

---

**MODELLING OF DENSE GAS-PARTICLE FLOWS  
USING KINETIC THEORY OF GRANULAR FLOW**

**J.A.M. KUIPERS**

**TWENTE UNIVERSITY  
THE NETHERLANDS**

---

# DENSE GAS-SOLID FLOWS

“shifting sands” (Tanzania)

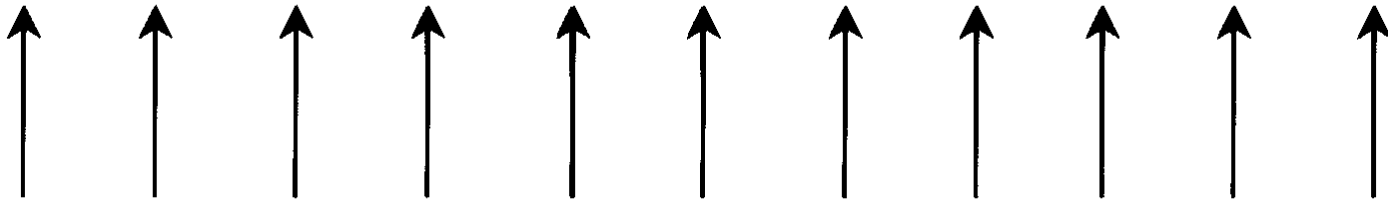
---



# DENSE GAS-SOLID FLOWS

clusters in co-current vertical gas-solid flows

---

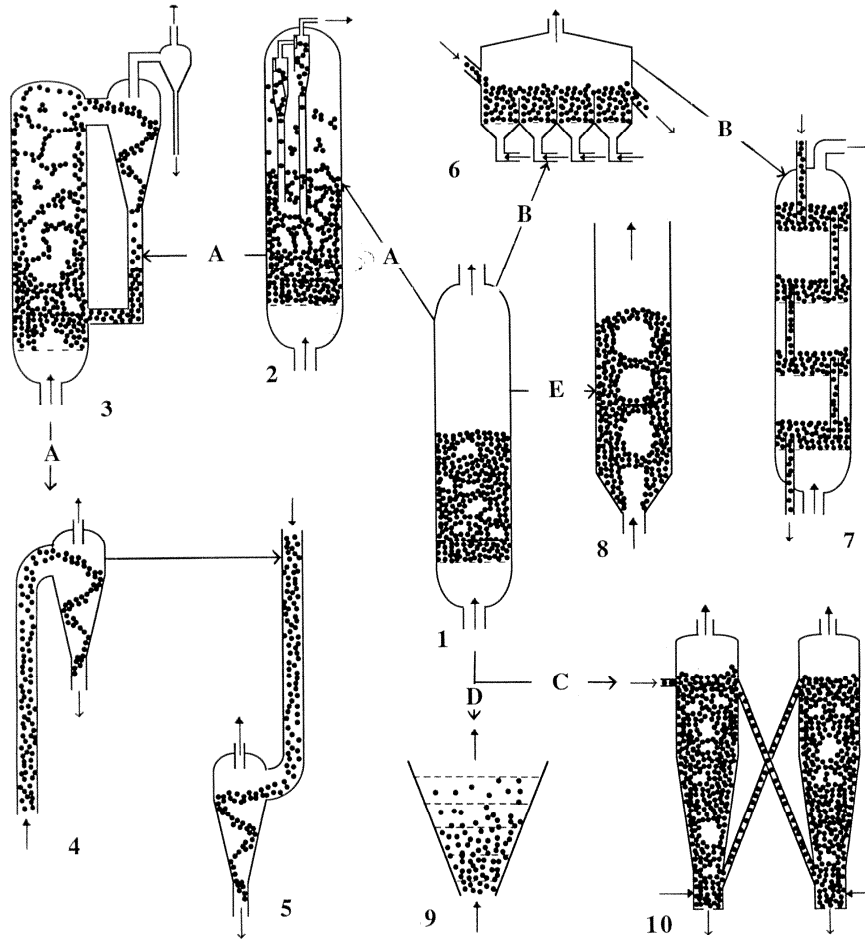


gas

---

# INTRODUCTION

## dense gas-particle flows in fluid bed family of contactors



- 1: bubbling bed
- 2: turbulent bed
- 3: circulating bed
- 4: riser
- 5: downer
- 6: lateral staged bed
- 7: vertical staged bed
- 8: spouted bed
- 9: floating bed
- 10: twin bed

# INTRODUCTION

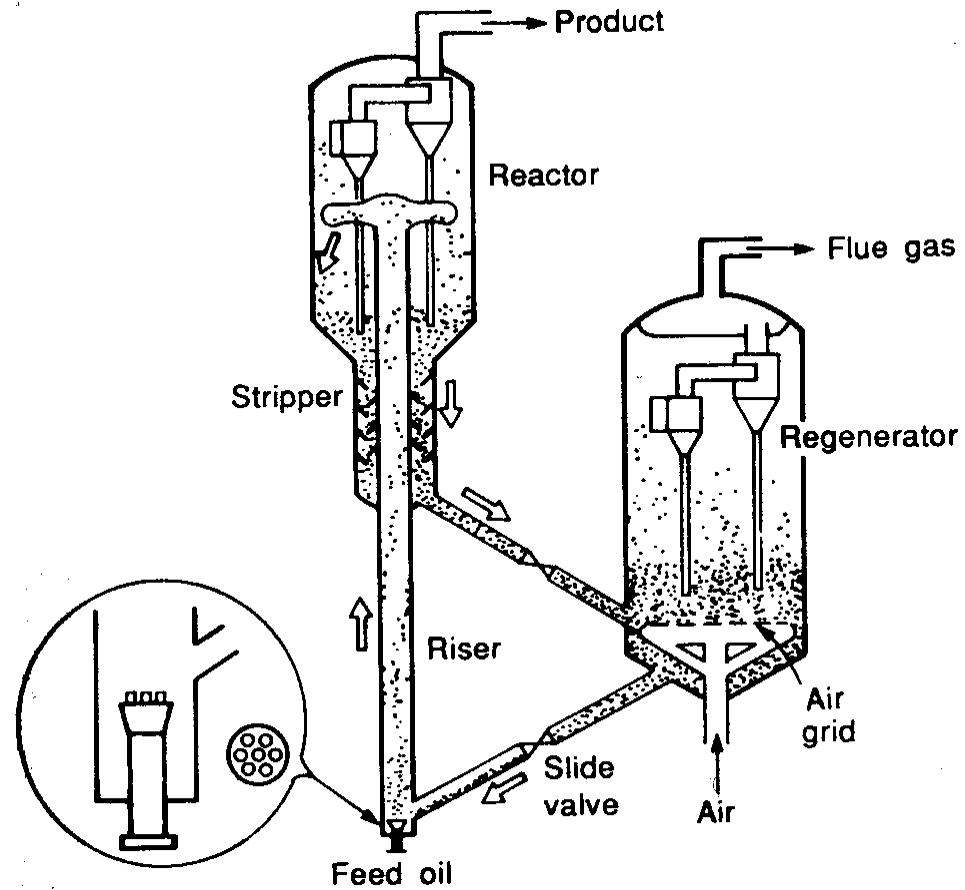
---

- APPLICATIONS OF FLUIDIZED SYSTEMS
    - + heat exchange and drying
    - + coating and granulation
    - + gas purification via adsorption
    - + chemical synthesis (acrylonitrile, maleic and phthalic anhydride)
    - + polymerization of lower olefines (propylene)
    - + Fischer-Tropsch synthesis
    - + Fluid Coking and Flexi-Coking
    - + combustion and incineration
    - + Fluid Catalytic Cracking (FCC)
-

