An Introduction to Granular Matter

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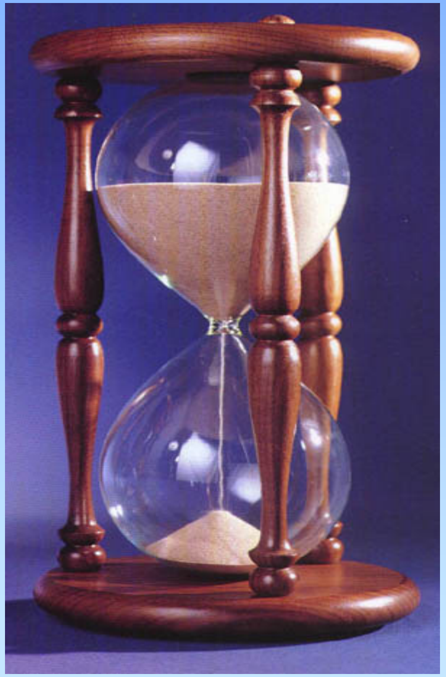
GRANULAR MATTER is everywhere:

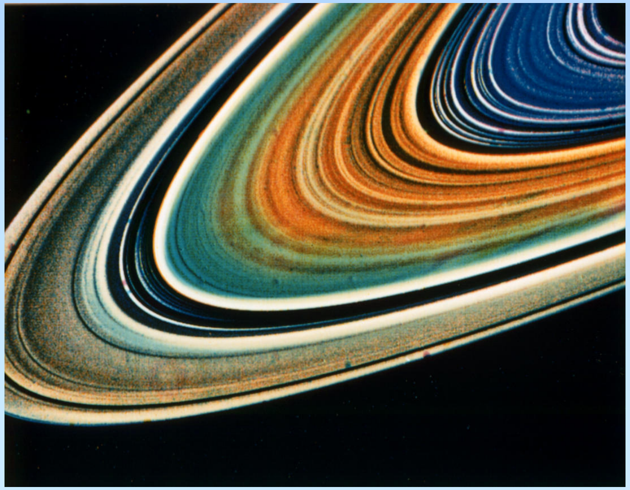
- in nature: beach, soil, snow, desert, mountains, sea floor, Saturn's rings, asteroids ...
- in industry: mining, pharmaceutical, food, construction, chemical ...

Granular Matter can behave like...

... a solid



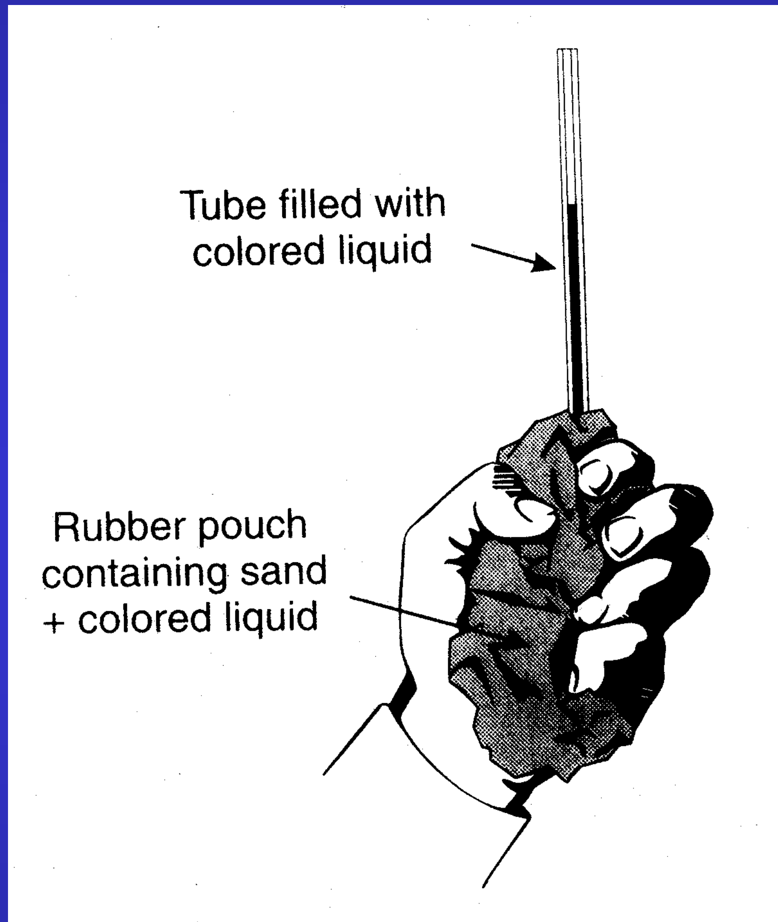
... a liquid



... or a gas

When solid,
Granular Matter is a <u>special</u> solid

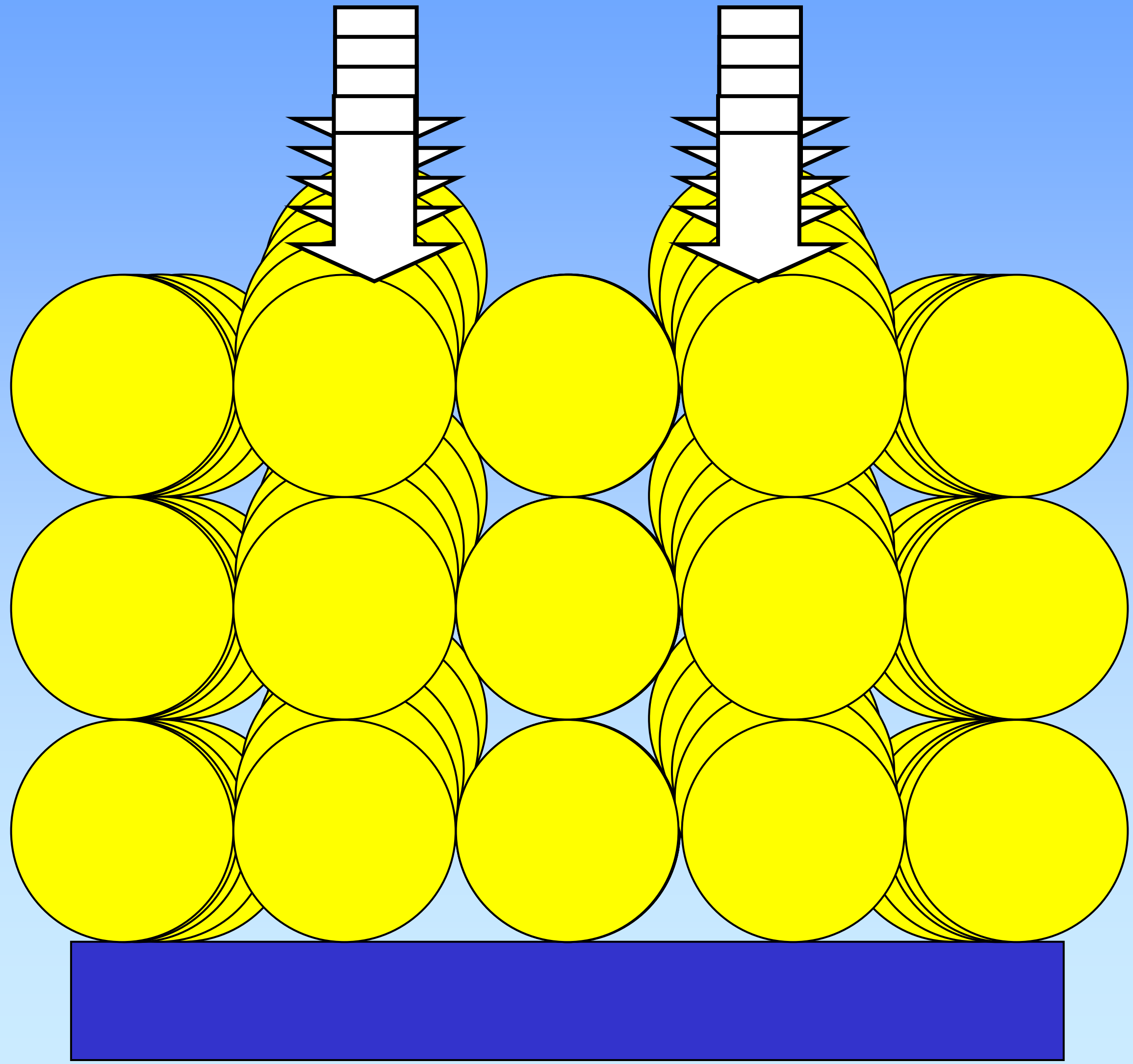
Reynolds dilatancy



Osborne Reynolds (1885):

“A strongly compacted granular medium dilates under pressure”.

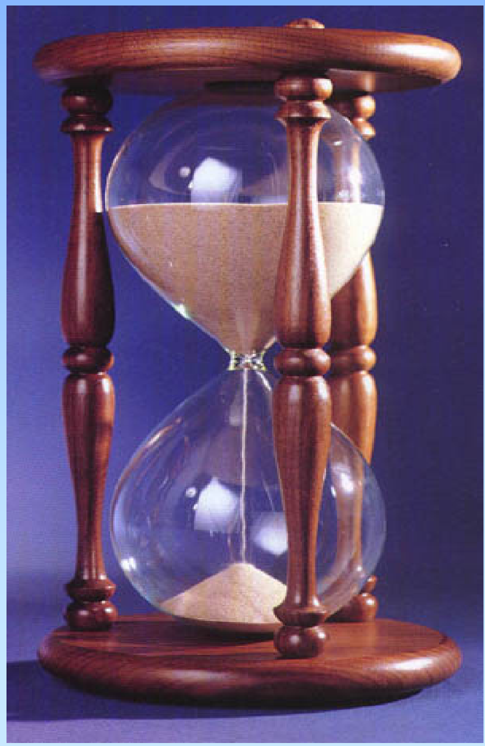
What causes the dilatancy ?



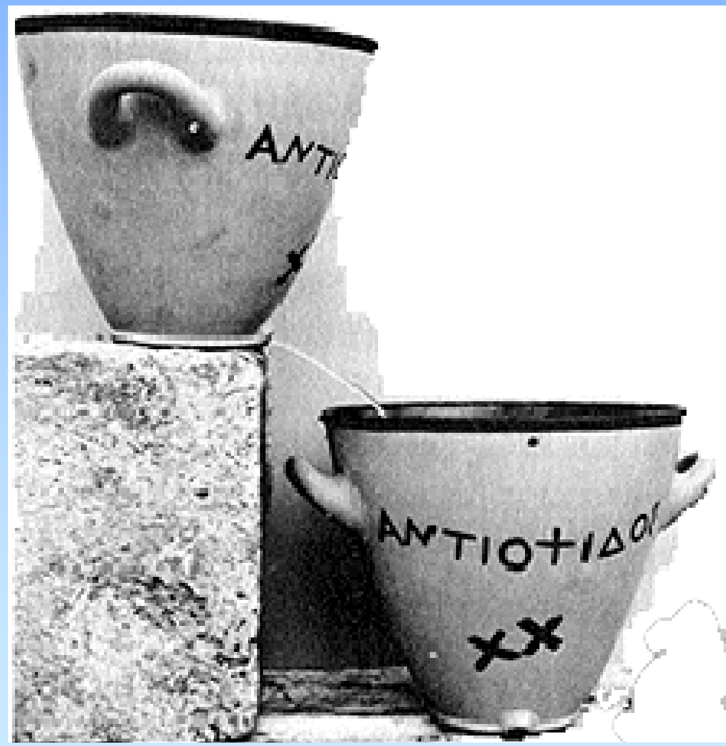
$f = 0.907$

$f = 0.785$

When it behaves like a liquid, Granular Matter is a special liquid

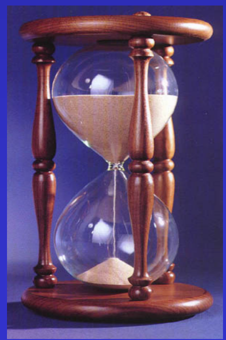


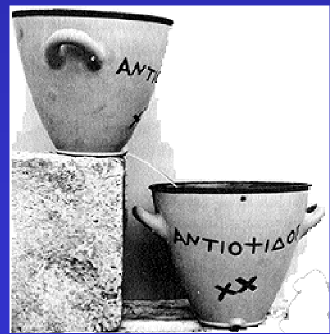
$\overset{?}{=}$

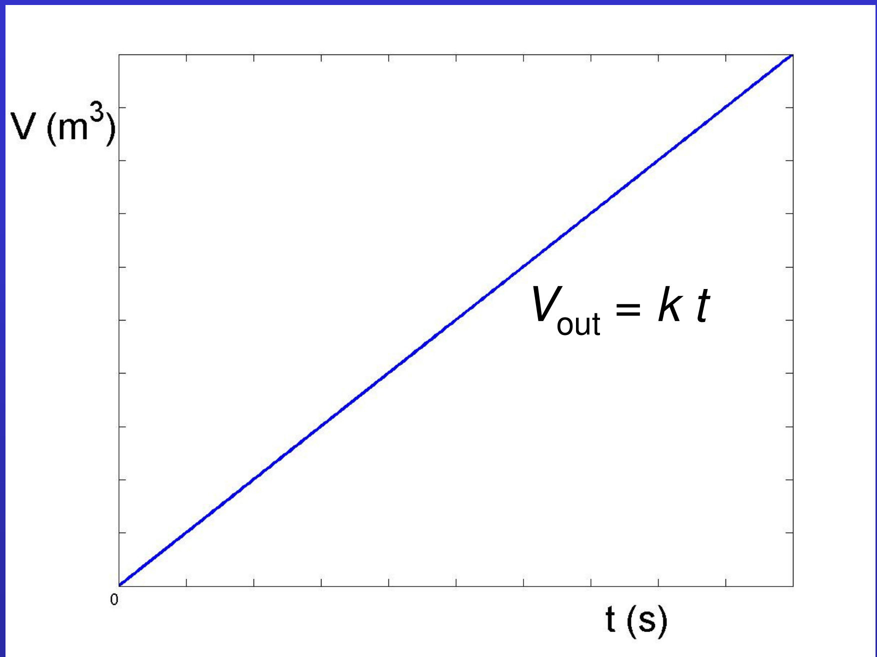


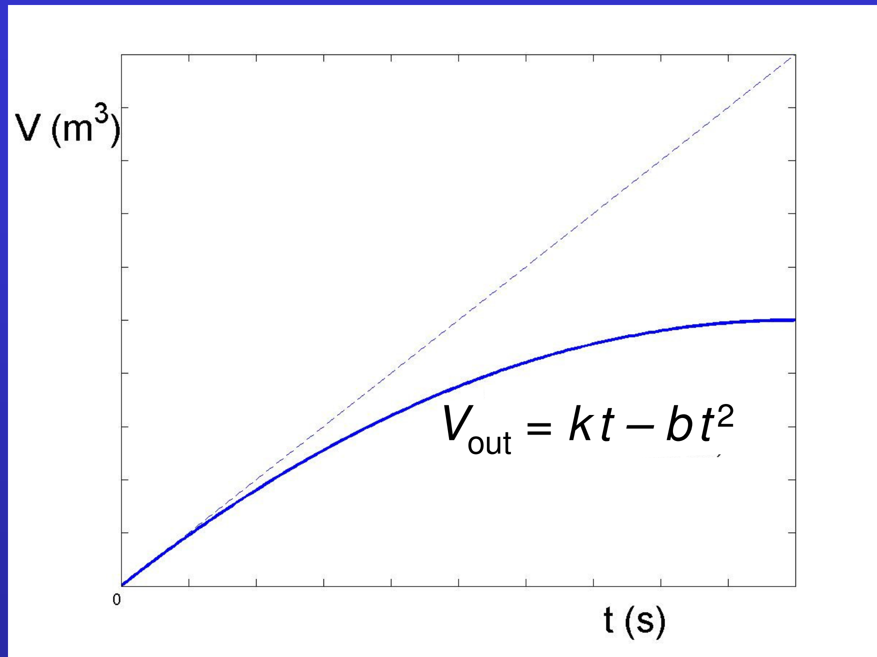
hourglass - sand

hourglass – water "klepsydra"

