

Granular Flows



Collisional flow

Inertial flow

Plastic flow

Solid

Slow Granular Flows

Shear Bands

Classical Picture based on Friction

Experiments on Wide Shear Zones

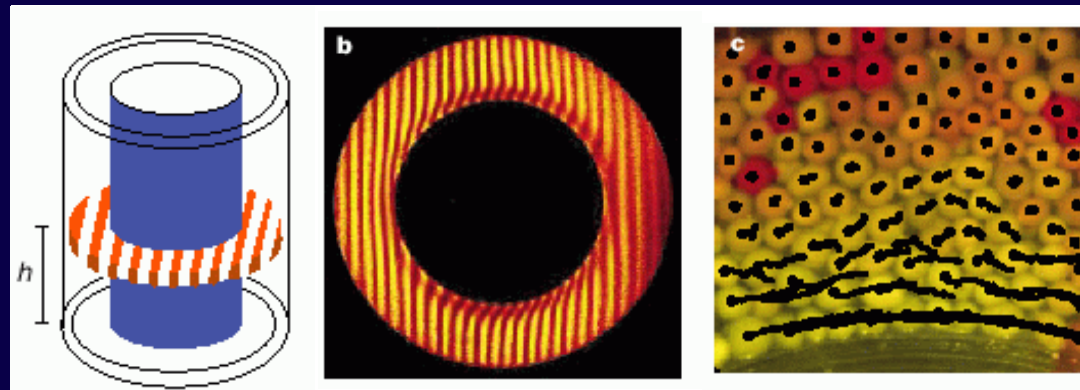
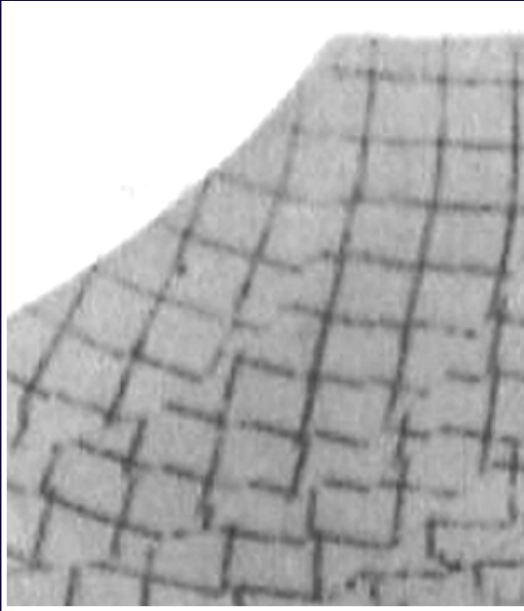
Comparison Experiments – Classical Picture

New Framework: Shear Free Sheets

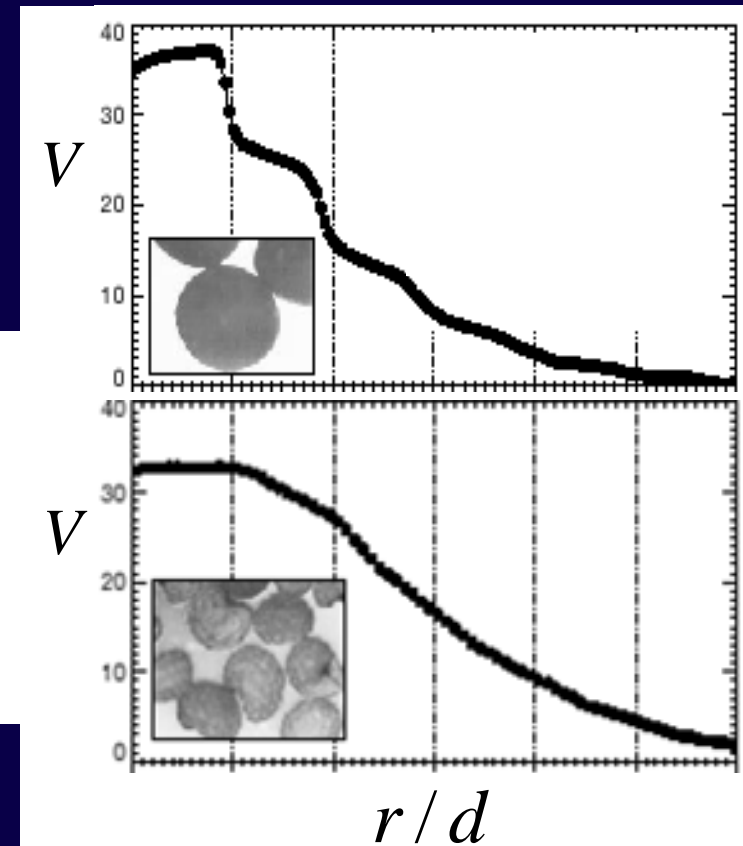
Experiments: Melting Sand by Stirring

Outlook

Shear Bands & Grain Details



D.M. Mueth *et al.*,
Nature **406**, 385 (2000)