# INTRODUCTION

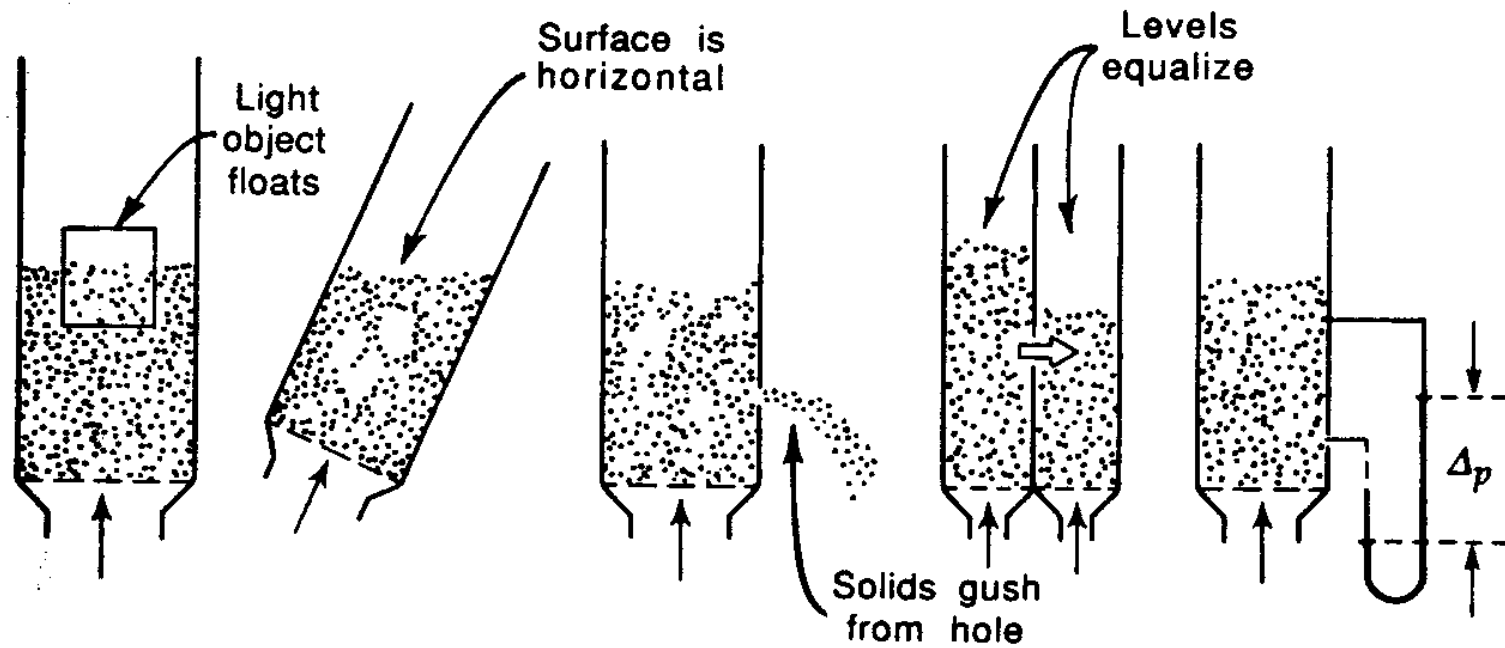
## Fluid Catalytic Cracking (FCC) unit

---



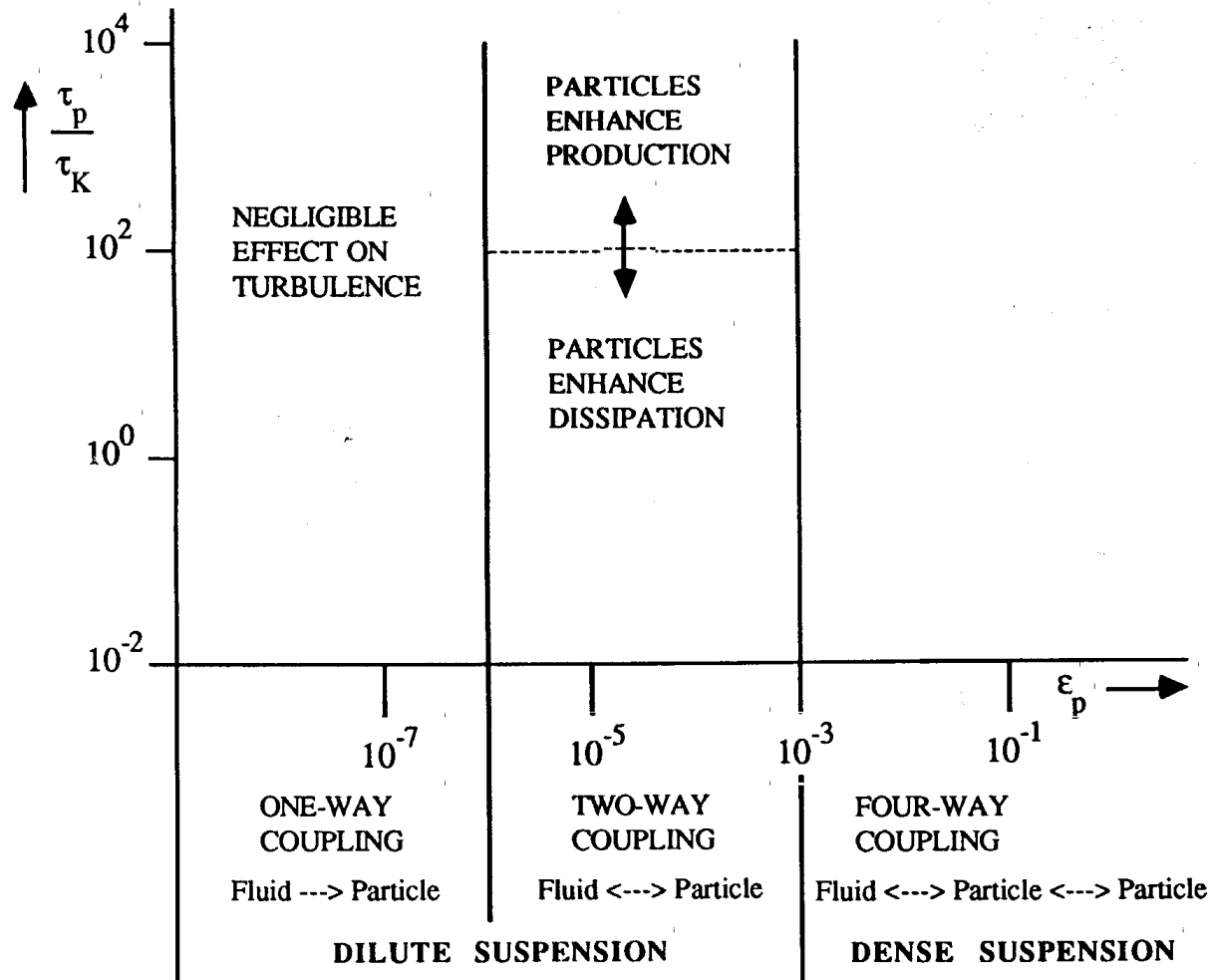
# ELEMENTARY PROPERTIES OF GAS-FLUIDIZED BEDS

## dense beds



# FLOW REGIMES + KEY PHENOMENA

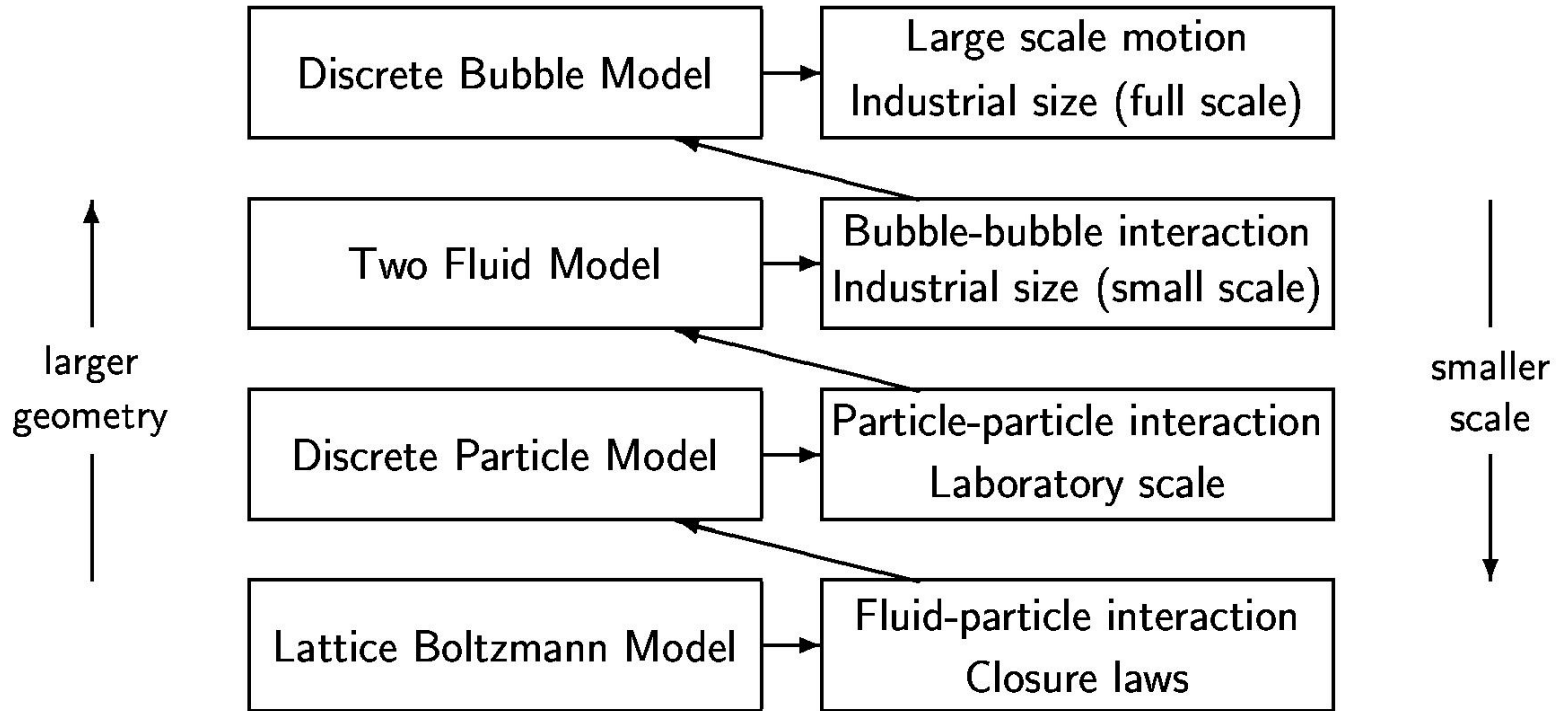
## map of flow regimes in particle-laden flows





# MULTI LEVEL MODELLING

- MULTI LEVEL MODELLING OF DENSE GAS-SOLID FLOWS



Van der Hoef et al., CES (2004)

# DBM SIMULATION

## industrial size column

---

- SIMULATION CONDITIONS

### Industrial scale column:

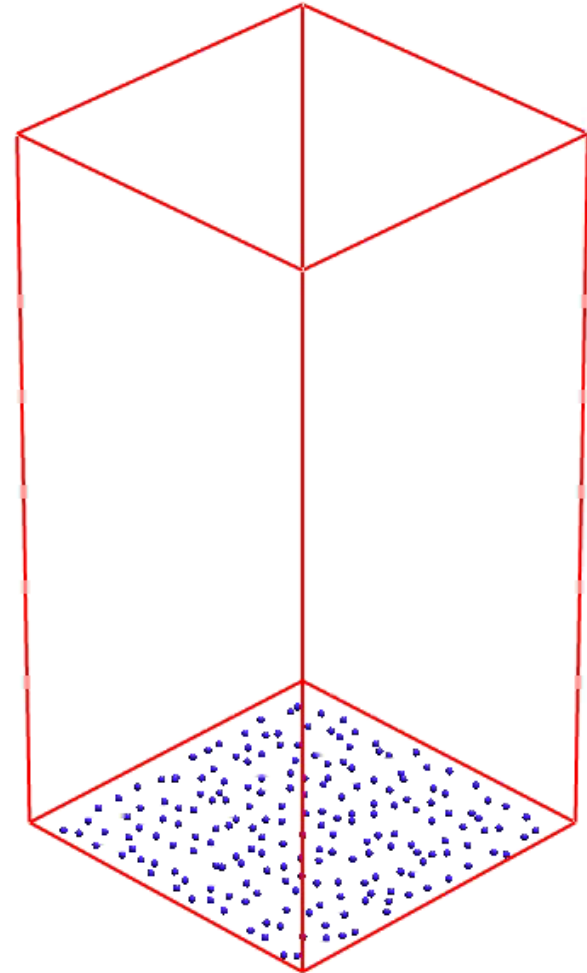
- Dimensions: 4 m x 4 m x 8 m
- Gas velocity:  $2.5U_{mf}=0.25$  m/s

### Emulsion phase properties:

- Density: 400 kg/m<sup>3</sup>
- Viscosity: 0.1 Pa.s

### Bubble properties:

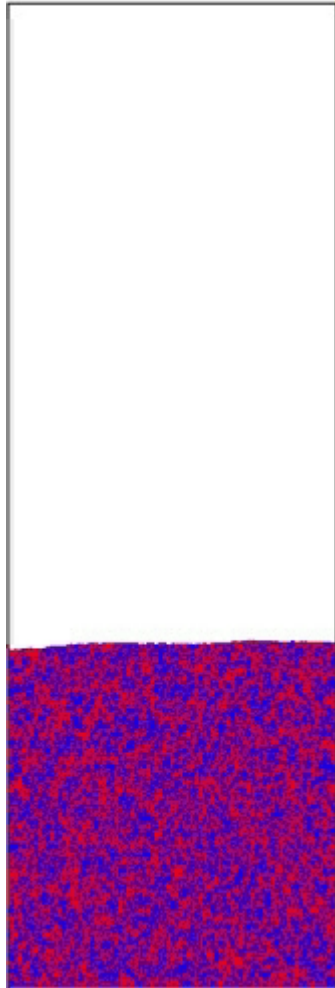
- Initial bubble size: 8 cm
- Maximum bubble size: 80 cm
- Typically ~ 5000 bubbles



