the area under all curves integrates to 1.

ticles accumulate at the bottom. In Fig. 5(b), we show a binary gas in the absence of gravity, subjected to a thermal gradient. Now the particles particles accumulate against the upper, cold wall. Therefore, neither segregation mechanism relies on the dissipation of energy during collisions. This dissipation serves only to set up the necessary gradients which drive the segregation of the particles (see also the paper by Luding, Strauß, and McNamara in this proceedings).

To use this method to mix granular materials, the particles could be put into a chamber like the one shown in Fig. 1. To obtain the proper value of $m_A gH/T$, it is perhaps most convenient to adjust the height of the container H. It is also possible to control T by changing the vibration velocity V [12]. One possible disadvantage is that only a small amount of material can be mixed at one time. It also may be difficult in practice to adjust H or T correctly. Replacing the periodic boundaries with side walls may also introduce new effects.

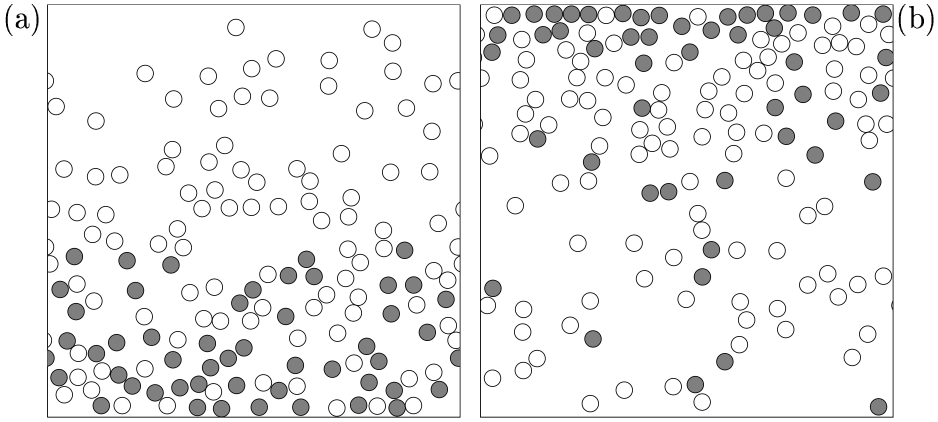

Figure 5. (a) A simulation with $N = 150$, $N_A = 50$, $L = H = 50a$, $m_A/m_B = 2$, $g = 1$, $r = 1.0$. All the walls are stationary, so no energy is added or subtracted. Note that the dense particles sink to the bottom. (b) same as except that $g = 0$ and when a particle touches the upper or lower wall, a new velocity is selected from a Maxwellian velocity distribution with a certain temperature. The temperature of the lower wall is 160 times as large as the temperature of the upper wall. Smaller temperature gradients also sort the particles by density, but it is much less visually striking.

Acknowledgements

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