# FLUCTUATIONS AND FLOW FOR GRANULAR SHEARING

*Results from experiment and simulation*

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**Abstract.**

We present results from both simulation and experiment on a 2D granular shear-cell. The experiments determine the position of disks and their orientations over time, as well as the force on individual disks. We use computerized particle tracking techniques to achieve the former and photoelasticity to achieve the latter. The simulations use MD force laws and efficient algorithms to simulate as closely as possible the experimental system. We measure the radial dependence of velocities and their distributions. In particular we find that the azimuthal velocity decays exponentially to some background level within a distance of about 7 disk diameters from the shearing wheel. Experimentally the distribution of azimuthal velocities is found to have a complex, roughly bimodal distribution close the shearing wheel which is indicative of a complex combination of slip, no-slip, and rolling processes at the boundary, and a more exponential distribution away from the shearing surface. The distribution of stresses shows a falloff which is approximately exponential at large forces, although it is probably not possible to determine which among competing models for force distributions best fits these results. The model can capture most but not all of the features seen in the experiment. The mean velocity profile, the qualitative nature of force chains and the distribution of velocities far from the shearing surface are well captured in the simulations. The velocity and the force distributions from the experiment and simulation differ however.

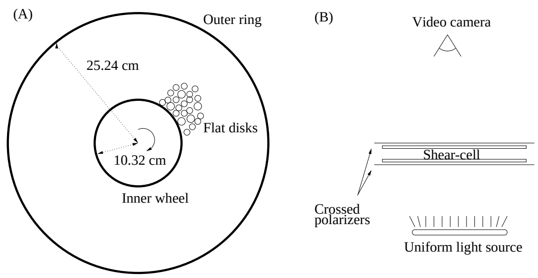


*Figure 1.* (A) top and (B) side view of the 2D Couette shear-cell.

## 1. Introduction

Granular systems have captured much recent interest because of their rich phenomenology and important range of applications. Many examples are given in this book and for other reviews see e.g. ref. [1]. One particular problem concerns the statistics and fluctuations in slowly evolving systems. Such systems, which may be considered as quasi-static, can exhibit strong spatio-temporal fluctuations in local force and velocities.

Here we consider results from simulations and experiments in a 2D granular system consisting of disks subject to steady shearing in a flat Couette geometry[2, 3]. We measure the shear-induced velocity profiles, the velocity distributions, and the local forces on grains.

In the following we briefly present the system and the simulational and experimental methods. In the presentation of the results we focus on the comparison of simulation and experiment. Thus, several aspects of the results, such as changing the packing fraction and the shear-rate, will not be discussed here. An extensive description of methods and results can be found elsewhere[4, 5].

## 2. The 2D Couette system

The system, sketched in Fig. 1, consists of an inner shearing wheel of diameter $D$, rotating with angular frequency $\Omega$, and a fixed outer ring. Flat disks are confined between these rings and two smooth horizontal Plexiglas sheets. The side of the wheel and the inner surface of the ring are coated with plastic 'teeth' spaced 0.7 cm apart and 0.2 cm deep.

We use a bimodal distribution of disks, with roughly 400 larger disks of diameter 0.9 cm, and roughly 2500 smaller disks of diameter 0.74 cm. A bimodal distribution is useful, since it limits the formation of hexagonally ordered packings over large regions. We use the diameter $d$ of the smaller disks as a convenient lengthscale and a polar coordinate system $(r,\Theta)$ with origin in the center of the shearing wheel. The distance from the inner wheel is thus $R = r - D/2$, and the dimensionless tangential velocity is $V_\Theta = \dot{\Theta}/\Omega$. Thus, $V_\theta = 1$ indicates that the disks are being dragged by the wheel, moving together like a solid body.

## 3. Experimental and simulational technique

In the experiment the individual disks were marked so that their position and orientation could be tracked from frame to frame using a video/frame-grabber system and some image analysis. The disks were made of a photo-elastic polymer which under stress changes the polarization of light coming from below [see Fig. 1(B)]. By using crossed polarizers under and above the shear-cell we get information about the stress level of each disk. An unstrained photo-elastic sample is dark when viewed between crossed polarizers, but becomes light if the stress is large enough. Further strain can actually cause the sample to become dark again. Here, we exploit this fact to obtain a reasonably quantitative measure of the forces. We do so by digitizing the intensity $I$ of the image obtained through crossed polarizers, and by then computing $G = |\nabla I|^2$. $G$ grows monotonically with stress, and a calibration shows a reproducible linear dependence[5].