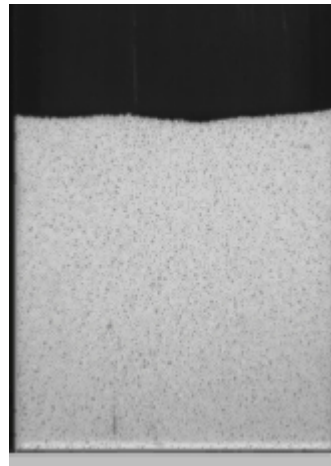
# DPM + KTGF SIMULATION

bubble formation: 15 cm bed

---



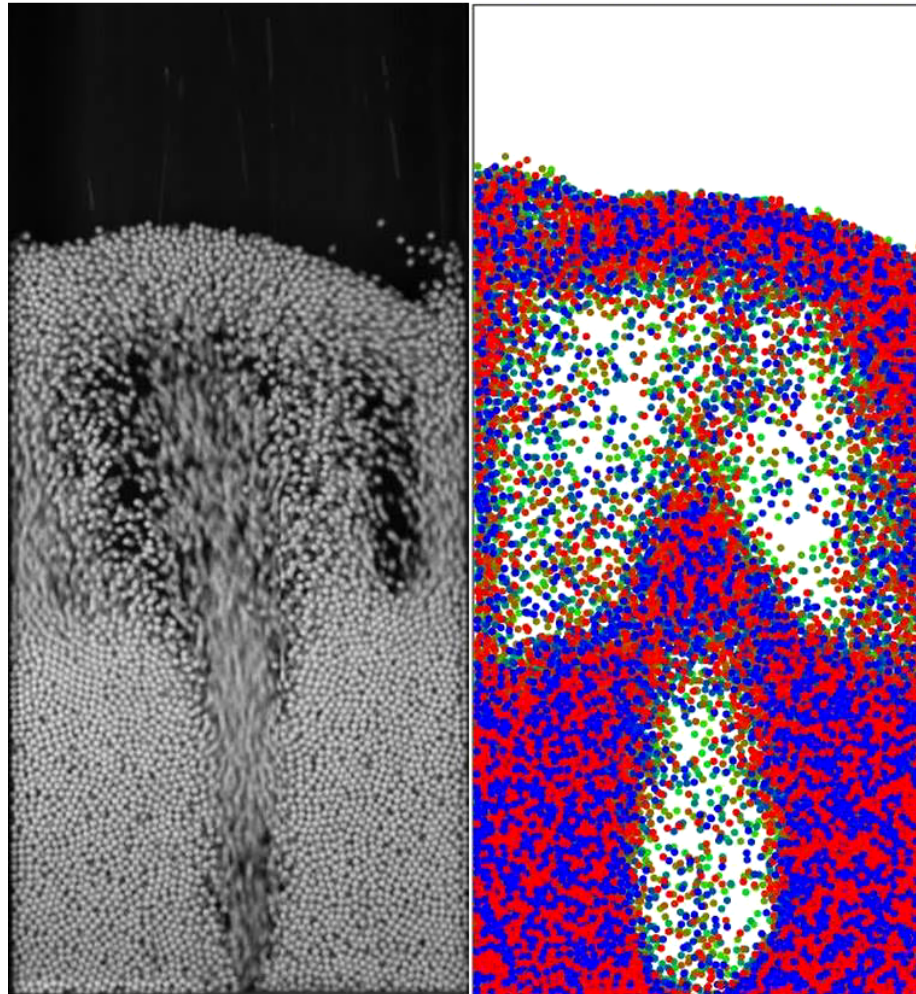
$W=0.15$  m  
 $d_p=1.5$  mm  
 $\rho_s=2526$  kg/m<sup>3</sup>  
 $U_b=0.85$  m/s  
 $U_j=15.0$  m/s  
 $N_p=120000$