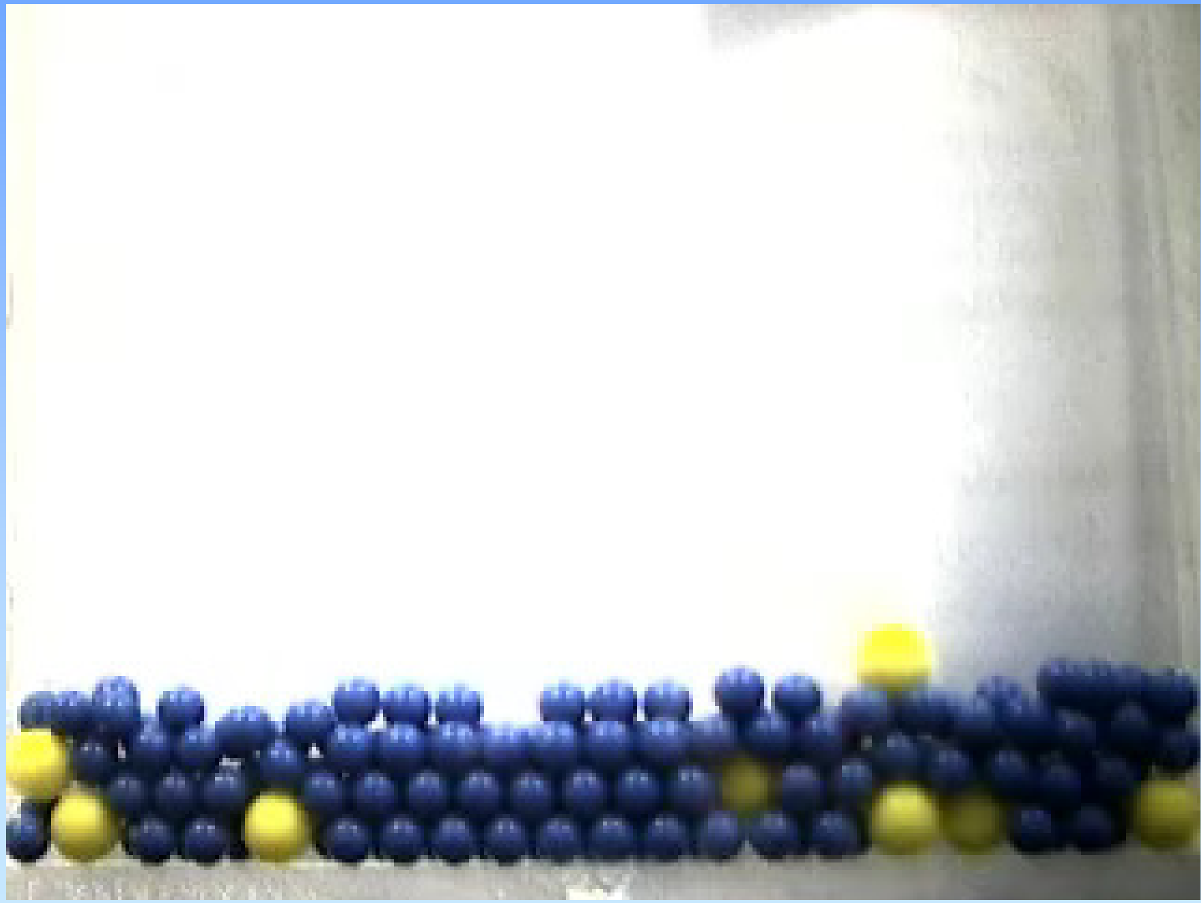








Vibrated bidisperse mixture

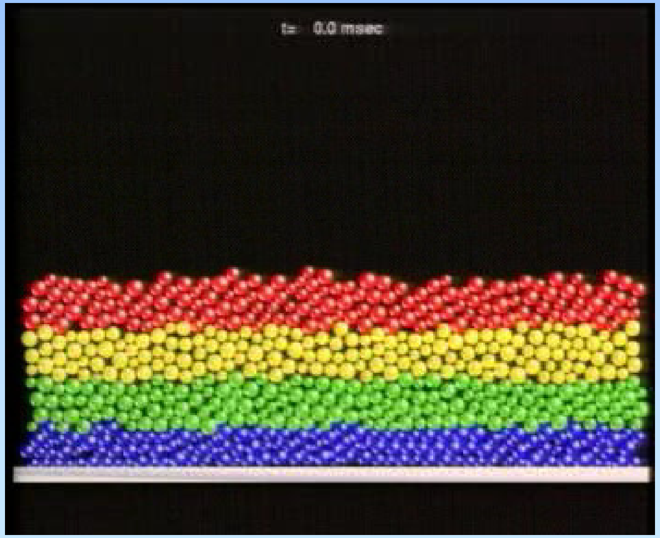


Segregation !

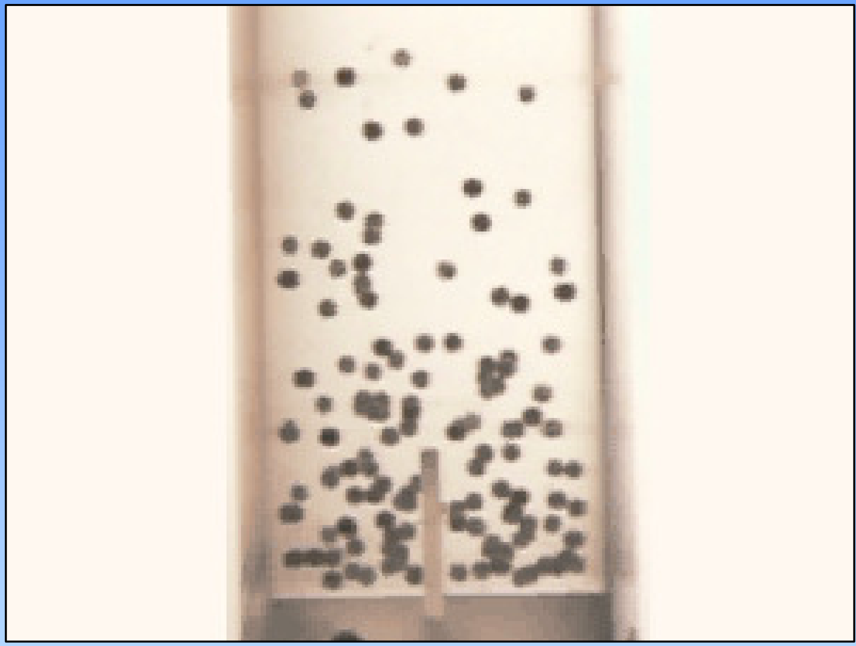
“Brazil Nut Effect”

Three explanations BNE

1. **percolation:** small grains percolate the empty spots between the large ones.
2. **exclusion:** while vibrating small grains fill space below the large ones, not vice versa.
3. **convection:** interaction with walls trigger convection rolls.



large grains can follow the upward, but not the downward flow.



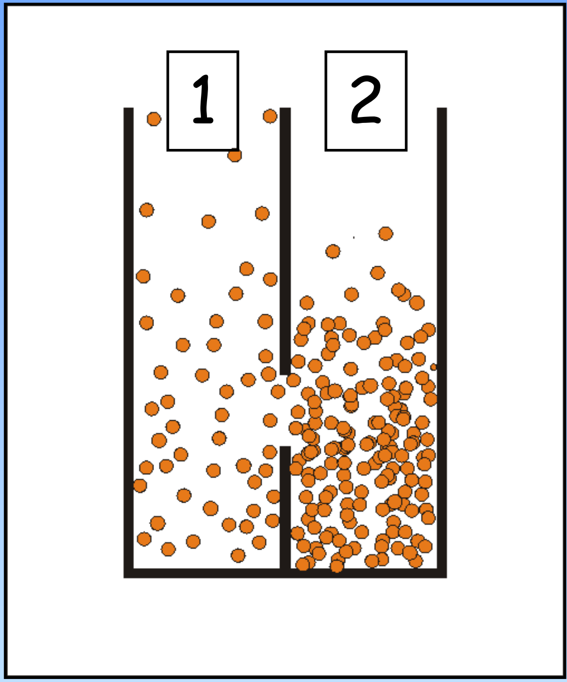
Driving strength: high low

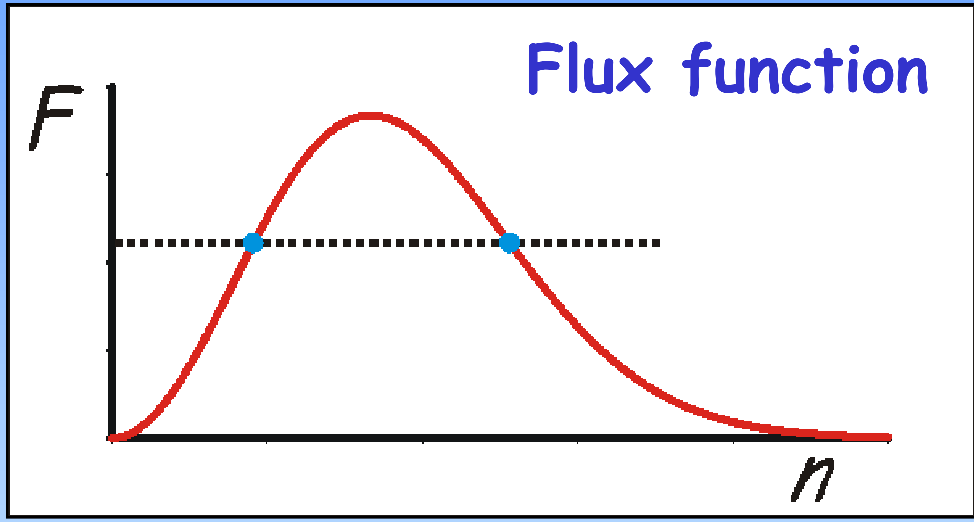
Reason clustering:

Inelastic collisions

And if Granular Matter behaves like a gas, it is a <u>special</u> gas

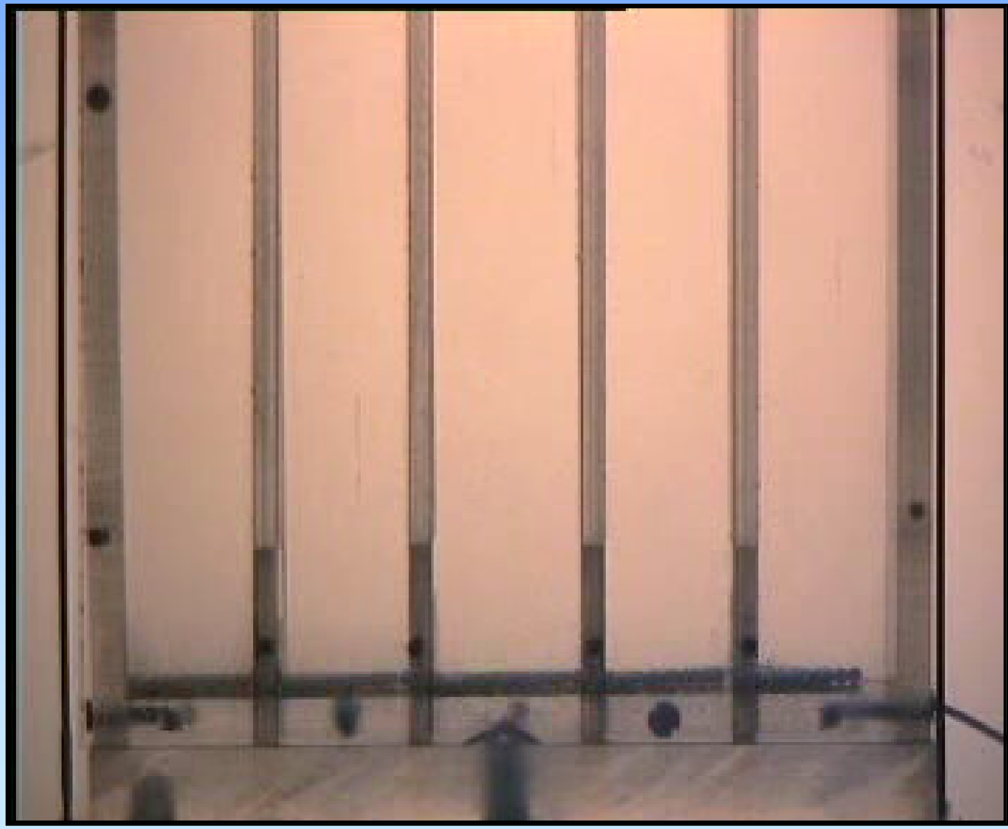
Flux model



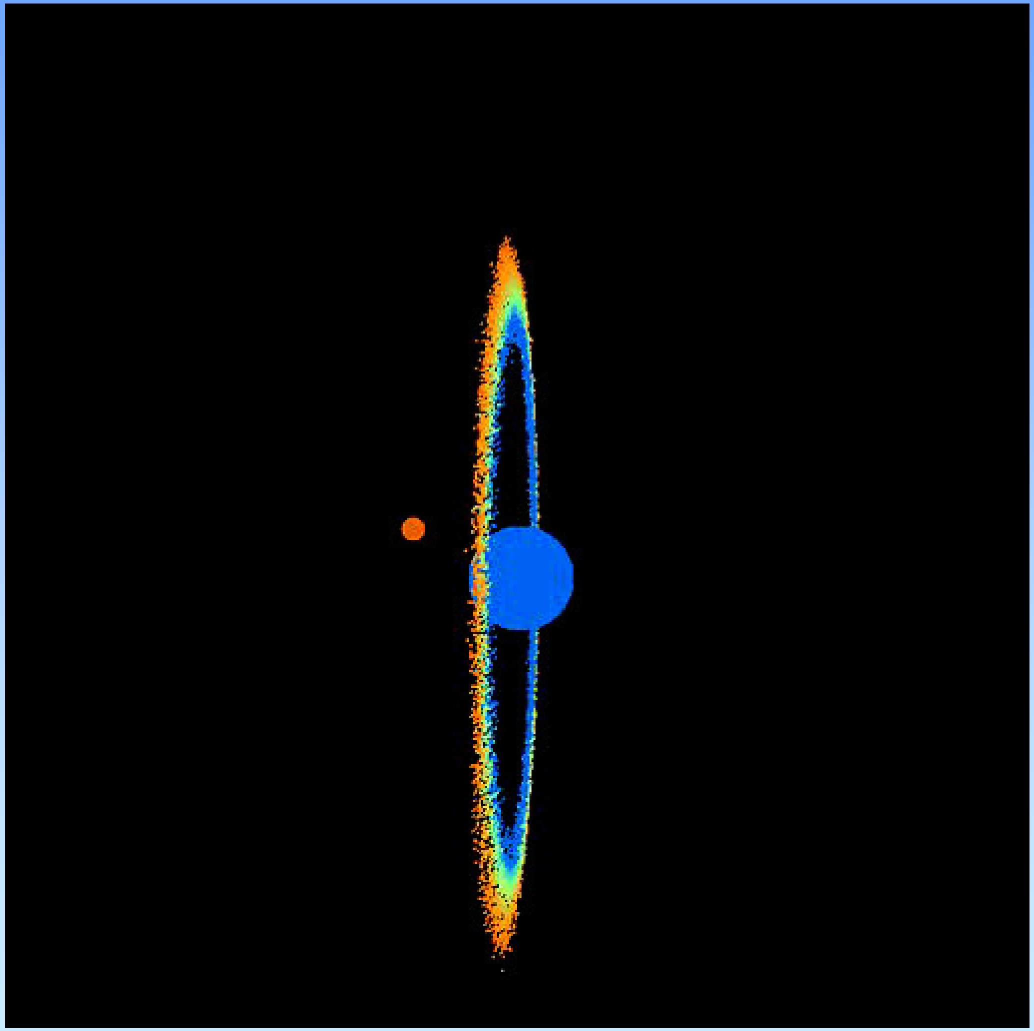


$$\frac{dn_1}{dt} = F_{2\rightarrow 1} - F_{1\rightarrow 2}$$

In five compartments:



Planet with rings



Phenomena

Granular Solid:

Packing density, dilatancy, force chains, compactification, pressure saturation (RJ law)

Granular Fluid:

Arching, blocking, convection, segregation

Granular Gas:

Clustering, non-equipartion

Why does Granular Matter behave so differently from other solids and fluids we know ?