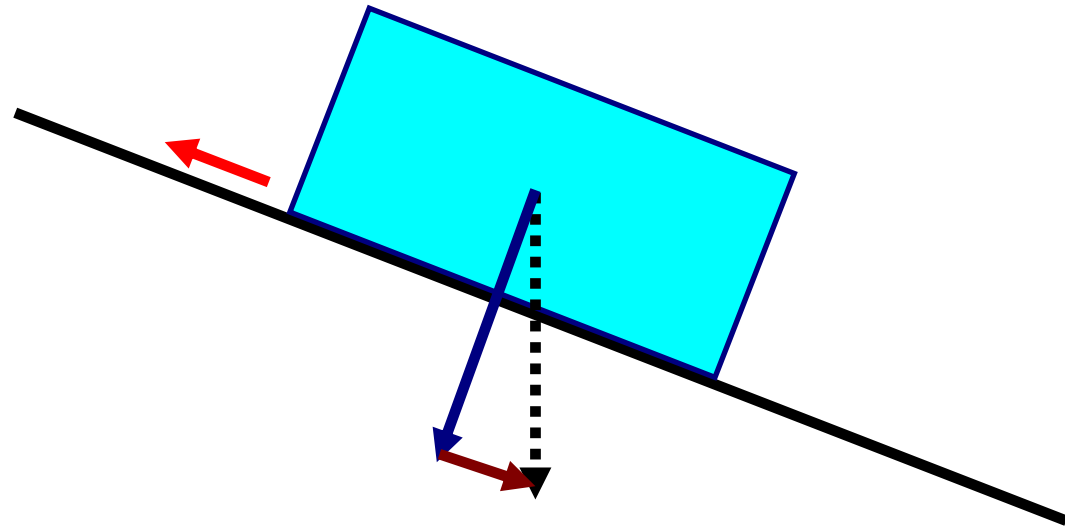
Granular Media: Analogy with Friction



Yield Criterion



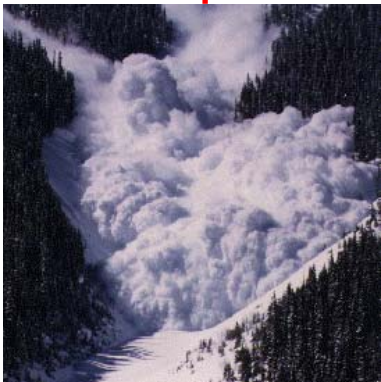
$$\frac{\text{Shear Force}}{\text{Normal Force}} \leq \mu$$



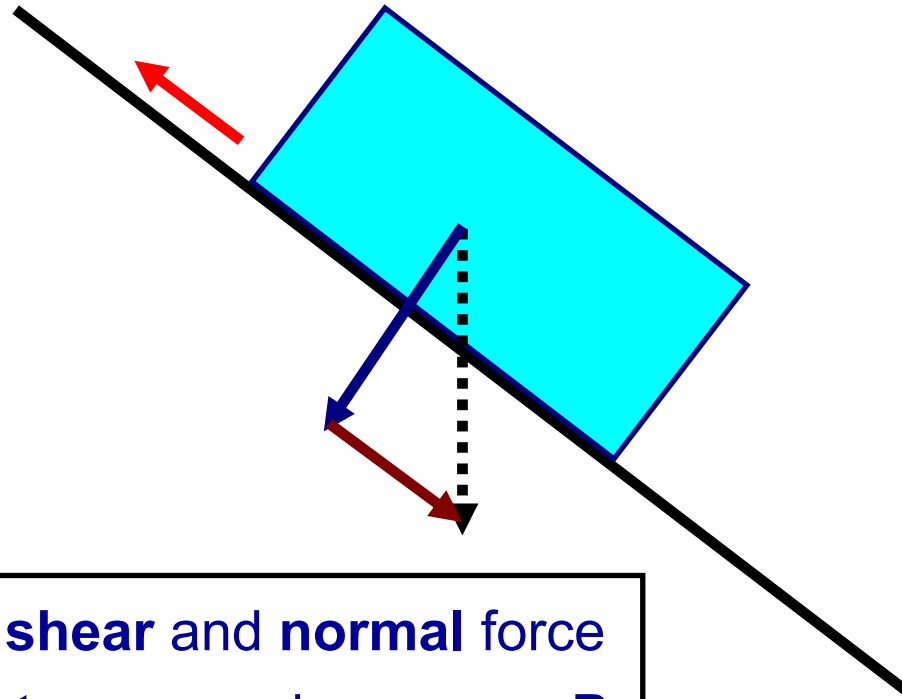
Granular Media: Analogy with Friction



Yield Criterion

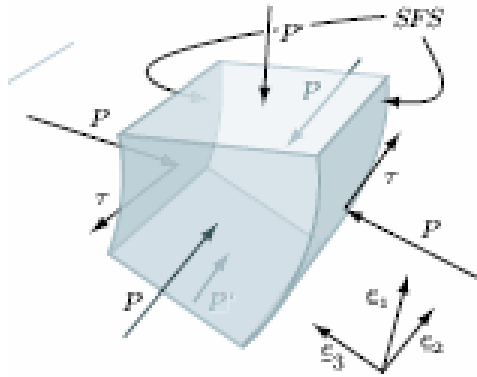


$$\frac{\text{Shear Force}}{\text{Normal Force}} > \mu$$



Translate **shear** and **normal** force
to **shear stress** τ and **pressure** P

