Hypoplastic constitutive modeling

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Thanks to:
K. Nuebel (Zueblin, Stuttgart)
J. Tejchman (Gdansk)
Andres A. Pena O. (Stuttgart)

DFG, FOM

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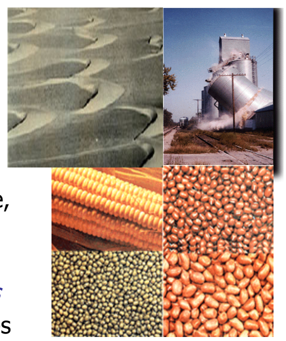
Granular Materials



Real:
- sand, soil, rock,
- grain, rice, lentils,
- powder, pills, granulate,
- … and many others …

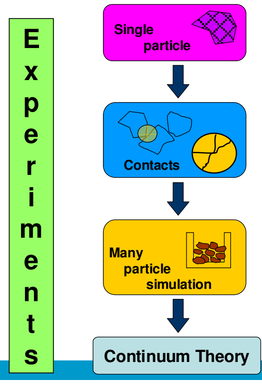
Model Granular Materials
- steel/aluminum spheres
- spheres with dissipation/friction/cohesion

Why ?

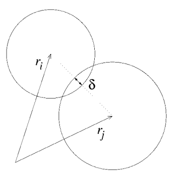
- \`Many particle' simulations work
  for small systems only (10^4- 10^6)
- Industrial scale applications rely on FEM
- FEM relies on continuum mechanics
  + constitutive relations
- \`Micro-macro' can provide those !
- Homogenization/Averaging

Contents

- Introduction
- Results DEM
- Micro-Macro transition
- Anisotropy
- Outlook



Discrete particle model



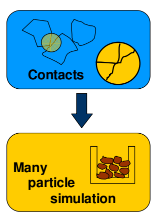
Equations of motion

$$m_i \frac{d^2 \vec{r}_i}{dt^2} = \vec{f}_i \qquad I_i \frac{d\vec{\omega}_i}{dt} = \vec{t}_i$$

Forces and torques:

$$\vec{f}_i = \sum_c \vec{f}_i^{\,c} + \sum_w \vec{f}_i^{\,w} + m_i g$$

$$\vec{t}_i = \sum_c \vec{r}_i^{\,c} \times \vec{f}_i^{\,c}$$



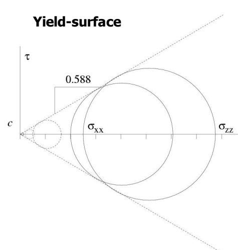
Overlap $\delta = \frac{1}{2}\left(d_i + d_j\right) - \left(\vec{r}_i - \vec{r}_j\right) \cdot \vec{n}$

Normal $\hat{n} = \vec{n}_{ij} = \frac{\left(\vec{r}_i - \vec{r}_j\right)}{\left|\vec{r}_i - \vec{r}_j\right|}$

