the granular solid

jacco snoeijer - university of twente

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an old problem...

a wise man who built his house on the rock

a foolish man who built his house on the sand...
and the house fell

(Matthew 7:24-27)

an old problem...

a wise man who built his house on the rock

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(Matthew 7:24-27)

soil mechanics

continuum (large scale) models:

stress and strength

book by R.M. Nedderman 1992

grain level physics ?

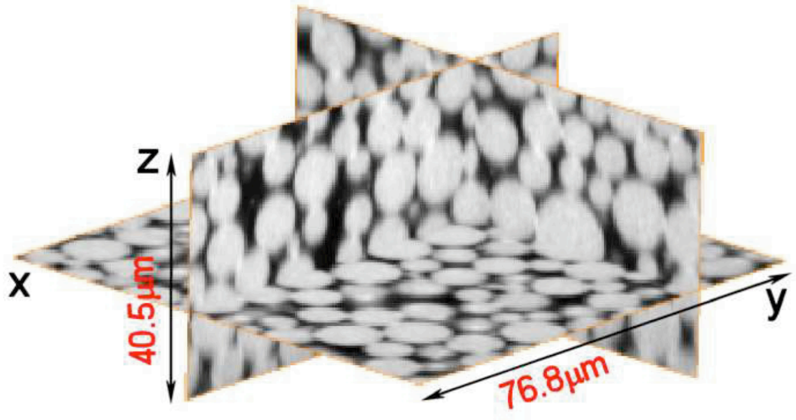
this lecture...

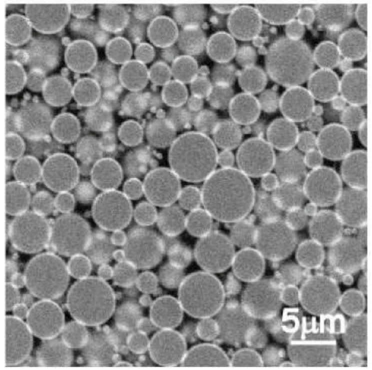
- granular packing as unusual solid
- experimental motivation
- some theoretical ideas

general question: can we understand <u>macroscopic</u> from <u>microscopic</u> behavior?

looking inside: emulsions

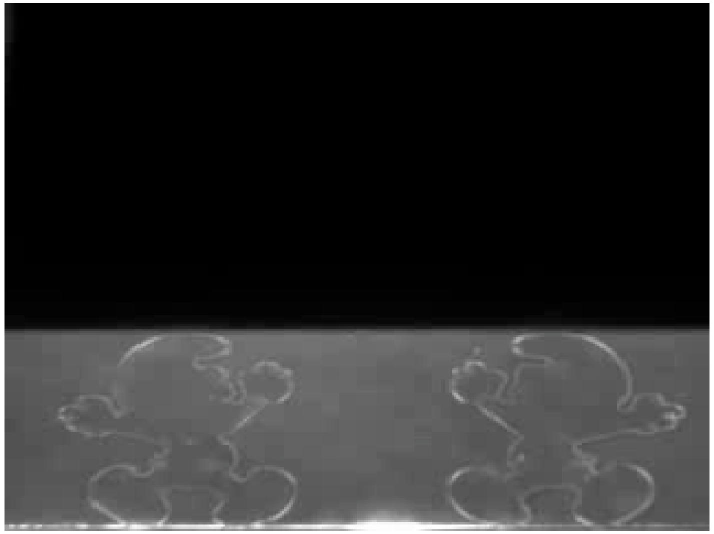
packing of oil droplets dispersed in liquid

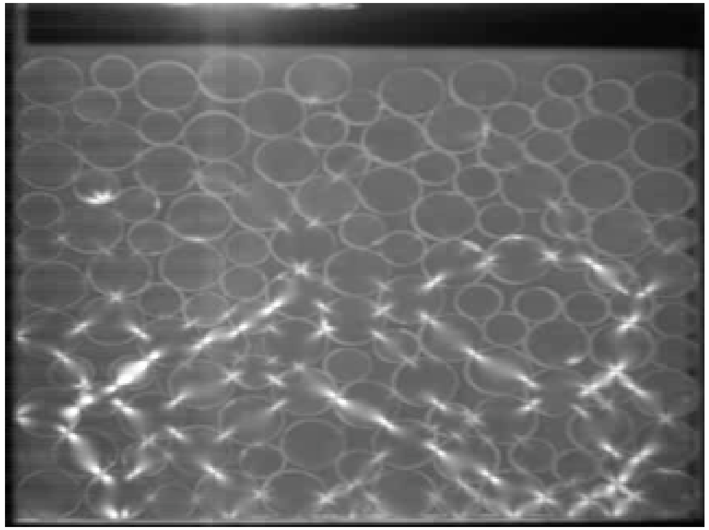




Brujic et al., Phys. A 2003
Zhou et al., Science 2006

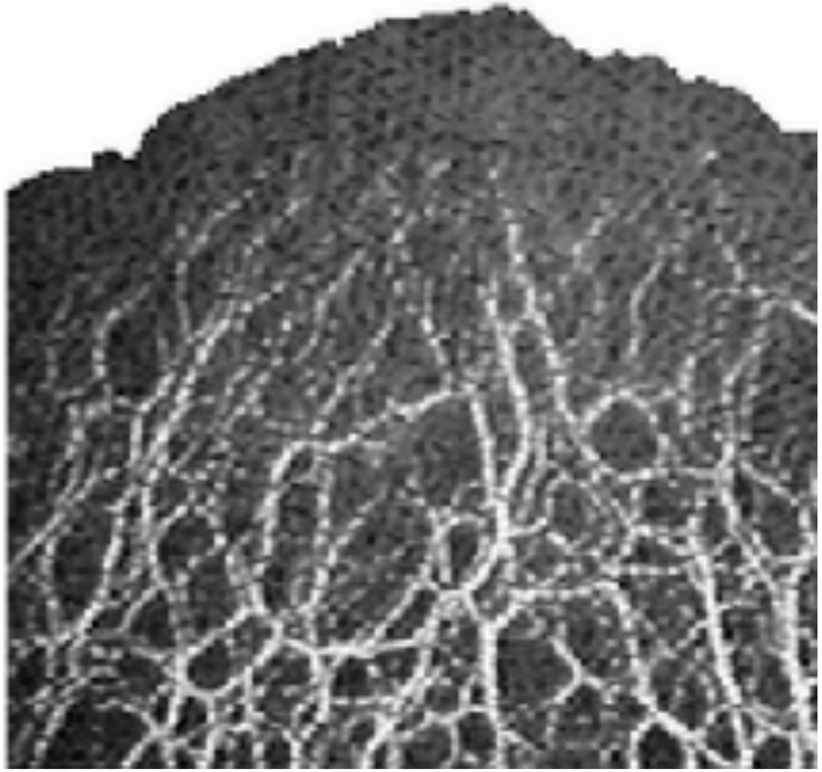
photo elastics: stress





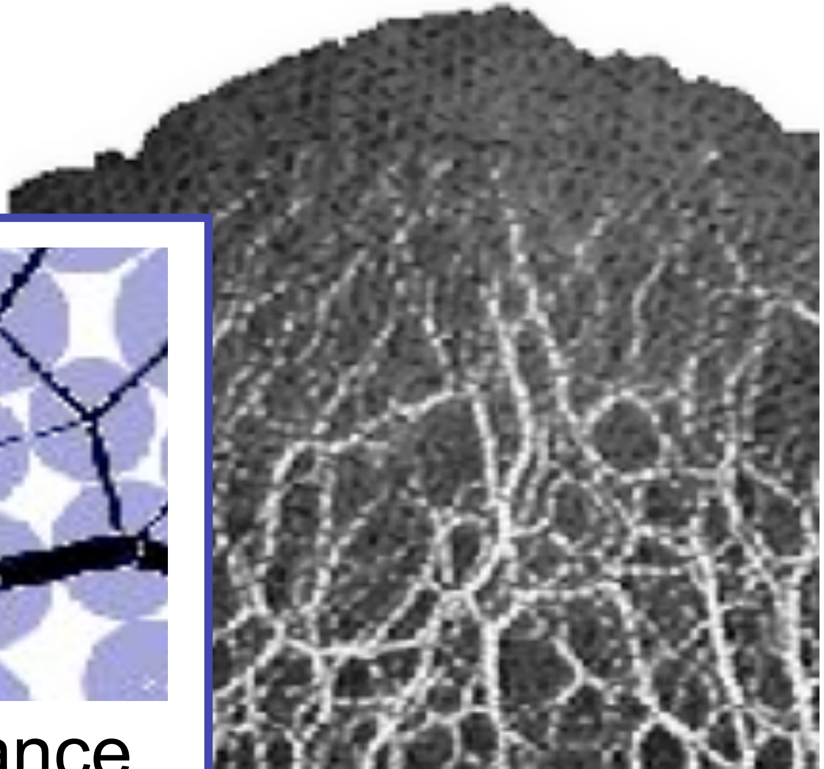
thanks to Martin van Hecke

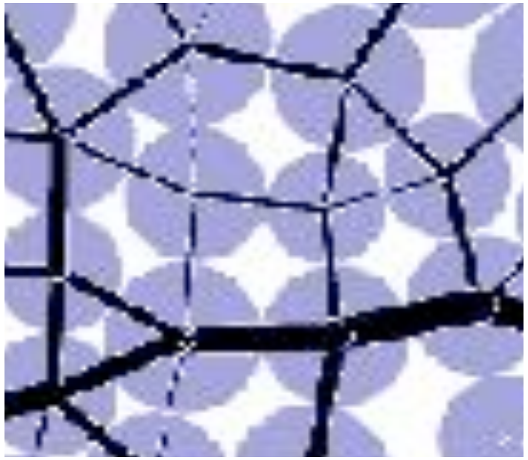
force networks



R.P. Behringer *et al.*

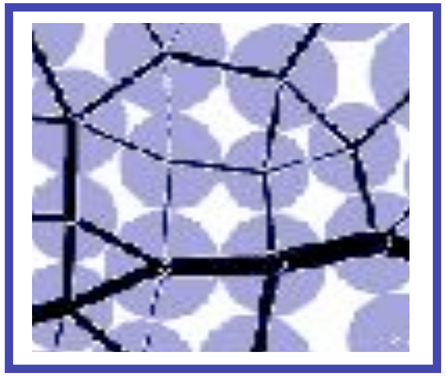
force networks

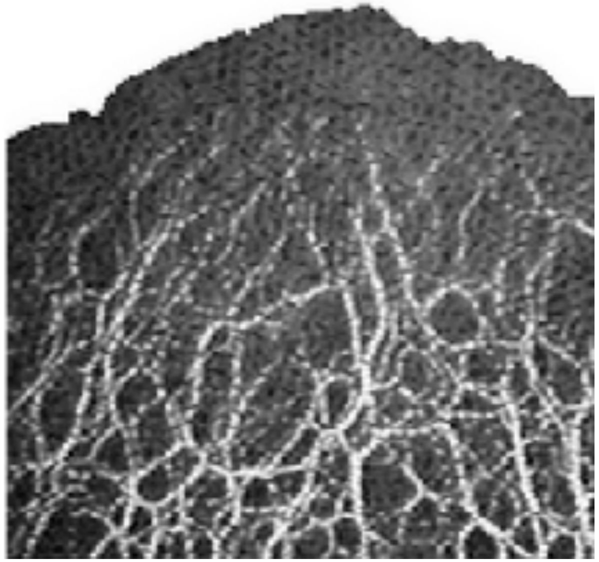




force balance

force networks





1. inhomogeneity
2. structure: ‘force propagation’

force propagation

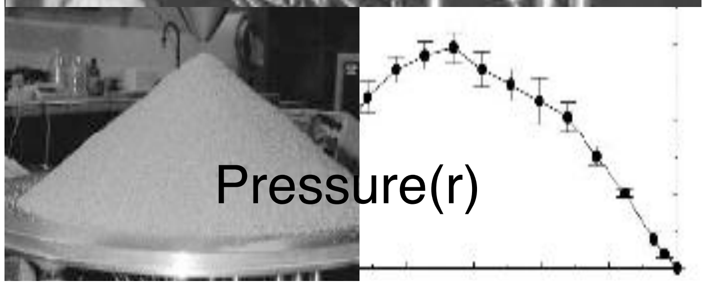
hopper

Pressure(r) ?