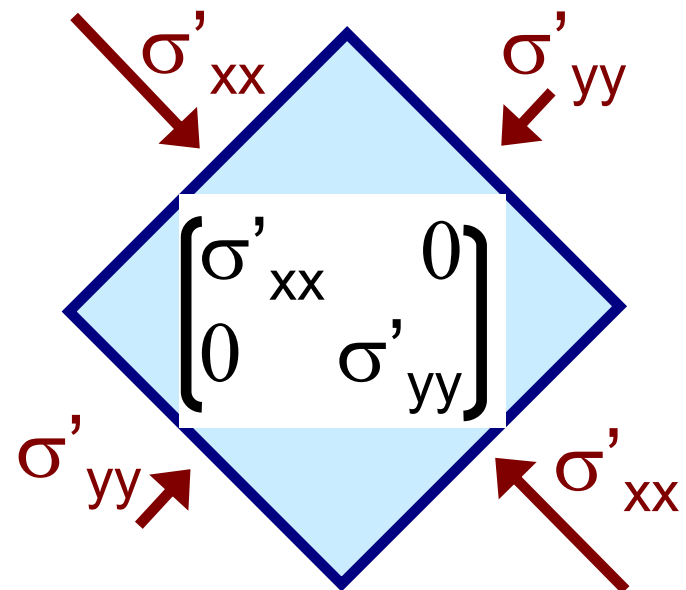
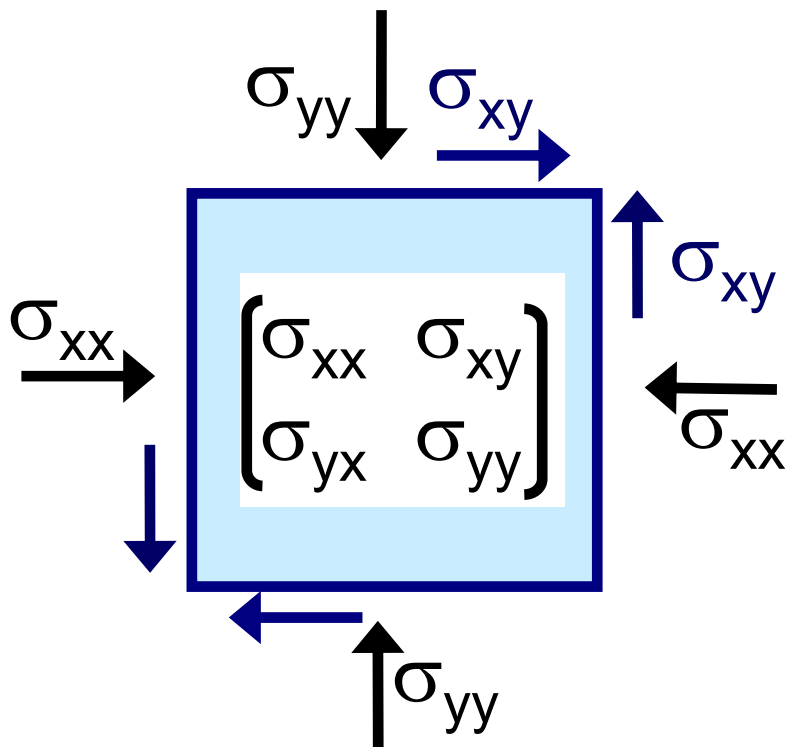
Intermezzo: Tensors



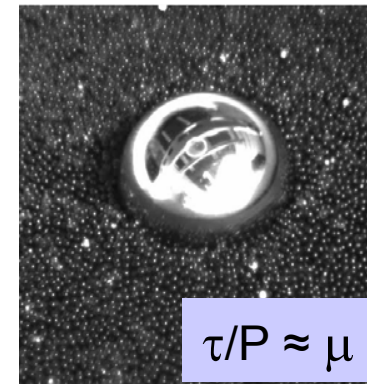
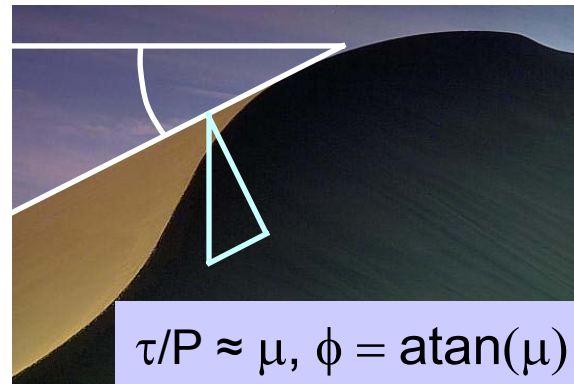
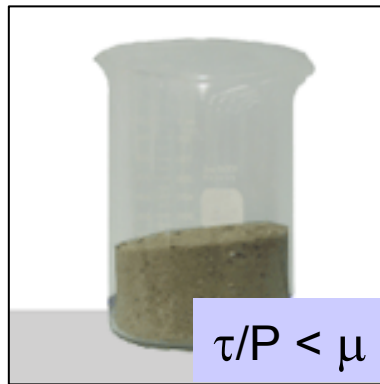
Stress = Force / Area