We simulate the 2D shear-cell using a molecular dynamics (MD) algorithm [6, 7], see also the contribution of L. Brendel in this book. The number of disks, their sizes and material parameters and the geometry of the shear cell are exactly the same as in the experiment. The rough surface of the boundaries is modeled by sticking small semi-circles of radius 1 mm on the cylindrical walls. We also explored different force laws for the friction between the disks. We had to account for a small friction between the disks and the bottom plate to get agreement with the experiment[4].

## 4. Kinematic results

First we present the results of the mean azimuthal velocities as function of their distance to the shearing-wheel. Since there is no net flow in the radial direction, the radial velocity profile will fluctuate with zero mean. In Fig. 2(A) we show an example of the azimuthal velocity $V_\theta$ for both simulations and experiments. We use the normalized values, $V_\theta$, since in the very slow quasi-static limit, this quantity should be independent of $\Omega$[8]. We find a roughly exponential decay of $V_\theta$ to about $r/d = 7$ where the profile saturates at some background level. The mean profiles for the spin $S$ (angular velocity of individual disks) are shown in Fig. 2(B). $S$ has been normalized by $\Omega D/d$ which is the gear-ratio for rotation of a disk on the inner wheel. Thus, $S/(\Omega D/d) = 1$ means that disks are rolling perfectly on the shear-wheel. The disks nearest to the inner wheel rotate, on average,

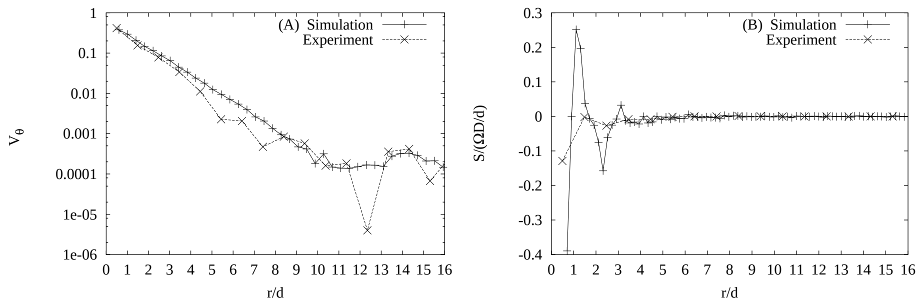


*Figure 2.* Mean normalized $V_\theta$ and $S$ vs. distance to the shearing wheel.

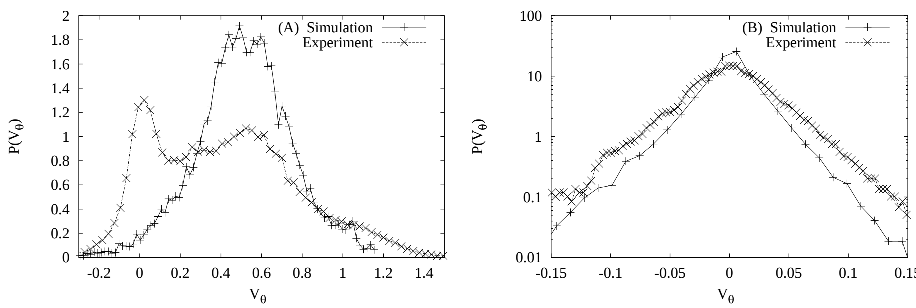


*Figure 3.* Distribution of $V_\theta$ for: (A) $0 < r/d < 1$, (B) $9 < r/d < 10$.

backwards, i.e., in the direction opposite to the wheel. However, the next layer rotates in the reverse direction. These oscillations damp very quickly with the distance from the wheel.

The simulation reproduces the experimental results very well in the case of the azimuthal velocity profile. For the spins the general trend is reproduced but the overall amplitudes are too high in the simulation. This maybe, in part, due to the fact that the friction between disks and bottom plate does not affect the rotation of the disks but only their translational movement.