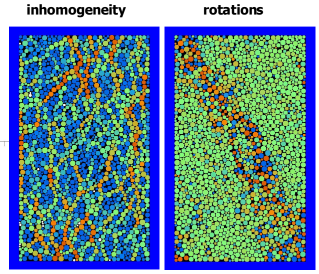
Material behavior of granular media

Non-Newtonian Flow behavior under slow shear

Yield-surface



inhomogeneity **rotations**



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Biaxial box set-up

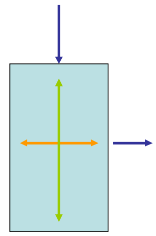
- Top wall: strain controlled

$$z(t) = z_f + \frac{z_0 - z_f}{2}\left(1 + \cos\omega t\right)$$

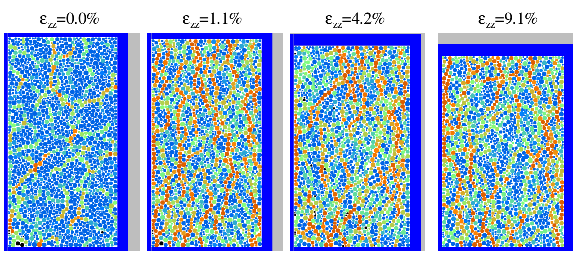
- Right wall: stress controlled

$$p = \text{const.}$$

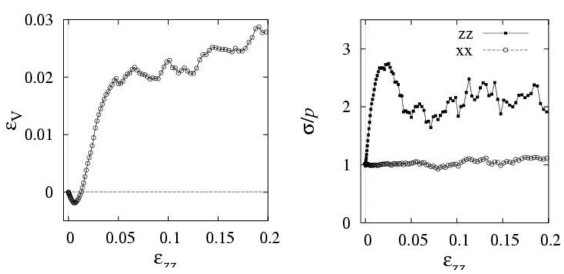
- Evolution with time … ?



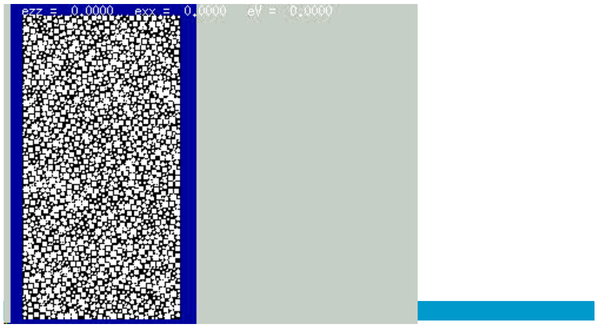
Simulation results



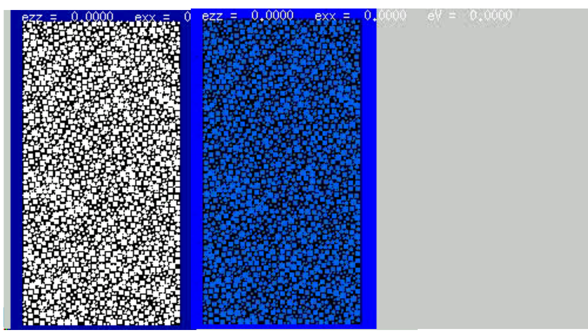
Bi-axial compression with p_x=const.



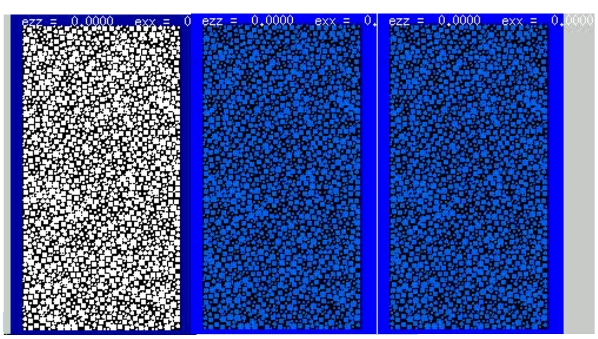
Bi-axial box (stress chains)



Bi-axial box (kinetic energy)



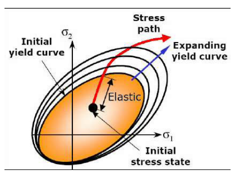
Bi-axial box (rotations)



Constitutive modeling: disadvantages

- ***Many*** constitutive models on the market
- Typically a ***large number*** of paramters
- Calibration with experiment is ***difficult***
- Little microscopic ***insight/motivation***
- Limited range of ***applicability***

Constitutive models:



- ***Gradient*** continua
- ***Micro-polar*** continua
- ***Elasto-(visco)-plastic*** models
- Hyper-, ***hypo-plastic***, -elastic models
- ***Challenges:***
- Cohesion, creep, cyclic loading, ageing, …

Basics: Hypoplasticity

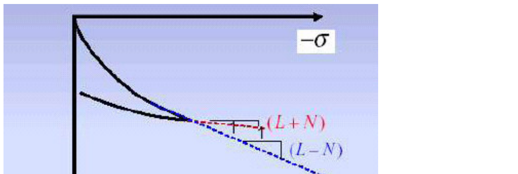
$$\Delta\sigma = L(\sigma, e)\cdot\Delta\varepsilon + N(\sigma, e)\cdot\|\Delta\varepsilon\|$$