$$\sigma_{xy} = \sigma_{yx} = F_x / \text{Area}_y = F_y / \text{Area}_x$$

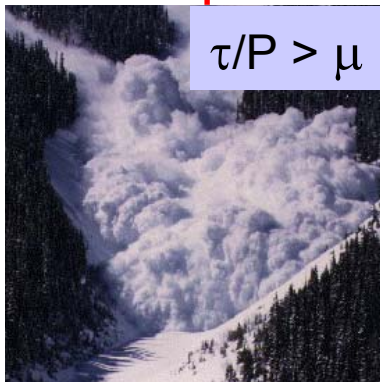
Principle Directions



Granular Media: Yield Criterion



Yield Criterion



Stresses \rightarrow Normal, shear components

Solid: $\tau/P < \mu_{\text{solid}}$

Flow: $\tau/P = \mu_{\text{sliding}} < \mu_{\text{solid}}$

Narrow shearbands follow naturally

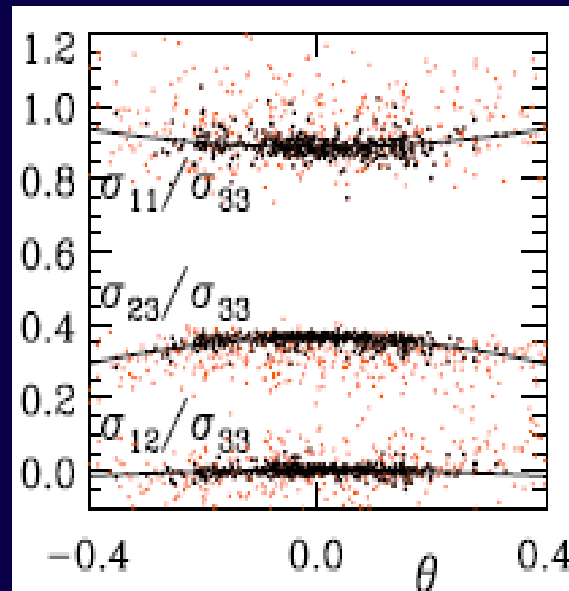
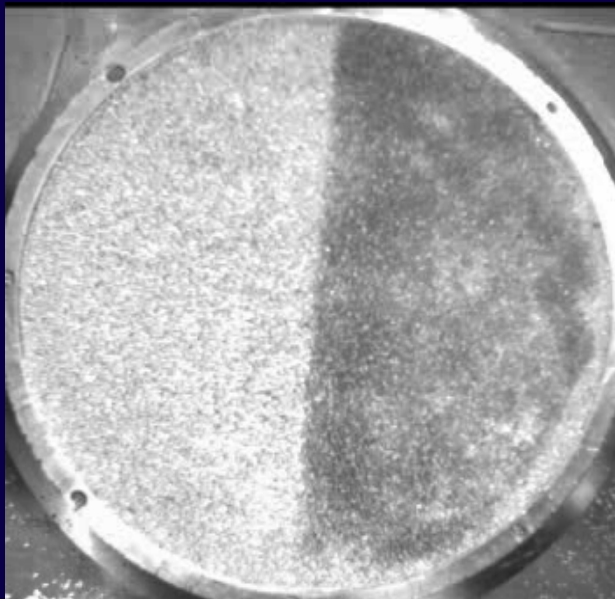
Granular Flows – Accepted Wisdom

Narrow shear bands
Grain details matter
Solid-like regions
Constant sliding friction.

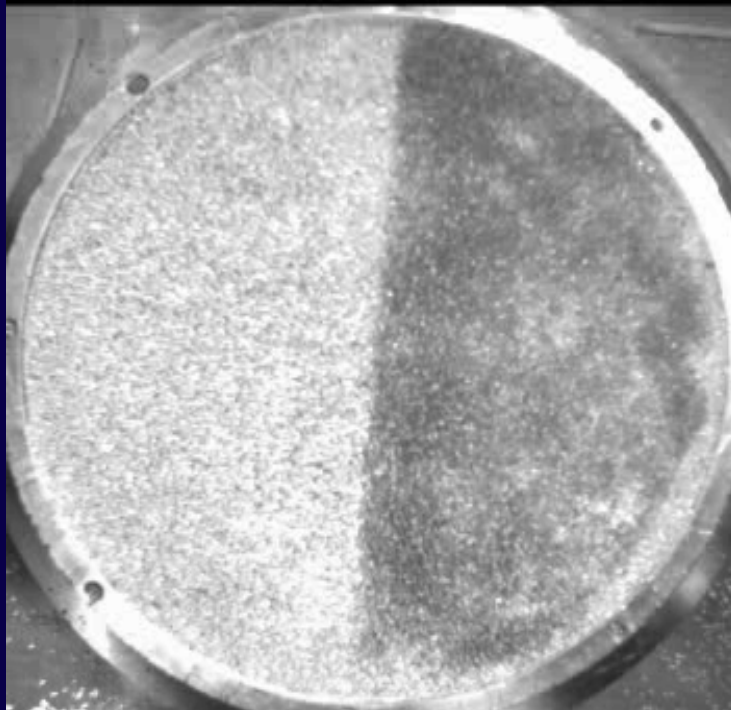


Kiri Nichol

And now ...