# DPM SIMULATION

spouted bed

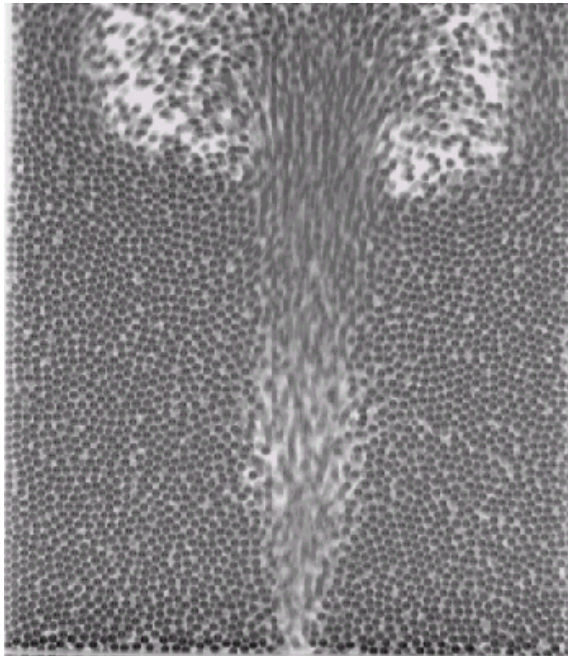
---



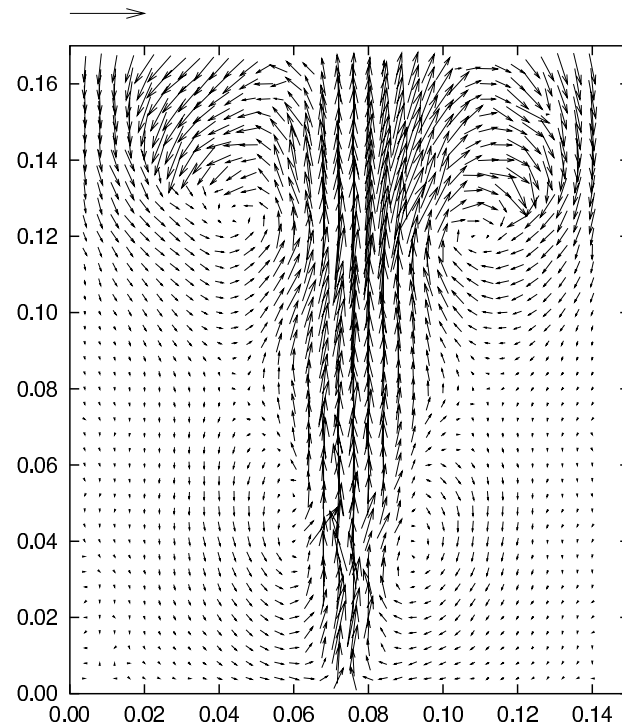
# DPM SIMULATION

## spouted bed

$$u_{sf} / u_{mf} = 16.0 \leftrightarrow u_{bf} / u_{mf} = 1.2$$



particle configuration



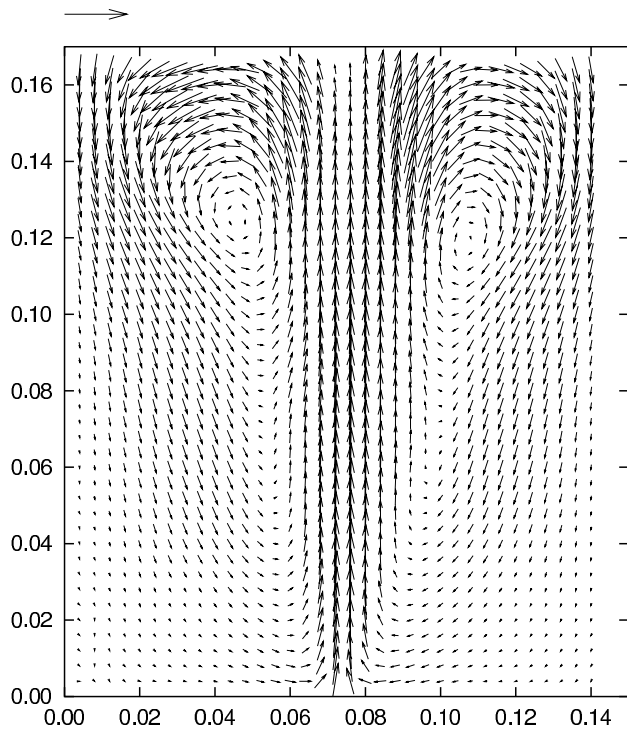
particle velocity map

# DPM SIMULATION

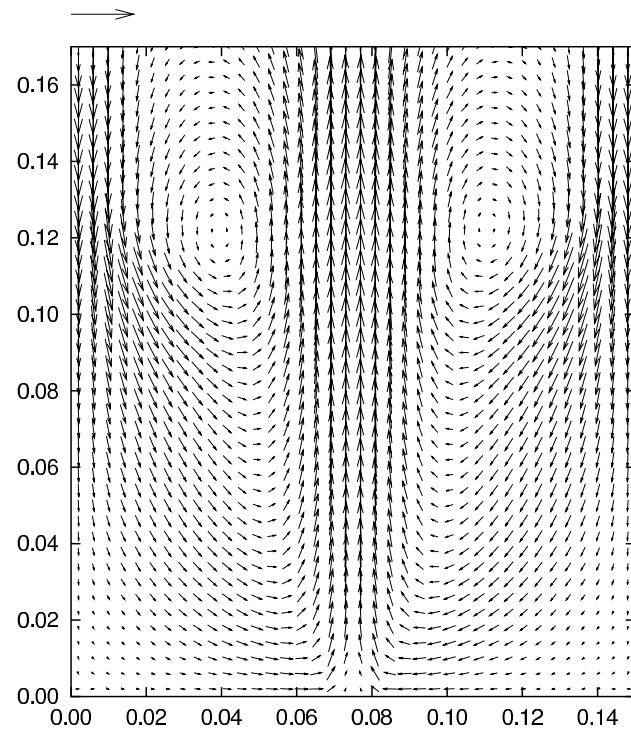
spouted bed

---

$$u_{sf} / u_{mf} = 16.0 \leftrightarrow u_{bf} / u_{mf} = 1.2$$



experimental



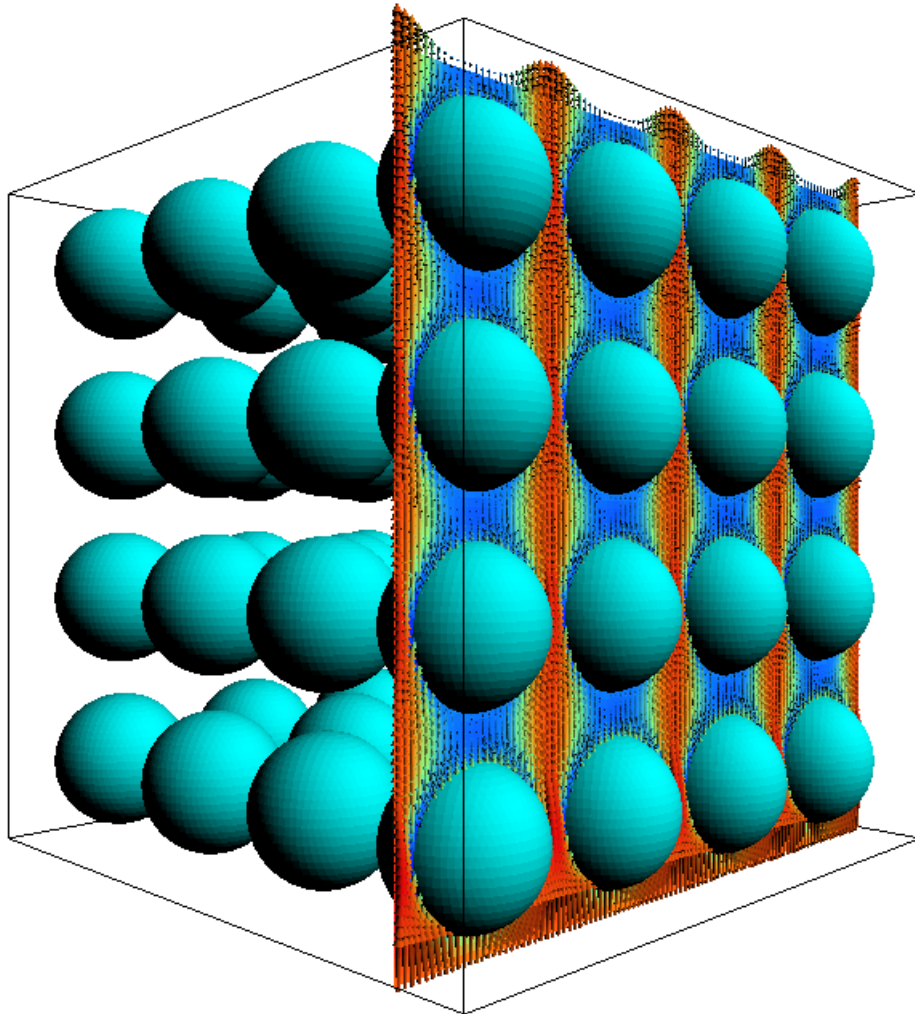
simulated

---

## DNS (IBM)

flow through cubic array of 64 particles at  $Re_p=2.0$

---



$100^3$  Eulerian grid

$$N=(d_p/h)=20$$

Dimensionless drag

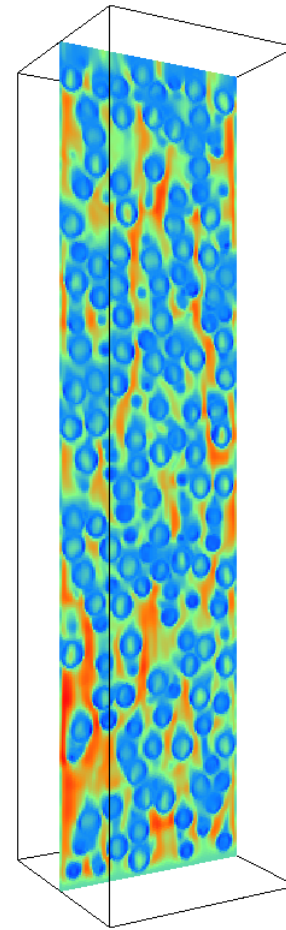
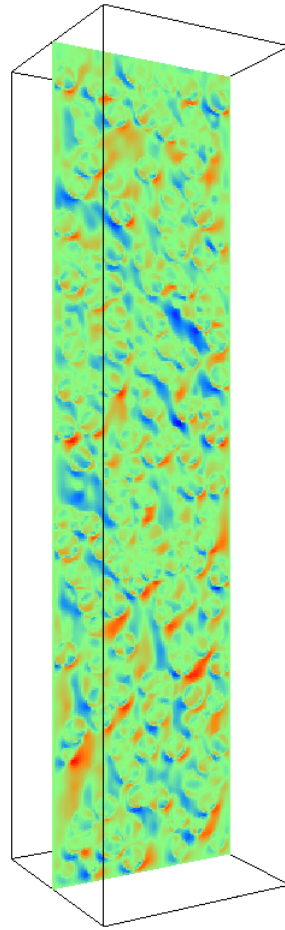
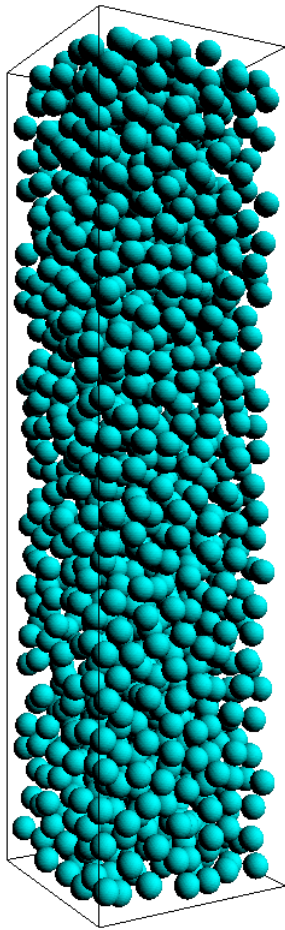
$F=10.9$  (computed)

$F=10.2$  (analytical)

## DNS (IBM)

flow through random array of 1326 particles at  $Re_p=120$

---

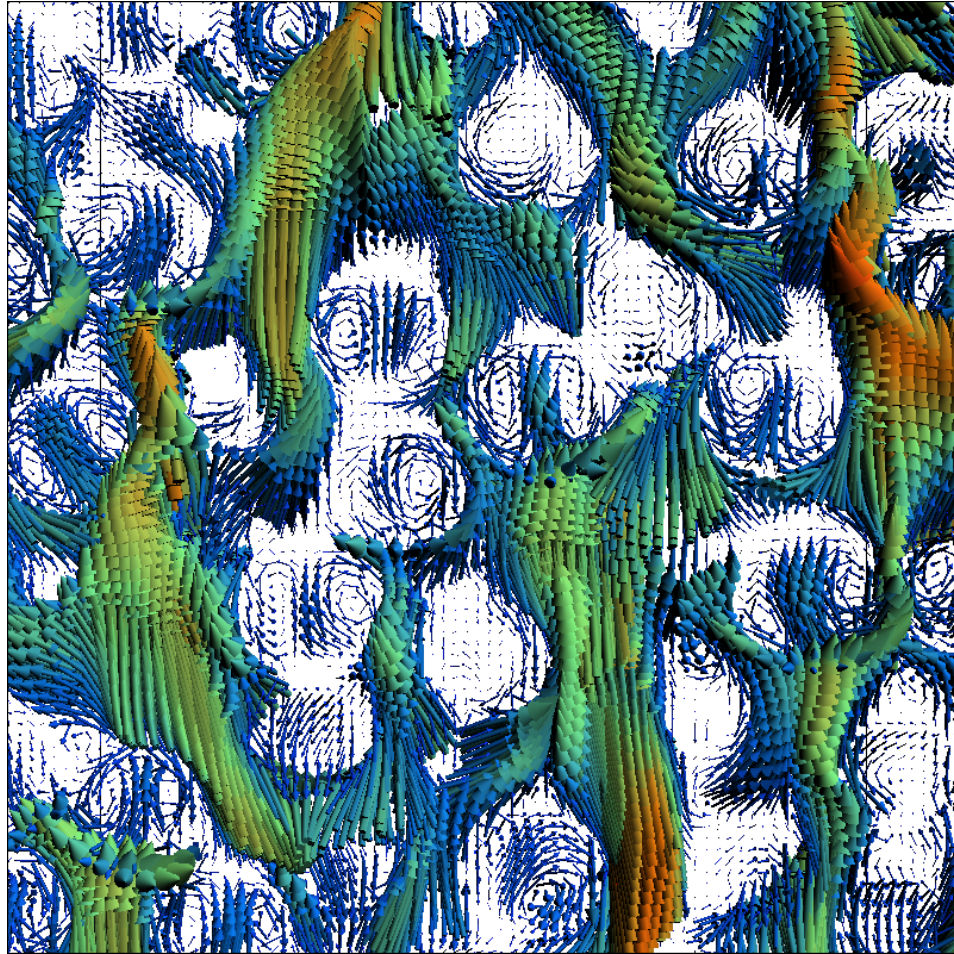




DNS (IBM)

flow through random array of 1326 particles at  $Re_p=120$

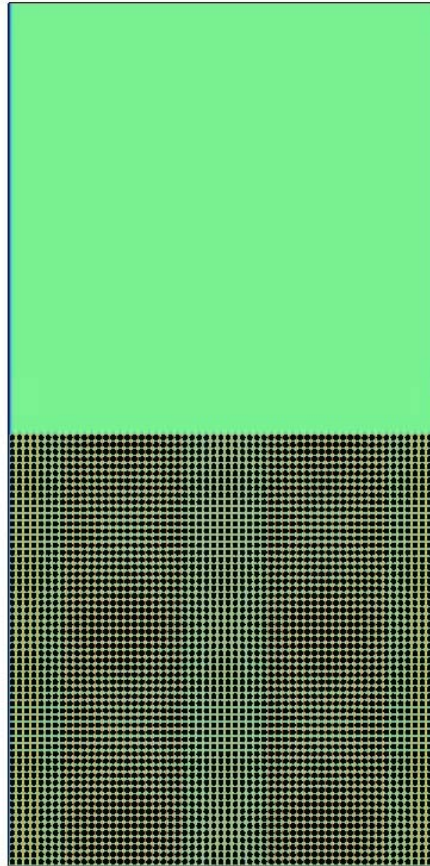
---



# DNS (IBM)

## fluidization of 3600 discs

---



# KINETIC THEORY BASED MODELS

---

- BASIC FEATURES

  - + statistical mechanical description of particle-particle encounters

- ADVANTAGES

  - + based on more fundamental description of particle-particle interaction compared to classical two-fluid model

- DISADVANTAGES

  - + incorporation of different particle properties (polydispersity) is quite difficult and leads to (many) additional equations (CPU limitations)

---

# KINETIC THEORY BASED MODELS

---

- LIMITATIONS AND PRESENT DIFFICULTIES
    - + nearly spherical particles
    - + not suited for dense gas-particle flows where (quasi-)static particle zones prevail (hoppers, fixed beds and moving beds)
    - + incorporation of detailed particle-particle interaction models difficult
    - + systems with broad distribution in physical properties (size, density)
    - + systems with (rapid) changes in particle size (polymerization)
-

# CONTINUUM MODEL BASED ON KINETIC THEORY

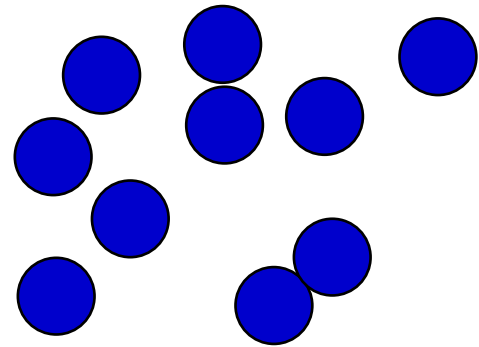
---

- DEFINITION OF PARTICLE VELOCITIES

+ instantaneous particle velocity:  $\bar{c}$

+ ensemble averaged particle velocity:  $\bar{v}$

+ fluctuating particle velocity:  $\bar{C} = \bar{c} - \bar{v}$



- DISTRIBUTION OF FLUCTUATING VELOCITIES (KTG)

$$f = n \left( \frac{m}{2\pi kT} \right)^{3/2} \exp\left( -\frac{mC^2}{2kT} \right)$$

Maxwell's velocity distribution

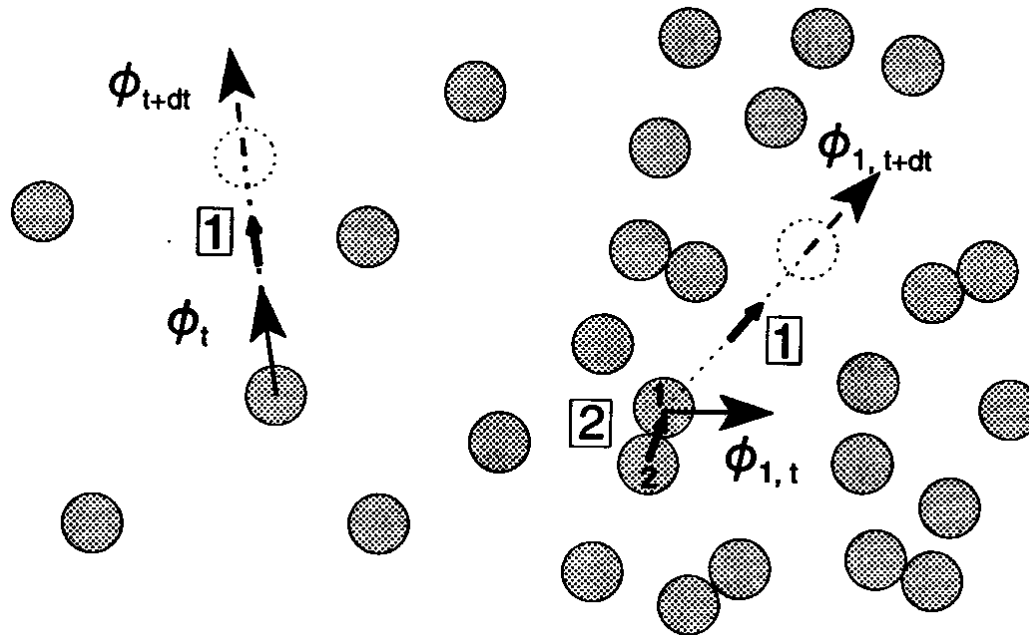
number density

Boltzmann constant

# CONTINUUM MODEL BASED ON KINETIC THEORY

---

- TRANSPORT MECHANISMS FOR PARTICLE PROPERTY  $\phi$



## CONTINUUM MODEL BASED ON KINETIC THEORY

---

- BOLTZMANN EQUATION IN TERMS OF  $\bar{c}$

$$\frac{\partial f}{\partial t} + (\bar{c} \bullet \frac{\partial f}{\partial \bar{r}}) + (\bar{F} \bullet \frac{\partial f}{\partial \bar{c}}) + f(\frac{\partial}{\partial \bar{c}} \bullet \bar{F}) = (\frac{\partial_e f}{\partial t})$$