r →

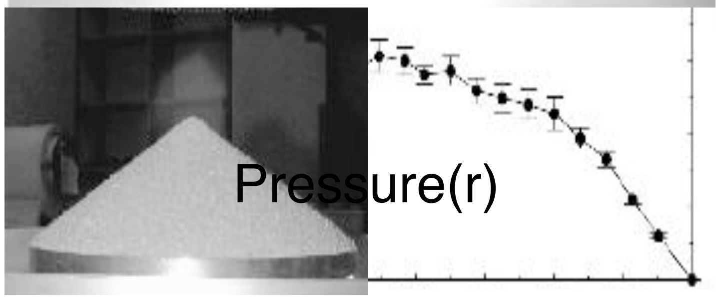
force propagation

hopper



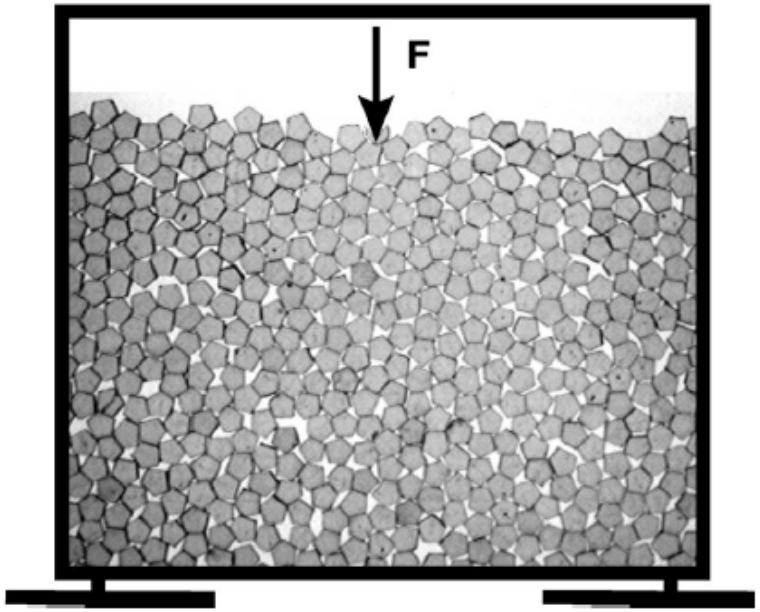
L. Vanel *et al.,* PRE **60**, R5040 ('99)

homogeneous 'rain'



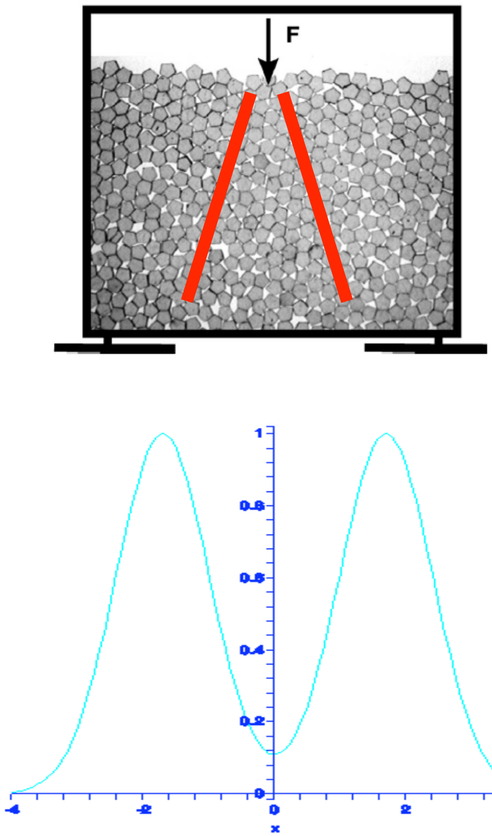
recent papers: Atman et al. 2005, Mullin 2007

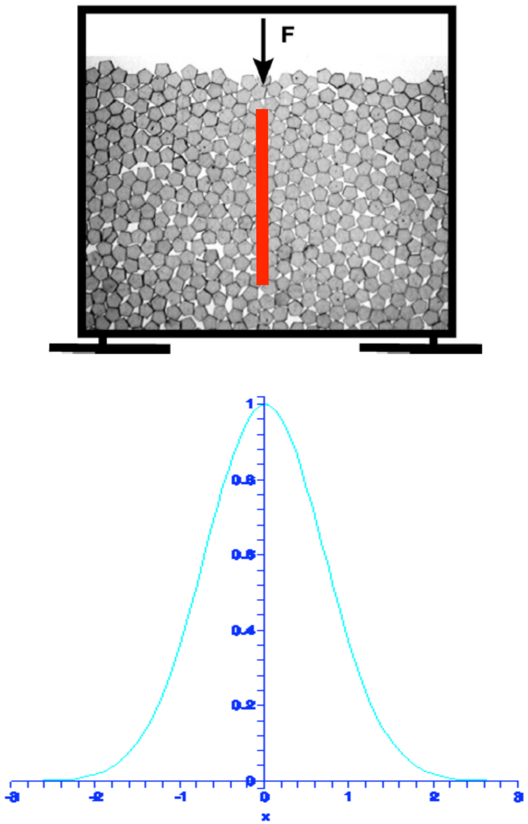
response to local force ?



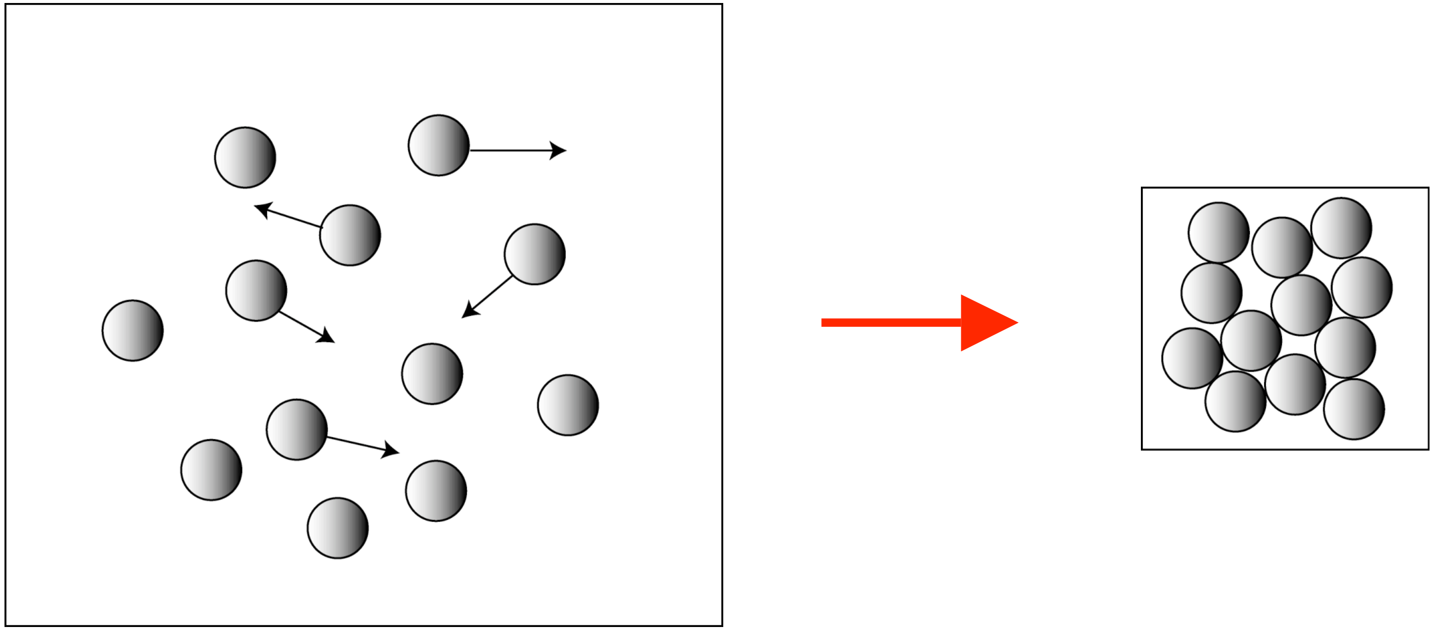
response to local force ?

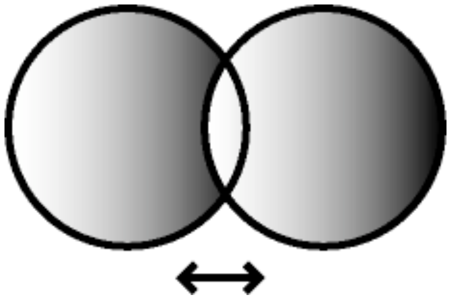
force propagation vs 'usual' elastic behavior





molecular dynamics

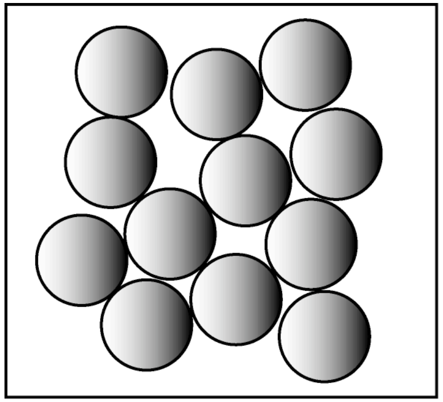


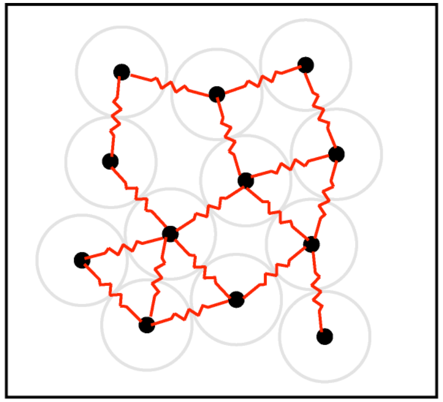


repulsive interaction:

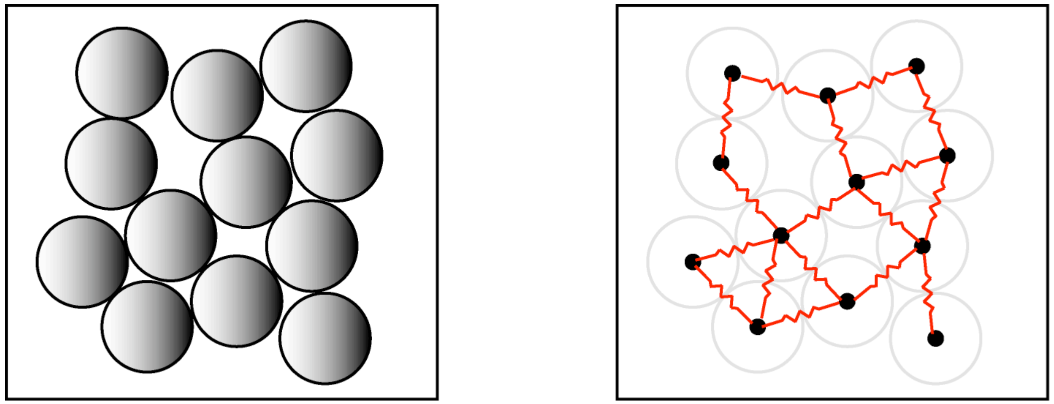
nonlinear spring

response: mass-spring system





response: mass-spring system



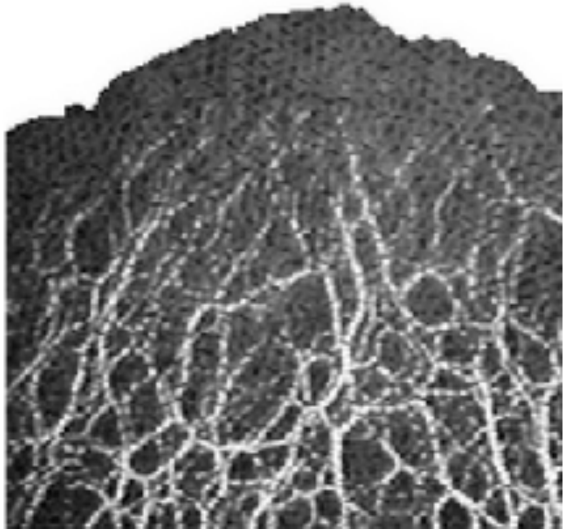
static granular:

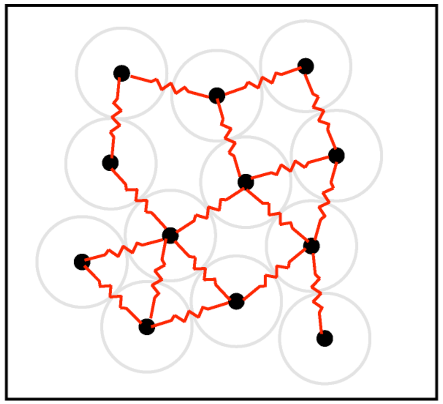
friction, no attraction

amorphous molecular solid:

no friction, attraction, thermal fluctuations

two visions on static granulars





force propagation -- elastic behavior

theory: parameters

grain properties:

shape, sizes, friction, hardness, etc.

theory: parameters

grain properties:

shape, sizes, friction, hardness, etc.

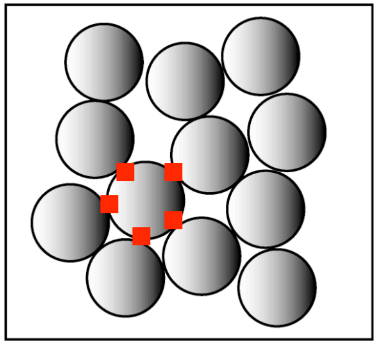
packing properties - local geometry:

of contacts ←

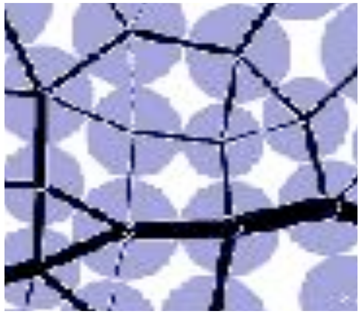
contact distribution

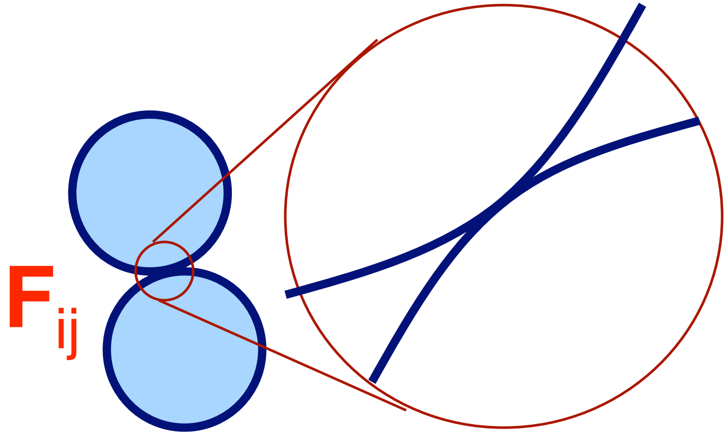
density

correlations, ... ???

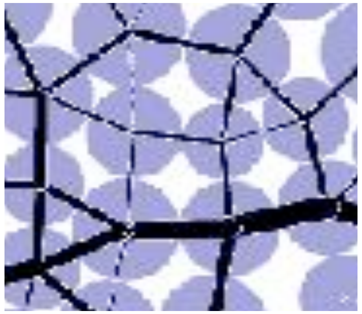


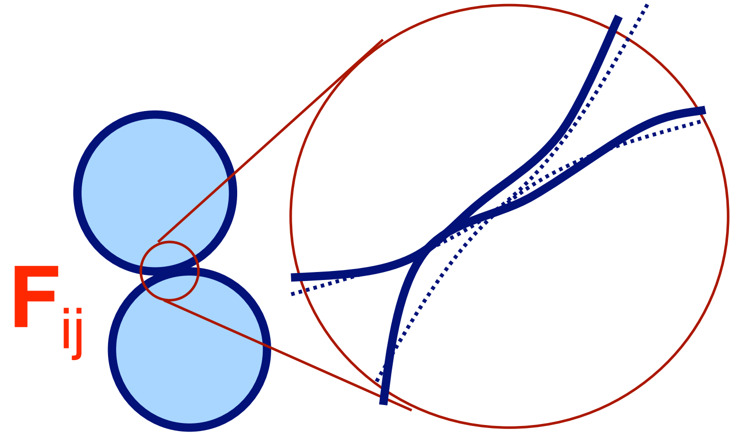
force balance



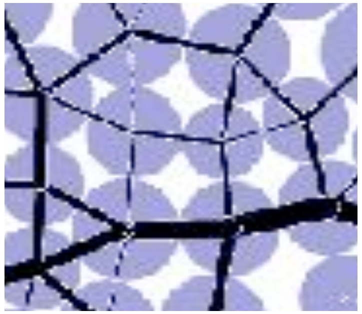


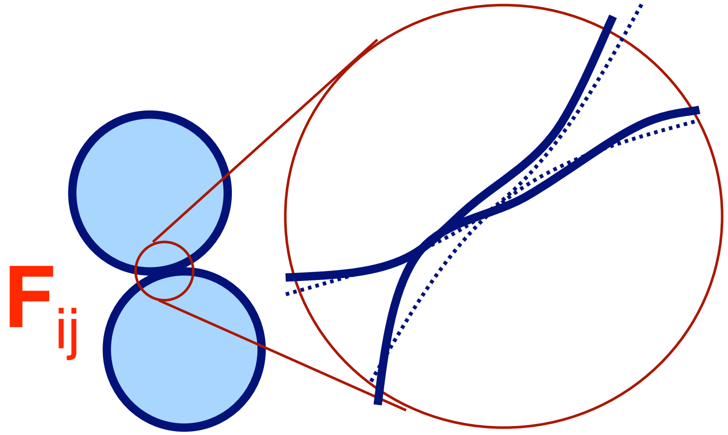
force balance





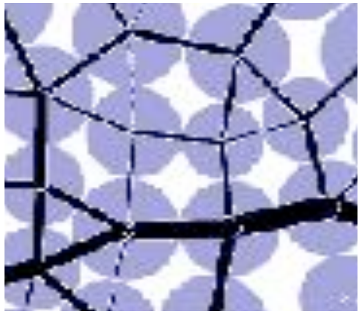
coordination number: z

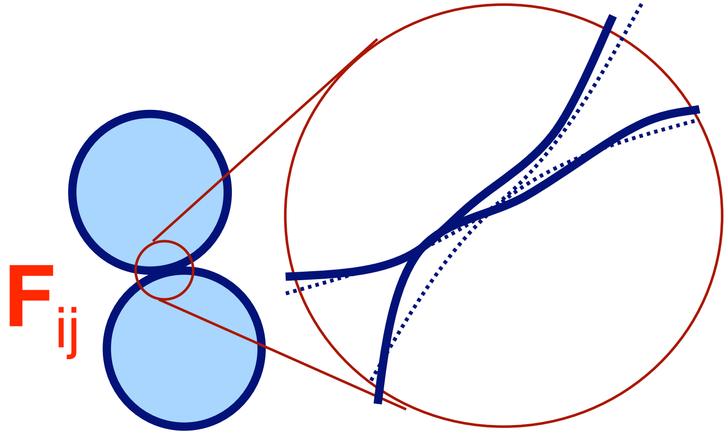




treat $\mathbf{F}_{ij}$ as 'unknown variable': $zN/2$ unknowns

coordination number: z



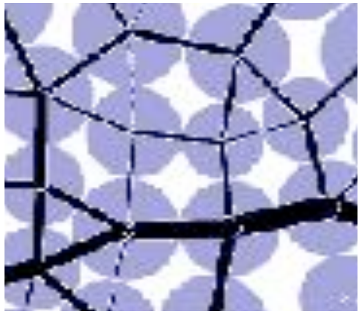


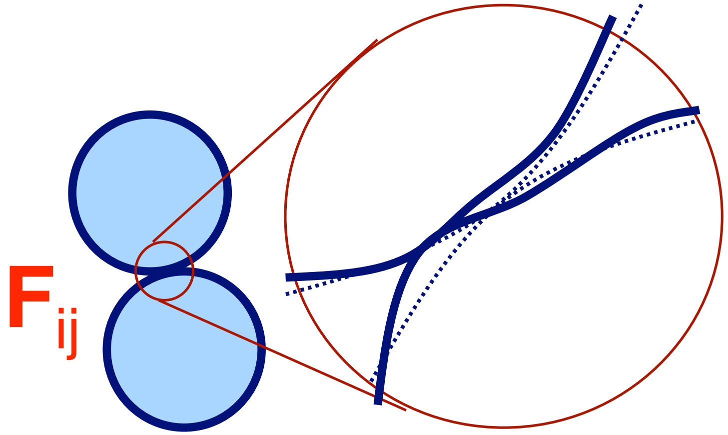
treat F_{ij} as 'unknown variable': zN/2 unknowns

mechanical equilibrium: 2N equations

(2 dimensions, frictionless particles)

coordination number: z





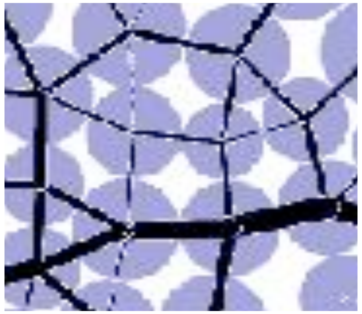
treat $\mathbf{F}_{ij}$ as 'unknown variable': $zN/2$ unknowns