Smooth Granular Flows



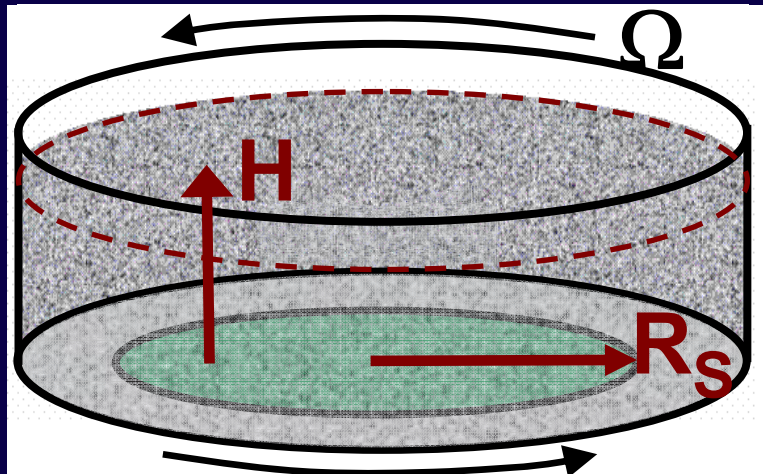
Surface Velocity Profiles

Rate independent

Short transients

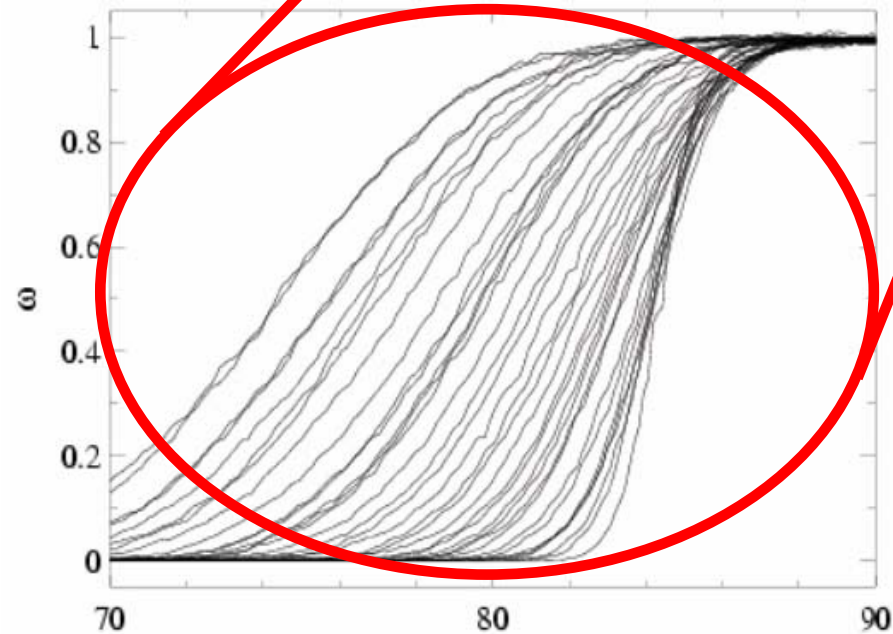
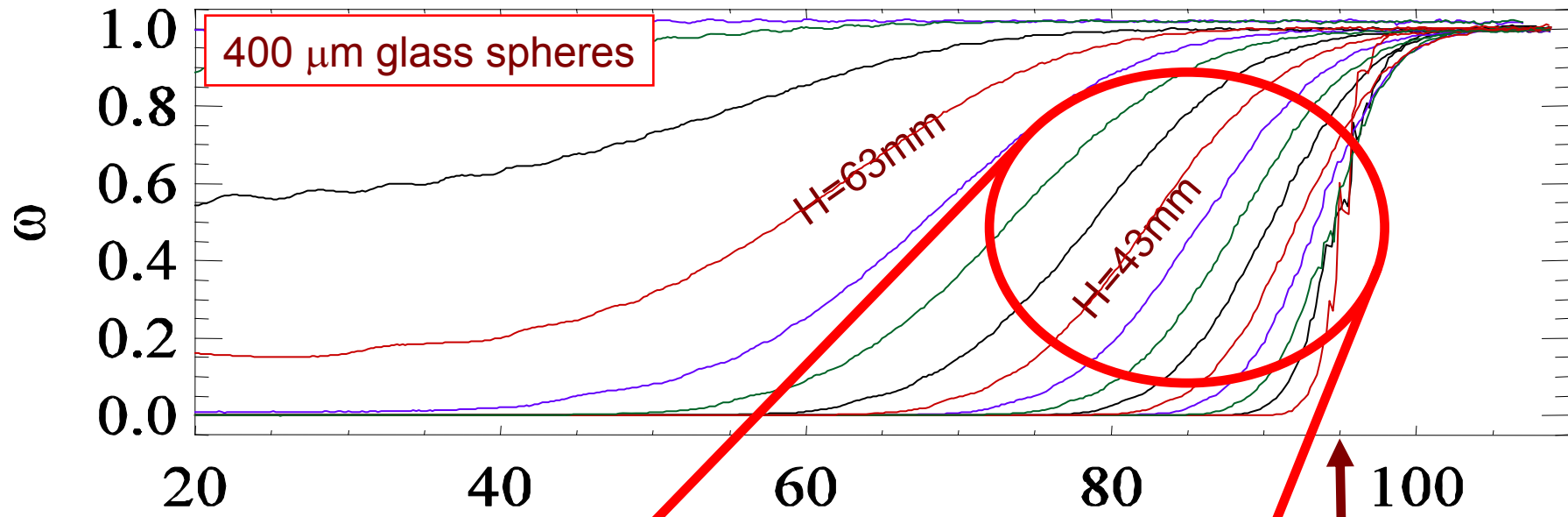
Azimuthal