1. GM is athermal

Definition:

Granular Matter =

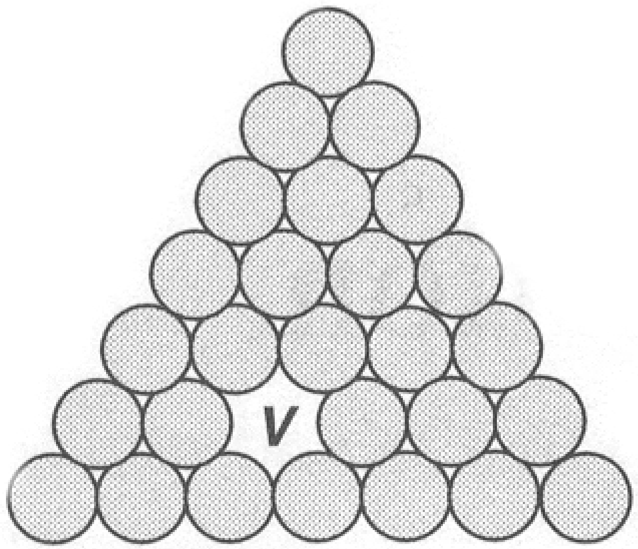
many body system in which the typical particle size > 100 μm

$$\tfrac{1}{2} m v_{thermal}^2 = \tfrac{3}{2} k_B T \Longrightarrow$$ **(at room temperature)**

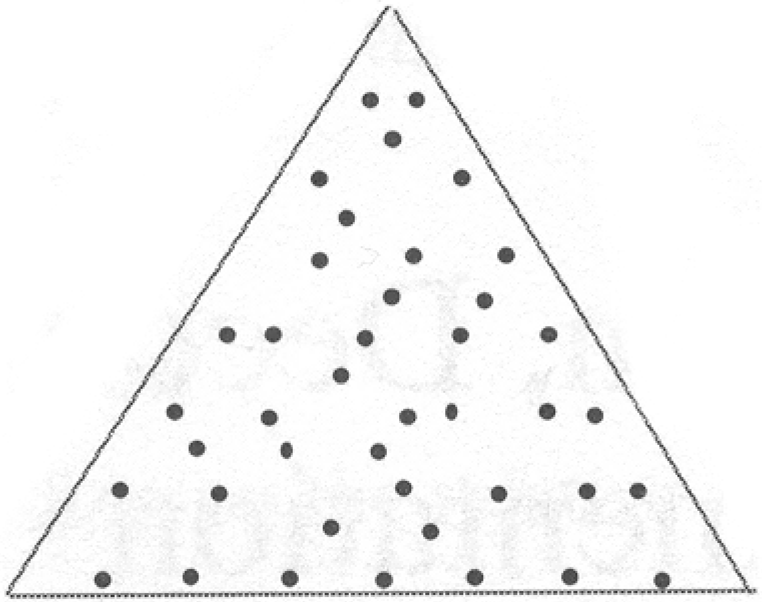
$$v_{thermal} = \sqrt{\frac{3 k_B T}{\frac{4}{3}\pi r^3 \rho}} = \sqrt{\frac{10^{-20}}{10^{-8}}} = 10^{-6}\ \mathrm{m/s}$$

Thermal energy is negligible for such particles !

2. GM interacts through *contact forces*



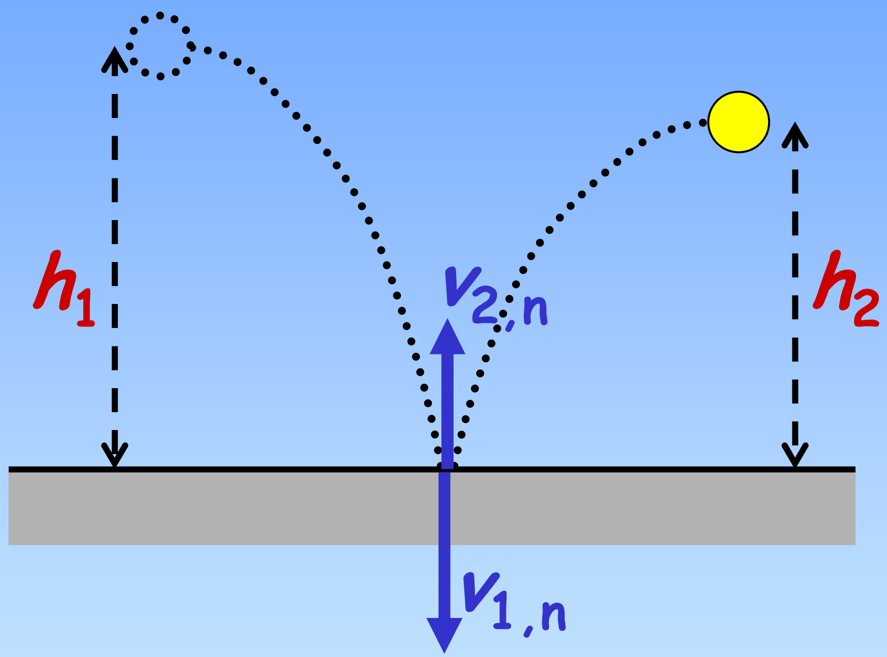
Cannon Ball Stack



Contact Points

"Chaotic" network of contact points and forces !

3. GM interactions are dissipative



coefficient of normal restitution:

$$e = \frac{v_{2,n}}{v_{1,n}} \left(= \sqrt{\frac{h_2}{h_1}} \right)$$

Grains have many internal degrees of freedom through which kinetic energy is dissipated.

(sound, heat, deformation)

Implications

1. athermal ⇨ Thermodynamic *T* irrelevant

Define granular temperature $T_g = \left\langle \frac{1}{2} m v_{macro}^2 \right\rangle$

2. contact forces ⇨ Ordered molecular-scale structures do not occur

3. dissipation ⇨ Far-from-equilibrium system

Constant energy supply is necessary to keep systems "alive" (i.e. T_g>0)

Typical practical problems

Production and handling:

- cornflakes: filling
- pill production: mixing
- casting by sacrificial polystyrene

Nature (geophysics):

- dunes: movement
- avalanches: ranges, volume, prediction
- dikes: stability
- seismology

1.2.3. *Problems of Segregation*

The industry processes enormous quantities of granular materials year in and year out. Virtually every stage of its operations is at the mercy of segregation—a most irksome phenomenon that tends to separate the components of a mixture supposed

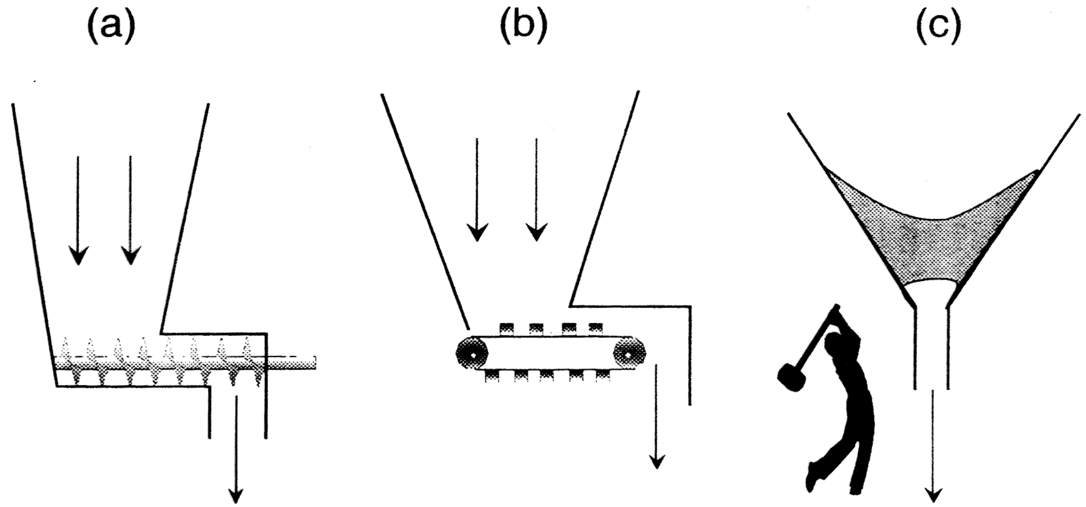
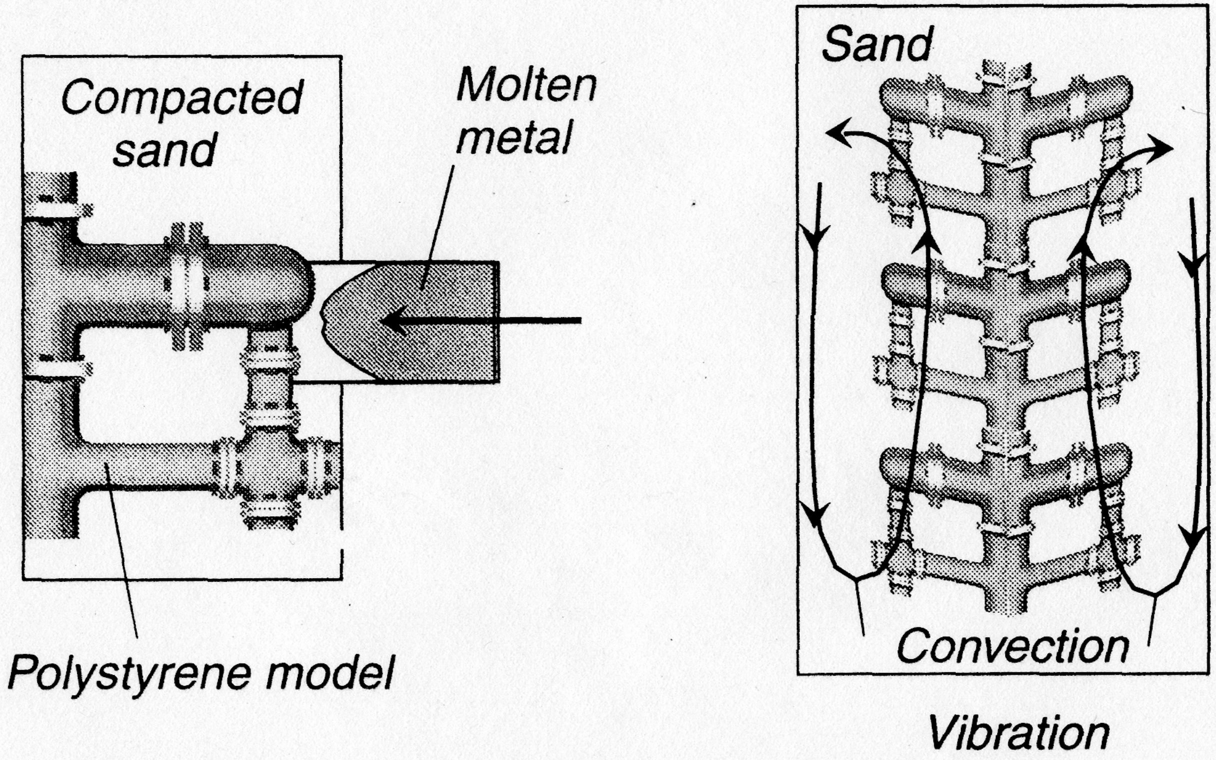


FIGURE 6. Three methods used in industry to prevent or remedy blockages by arch effect. They include (a) an Archimedes screw, and (b) a conveyor belt with a corrugated surface. In (c), a plant worker pounds an obstructed hopper with a sledge hammer to get the flow started again. The latter is the method of choice in industries producing low-value-added granular materials.

Casting by sacrificial polystyrene



Principle of casting by sacrificial polystyrene.

Granular packing (for spheres)

fluid

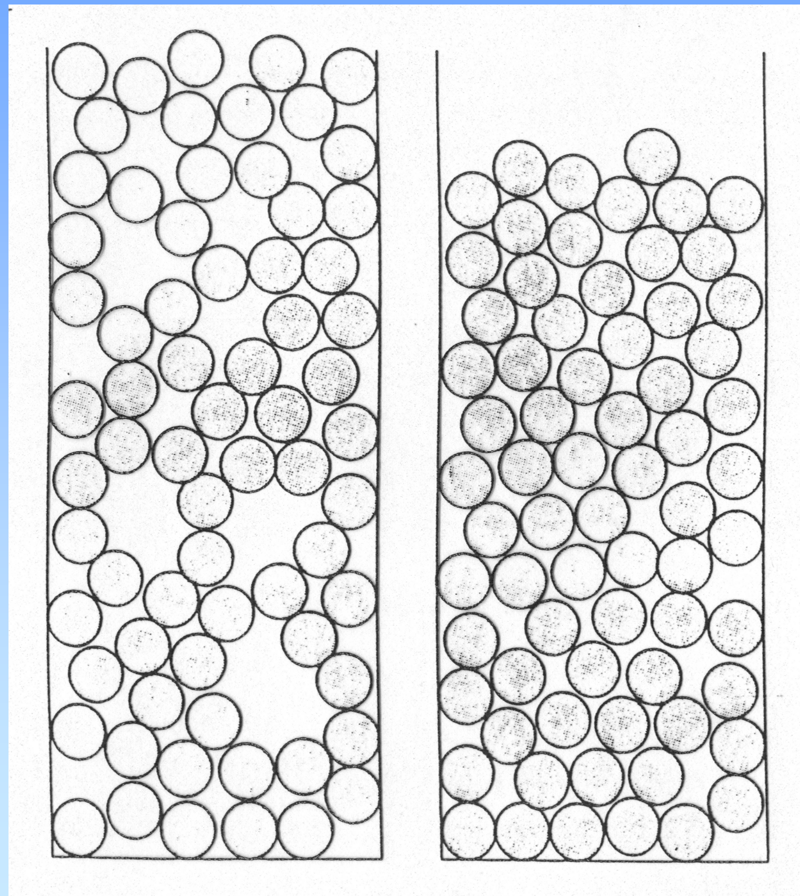
solid

0.57, RLP =
random loose packing

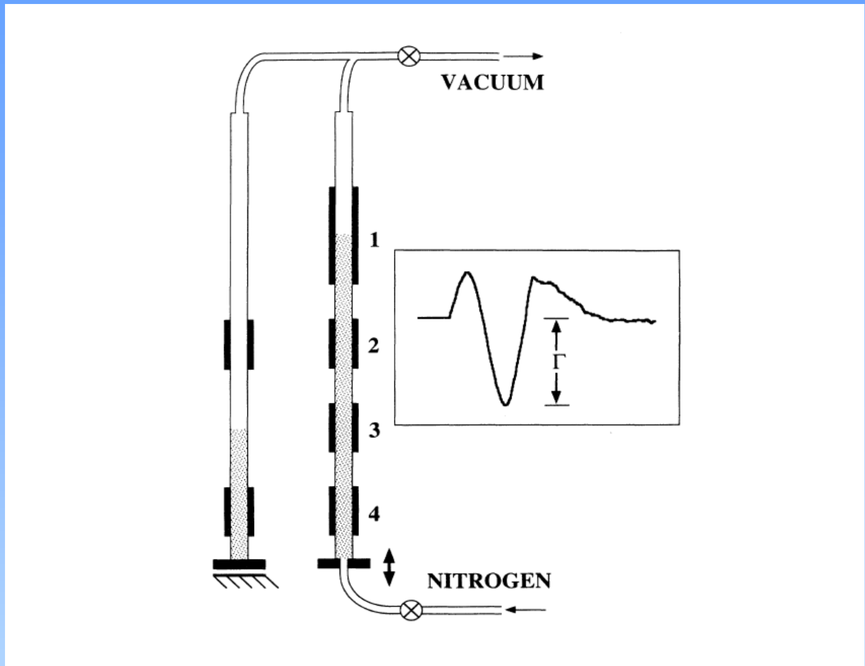
0.64, RCP =
random close packing

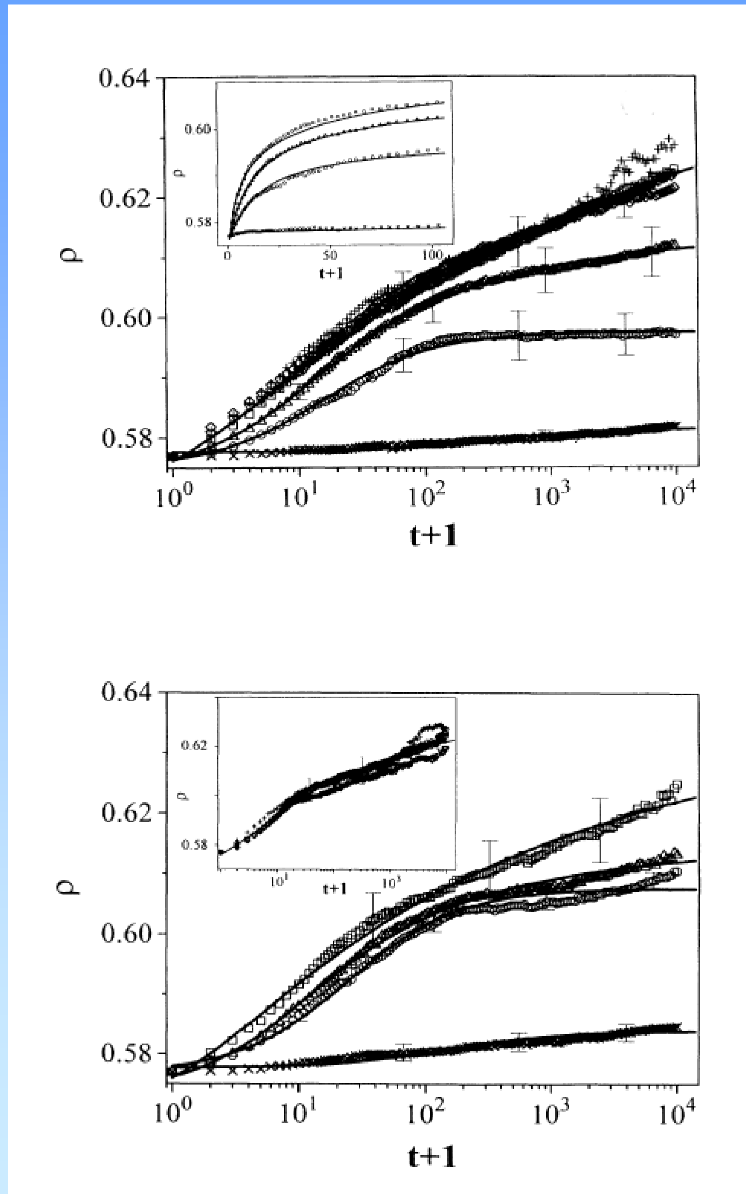
0.74, crystal =
perfectly hexagonal

solid fraction



Compactification experiment





$$\rho(t) = \rho_f - \frac{\rho_f - \rho_0}{1 + B\log(1 + t/\tau)}$$

$\rho(t=0) = \rho_0; \quad \rho(t \to \infty) = \rho_f$

regime 1: ***local*** **reorganization**

regime 2: ***global*** **reorganization**

Analogy: car-parking in street

Model (Ben-Naim):

- Initial state: randomly parked cars (no extra fit in)
- Start to move cars randomly. Whenever there is a large enough gap, a new car jumps in.