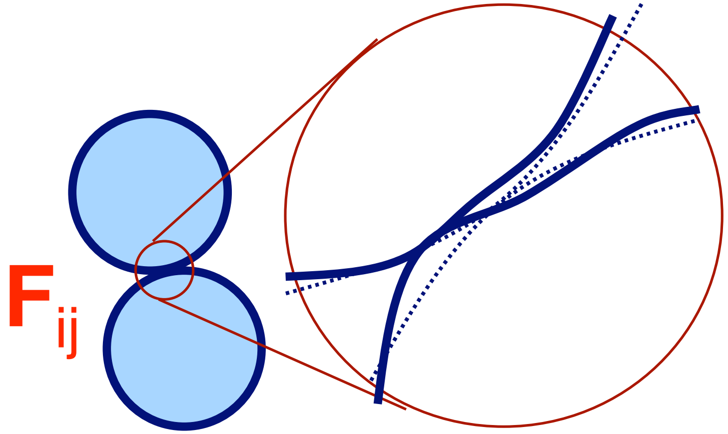
mechanical equilibrium: $2N$ equations

solutions exist if #unknowns $\geq$ #equations

$z \geq 4$

coordination number: z





z = 4: isostatic (unique force solution)

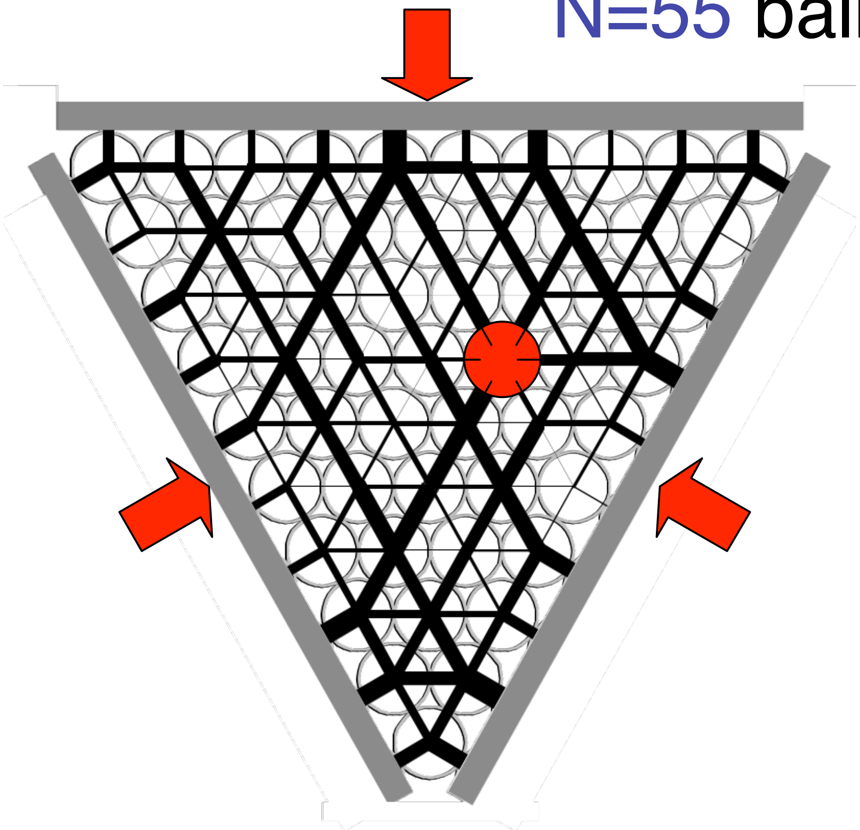
z > 4: hyperstatic (many possible force solutions)

z < 4: no equilibrium possible

snooker!

‘snooker’ packing

N=55 balls – repulsive forces

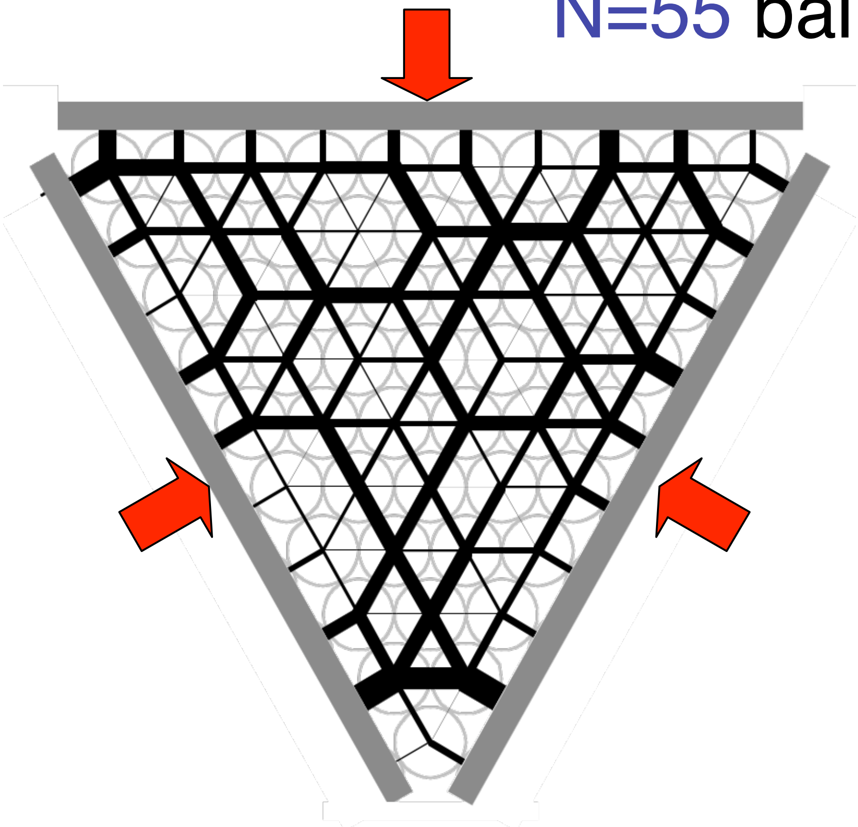


3N unknowns (forces)

2N eqs (balance x & y)

‘snooker’ packing

N=55 balls – repulsive forces



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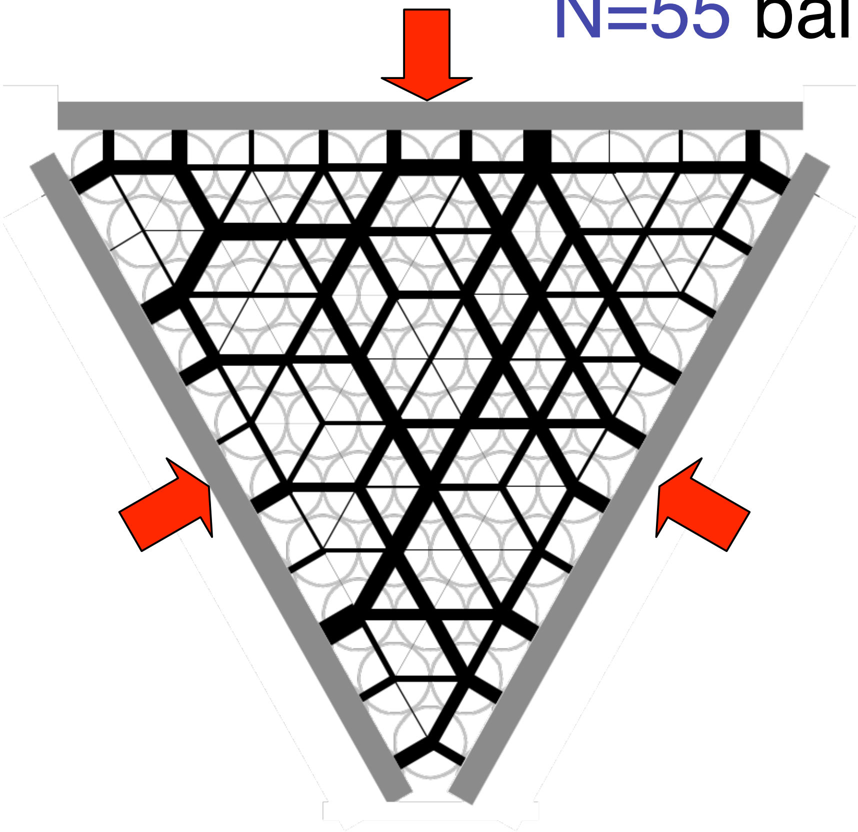

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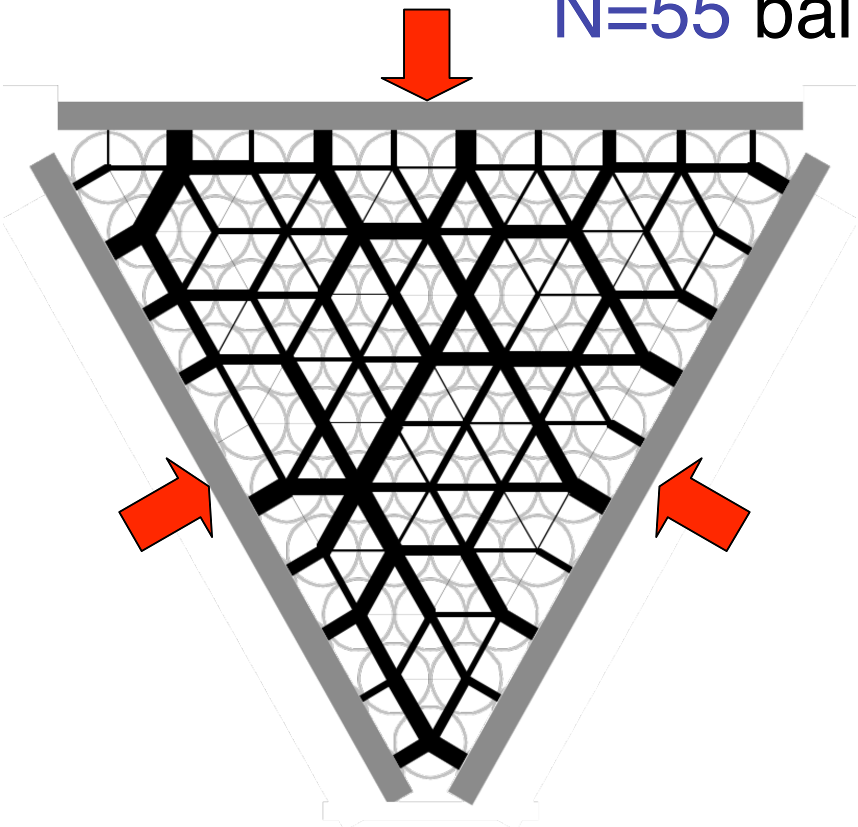

3N unknowns (forces)

2N eqs (balance x & y)

force statistics: P(f) ?