$$\omega(r) := \omega_{\text{dim}}(r) / \Omega$$

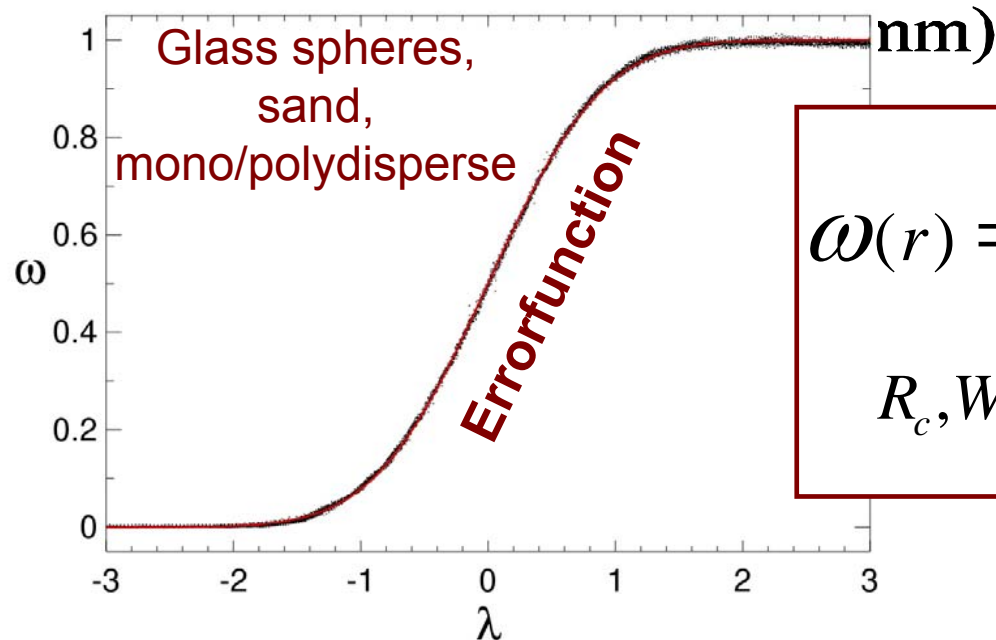
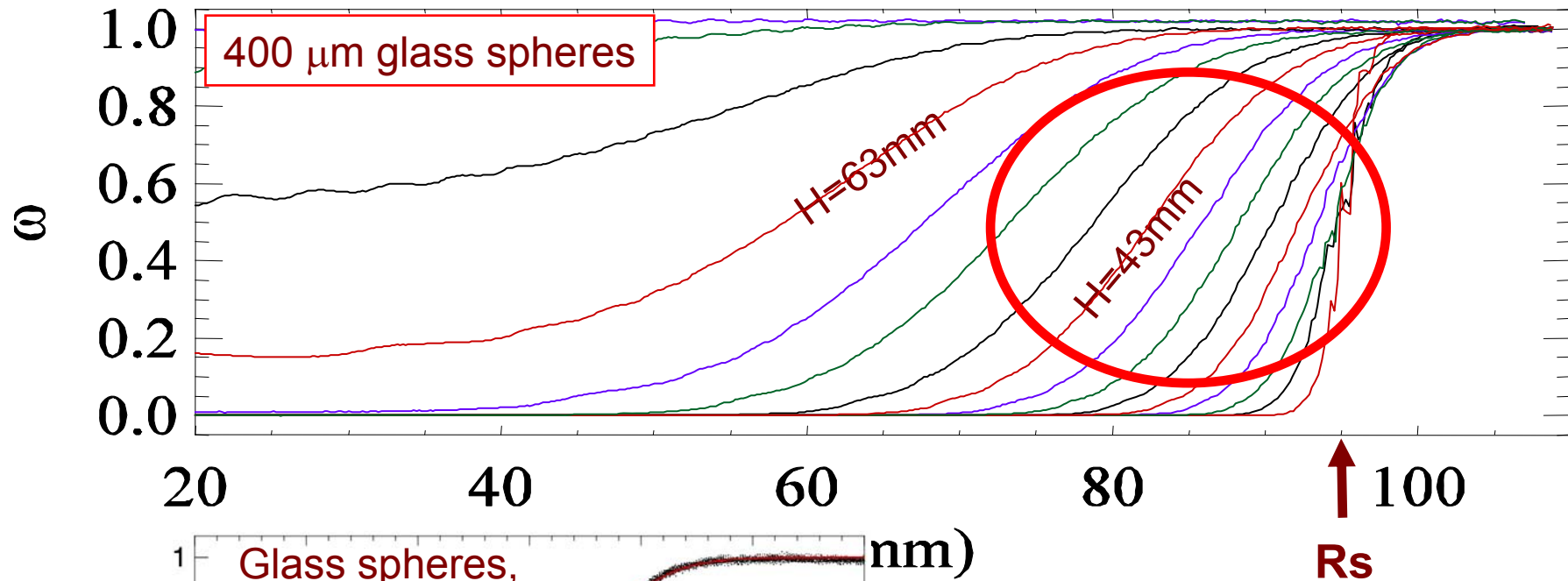


D. Fenistein, JW. van de Meent and MvH,
PRL. **92**, 094301 (2004); **96**, 118001 (2006).
D. Fenistein and MvH, *Nature* **425**, 256 (2003).