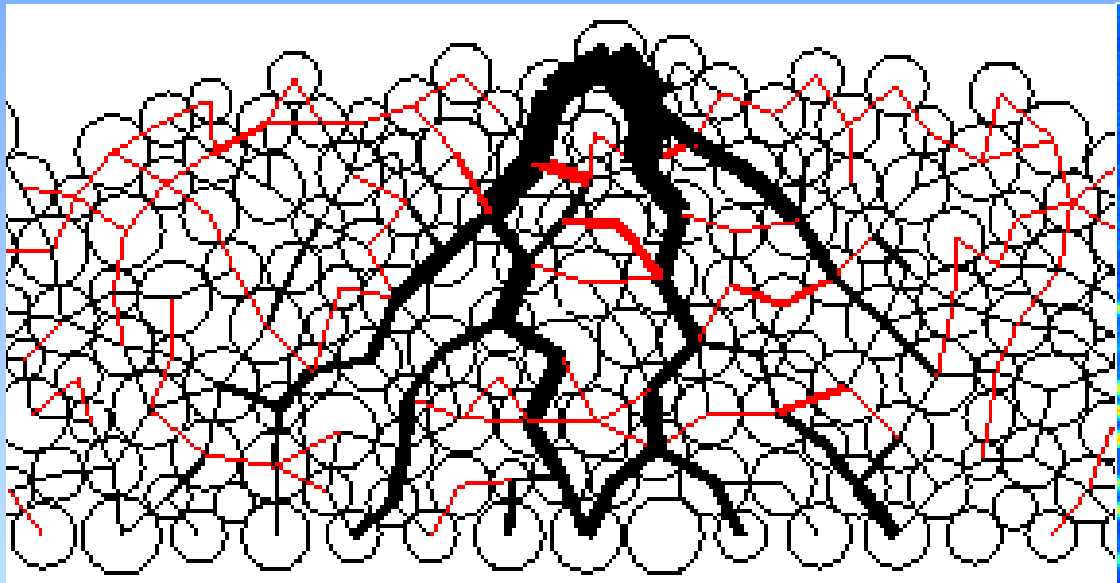
<u>regime 1</u>: movement of a single car creates gap

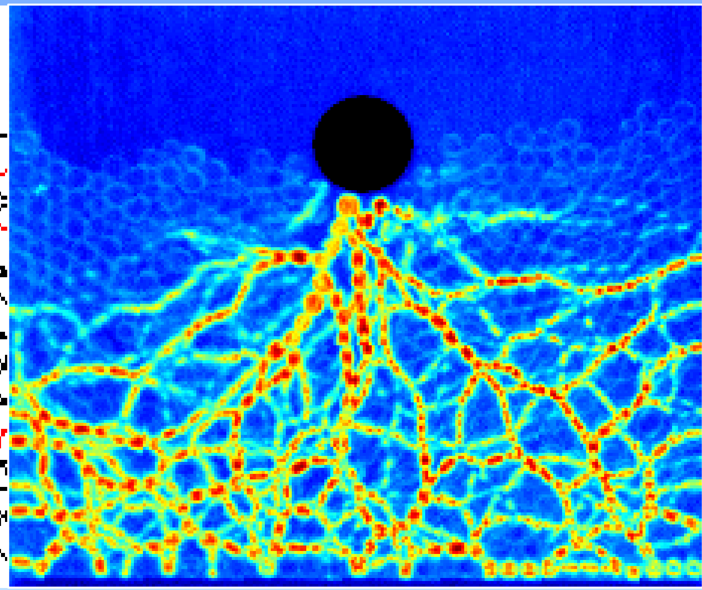
<u>regime 2</u>: more than one car has to move:

required time for gap to open grows exponentially:

$$\frac{\rho(t)-\rho_f}{\rho_f-\rho_0} \propto \frac{1}{\log(t/\tau)}$$

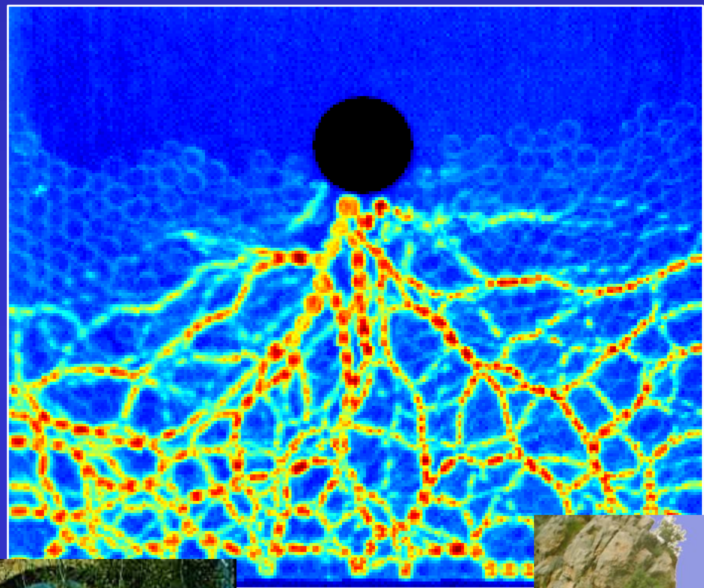
Force Chains

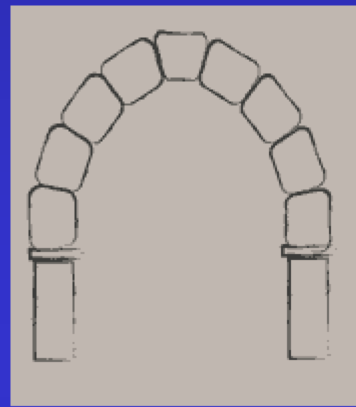


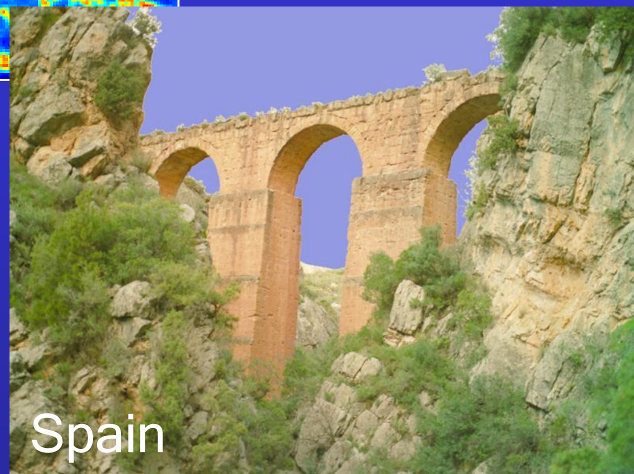


(Bob Behringer, Duke)

In stalling flow, force chains manifest themselves as arches

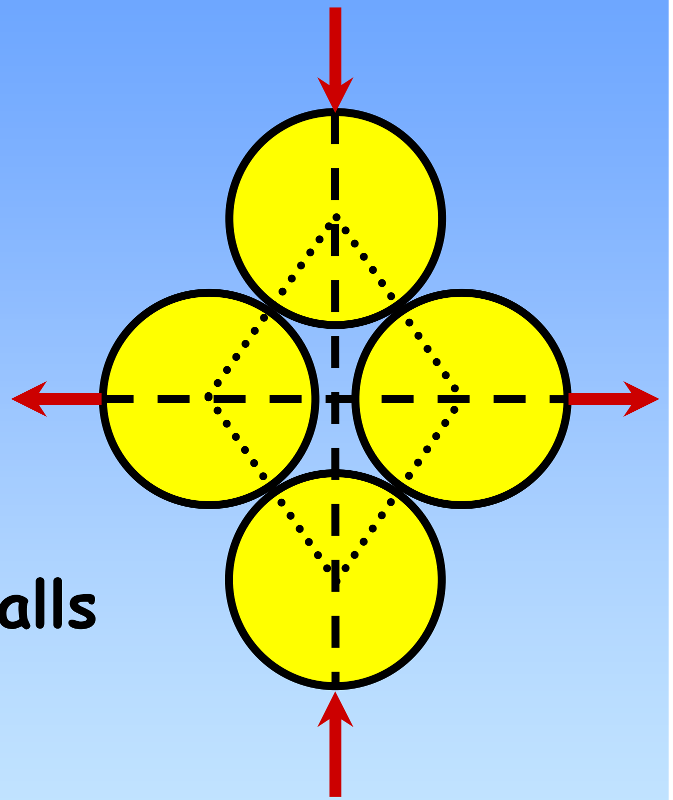






Importance of sidewalls: Rayleigh-Janssen model

Force parallelogram as unit cell
of a 2D granular medium



$\boxed{\text{vertical forces} \Rightarrow \text{horizontal forces}}$

balanced by sidewalls

Lord Rayleigh:

$$\boxed{p_h = K\, p_v}$$

K = coefficient of *redirection*

Importance of sidewalls: Rayleigh-Janssen model (2)

Slice experiences friction force with sidewalls:

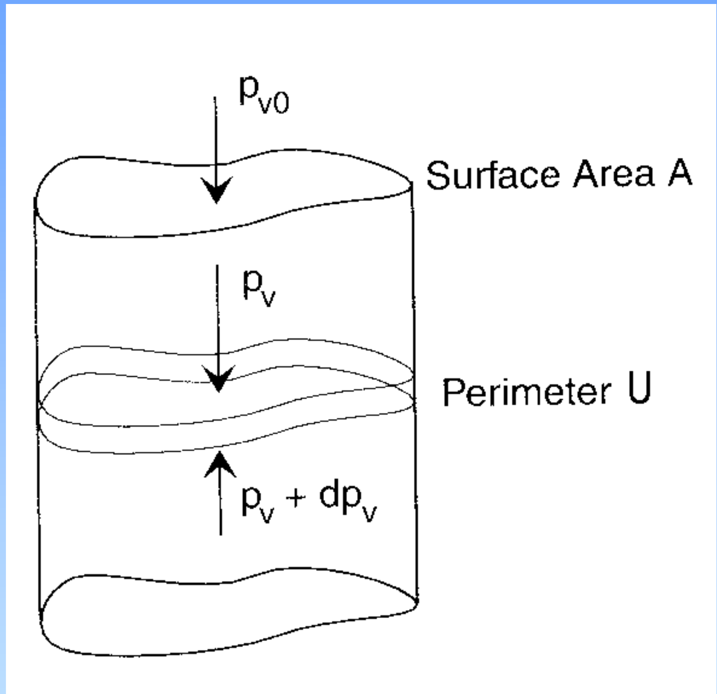
$$dF_{\text{friction}} = \mu_s\, p_h\, U\, dh$$

$$= \mu_s\, K p_v\, U\, dh$$

Vertical force balance on slice:

$$(p_v(h+dh) - p_v(h))\,A +$$

$$\mu_s\, K p_v\, U\, dh = \rho\, g\, A\, dh$$



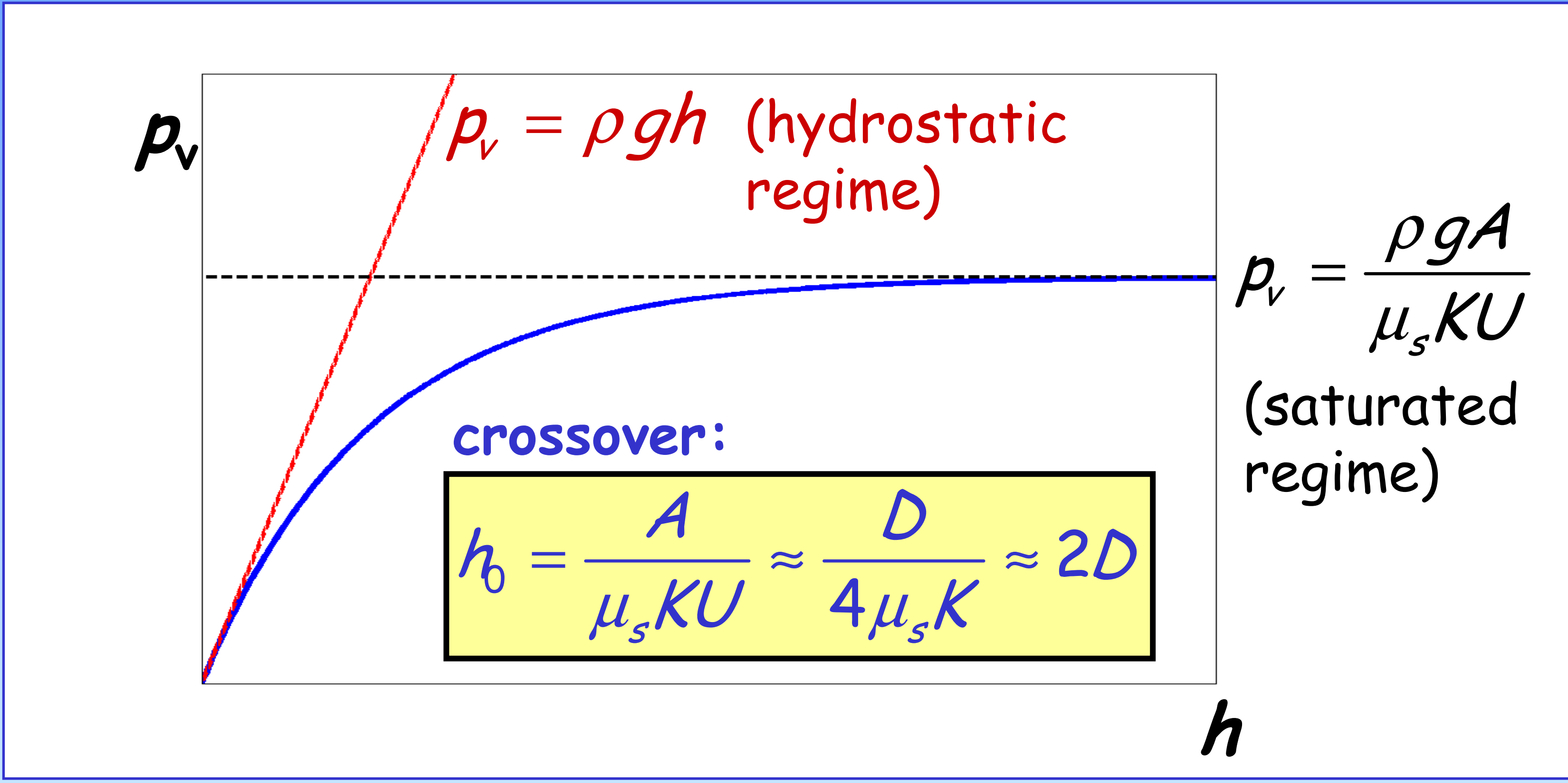
$$\boxed{\frac{dp_v}{dh} + \mu_s K \frac{U}{A} p_v = \rho\, g}$$