- BOLTZMANN EQUATION IN TERMS OF  $\bar{C} = \bar{c} - \bar{v}$

$$\frac{Df}{Dt} - (\frac{D\bar{v}}{Dt} \bullet \frac{\partial f}{\partial \bar{C}}) + (\bar{C} \bullet \frac{\partial f}{\partial \bar{r}}) - (\bar{C} \frac{\partial f}{\partial \bar{C}} : \frac{\partial \bar{v}}{\partial \bar{r}}) + (\frac{\partial}{\partial \bar{C}} \bullet (f\bar{F})) = (\frac{\partial_e f}{\partial t})$$

- SUBSTANTIAL DERIVATIVE:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\bar{v} \bullet \frac{\partial}{\partial \bar{r}})$$


---

## CONTINUUM MODEL BASED ON KINETIC THEORY

---

- MAXWELL TRANSPORT EQUATION FOR PROPERTY  $\phi$

$$\frac{D(n \langle \phi \rangle)}{Dt} + \left( \frac{\partial}{\partial \bar{r}} \bullet n \langle \phi \bar{C} \rangle \right) + n \langle \phi \rangle \left( \frac{\partial}{\partial \bar{r}} \bullet \bar{v} \right) -$$

$$n \left[ \left\langle \frac{D\phi}{Dt} \right\rangle + \left\langle \bar{C} \bullet \frac{\partial \phi}{\partial \bar{r}} \right\rangle + \left\langle \bar{F} \bullet \frac{\partial \phi}{\partial \bar{C}} \right\rangle - \left( \frac{D\bar{v}}{Dt} \bullet \left\langle \frac{\partial \phi}{\partial \bar{C}} \right\rangle \right) -$$

$$\left( \frac{\partial \bar{v}}{\partial \bar{r}} : \left\langle \bar{C} \frac{\partial \phi}{\partial \bar{C}} \right\rangle \right) ] = n \Delta \langle \phi \rangle = \iiint \phi \left( \frac{\partial_e f}{\partial t} \right) d\bar{C} =$$

- ENSEMBLE AVERAGE OF PROPERTY  $\phi$

$$\langle \phi \rangle = \iiint \phi f d\bar{C}$$

$$\begin{aligned} \phi &= 1 \\ \phi &= m\bar{C} \\ \phi &= \frac{1}{2} m(\bar{C} \bullet \bar{C}) = \frac{1}{2} mC^2 \end{aligned}$$



## MICRO BALANCE EQUATIONS

---

- CONTINUITY EQUATIONS

$$\frac{\partial}{\partial t}(\varepsilon_f \rho_f) + (\nabla \cdot \varepsilon_f \rho_f \bar{u}) = 0$$

$$\frac{\partial}{\partial t}(\varepsilon_s \rho_s) + (\nabla \cdot \varepsilon_s \rho_s \bar{v}) = 0$$

- MOMENTUM EQUATIONS

$$\frac{\partial}{\partial t}(\varepsilon_f \rho_f \bar{u}) + (\nabla \cdot \varepsilon_f \rho_f \bar{u} \bar{u}) = -\varepsilon_f \nabla p - (\nabla \cdot \varepsilon_f \tau_f) - \beta(\bar{u} - \bar{v}) + \varepsilon_f \rho_f \bar{g}$$

$$\frac{\partial}{\partial t}(\varepsilon_s \rho_s \bar{v}) + (\nabla \cdot \varepsilon_s \rho_s \bar{v} \bar{v}) = -\varepsilon_s \nabla p - (\nabla \cdot \varepsilon_s \tau_s) - \nabla p_s + \beta(\bar{u} - \bar{v}) + \varepsilon_s \rho_s \bar{g}$$

---

# MICRO BALANCE EQUATIONS

---

- GRANULAR TEMPERATURE EQUATION

$$\frac{3}{2} \left[ \frac{\partial}{\partial t} (\varepsilon_s \rho_s \Theta) + (\nabla \cdot \varepsilon_s \rho_s \Theta \bar{v}) \right] = -(p_s I + \varepsilon_s \tau_s) : (\nabla \bar{v}) + (\nabla \cdot \varepsilon_s \kappa_s \nabla \Theta) + \beta [\overline{\bar{c}_f \cdot \bar{c}_s} - 3\Theta] - \gamma$$

- ADDITIONAL EQUATIONS AND CLOSURES

+ phase densities

+ phase stress tensors and phase viscosities

+ pseudo Fourier energy flux and solids pseudo conductivity

+ solids pressure

+ interphase momentum exchange coefficient

+ covariance between fluid and solid fluctuating velocities

↑  
for dense flows this term  
can safely be neglected

# CLOSURE OF MICRO BALANCE EQUATIONS

## interphase momentum transfer coefficient

---

- Ergun equation ( $\varepsilon_f < 0.8$ ):

$$\beta = 150 \frac{(1 - \varepsilon_f)^2}{\varepsilon_f} \frac{\mu_f}{d_p^2} + 1.75(1 - \varepsilon_f) \frac{\rho_f}{d_p} |\bar{u} - \bar{v}|$$

- Wen and Yu equation ( $\varepsilon_f > 0.8$ ):

$$\beta = \frac{3}{4} C_d \frac{\varepsilon_f \varepsilon_s}{d_p} \rho_f |\bar{u} - \bar{v}| \varepsilon_f^{-2.65}$$

exponent depends on  
particle Reynolds number  $Re_p$

- Drag coefficient:

$$C_d = \frac{24}{Re_p} [1 + 0.15(Re_p)^{0.687}]$$

$$Re_p = \frac{\varepsilon_f \rho_f |\bar{u} - \bar{v}| d_p}{\mu_f}$$

$$C_d = 0.44$$

$Re_p < 1000$

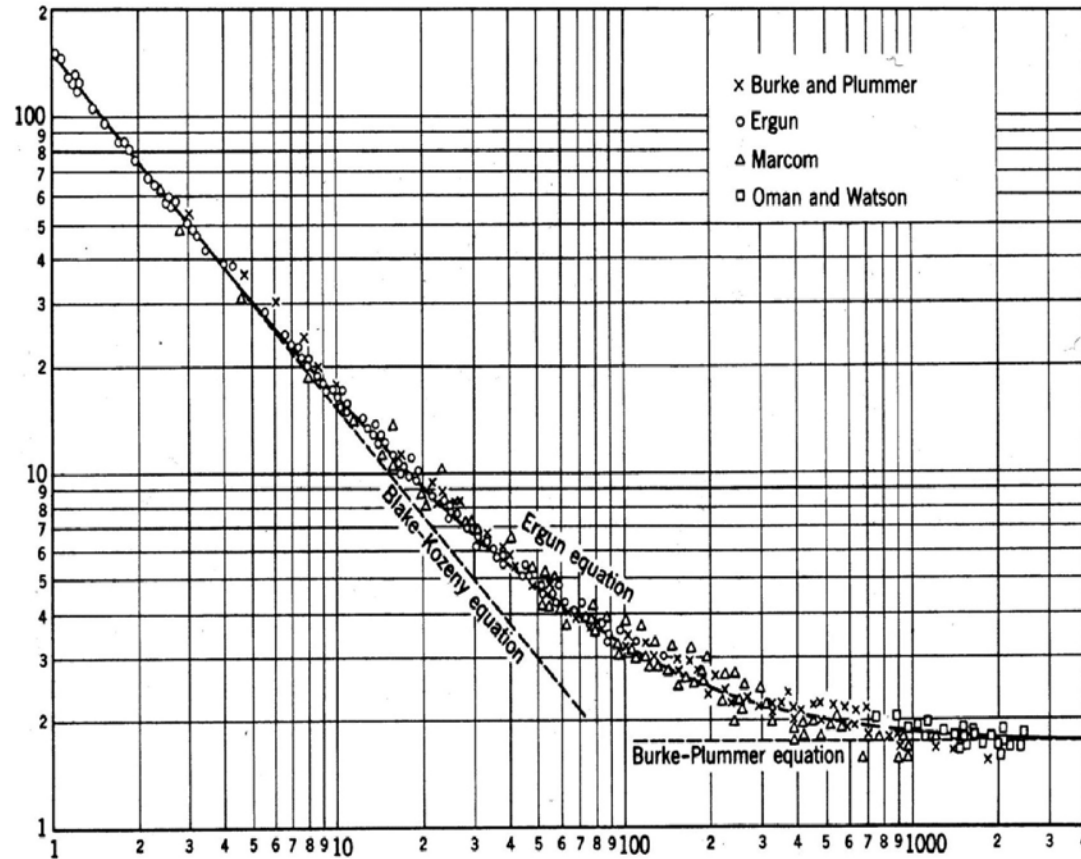
$Re_p > 1000$

---

# DRAG CLOSURE

## Ergun equation

$$F * \left( \frac{18\epsilon}{Re} \right)$$



$$Re/(1-\epsilon)$$

## CONTINUUM MODEL BASED ON KINETIC THEORY

---

- SUMMARIZING THE KEY FEATURES OF KTGF
    - + particles interact through binary nonideal collisions (no friction)
    - + non-ideal particle-wall collisions are accounted for
    - + departure from Maxwellian (velocity) distribution function is small
    - + one additional equation (“granular temperature equation”)
    - + closures for solids viscosity, pressure and pseudo conductivity
    - + random granular motion and no collective granular motion
-

# NUMERICAL SOLUTION METHOD

---

- KEY FEATURES

+ explicit treatment of convection and diffusion terms

$$\left[ \frac{|u_x|}{\Delta x} + \frac{|u_y|}{\Delta y} + \frac{|u_z|}{\Delta z} \right] \Delta t < 1$$

stability  
conditions

$$v_f \Delta t < \frac{1}{2} \left[ \frac{1}{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}} \right]$$

$$\left[ \frac{|v_x|}{\Delta x} + \frac{|v_y|}{\Delta y} + \frac{|v_z|}{\Delta z} \right] \Delta t < 1$$

$$v_s \Delta t < \frac{1}{2} \left[ \frac{1}{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}} \right]$$

+ implicit treatment of porosity pressure in momentum equations

+ staggered computational mesh

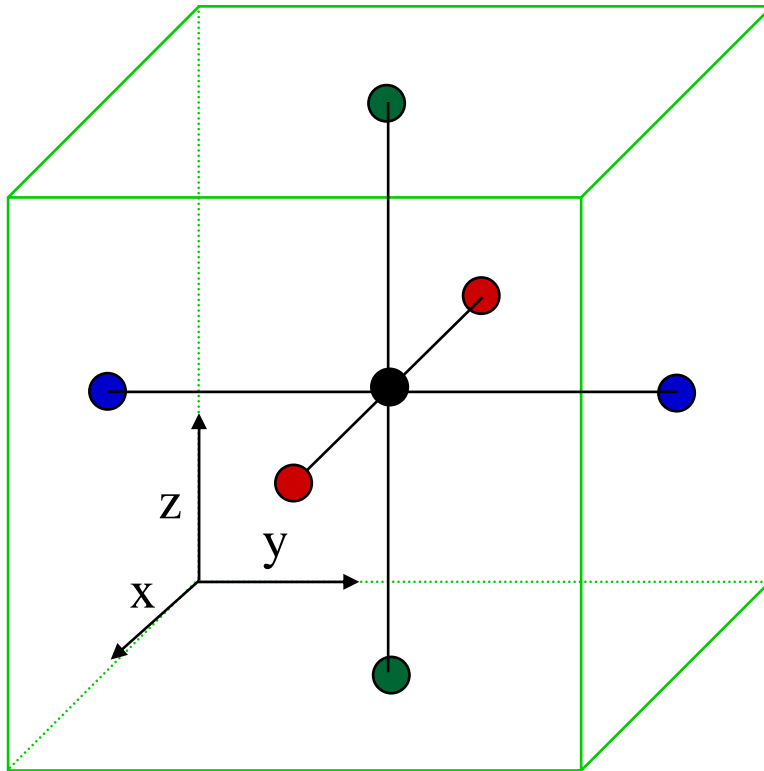
+ high order schemes for convection for mass and momentum