Shallow Layers



Shallow Layers

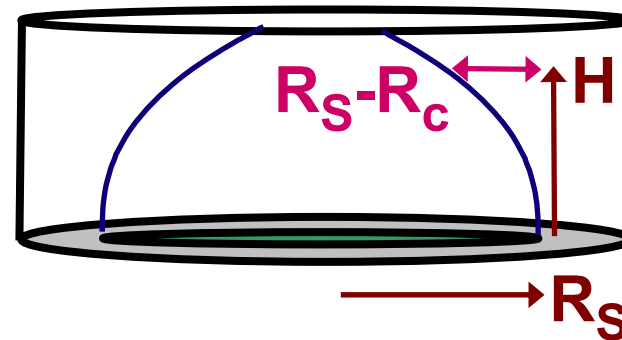
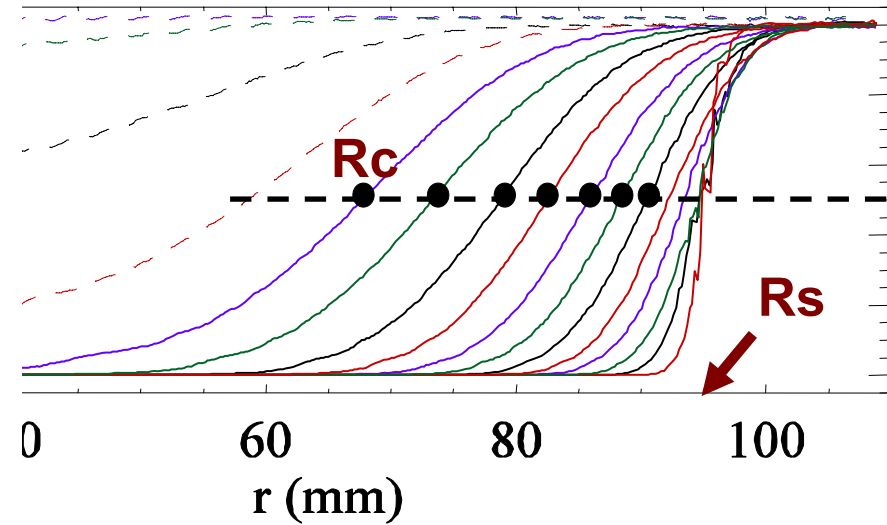
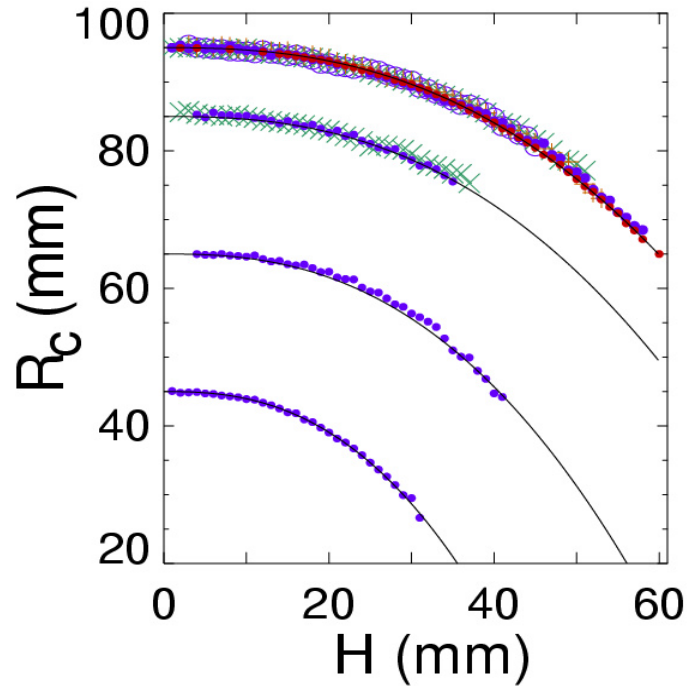


$$\omega(r) = \frac{1}{2} + \frac{1}{2} \operatorname{erf}\left(\frac{r - R_c}{W}\right)$$