Integration gives:

$$\boxed{p_v = \frac{\rho g A}{\mu_s K U}\left(1 - \exp\left(-\mu_s K \frac{U}{A} h\right)\right)}$$

Janssen's equation

Importance of sidewalls: Rayleigh-Janssen model (2)



$$p_v = \frac{\rho g A}{\mu_s K U}\left(1 - \exp\left(-\mu_s K \frac{U}{A} h\right)\right)$$

Janssen's equation

Effective weight of granulate in silo

$$\boxed{\chi = \frac{\mu_s K U}{A} h}$$ (decompaction parameter)

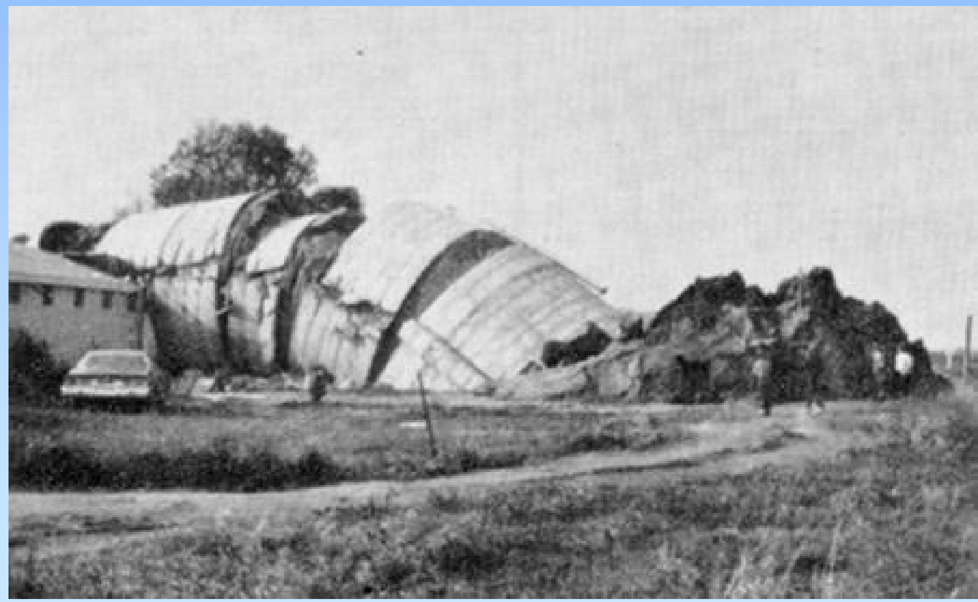
Effective weight on bottom = $F_v(h)$ = $p_v(h)\,A$

$$\boxed{F_v(h) = mg\frac{1-\exp(-\chi)}{\chi} \approx \frac{mg}{\chi}}$$ (for large χ, i.e., large h)

What happens to the remaining weight?

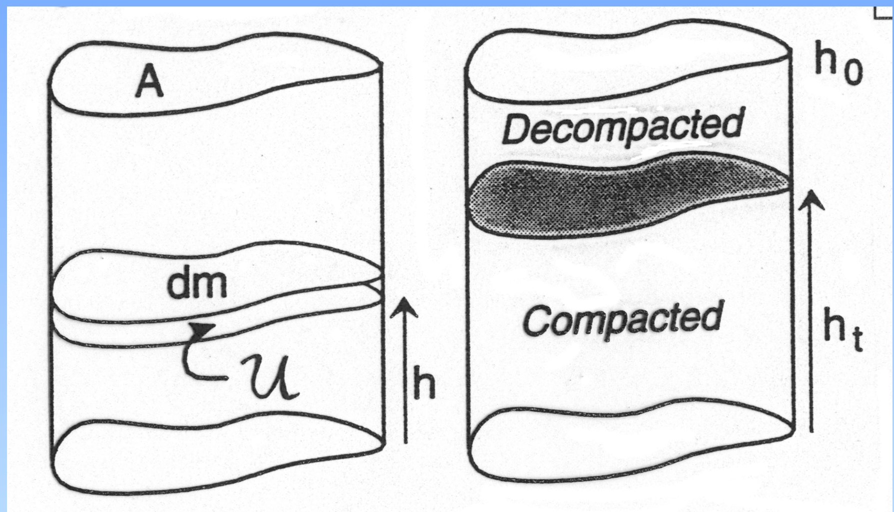
Collapsing silos

Walls take this weight!



Decompactification through shaking

Shaking: *a* sin(ωt); dim.less acceleration: $\Gamma = \dfrac{a\omega^2}{g}$



vertical shaking

"Decompacted" means: acceleration overcomes friction

Force balance: acceleration – gravity = (wall) friction:

$$\boxed{\Gamma\, g\, dm - g\, dm = dF_{\text{friction}}}$$

Sand moves freely if lhs > rhs !

Decompactification through shaking (threshold calculation)

Total height of stack: h_0

Threshold condition lhs>rhs fullfilled from h_t **(<** h_0**) on.**

$$(\Gamma - 1) = \frac{1}{g}\frac{dF_{\text{friction}}}{dm} = \frac{\mu_s K\, p_v(h_t) U\, dh}{g\,\rho\, A\, dh} = \frac{\mu_s K U}{\rho\, g\, A} p_v(h_t)$$

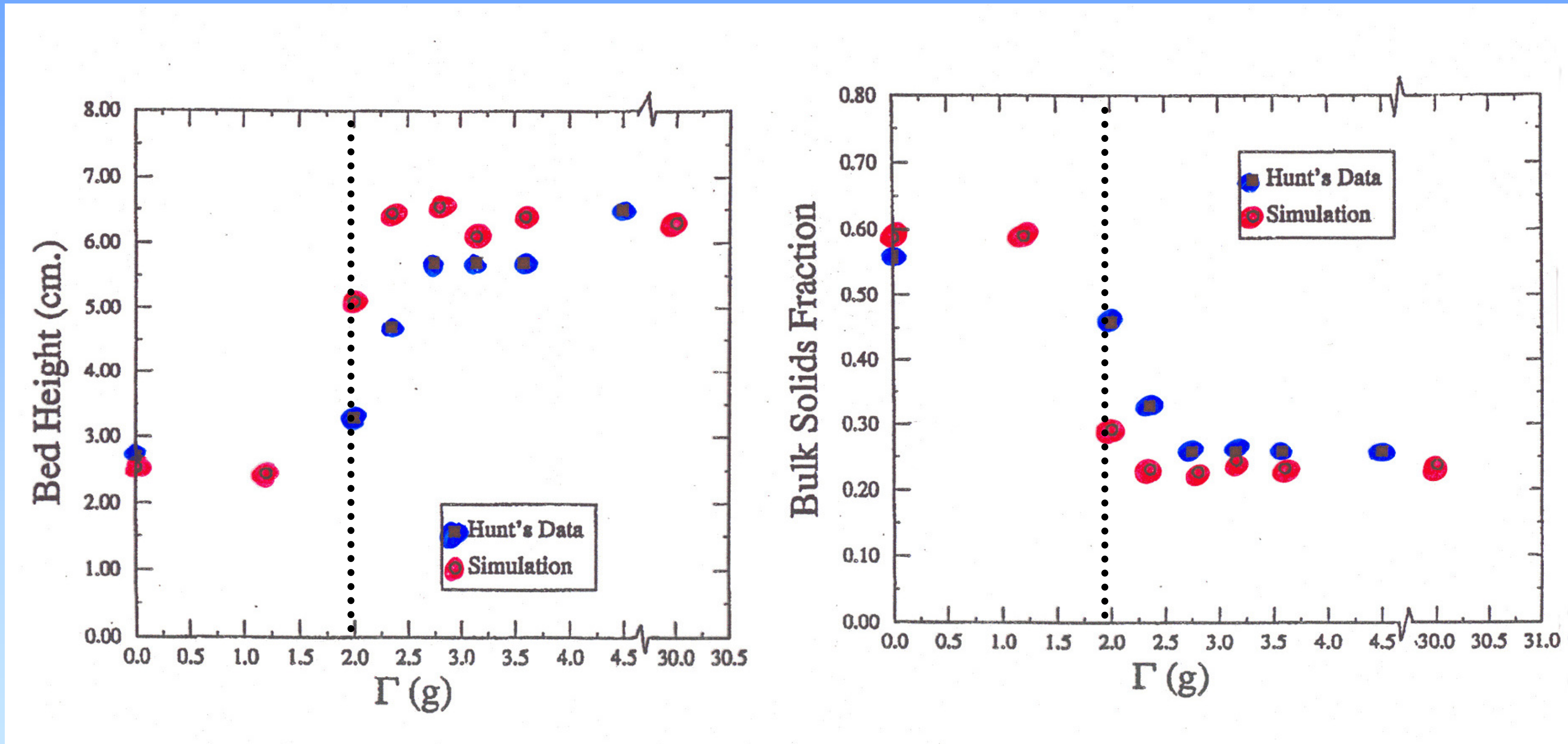
$$p_v(h_t) = \frac{\rho g A}{\mu_s K U}\left(1 - \exp\left(-\mu_s K \frac{U}{A}(h_0 - h_t)\right)\right)$$

$$\boxed{\Gamma = 2 - \exp\left(-\frac{K \mu_s U}{A}(h_0 - h_t)\right)}$$

$\Gamma = 1$ **means:** $h_t = h_0$**: nothing can be fluidized**

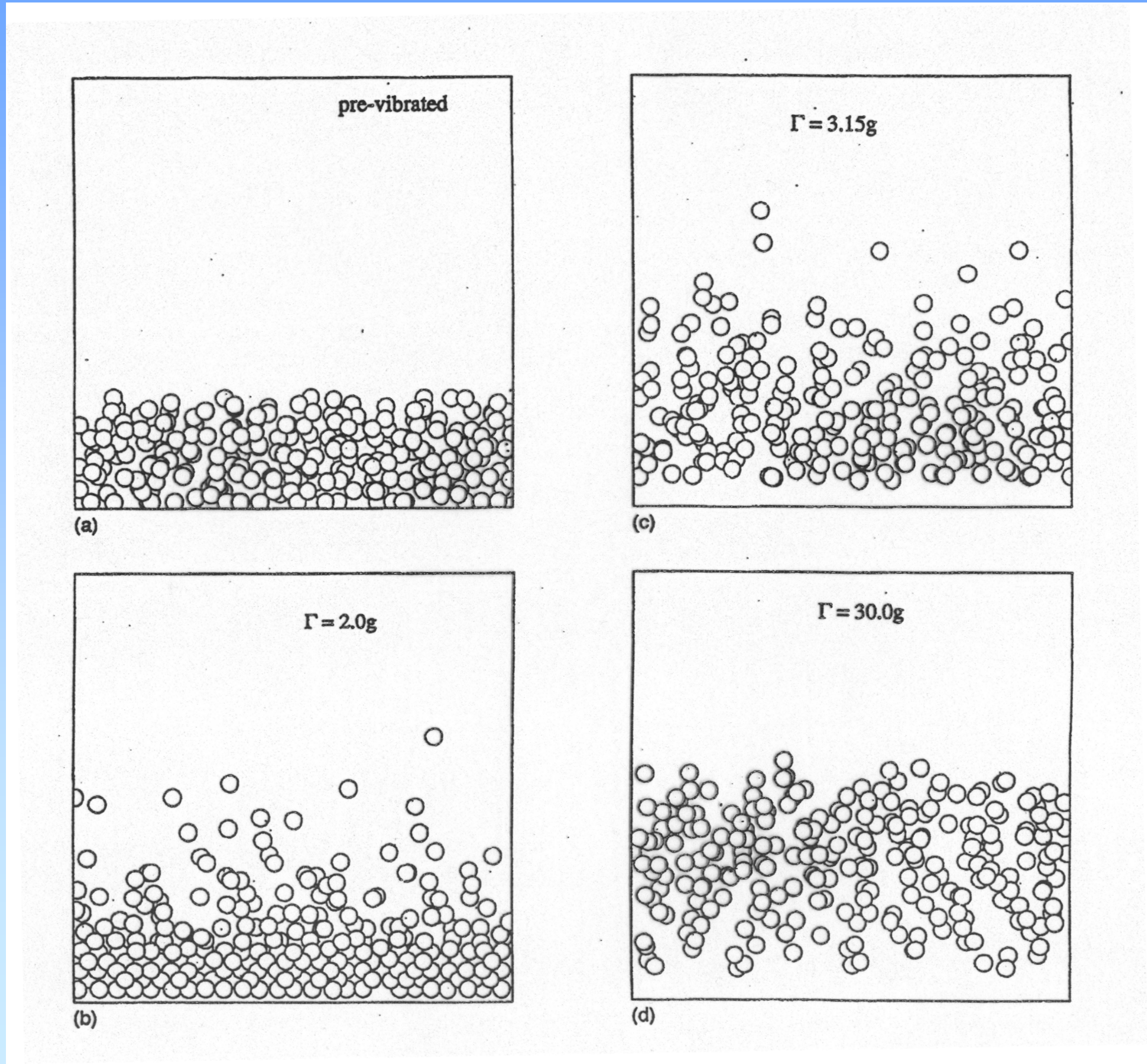
$\Gamma = 2$ **or larger: all can be fluidized**

Decompaction: Experiment, Simulation and Theory

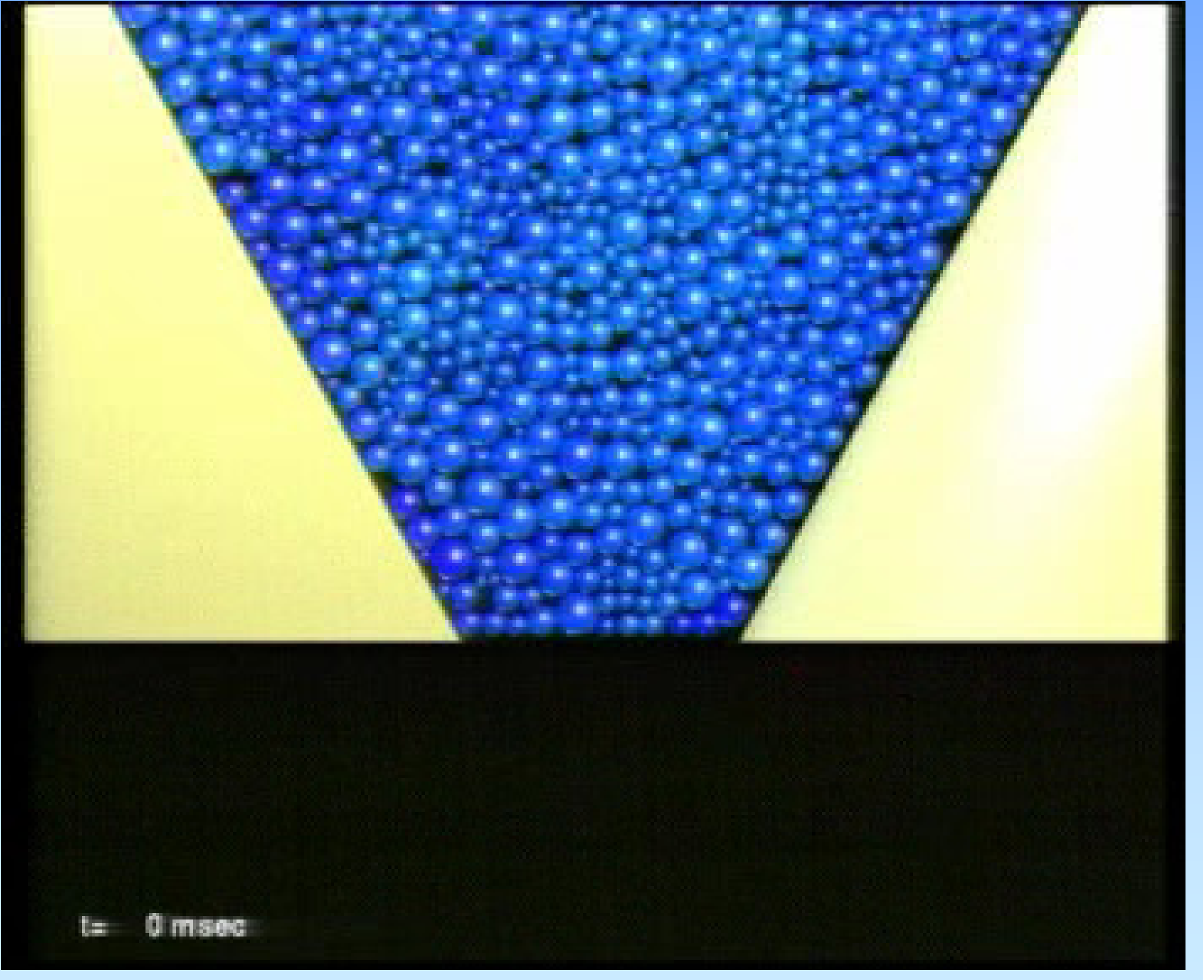


Lan & Rosato, Phys. Fluids **7**, 1818 (1995)