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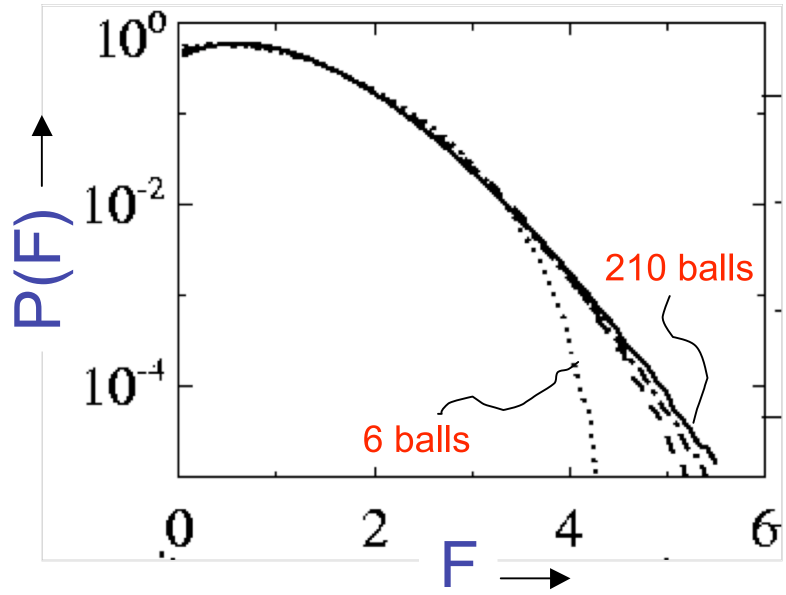
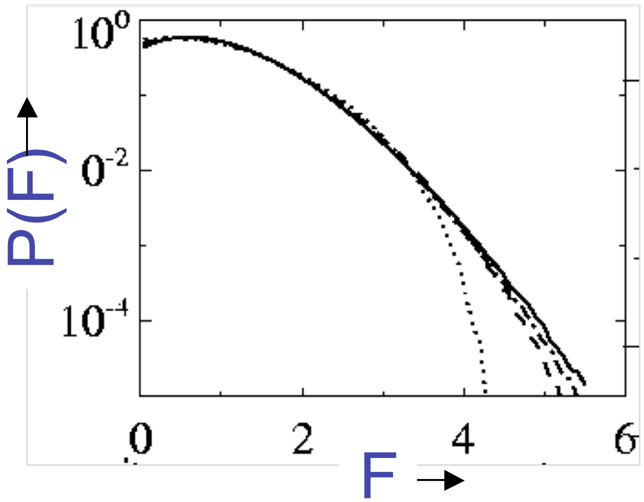


Snoeijer, Vlugt, van Hecke, van Saarloos, Phys. Rev. Lett. 2004
van Eerd, Ellenbroek, van Hecke, Snoeijer, Vlugt, Phys. Rev. E 2007

Tighe, Socolar, Schaeffer, Mitchener, Huber, Phys. Rev. E 2005

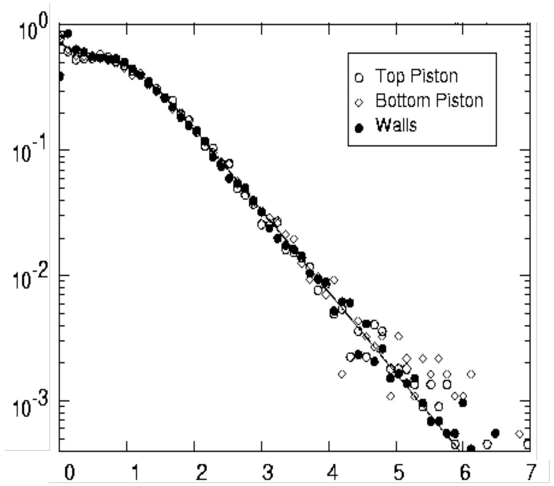
force statistics: P(f) ?

Theory: snooker



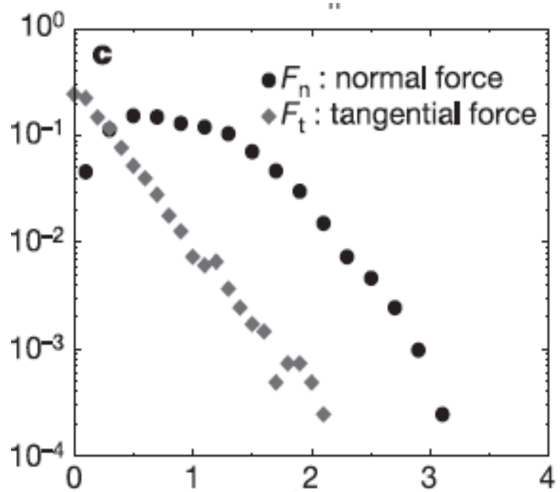
Experiments

glass beads (carbon paper)



Liu *et al.*, Science 1995
D. Blair *et al.*, PRE 2001

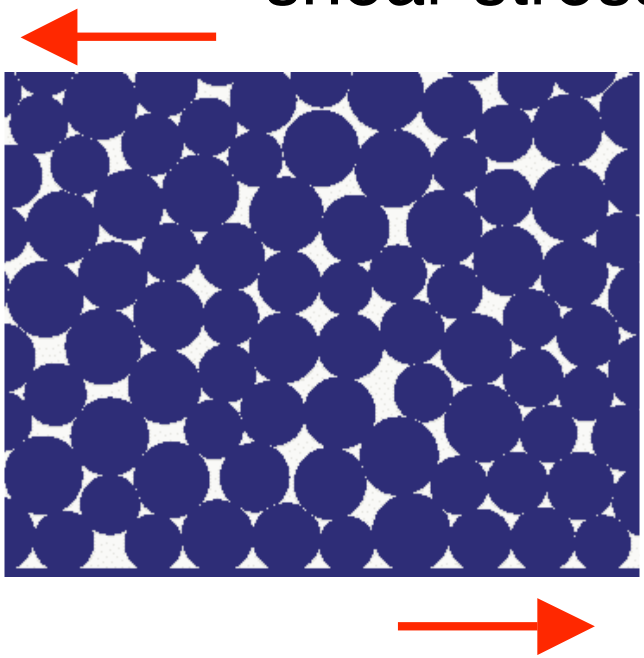
photo-elastic grains



Majmudar & Behringer, Nature 2005

shear stress

shear stress $\tau = \sigma_{xy} / \sigma_{xx}$



$$\sigma_{xx} = \Sigma_i F_i r_i \cos^2 \phi_i$$

$$\sigma_{yy} = \Sigma_i F_i r_i \sin^2 \phi_i$$

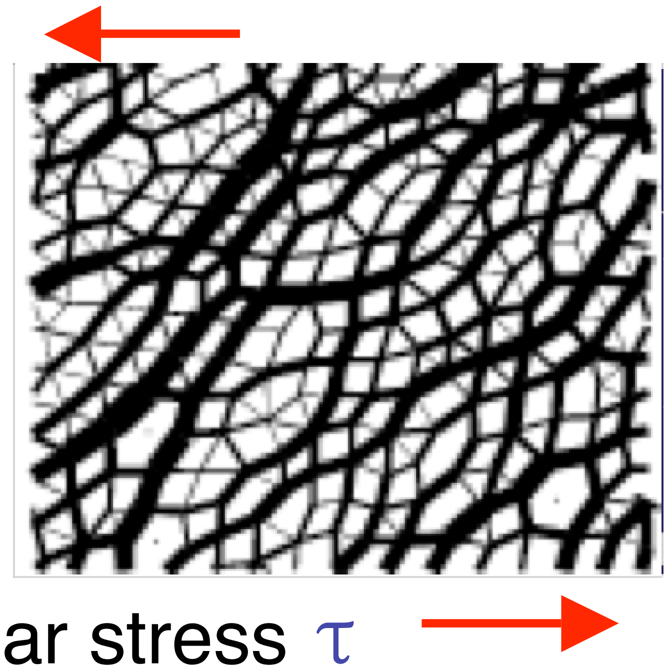
$$\sigma_{xy} = \Sigma_i F_i r_i \cos \phi_i \sin \phi_i$$

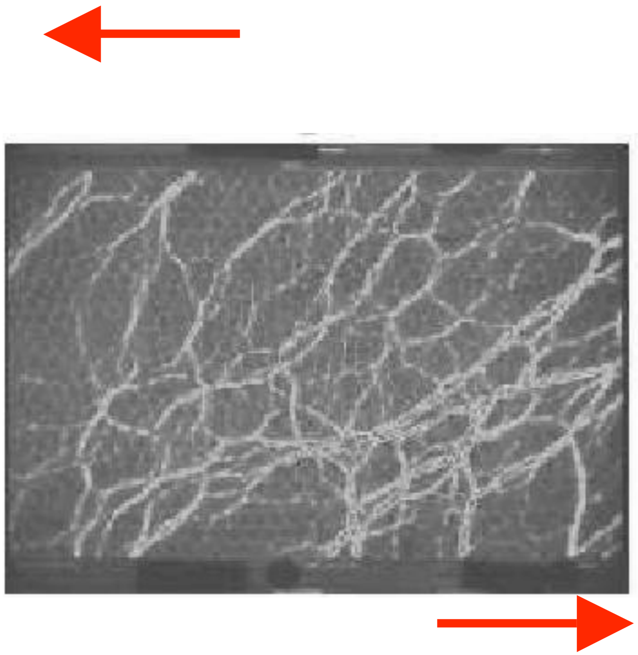
different solutions have different shear stress

shear stress

Theory

Experiment





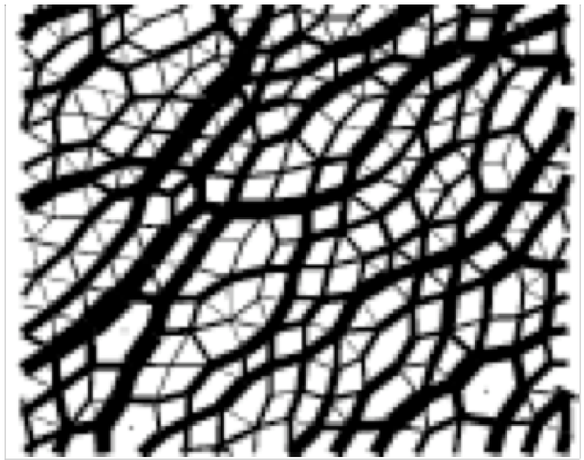
shear stress τ

J. Geng *et al.,* Physica D ('03)
Atman *et al.,* Eur. Phys. J. E ('05)

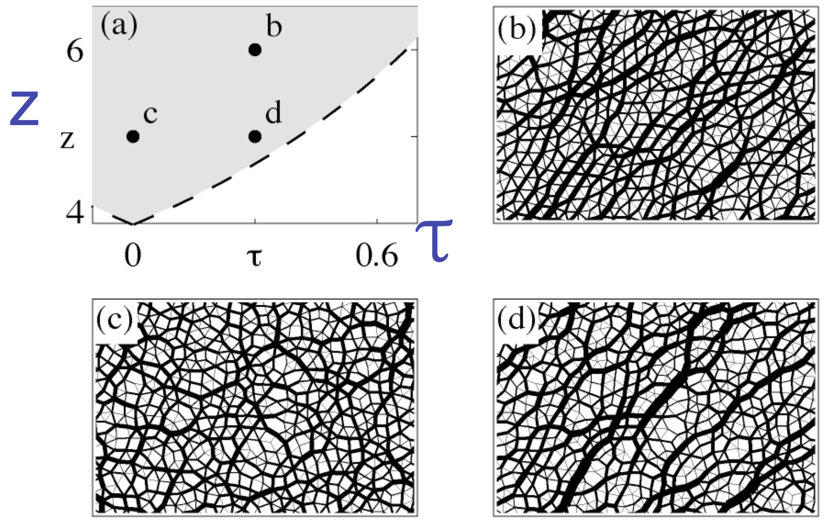
force anisotropy

observation:

no more solutions beyond critical τ!



observation:



scaling with z:

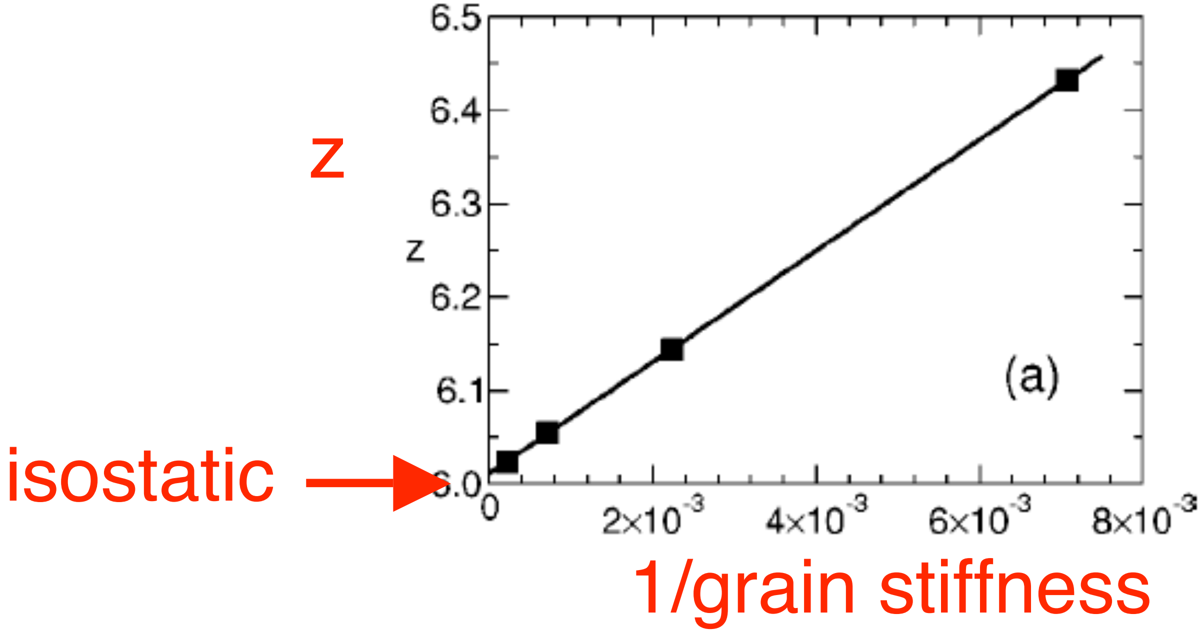
$$\tau_{\max} \approx 2\,\frac{z - z_c}{z}$$

Snoeijer, Ellenbroek, Vlugt, van Hecke Phys. Rev. Lett. 2006

do isostatic packs exist?

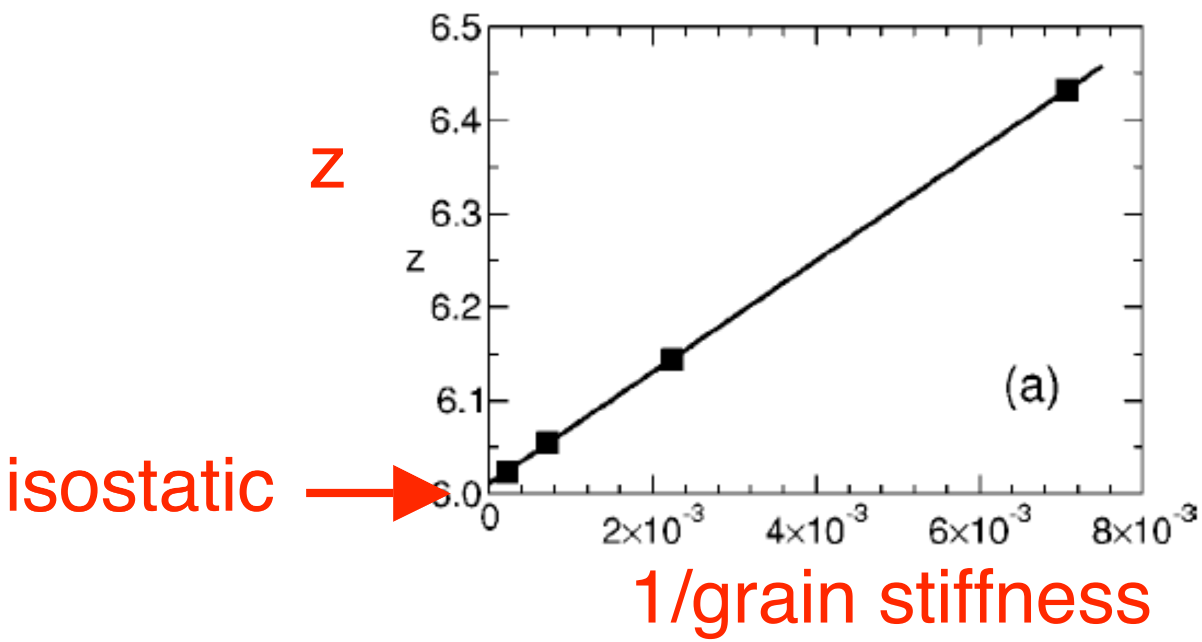
do isostatic packs exist?

Silbert et al. PRE 2002



do isostatic packs exist?

Silbert et al. PRE 2002

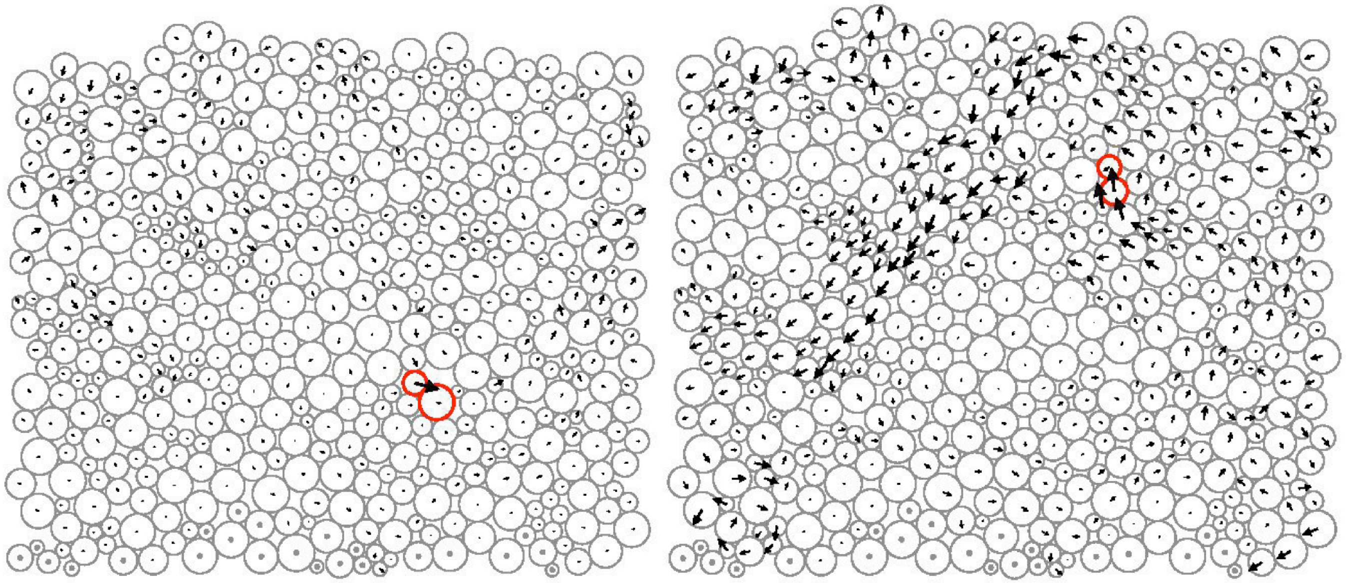


infinitely rigid (frictionless) grains: isostatic

Tkachenko and Witten PRE 1998

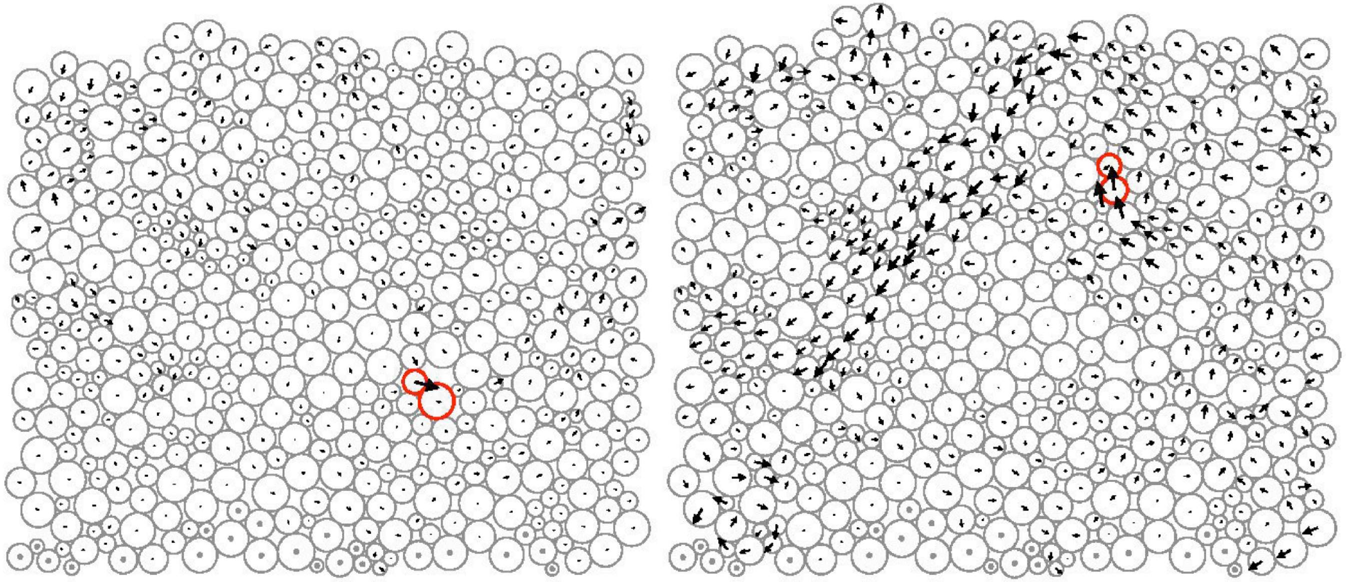
near isostatic packs...

removing 1 contact gives free motion



near isostatic packs...