---

# NUMERICAL SOLUTION METHOD

---

- DEFINITION OF EULERIAN VARIABLES

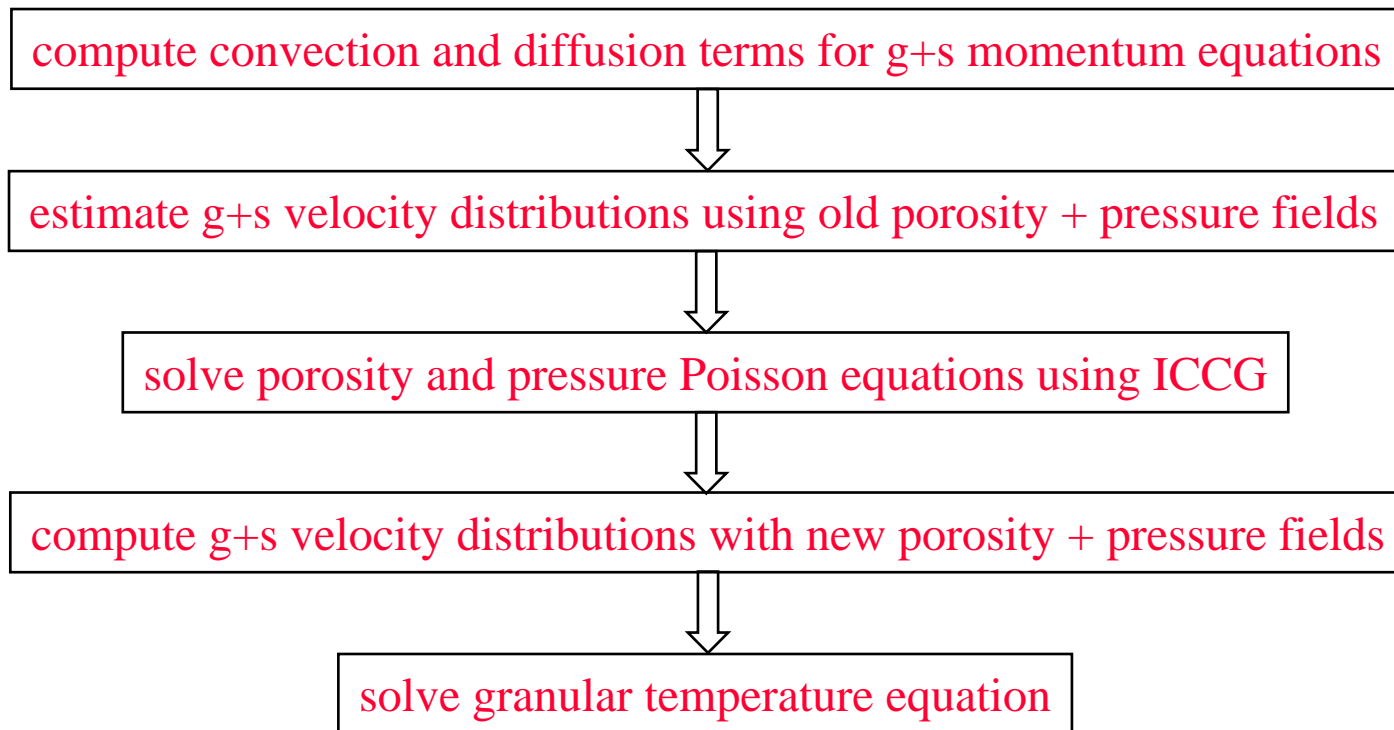


- scalar variables
  - x-velocity component
  - y-velocity component
  - z-velocity component
-

## NUMERICAL SOLUTION METHOD

---

- OVERALL COMPUTATIONAL STRATEGY PER CYCLE



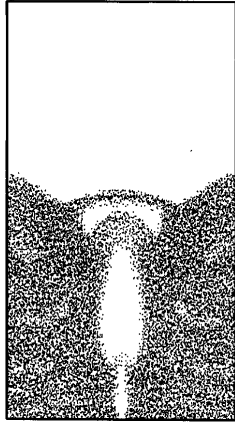


# RESULTS OF KTGF MODEL

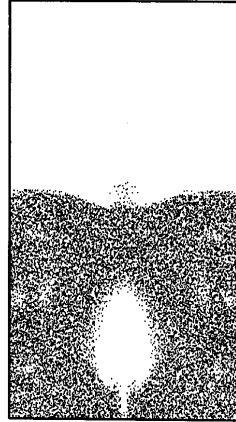
## bubble formation at a jet using 3D model

---

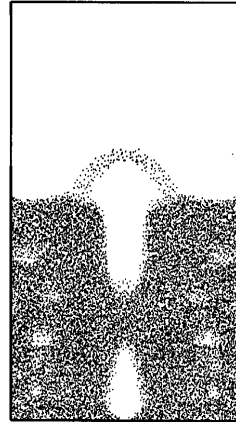
t = 1.0000 s.



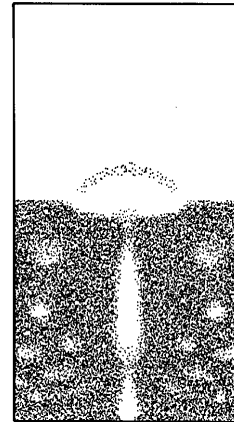
t = 2.0000 s.



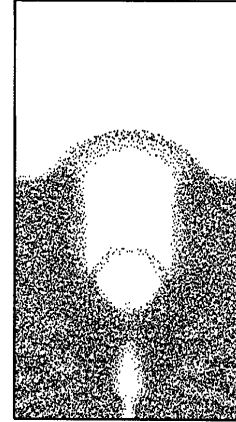
t = 3.0000 s.



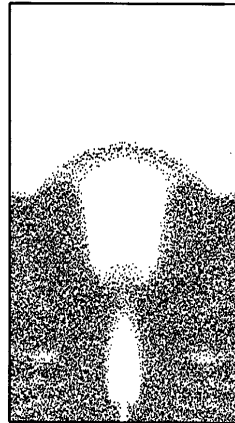
t = 4.0000 s.



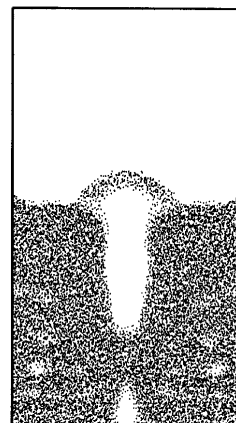
t = 5.0000 s.



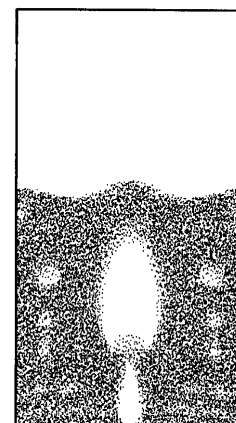
t = 6.0000 s.



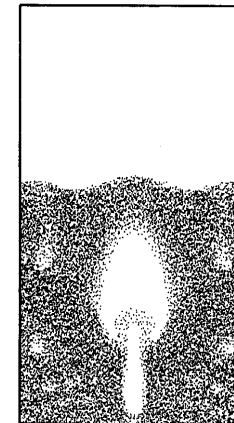
t = 7.0000 s.



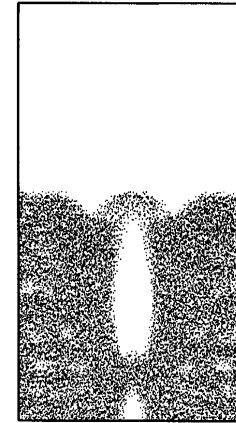
t = 8.0000 s.



t = 9.0000 s.



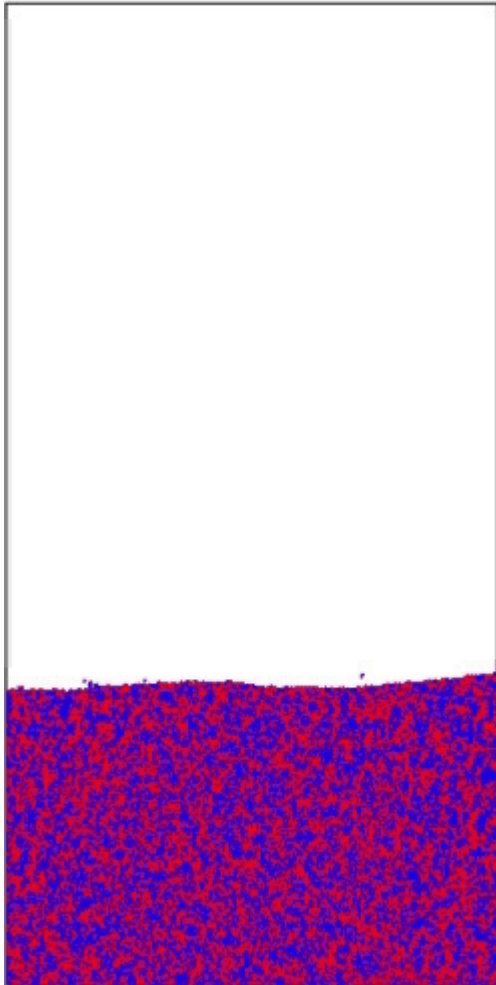
t = 10.0000 s.



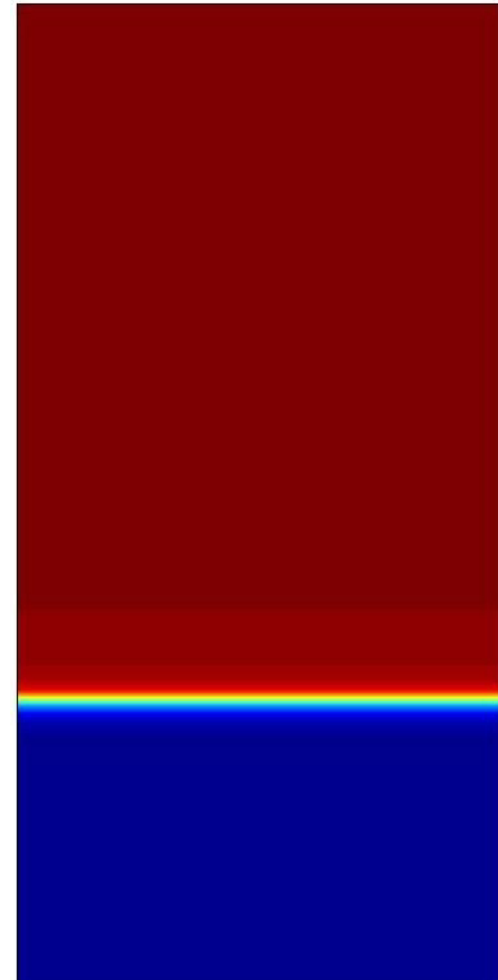
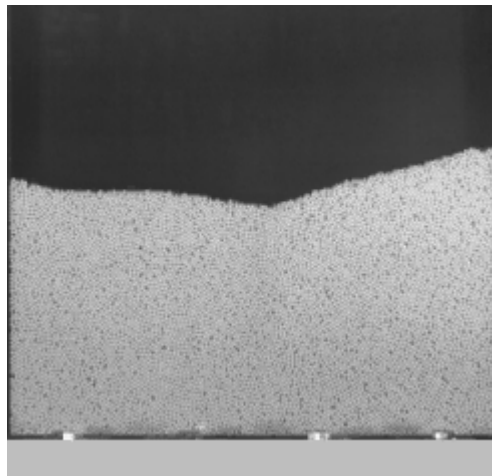
# RESULTS OF KTGF MODEL: MONODISPERSE SYSTEMS