Linear relation *Nonlinear relation*

Assumptions:
- state of the material is described by: stress and void ratio
- simple incremental formulation for all states, and also for loading and unloading

Strain increment $\Delta\varepsilon$, Stress increment $\Delta\sigma$, constitutive functions L, N of stress and void ratio e

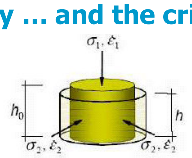
Basics: Hypoplasticity



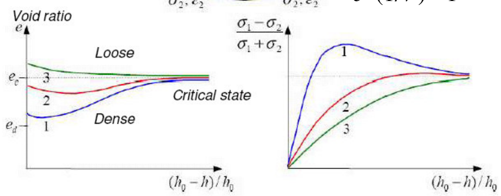
$$\Delta\sigma = L(\sigma,e)\cdot\Delta\varepsilon + N(\sigma,e)\cdot\|\Delta\varepsilon\|$$

Hypoplasticity … and the critical state

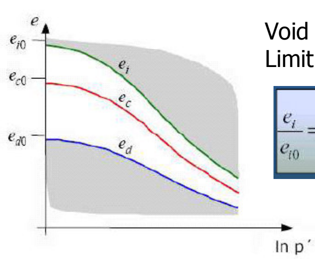
Volume fraction:
$\nu = V_{solid}/(V_{void}+V_{solid})$



Void ratio:
$e = V_{void}/V_{solid}$
$e = (1/\nu) - 1$



Hypoplasticity … and its limit states



Void ratios: e_i, e_c, e_d
Limit void ratios: e_{i0}, e_{c0}, e_{d0}

$$\frac{e_i}{e_{i0}} = \frac{e_c}{e_{c0}} = \frac{e_d}{e_{c0}} \exp\left[-\left(\frac{3p'}{h_s}\right)^n\right]$$

Hardness: h_s, power n

Hypoplasticity

Stress rate tensor $\mathring{\mathbf{T}} = \mathcal{L}(\mathbf{T},e):\mathbf{D} + \mathbf{N}(\mathbf{T},e)\,\|\mathbf{D}\|$

Linear relation — *Nonlinear relation*

Evolution of ***e*** $\dot{e} = (1+e)\cdot \mathrm{tr}\mathbf{D}$

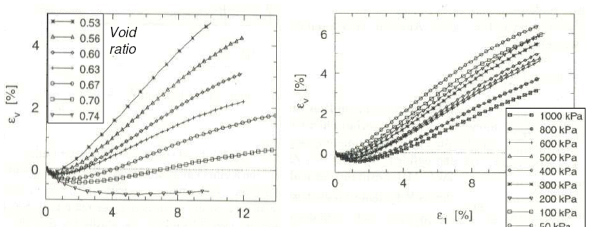
$$\mathcal{L} = f_s \cdot \frac{1}{\mathrm{tr}(\hat{\mathbf{T}}^2)} \cdot \left(F^2 \mathcal{I} + a^2 \hat{\mathbf{T}}^2\right) \qquad \mathbf{N} = f_s \cdot f_d \cdot \frac{a \cdot F}{\mathrm{tr}(\hat{\mathbf{T}}^2)} \cdot \left(\hat{\mathbf{T}} + \hat{\mathbf{T}}^*\right)$$

Pressure factor — *Critical State factor* — *Density factor* — *Limit condition Matsuoka-Nakai*

$$\hat{\mathbf{T}} = \mathbf{T}/\mathrm{tr}\mathbf{T} \qquad a = \frac{\sqrt{3}\cdot(3-\sin\varphi_c)}{2\cdot\sqrt{2}\cdot\sin\varphi_c} \qquad \hat{\mathbf{T}}^* = \hat{\mathbf{T}} - \tfrac{1}{3}\mathbf{1}$$