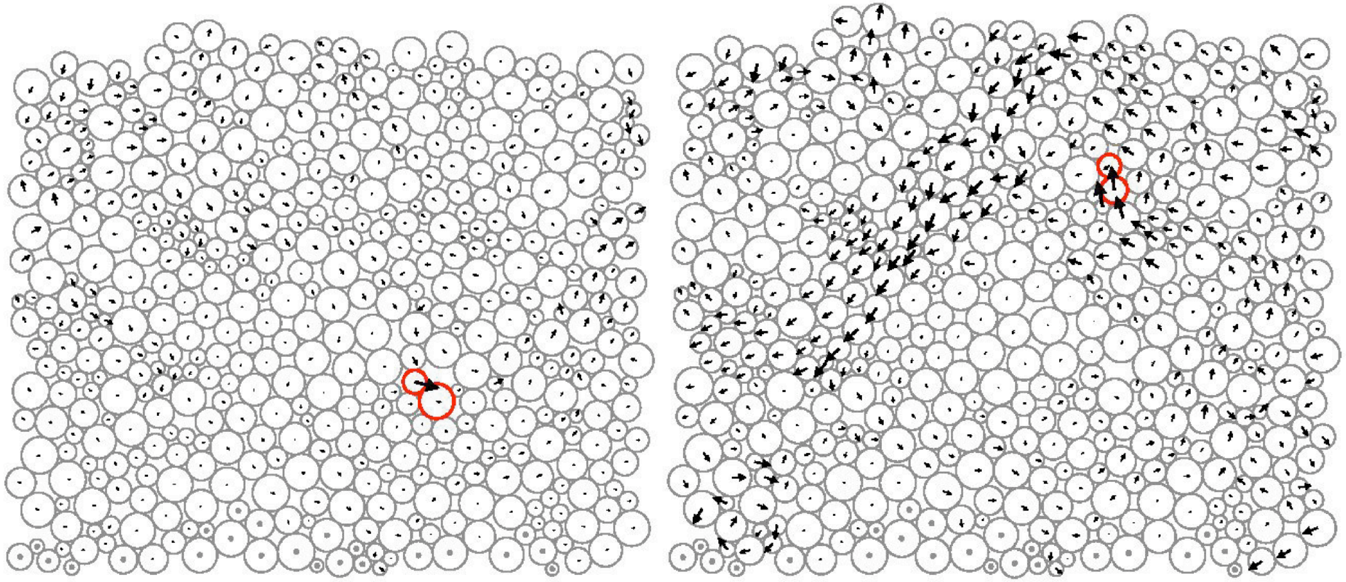
removing 1 contact gives free motion



non-local perturbation: ‘propagation’

near isostatic packs...

removing 1 contact gives free motion

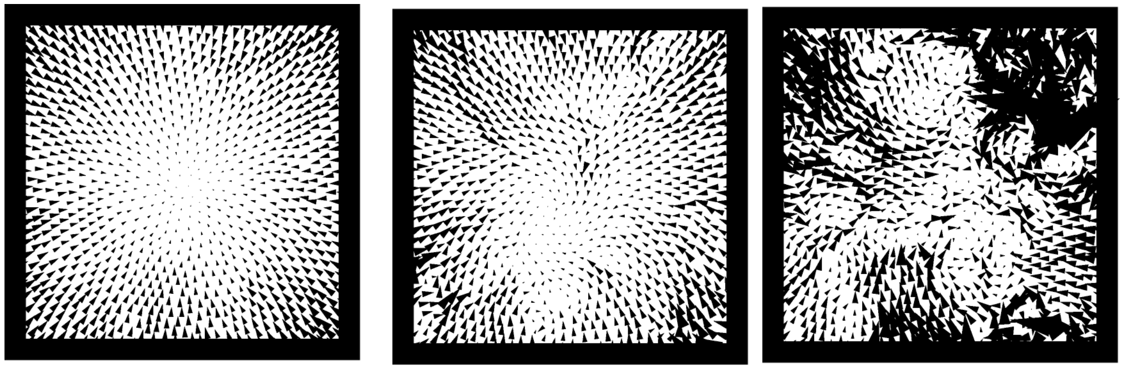


Wyart, Silbert, Nagel and Witten, 2005:

many ‘soft modes’: very different from elastic modes

near isostatic packs...

apply global compression

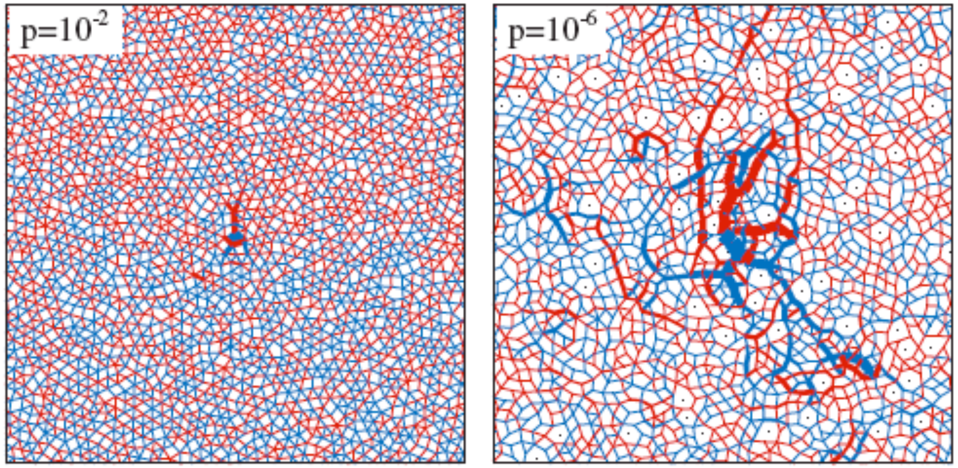


decreasing $z\text{-}z_c$ →

Wouter Ellenbroek, PhD thesis 2007

near isostatic packs...

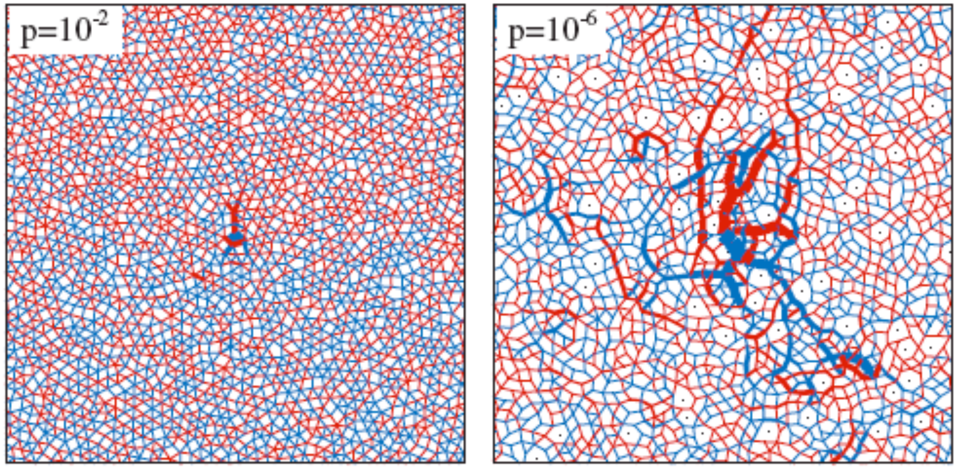
point force on particle



Ellenbroek, Somfai, van Hecke, van Saarloos, Phys. Rev. Lett. 2006

near isostatic packs...

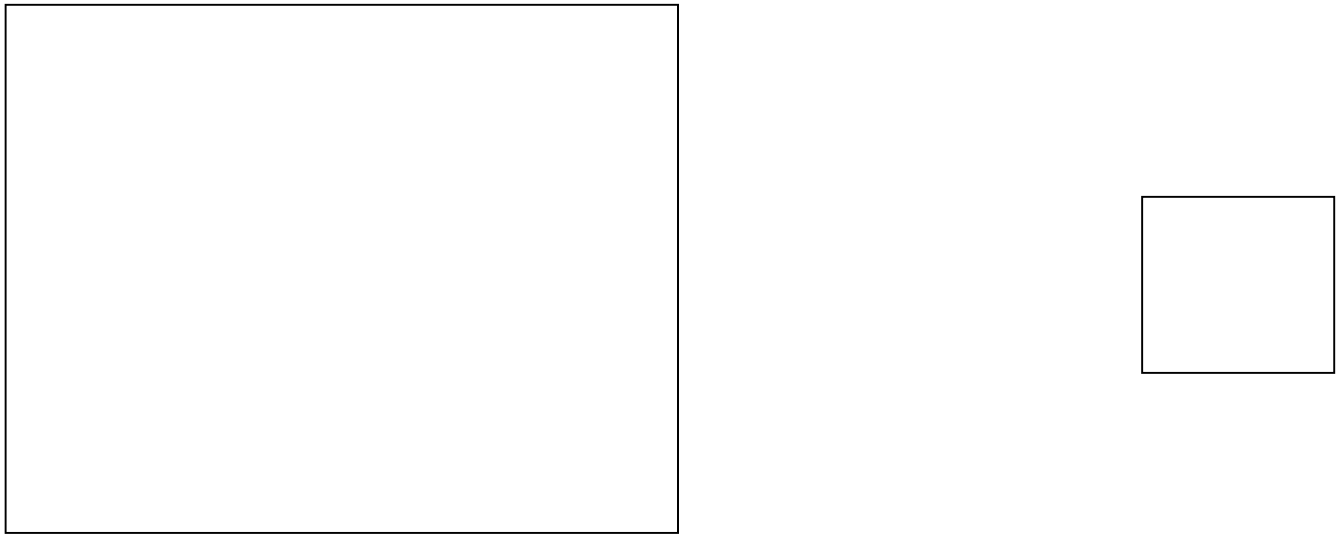
point force on particle



coarse graining length scale $\sim d/(z-z_c)$

Ellenbroek, Somfai, van Hecke, van Saarloos, Phys. Rev. Lett. 2006

final argument...



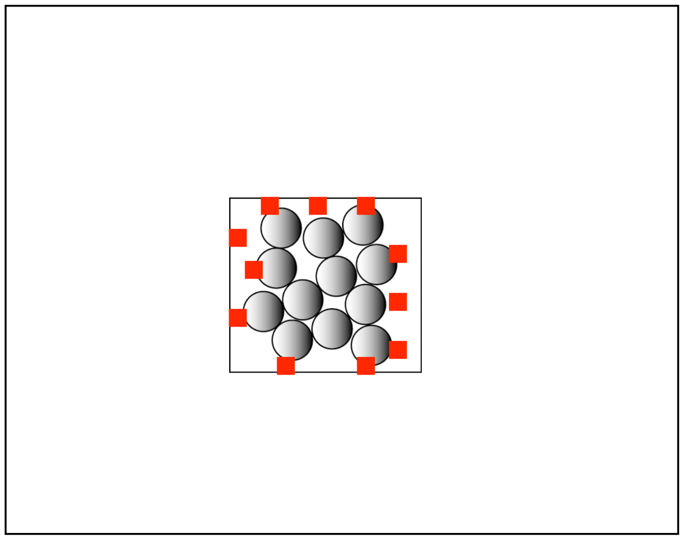
box of size LxL

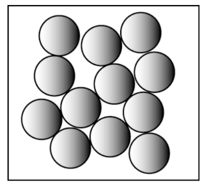
box of size λxλ

#contacts = $z/2\ \rho\ L^2$

$z/2\ \rho\ \lambda^2$

final argument...