bubble formation: 30 cm bed

---



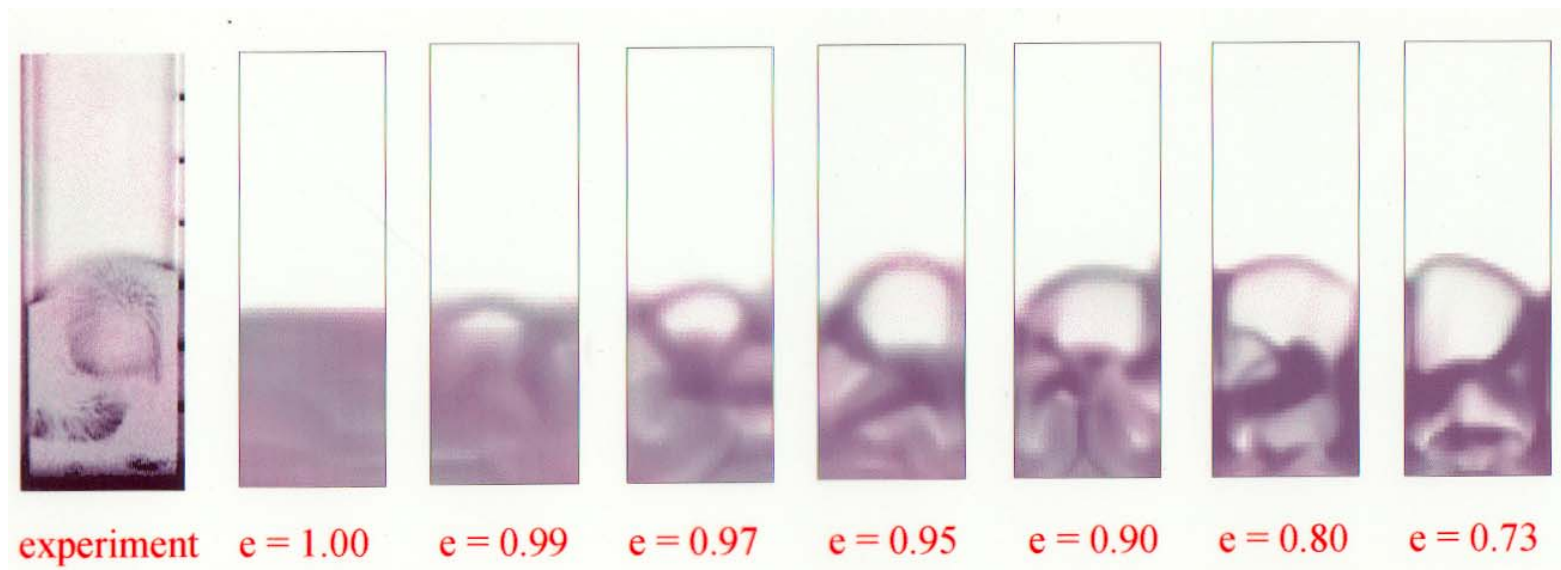
$W=0.30$  m  
 $d_p=2.5$  mm  
 $\rho_s=2526$  kg/m<sup>3</sup>  
 $U_b=1.20$  m/s  
 $U_j=20.0$  m/s  
 $N_p=60000$



# RESULTS OF KTGF MODEL

## effect of restitution coefficient on bed dynamics

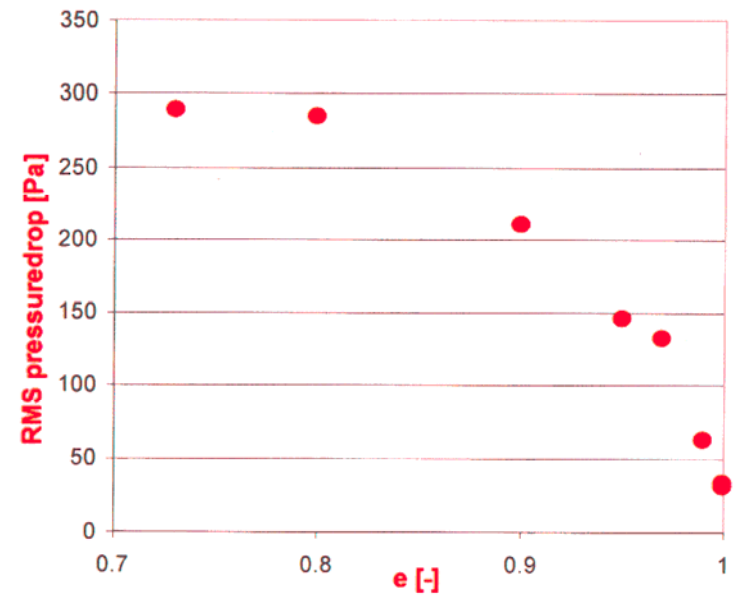
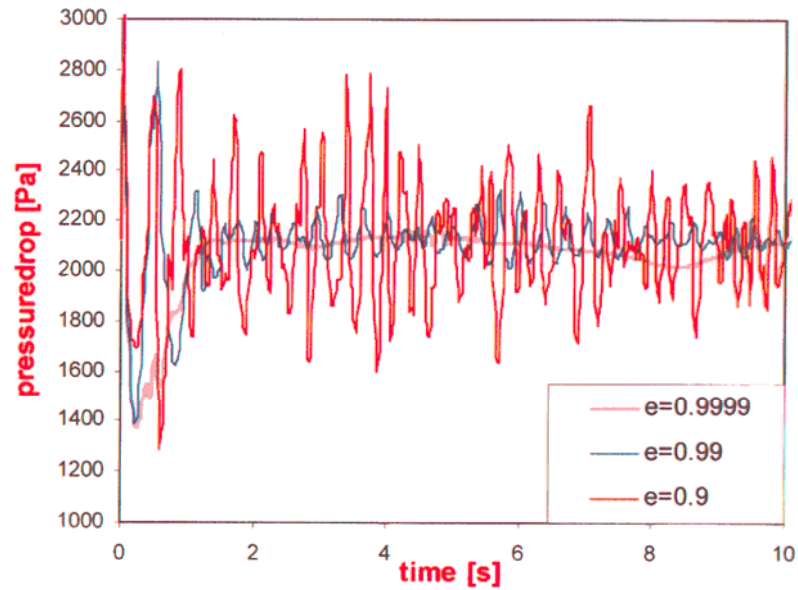
---



# RESULTS OF KTGF MODEL

## effect of restitution coefficient on bed dynamics

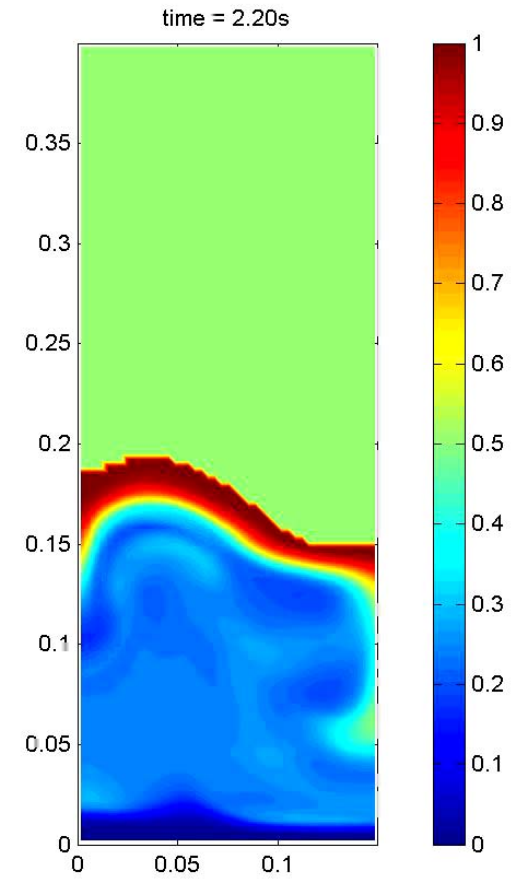
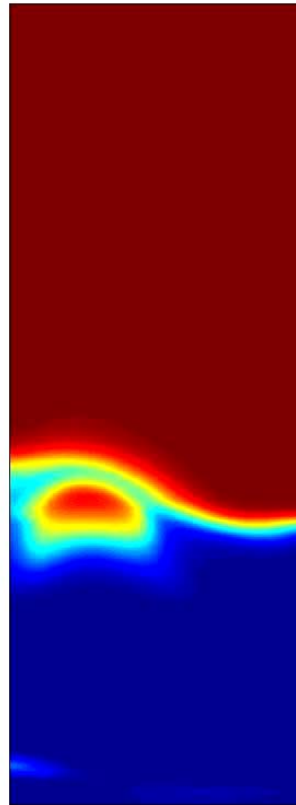
---



# RESULTS OF KTGF MODEL: BIDISPERSE SYSTEMS

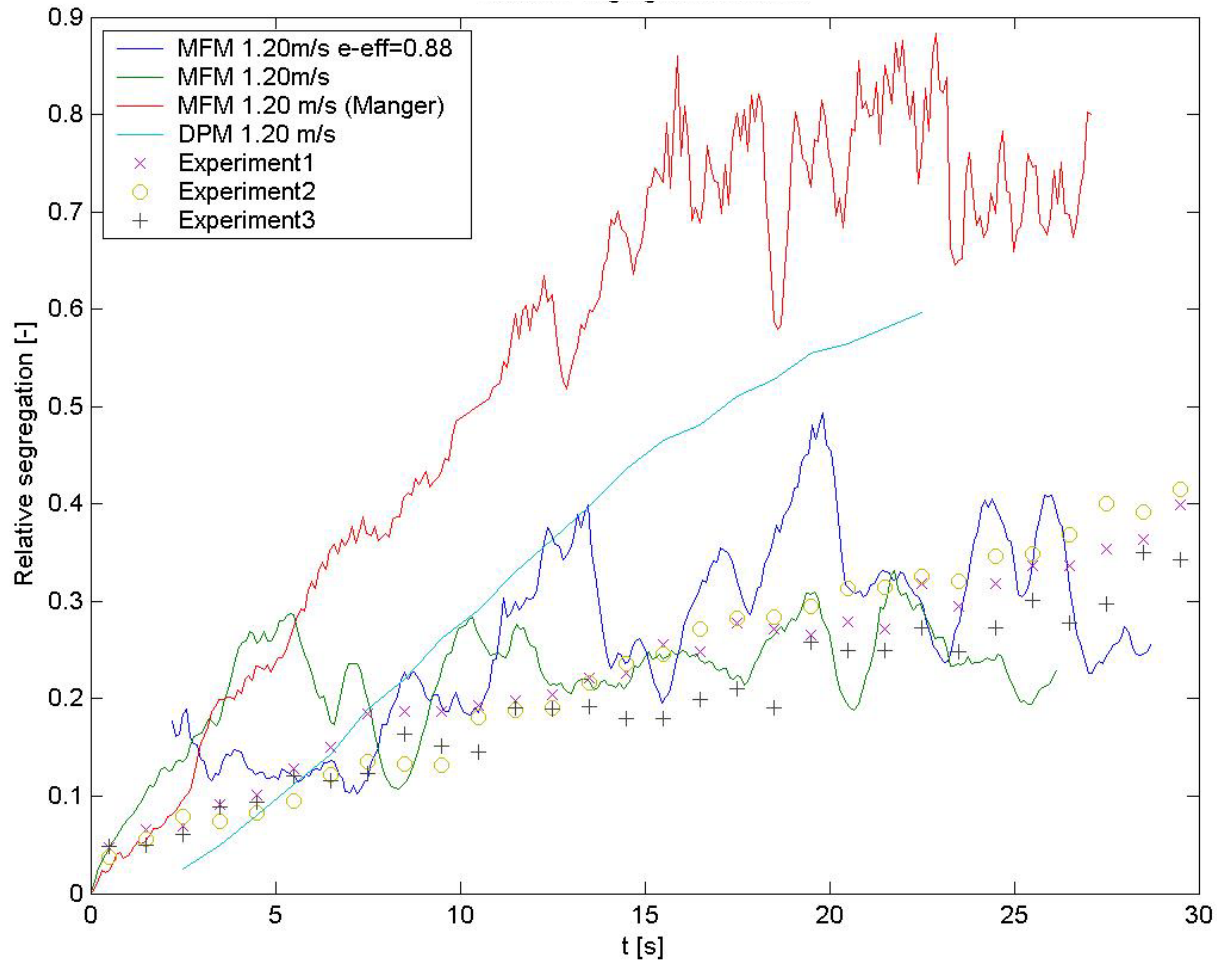
## segregation rate in bidisperse systems