Density factor – Pressure factor

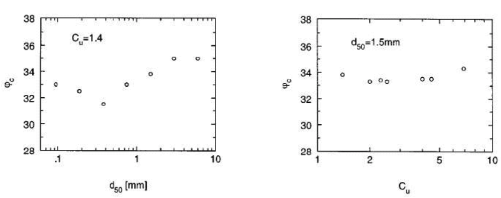
$$f_d = \left(\frac{e-e_d}{e_c-e_d}\right)^{\alpha} \qquad f_s = \frac{h_s}{n}\left(\frac{e_i}{e}\right)^{\beta} \cdot \frac{1+e_i}{e_i} \cdot \left(\frac{-\mathrm{tr}\mathbf{T}}{h_s}\right)^{1-n} \cdot \left[3 + a^2 - a\cdot\sqrt{3}\cdot\left(\frac{e_{i0}-e_{d0}}{e_{c0}-e_{d0}}\right)^{\alpha}\right]$$



Hypoplasticity: parameter calibration

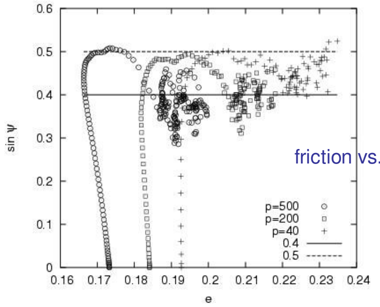
Critical friction angle: ϕ_c

- Angle of repose of a sandpile
- Shear- or tri-axial tests



Herle & Gudehus 1999

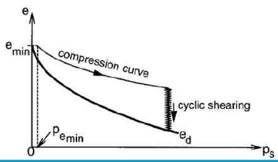
Micro-macro transition



Hypoplasticity: parameter calibration

Limit void ratios: e_{i0}, e_{c0}, e_{d0}

- Minimum density: e_{i0} — Prepare smoothly
- Critical density: e_{c0} — Shear tests …
- Maximum density: e_{d0} — Cyclic shear …



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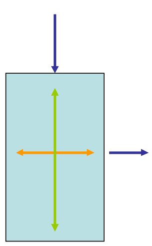
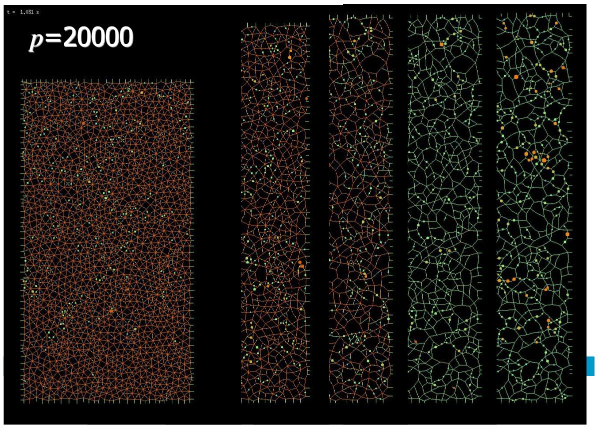
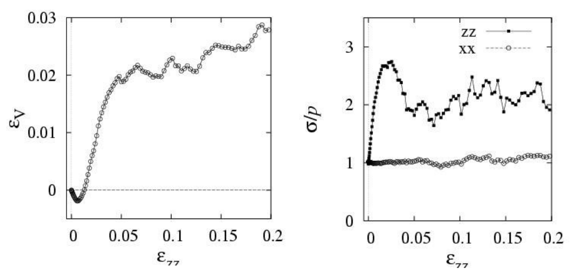
- Evolution with time … ?

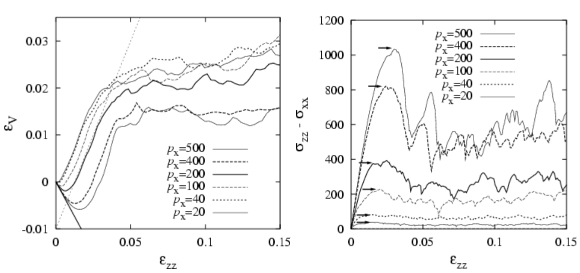




Bi-axial compression with p_x=const.