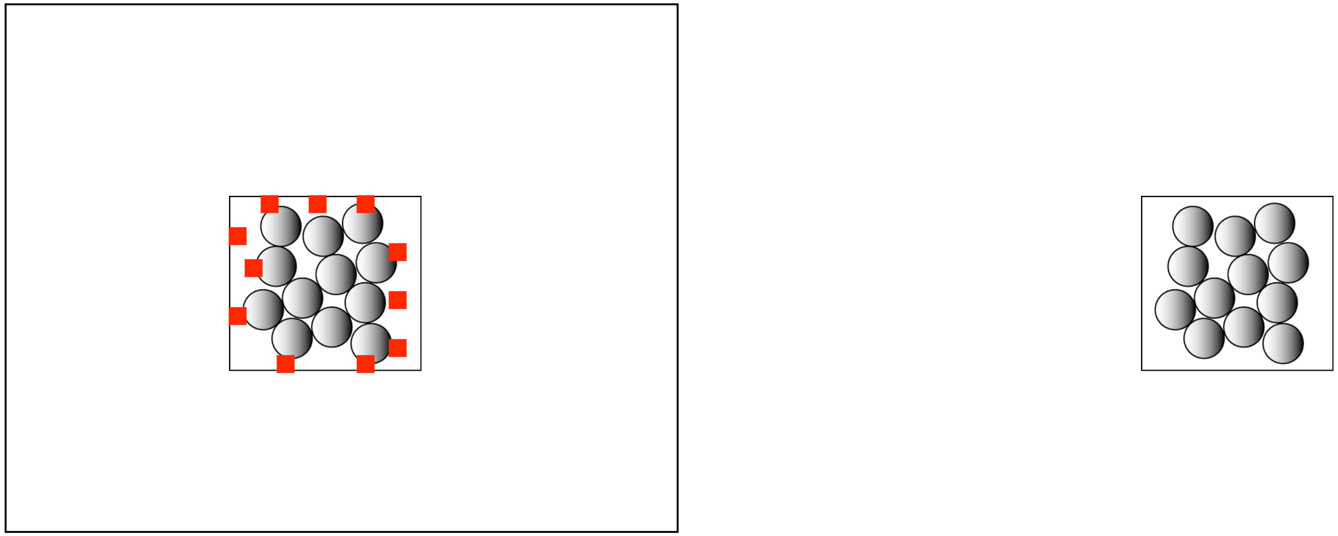
box of size LxL

#contacts = $z/2 \, \rho \, L^2$

box of size $\lambda x \lambda$

$z/2 \, \rho \, \lambda^2 - \alpha \, \lambda$

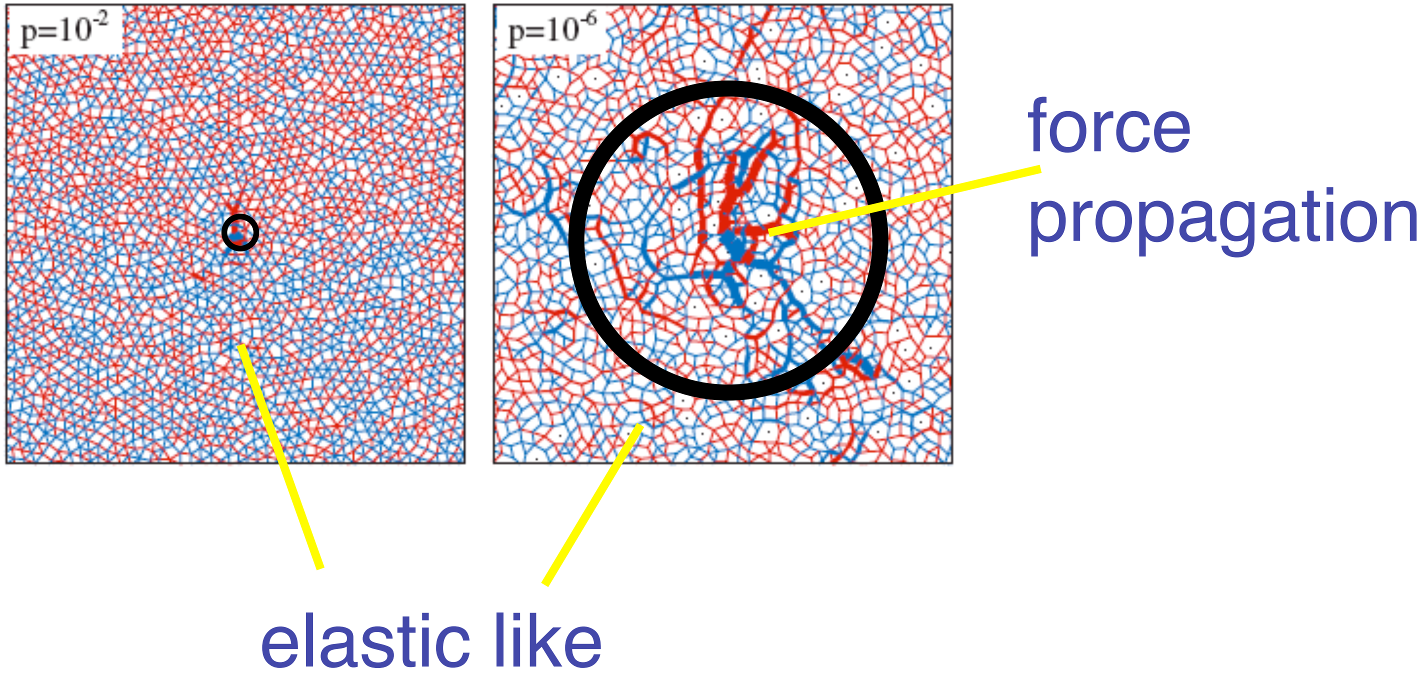
final argument...



isostatic: $z/2\ \rho\ \lambda^2 - \alpha\ \lambda = 2\ \rho\ \lambda^2$

$\lambda \sim 1/\,(z\text{-}z_c)$

length scale



conclusion

static grains are challenging:

- micro to macro -> dip
- coordination number: iso vs hyperstatic
- unusual elasticity and sound propagation
-...

conclusion

static grains are challenging:

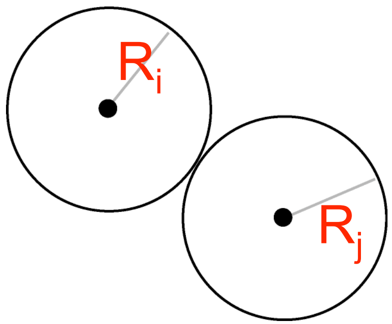
- micro to macro -> dip
- coordination number: iso vs hyperstatic
- unusual elasticity and sound propagation
- ...

other problems:

- quasi-statics - rearrangements:
  flow or weak vibrations

another counting argument...

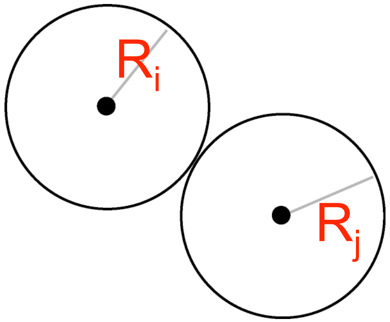
infinitely hard grains
at contact:



$$\sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2} = \left(R_i + R_j\right)$$

another counting argument...

infinitely hard grains
at contact:



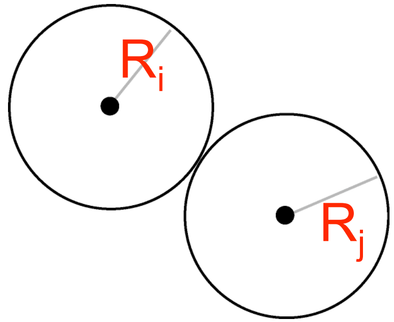
$$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} = (R_i + R_j)$$

positions x_i and y_i 'unknown variables': 2N unknowns
contact condition: zN/2 equations

#unknowns ≥ #equations $z \leq 4$

another counting argument...

infinitely hard grains
at contact: $z \leq 4$



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force balance (frictionless case):

$z \geq 4$

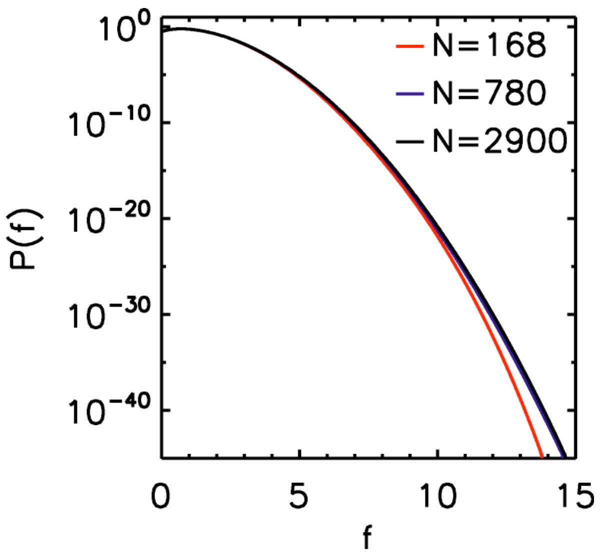
Tkachenko and Witten, 1998:

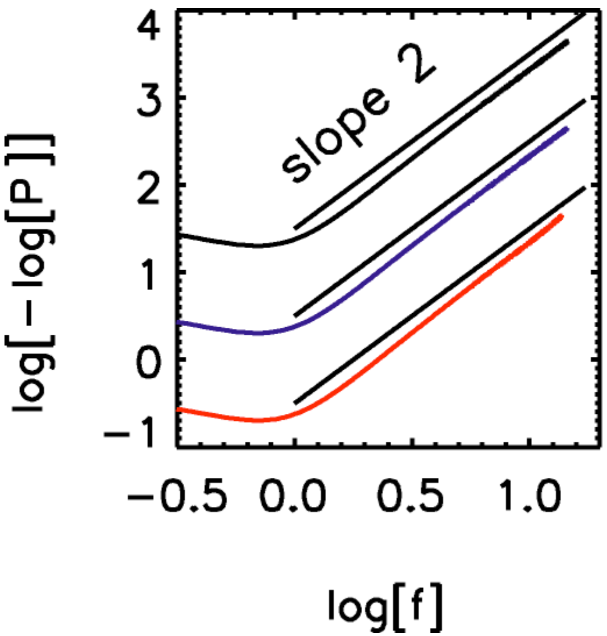
frictionless hard grains have $z = 4$

ensemble P(f)

2D hexagonal lattice

$$P(f) \propto \exp\left(-f^2\right)$$



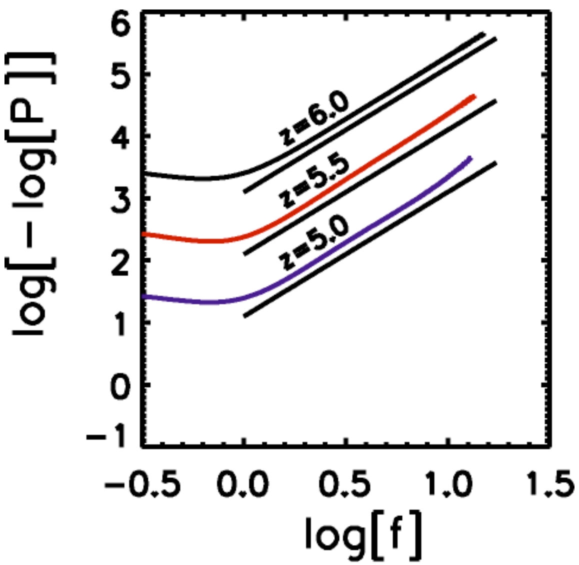


van Eerd, Ellenbroek, van Hecke, Snoeijer, Vlugt, submitted to Phys. Rev. E

ensemble P(f)

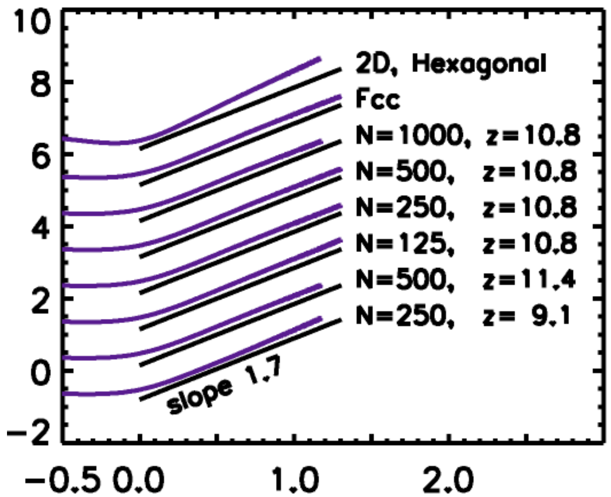
2D disordered packs

$$P(f) \propto \exp\left(-f^2\right)$$



3D disordered packs

$$P(f) \propto \exp\left(-f^{1.7}\right)$$



van Eerd, Ellenbroek, van Hecke, Snoeijer, Vlugt, submitted to Phys. Rev. E