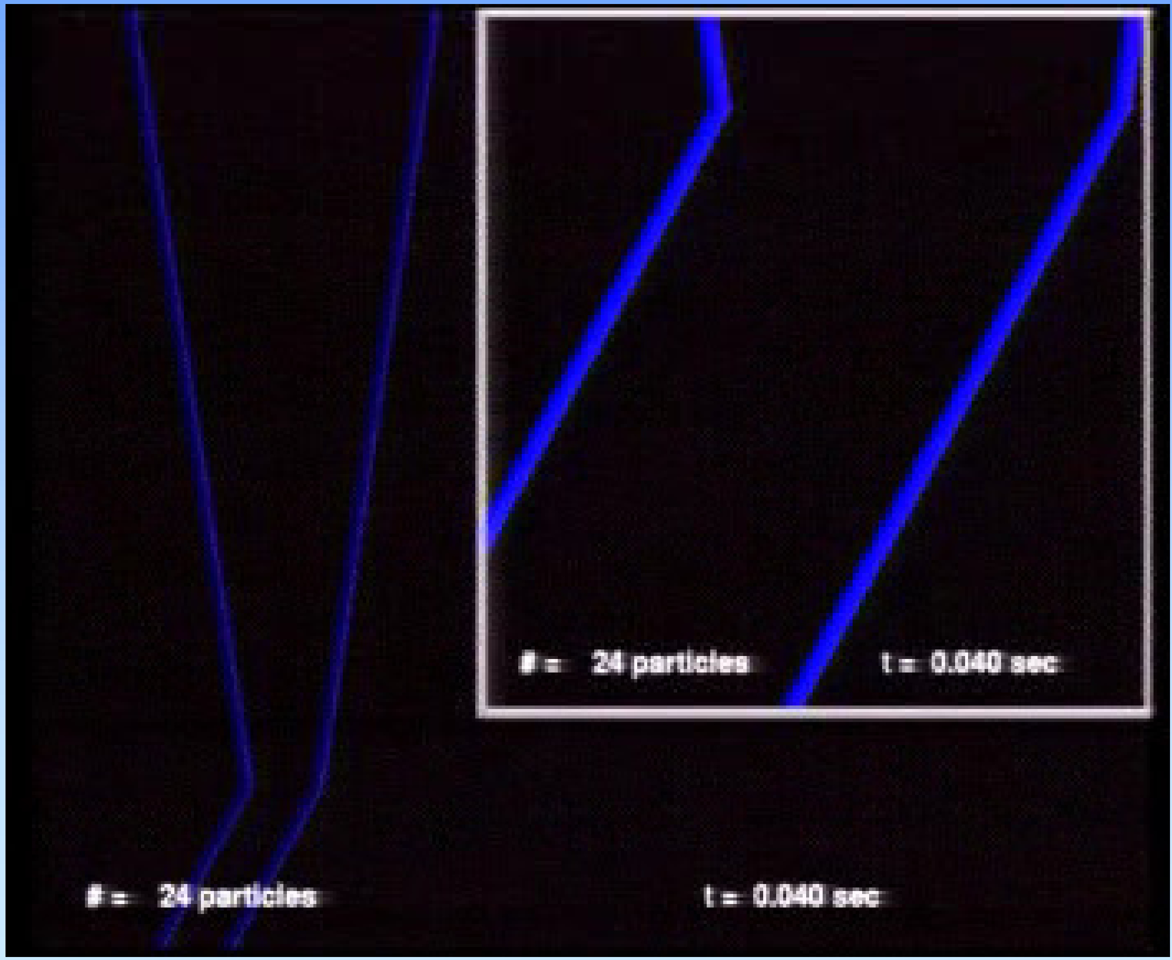
Decompaction: Experiment, Simulation and Theory



Granular matter in a hopper...



... and in a funnel



Is a general hydrodynamic description of granular matter possible ?

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Breakdown of Hydrodynamics in a One-Dimensional System of Inelastic Particles

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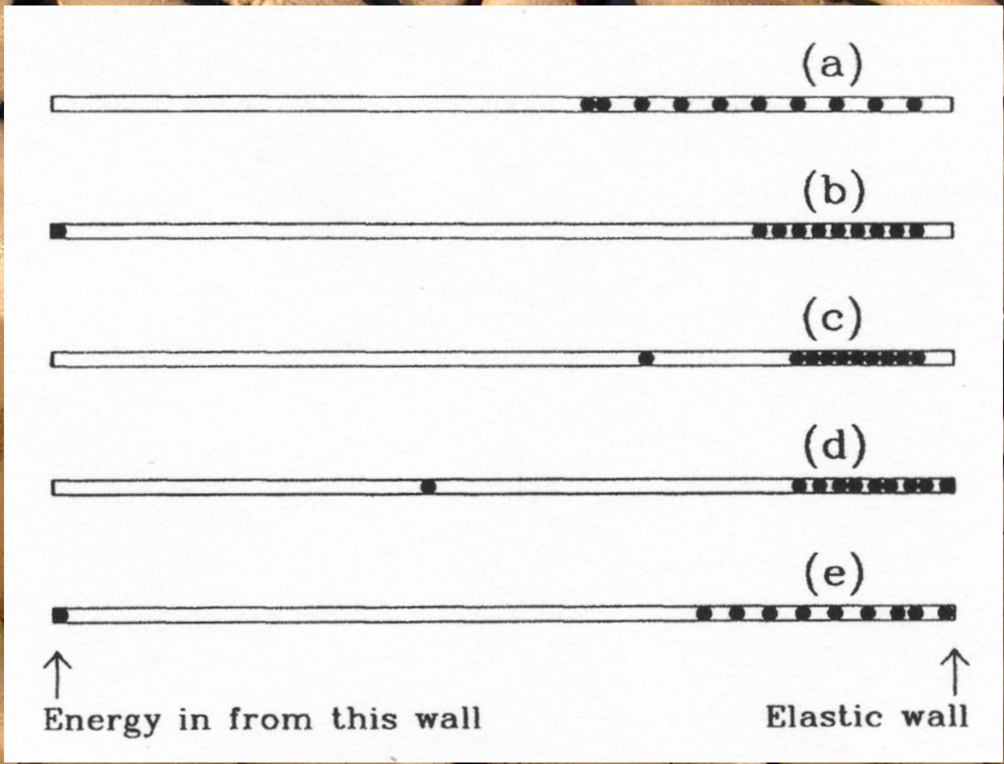
(Received 15 August 1994)



A) Hydrodynamic approach

Coarse graining over small intervals Δx, Δt to define macroscopic quantities:

density:

$$\rho(x,t) = \left\langle \sum_i \delta(x_i(t) - x) \right\rangle_{\Delta x,\Delta t}$$

velocity:

$$u(x,t) = \left\langle \sum_i v_i(t)\ \delta(x_i(t) - x) \right\rangle_{\Delta x,\Delta t}$$

temperature:

$$T(x,t) = \left\langle \sum_i (v_i(t) - u(x,t))^2\ \delta(x_i(t) - x) \right\rangle_{\Delta x,\Delta t}$$

Assuming local "thermal" equilibrium, one can derive mass, momentum, and energy conservation laws:

Conservation laws

In the dilute limit, using the ideal gas law:

$$\cancel{\partial_t \rho} + \cancel{\partial_x (\rho u)} = 0$$

$$\rho \cancel{\partial_t u} + \rho \cancel{u \partial_x} u = -c_1 \partial_x (\rho T)$$

$$\cancel{\rho \partial_t T} + \rho \cancel{u \partial_x} T + c_1 \rho \cancel{T \partial_x} u - c_2 \partial_x^2 (T^{3/2}) = -c_3 \varepsilon \rho^2 T^{3/2}$$

$\varepsilon = (1-e)/2$ expresses *inelasticity*

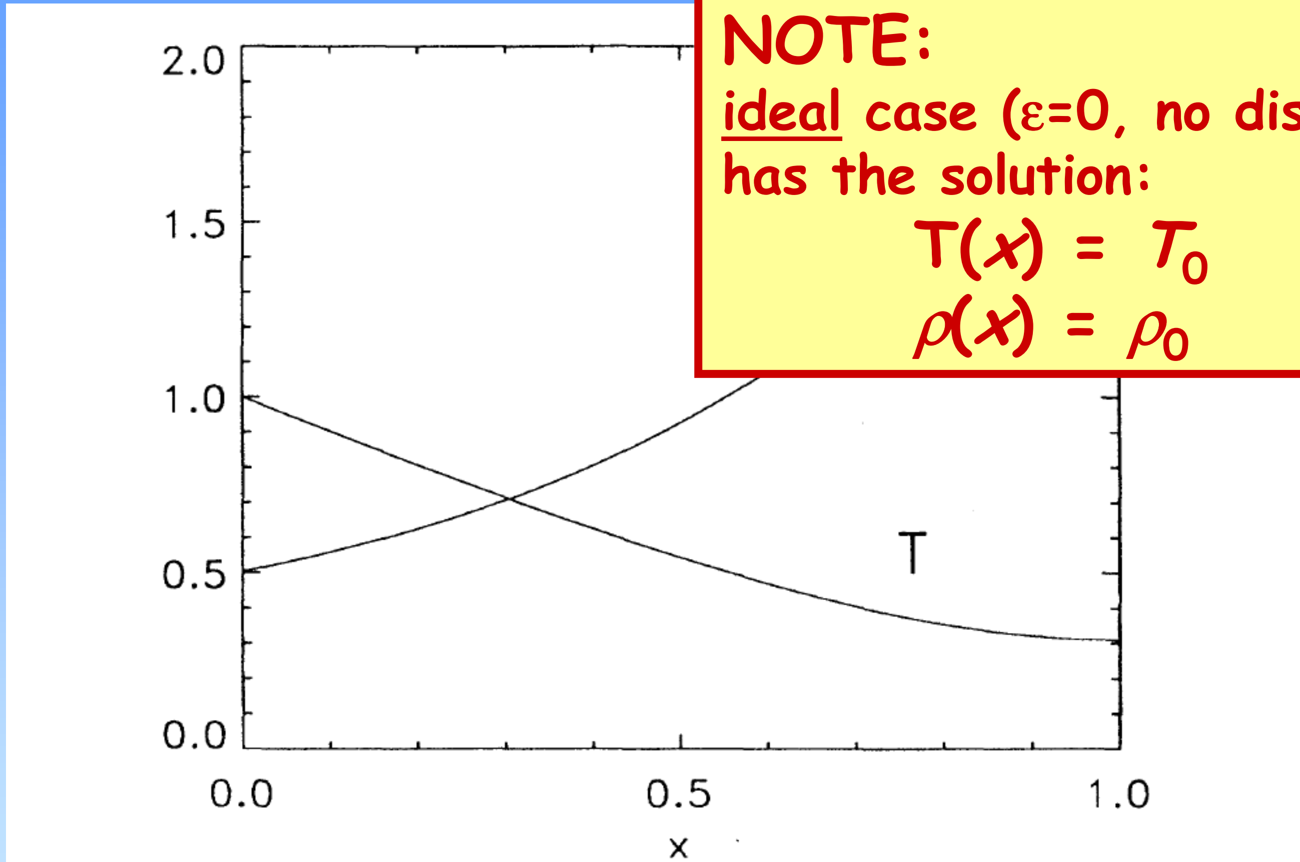
In the stationary limit (u=0, d_t=0) this becomes:

$$\rho T = \text{constant}$$

$$\partial_x^2 (T^{3/2}) = \frac{c_3 \varepsilon}{c_2} \rho^2 T^{3/2}$$

These equations can be solved analytically:

Hydrodynamic solution:



NOTE:
<u>ideal</u> case (ε=0, no dissipation) has the solution:

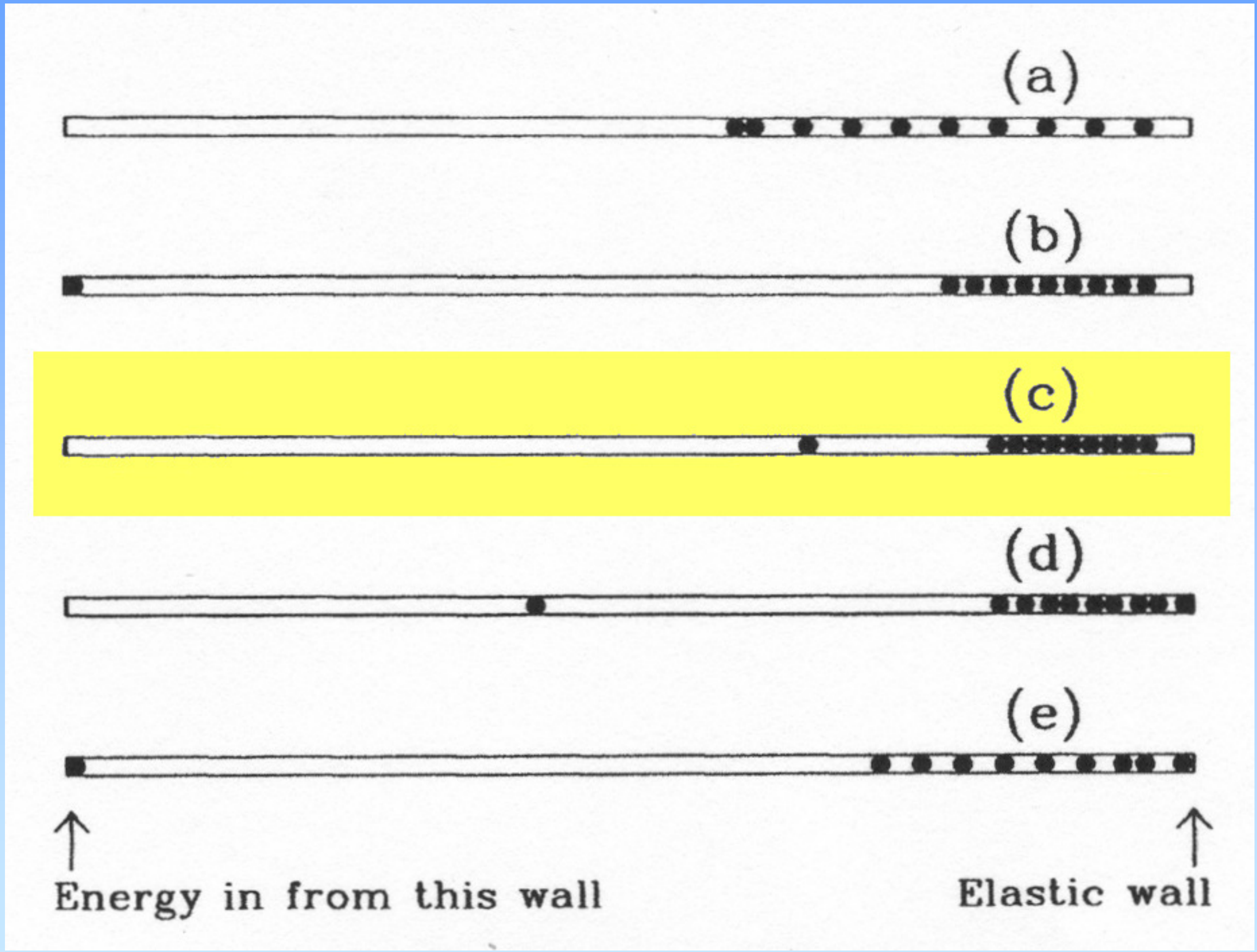
$$T(x) = T_0$$
$$\rho(x) = \rho_0$$

Using the boundary conditions:

$T(0) = T_0$ [constant T at left border]

$\partial_x T(1) = 0$ [elastic wall (no heat flux) at right border]

Particle dynamics solution:



(using MD simulations)

B) Discrete description

2-particle collision with

* **momentum conservation:**

$$v_1 + v_2 = v_1' + v_2'$$

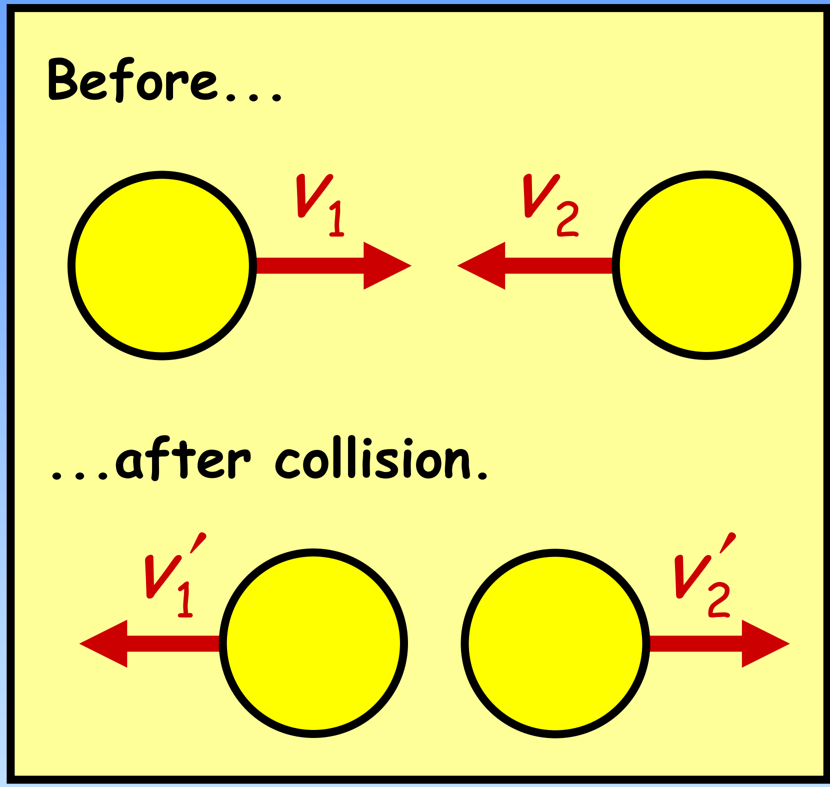
* **energy dissipation:**

$$v_1' - v_2' = -e(v_1 - v_2)$$

This implies:

$$\boxed{\begin{aligned} v_1' &= \varepsilon v_1 + (1-\varepsilon) v_2 \\ v_2' &= (1-\varepsilon) v_1 + \varepsilon v_2 \end{aligned}}$$

with: $\varepsilon = (1-e)/2$



Ideal case $\varepsilon = 0$:

$v_1' = v_2;\ \ v_2' = v_1$, exchange of velocities.