Pressure dependence



Results for friction μ=0.5 and different p_x *and* k_c*=0*

Micro-macro transition

Density vs. isotropic stress (=pressure)

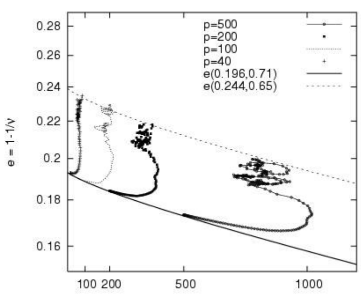
$$\frac{p}{p_0} = (\nu - \nu_0)^{4/3} \qquad p_0/k \approx 1.2 \text{ and } \nu_0 \approx 0.84$$

$$\left(\frac{p}{p_0}\right)^{3/4} = \nu - \nu_0 \qquad \Rightarrow \qquad \frac{e}{e_0} \approx 1 - \frac{1}{(1-\nu_0)\nu_0}\left(\frac{p}{p_0}\right)^{3/4}$$

$$\frac{e}{e_0} = \exp\left(-\left(\frac{p}{h_s}\right)^n\right) \approx 1 - \left(\frac{p}{h_s}\right)^n$$

… void ratio vs. pressure …

Micro-macro transition



void ratio vs. pressure

Hypoplasticity: parameters 2D

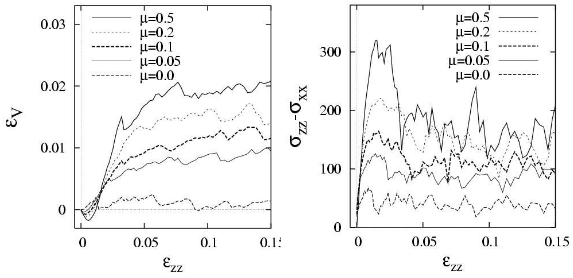
Hardness: h_s, $n \approx$ 0.66-0.75

Minimum density: $e_{i0} \approx$ 0.244

Critical density: $e_{c0} \approx$ 0.222

Maximum density: $e_{d0} \approx$ 0.196

Bi-axial: p_x=200 – varying friction



Hypoplasticity: parameter calibration

Exponents: α, β

Hypoplasticity parameters

8 Parameters

Critical friction angle: ϕ_c

Limit void ratios: e_{i0}, e_{c0}, e_{d0}

Hardness: h_s

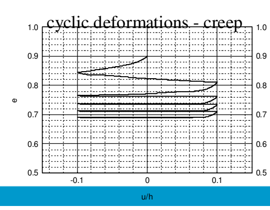
Exponents: α, β, n

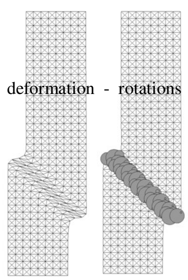
Hypoplasticity

Material	ϕ_c [°]	h_s [MPa]	n	e_{d0}	e_{c0}	e_{i0}	α	β
Hochstetten gravel	36	32000	0.18	0.26	0.45	0.50	0.10	1.9
Hochstetten sand	33	1500	0.28	0.55	0.95	1.05	0.25	1.0
Hostun sand	32	1000	0.29	0.61	0.96	1.09	0.13	2.0
Karlsruhe sand	30	5800	0.28	0.53	0.84	1.0	0.13	1.0
Lausitz sand	33	1600	0.19	0.44	0.85	1.0	0.25	1.0
Toyura sand	30	2600	0.27	0.61	0.98	1.1	0.18	1.1

Hypoplastic FEM model

+ successful tool – few parameters
- microscopic foundations ?
- extensions & parameter identification





Continuum Theory

Hypoplasticity – Limits

Micro-polar and Gradient extensions

Cyclic loading

Anisotropy

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