The probability density distributions $P(V_\theta)$ for both simulation and experiment are shown in Fig. 3 for two different ranges of $r/d$. For the innermost layer of grains [Fig. 3(A)] there is a significant difference between simulational and experimental distributions. For the simulation, $P$ shows a unimodal Gaussian-like distribution with a clearly defined mean and width. For the experiment, $P$ shows several bumps and peaks corresponding to details of the grain motion. Note that there is some motion in the negative direction, which occurs because grains slip backwards as a chain fails. For

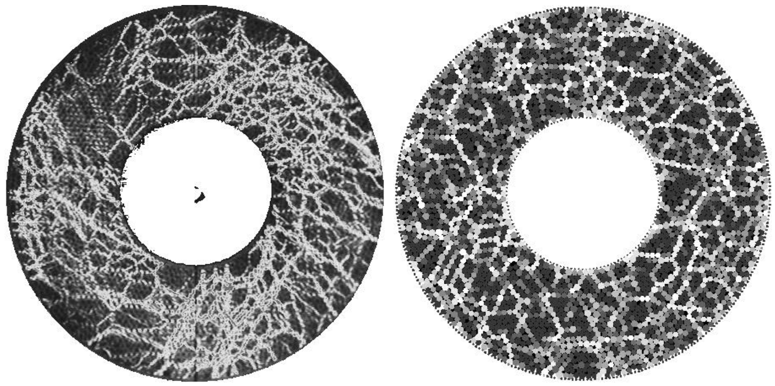

*Figure 4.* Pictures of the force-chains. (left) is an experiment showing the transparent light intensity. (right) is a simulation showing the potential energy stored in all contacts of one particle.

the same reason there is some motion faster than the shear-wheel for $V_\theta > 1$. There is a well defined peak at $V_\theta = 0$ that results from grains which slip relative to the inner wheel or roll without slipping against the wheel. The slipping grains remain at rest because of the weak friction with the bottom plate and a pinning effect from their exterior neighbors. The next peak is at $V_\theta/\Omega \sim 0.5$ corresponding to complex stick-slip-spin motion.

The distribution for $V_\theta$ narrows rapidly with distance away from the wheel for both simulation and experiment as can be seen in Fig. 3(B). Note that the distribution at this distance has a Gaussian-like peak but becomes rather exponential in the tails.

## 5. Force measurements

We show in Fig. 4, images of the force-chains obtained from experiment (left) and simulation (right). The force-chains are clearly visible in both images and the simulation seemingly captures some of the essential features of the experiment. In the simulation, we plot each disk as a unicolored dot which reduces the degree of detail.

We show an experimental stress distribution in Fig. 5(A). The stress $f$ is an average of $G$ over an area corresponding to $d^2$ and has been normalized to the mean stress. The distribution fall-off at $f \sim 10$ occurs because these forces are outside the useful range of the gradient technique, as we will explain elsewhere[5]. These data are consistent with the exponential fall-off predicted in the q-model and in the Contact Dynamics calculations of Radjai et al.[10]. However, the distribution at low forces appears to be consistent with a variation of the q-model[9] suggested by M. Nicodemi[11]

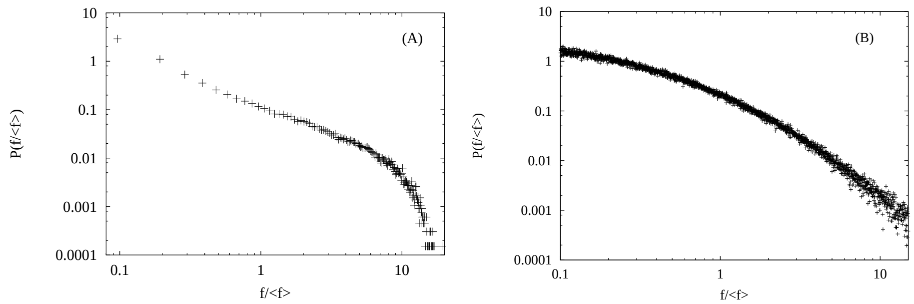


*Figure 5.* Distribution of forces found in (A) experiment and (B) simulation.

in which horizontal correlations are built in due to slip between grains.

In Fig. 5(B) we show the force distributions from the simulation. We observe a weaker decrease of the probability for large forces.

The photoelastic experiments described above are the only method of which we are aware in which the forces are determined within the sample and not at the boundary. Consequently, it is very important to determine the limits of the photoelastic technique. Ongoing experimental and numerical work will further explore the limits of validity for this technique.

CTV and RPB are indebted to the E.S.P.C.I.-H.M.P. where much of this work was carried out. The work of DWH and RPB was supported by the NSF, Grant DMR-9321791, DMS-9504577, and by NASA, Grant NAG3-1917.

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