Finally all velocities will be given by the PDF of velocities on the left.

Uniform distribution of particles, consistent with continuum description.

Non-ideal case $\varepsilon > 0$:

Numerical result very different from continuum result!

1 fast particle $v_N \sim \sqrt{T_0}$ and (N-1) slow particles, clustering to the right and dissipating energy. Fast particle transports energy from left to right.

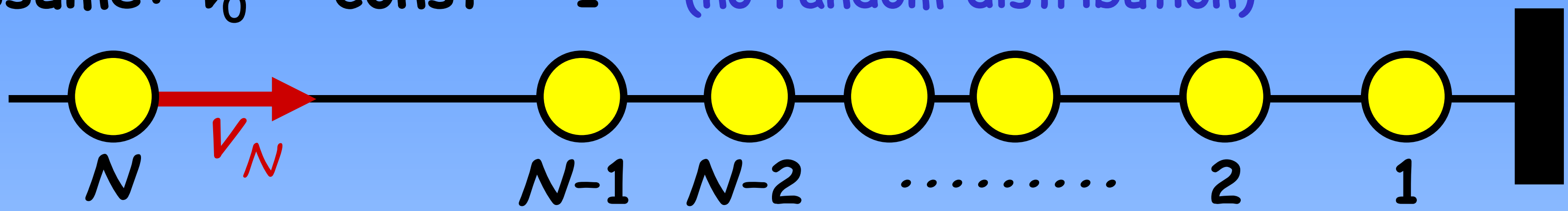
No longer local “thermal” equilibrium !

Breakdown of continuum approach !

Velocity center of

$$v_1' = \varepsilon v_1 + (1-\varepsilon) v_2$$
$$v_2' = (1-\varepsilon) v_1 + \varepsilon v_2$$

assume: v_0 = const = 1 (no random distribution)



before first collision:

$$v_N = v_0 = 1,\ v_i = 0 \ \text{for} \ i < N$$

after first collision (between N and N-1):

$$v_N = \varepsilon, \ \ v_{N-1} = 1-\varepsilon, \ \ v_i = 0 \ \text{for} \ i < N-1$$

after second collision (between N-1 and N-2):

$$v_N = \varepsilon,\ v_{N-1} = (1-\varepsilon)\varepsilon,\ v_{N-2} = (1-\varepsilon)^2,\ v_i = 0 \ \text{for} \ i < N-2$$

...

after (N-1)th collision (between 2 and 1):

$$v_N = \varepsilon,\ v_{N-1} = (1-\varepsilon)\varepsilon,\ v_{N-2} = (1-\varepsilon)^2\varepsilon,\ \ldots,$$
$$v_2 = (1-\varepsilon)^{N-2}\varepsilon,\ v_1 = (1-\varepsilon)^{N-1}$$

Velocity center of mass (2)

Mean velocity of particles N,N-1,...,3,2:

$$
\begin{aligned}
v_{\text{CM-cluster}} &= v_N + v_{N-1} + v_{N-2} + \ldots + v_3 + v_2 \\
&= \varepsilon + (1-\varepsilon)\varepsilon + (1-\varepsilon)^2\varepsilon + \ldots + (1-\varepsilon)^{N-2}\varepsilon \\
&= \varepsilon \sum_{k=0}^{N-2} (1-\varepsilon)^k = 1-(1-\varepsilon)^{N-1} \\
&\approx 1 - \exp(-(N-1)\varepsilon)
\end{aligned}
$$

for large N

- **ε = 0, ideal case:**

  $$v_{\text{CM-cluster}} = 0$$

- **$\varepsilon \neq$ 0, real case:**

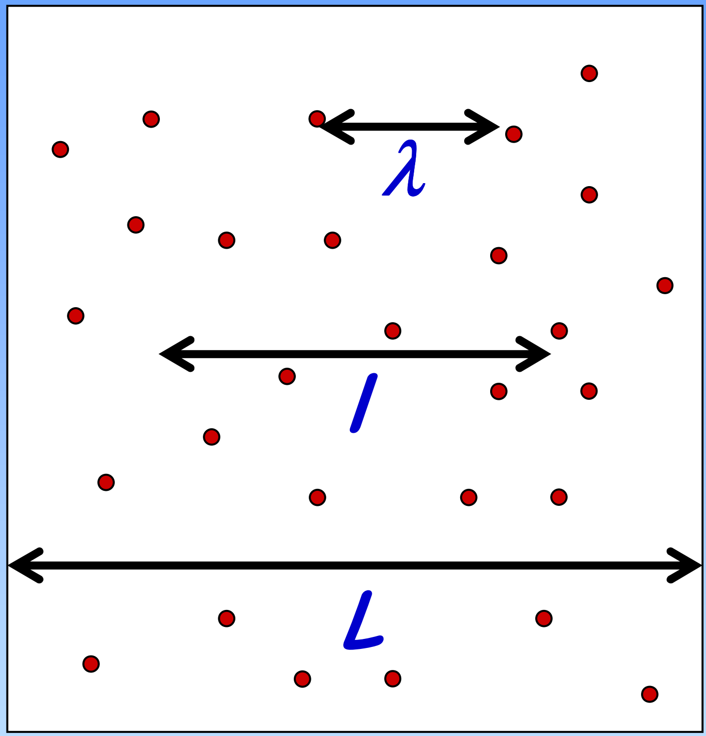
  $$v_{\text{CM-cluster}} > 0$$

drift of cluster towards wall !

In an isolated 1D case granular hydrodynamics does not work.

What about the general case?

Knudsen number



λ = mean free path

l = typical length at which macroscopic quantities vary

L = typical system size

$Kn = \lambda / L$ (global Knudsen number)

$Kn_{loc} = \lambda / l$ (local Knudsen number)

Hydrodynamics work if Kn<<1 !

Molecular system: local Kn <<1 **(not a Knudsen gas!)**

Granular system: local Kn large !

No separation of scales

No separation of length scales:

macroscopic quantities vary on the same scale as the mean free path !

Flowing systems:

mean velocity ~ "thermal" velocities = velocity fluctuations

⇓

No separation of time scales:

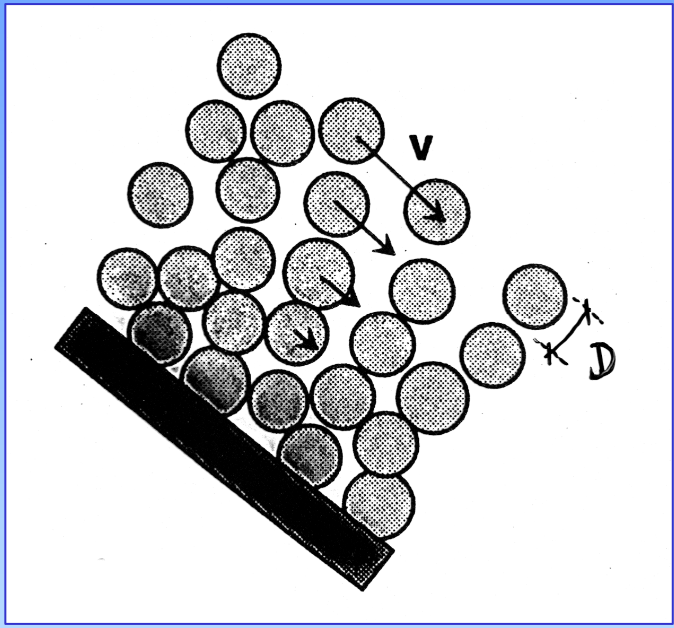
macroscopic quantities can change as fast as particle velocities

"Conclusion"

There are many reasons why granular hydrodynamics should NOT work...

The surprise is that nevertheless in many cases it DOES work !!!!

Influence o
the Bagnold n



$\gamma = \partial_z v =$

$D\gamma =$ typ
respect

$\lambda_e =$ cha
over wh
due to f

Ralph Bagnold

Friction force F_c: $F_c \approx \frac{\Delta E_k}{\lambda_e} \sim \frac{m(D\gamma)}{\lambda_e}$

Viscous damping force F_v: $F_v = 3\pi\eta D v \sim \eta\gamma D^2$

Bagnold number: $\boxed{B = \frac{F_c}{F_v} \sim \frac{m\gamma}{\lambda_e \eta}}$

B>450: granular regime
B<10: ambient fluid crucial

Bagnold number for a vertically vibrated granular gas

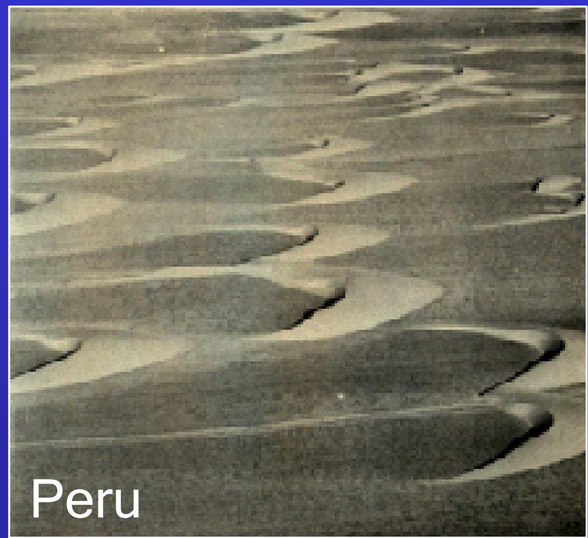
For a vertically vibrated granular gas the Bagnold number is defined as the ratio of gravity and viscous forces:

$$\boxed{B = \frac{F_z}{F_c} = \frac{mg}{3\pi\eta D v} = \frac{\rho_g D^2 g}{18\eta v}}$$

For a sand/glass particle ($\rho = 2.5 \cdot 10^3$ kg m^{-3}) in air ($\eta = 1.9 \cdot 10^{-5}$ Pa s) with a typical velocity of 1 m/s we have:

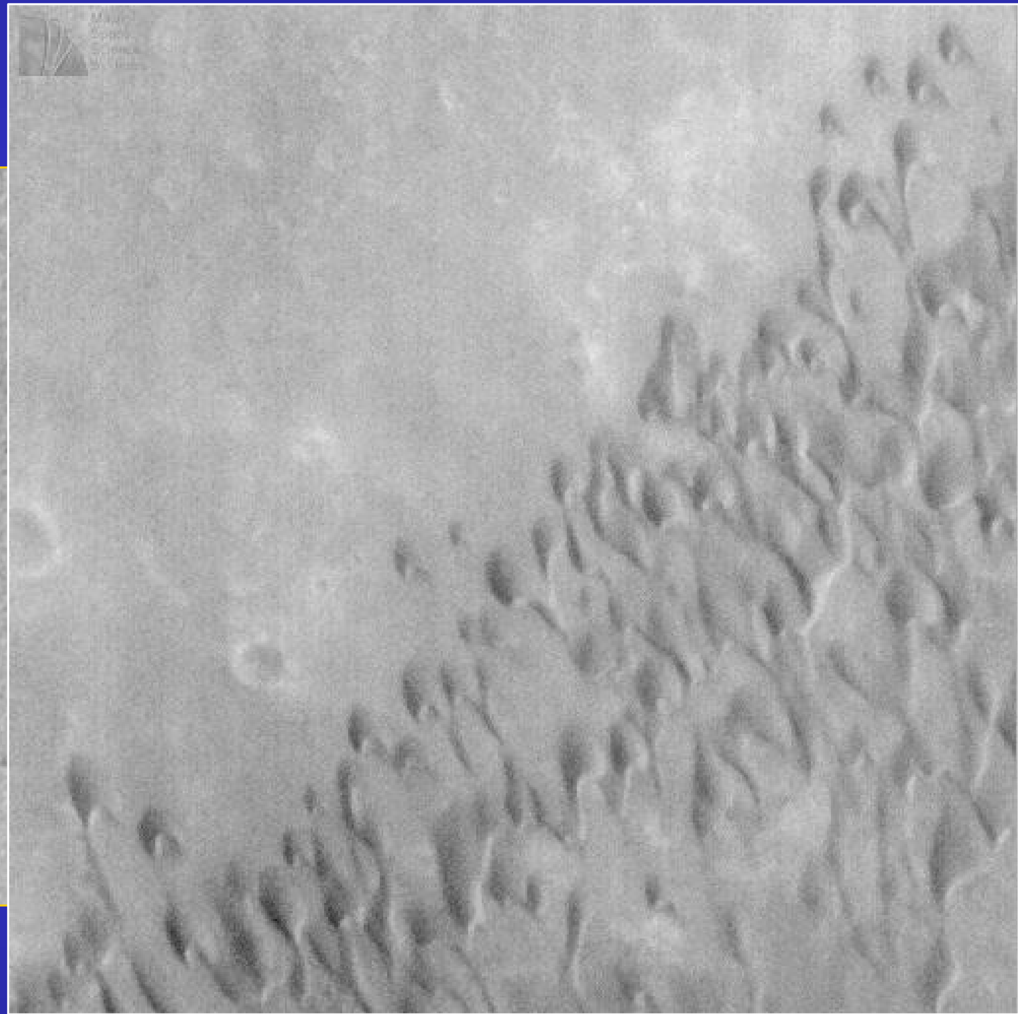
$$\left.\begin{array}{l} B \approx 7.3 \cdot 10^7 D^2 \\ B \approx 1 \end{array}\right\} \rightarrow D \approx 100\ \mu\text{m}$$

Sand dunes: “barchans”