---



# RESULTS OF KTGF MODEL: BIDISPERSE SYSTEMS

## segregation rate in bidisperse systems



## RESULTS OF KTGF MODEL

### effect of lateral segregation on riser reactor performance

---

- RISER FLOW

+ particle diameter (FCC)	40 $\mu\text{m}$
+ riser diameter	0.3 m
+ superficial gas velocity	6.3 m/s
+ solids mass flux	390 kg/(m <sup>2</sup> .s)

- FEATURES

- + two-fluid model incorporating kinetic theory of granular flow
  - + turbulence model: Prandtl mixing length model
  - + axi-symmetrical flow
-

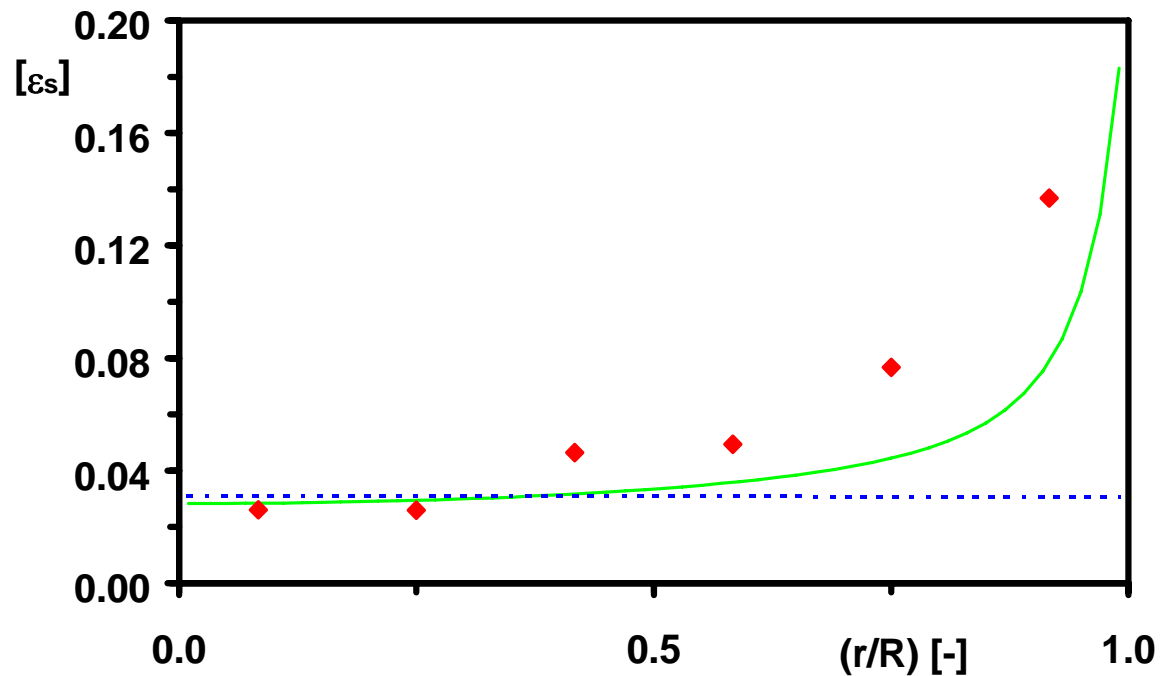
## RESULTS OF KTGF MODEL

### effect of lateral segregation on riser reactor performance

---

- RISER FLOW

RADIAL PROFILE OF SOLIDS VOLUME FRACTION  $\varepsilon_s$





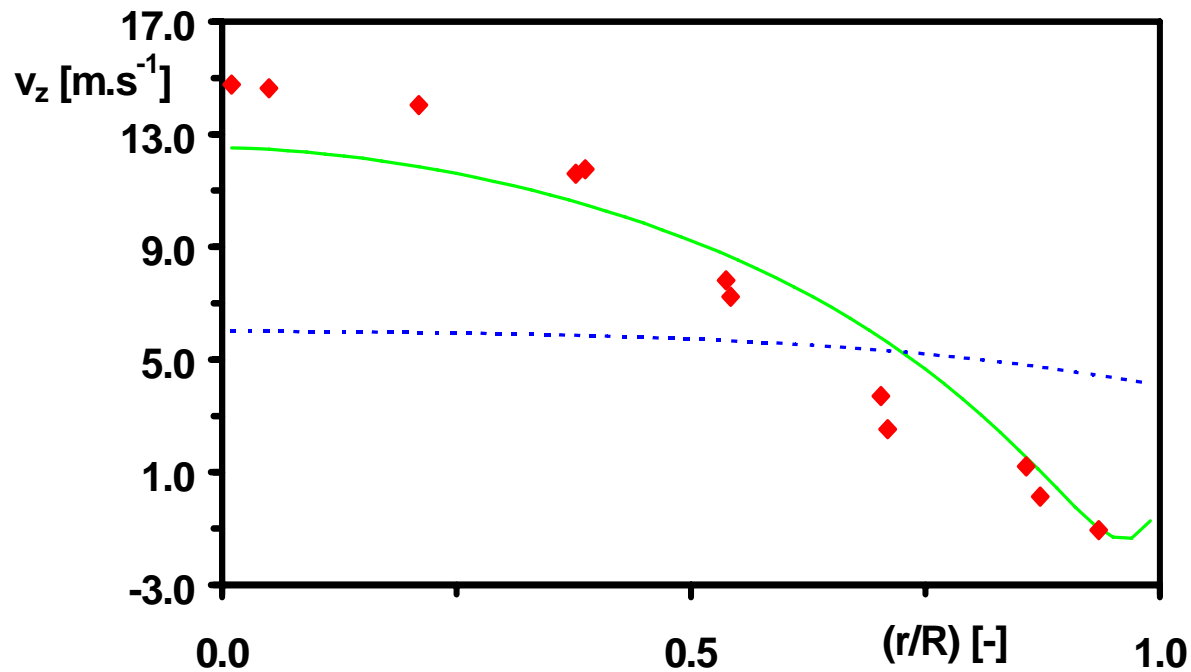
# RESULTS OF KTGF MODEL

## effect of lateral segregation on riser reactor performance

---

- RISER FLOW

### RADIAL PROFILE OF AXIAL SOLIDS VELOCITY $V_z$

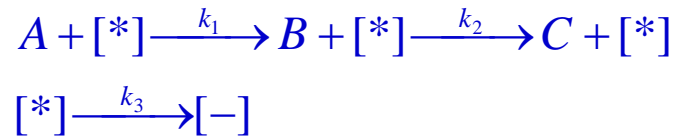


# RESULTS OF KTGF MODEL

## effect of lateral segregation on riser reactor performance

---

- REACTION SCHEME:



- KINETICS:

$$r_A = -\varepsilon_s[*]\rho_f k_1 x_A$$

$$r_B = +\varepsilon_s[*]\rho_f (k_1 x_A - k_2 x_B)$$

$$r_C = +\varepsilon_s[*]\rho_f k_2 x_B$$

$$r_* = -\varepsilon_s[*]\rho_s k_3$$

$$k_1 = 25 \text{ s}^{-1}$$

$$k_2 = 15 \text{ s}^{-1}$$

$$k_3 = 0.2 \text{ s}^{-1}$$

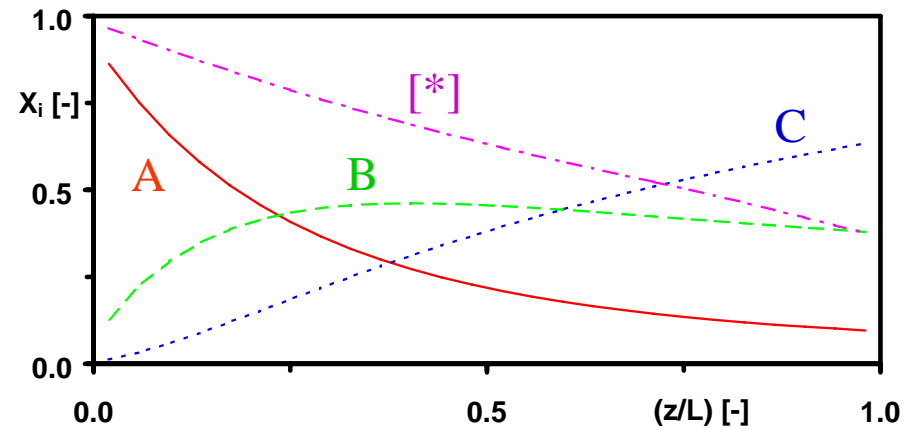
---

# RESULTS OF KTGF MODEL

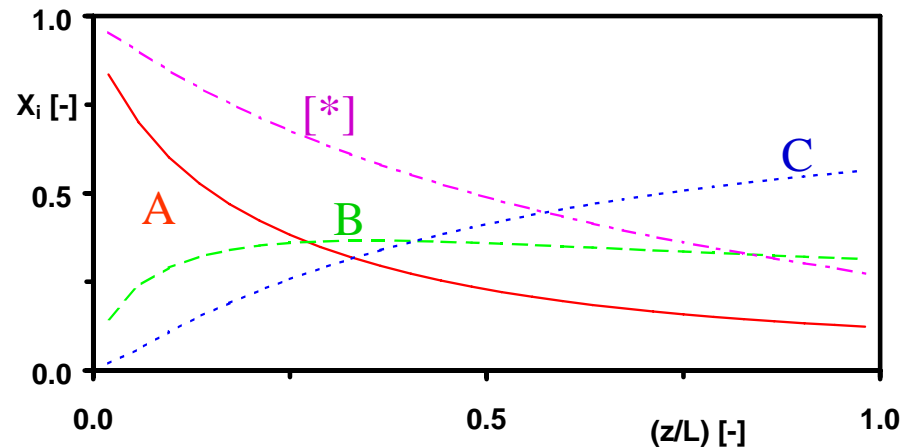
## effect of lateral segregation on riser reactor performance

---

- AXIAL PROFILE OF FLOW-AVERAGED FRACTIONS IN RISER



IDEAL



NON-IDEAL

---

## CONCLUSIONS

---

- BUBBLING BED

- + significant effect of  $e$  on bubble dynamics

- + dissipation level too low (friction is not included in KTGF)

- CFB RISER

- + radial segregation in dense riser flow can be predicted

- + significant effect of radial segregation on riser reactor performance

---