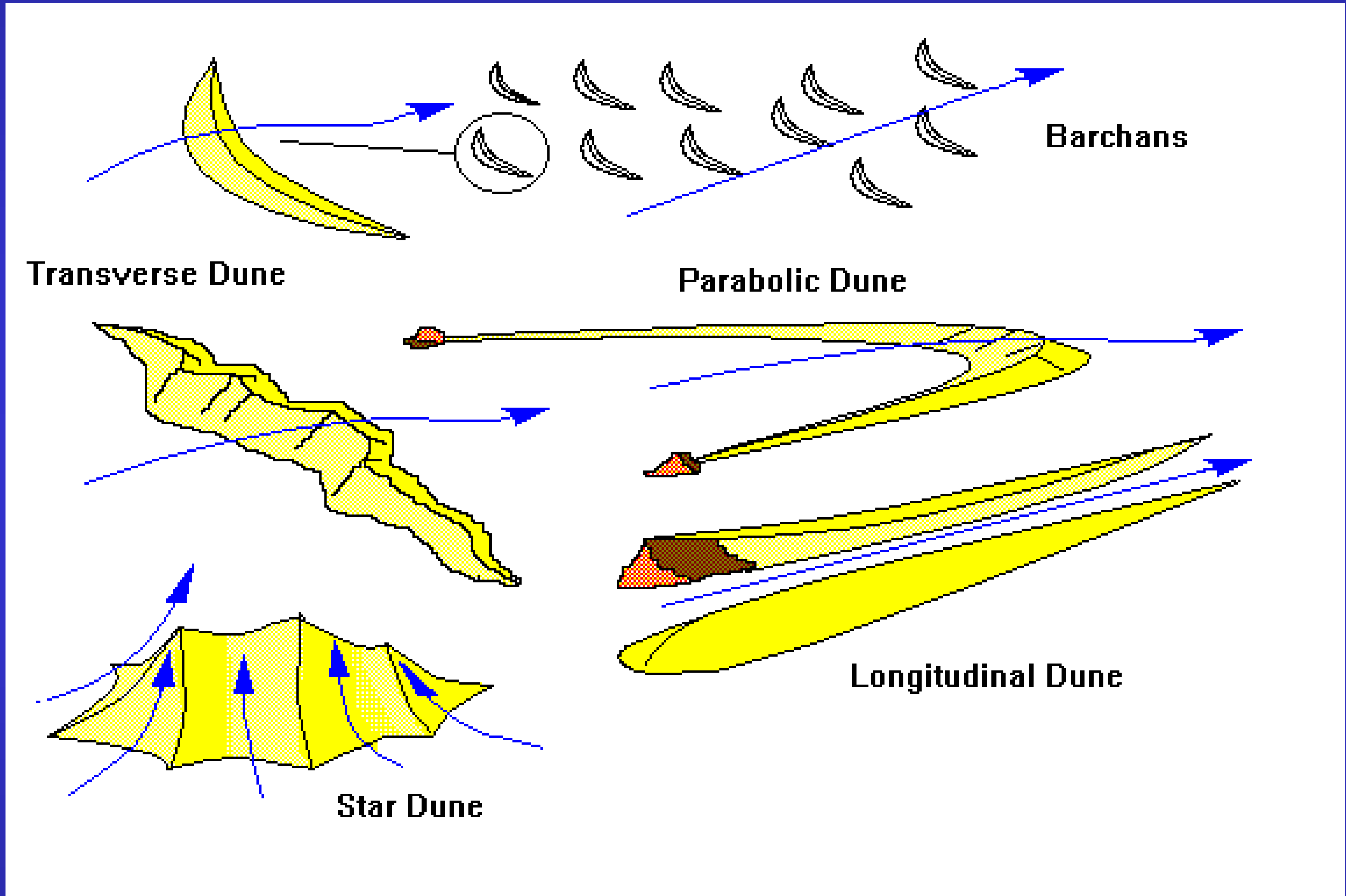


Sand dunes travel ...

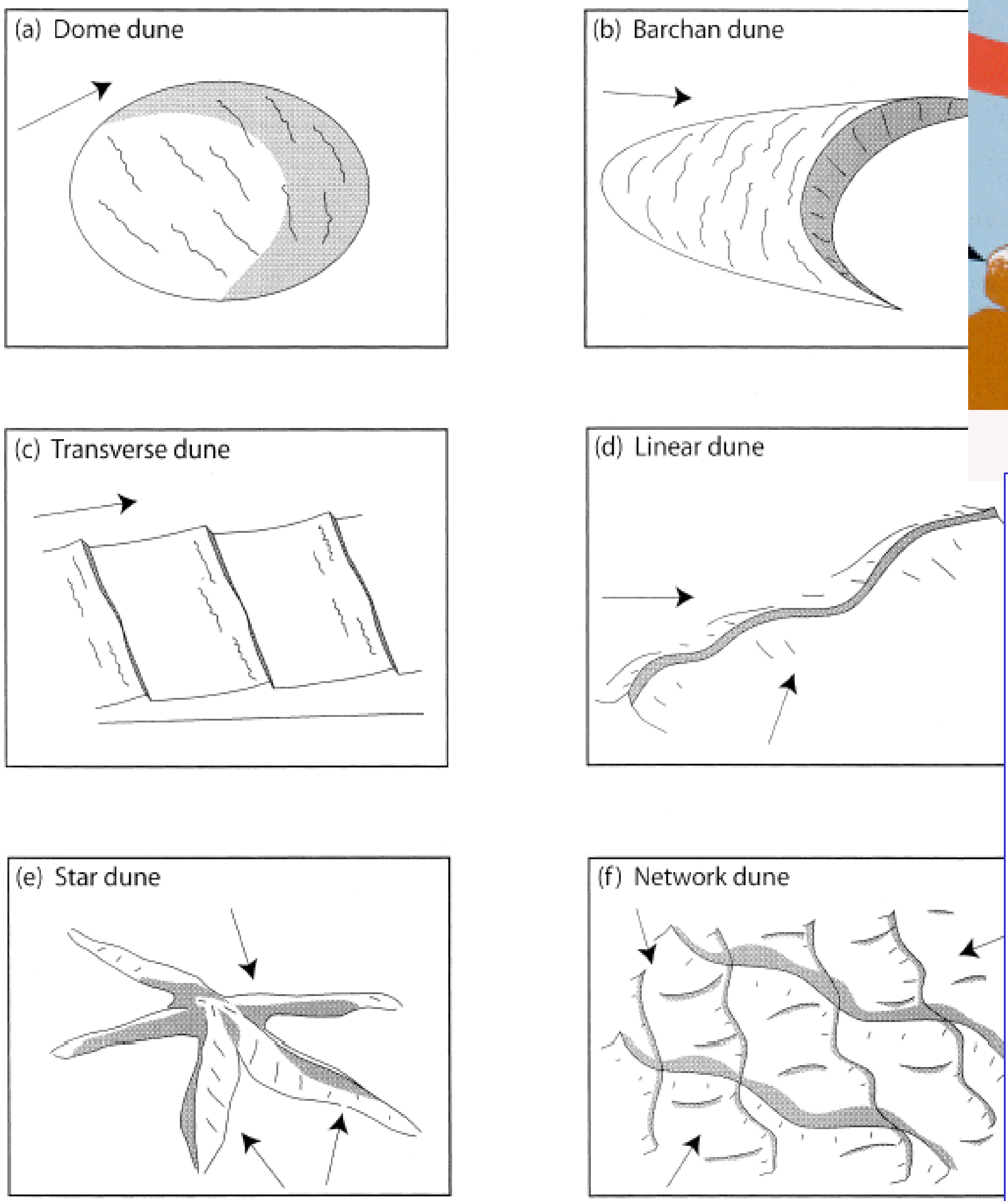
Barchans op Mars

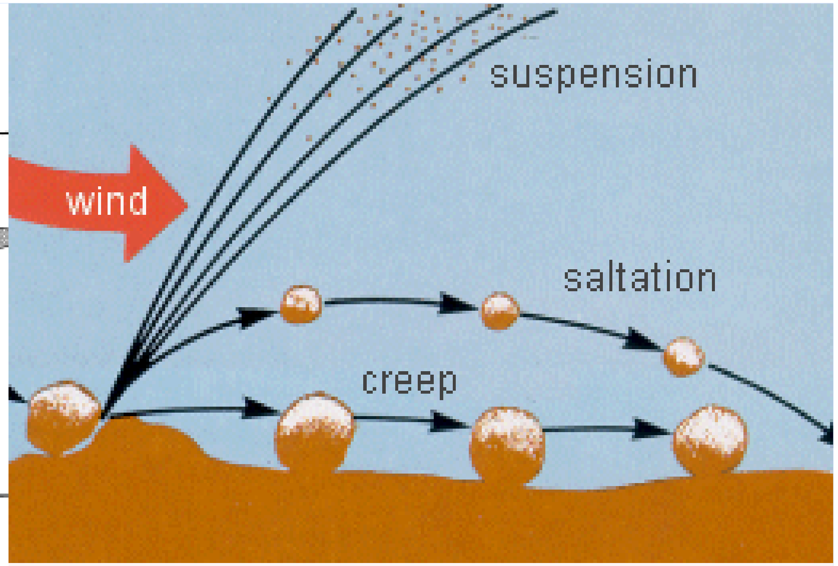


Photos from the Mars Orbiter Camera
(Mars Global Surveyor project)

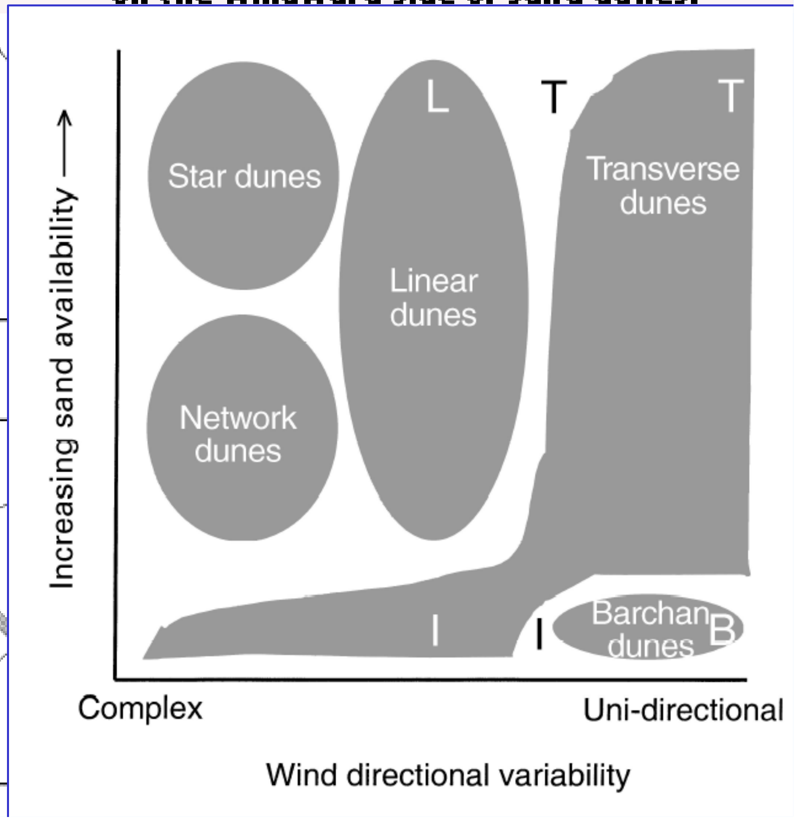


Barchans
Transverse Dune
Parabolic Dune
Longitudinal Dune
Star Dune

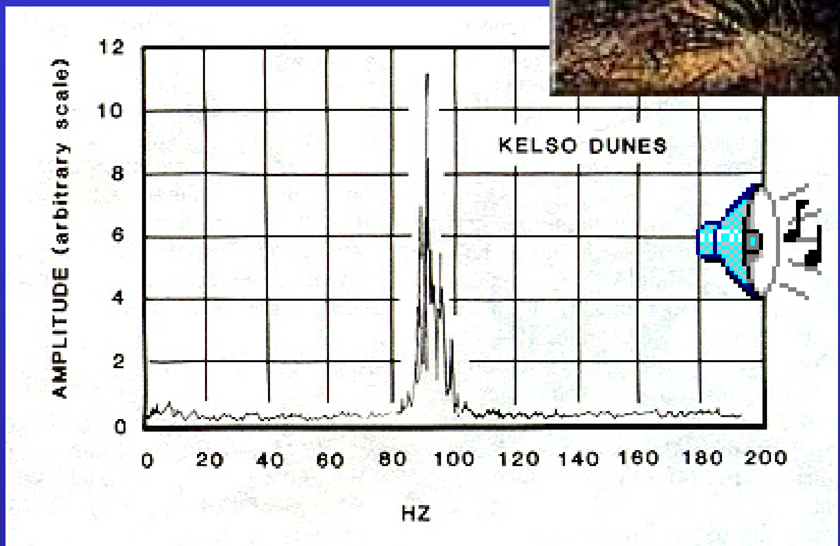




Wind causes saltation, or jumping grains, on the windward side of sand dunes.

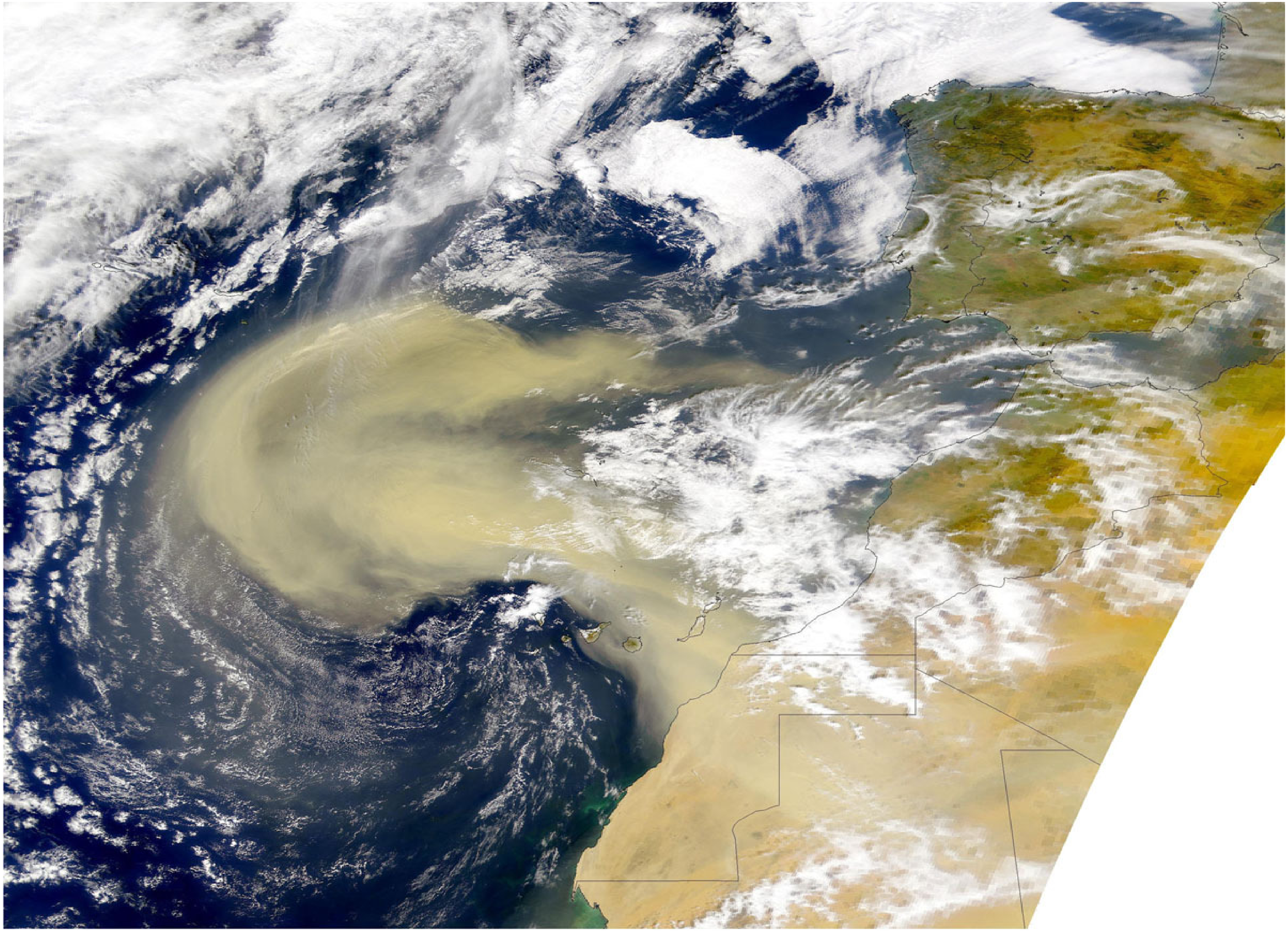


Singing sand



← Frequency spectrum of "booming sand" in the Kelso dunes, California.

Sand can also chirp or quack →



SeaWiFS Captures Massive Dust Storm February 26, 2000 SeaWiFS Project/ORBIMAGE

References

- H. Jaeger, S. Nagel, and R. Behringer, Rev. Mod. Phys. **68**, 1259 (1996).
- L.Kadanoff, Rev. Mod. Phys. **71**, 435 (1999).
- P.G. de Gennes, Rev. Mod. Phys. **71**, S374 (1999).
- I. Goldhirsch, Ann. Rev. Fluid Mech. **35**, 267 (2003).
- J. Duran, *Sands, powders, and grains: An introduction to the physics of granular matter* (Springer, New York, 1999).