$$R_c, W : \text{functions}(R_s, H, d, \dots)$$

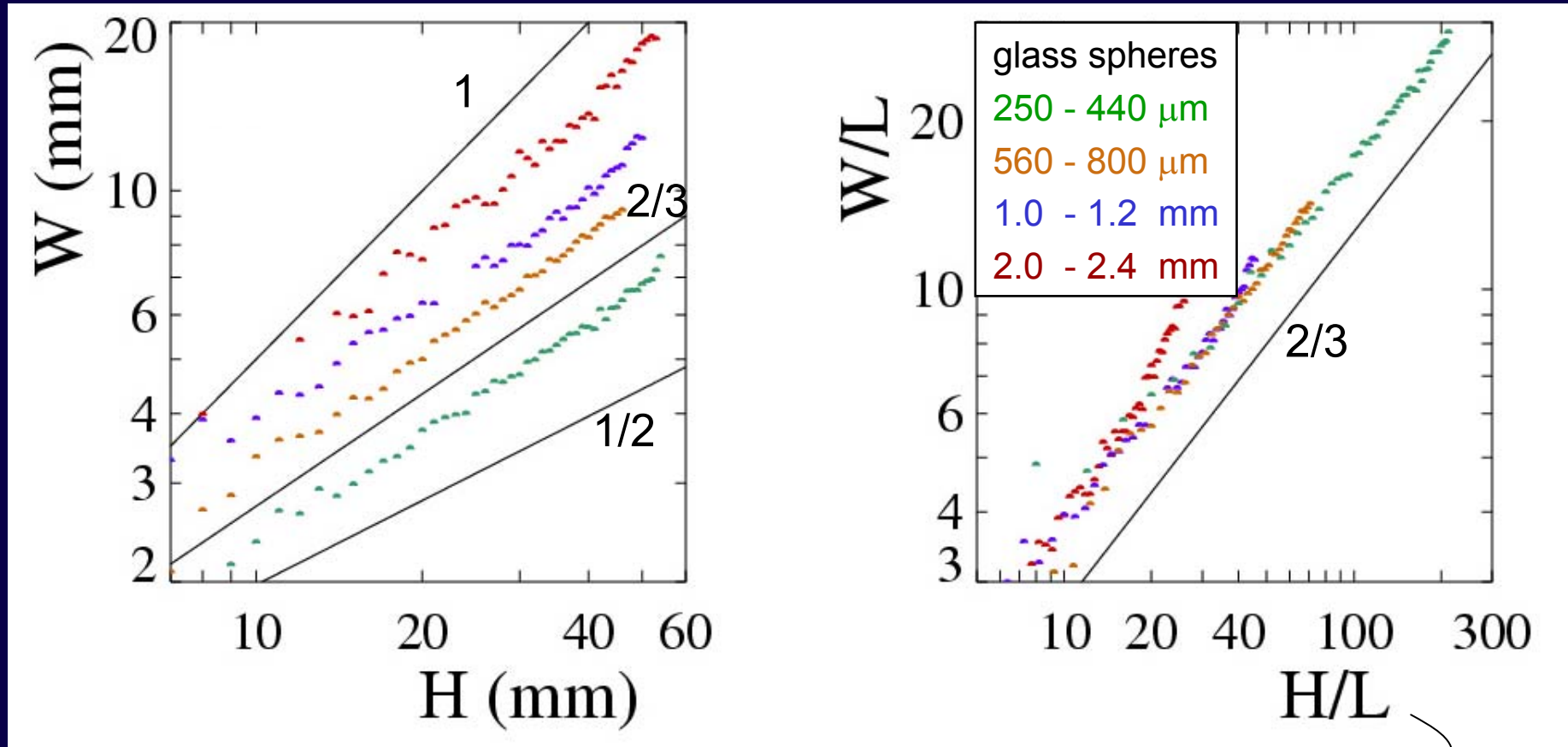
Shallow Layers: R_c

independent of grain size & shape



$$(R_s - R_c) / R_s = (H / R_s)^{5/2}$$

Shallow Layers: W



1.1 mm bronze: W 30% smaller

L : best fits, 0.25, 0.65, 1.1, 2.2 mm

$$W/L \sim (H/L)^{2/3}, L \approx d$$

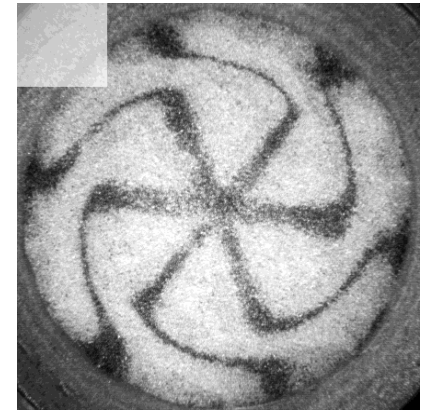
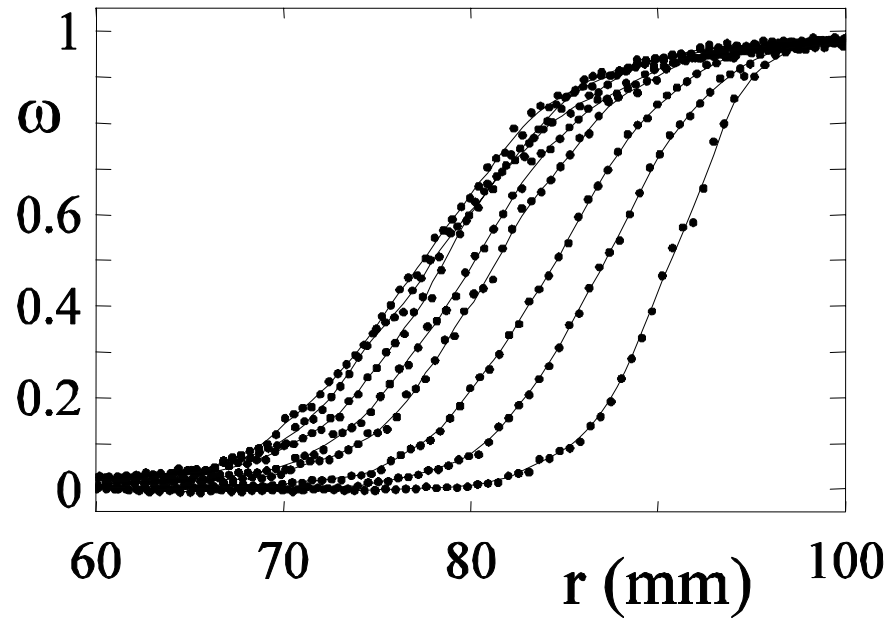
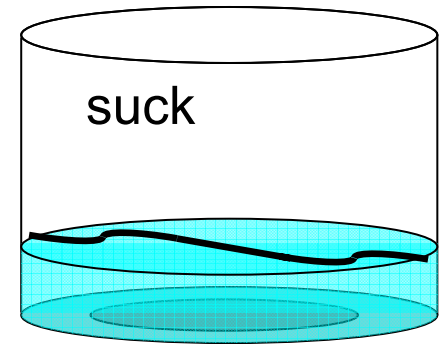
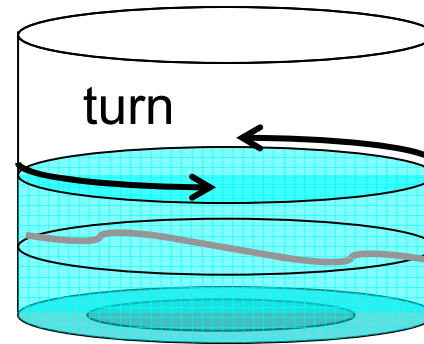
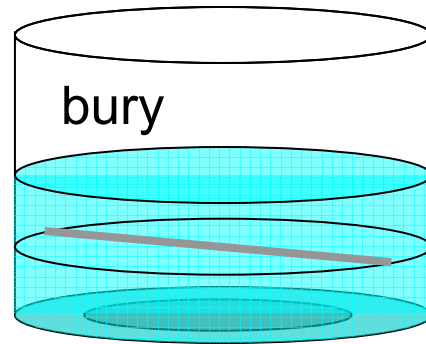
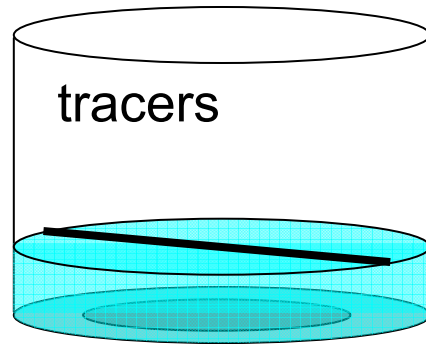
Wide & Universal Shear Zones

$$\omega(r) = 1/2 + 1/2 \operatorname{erf}[(r - R_c)/W]$$

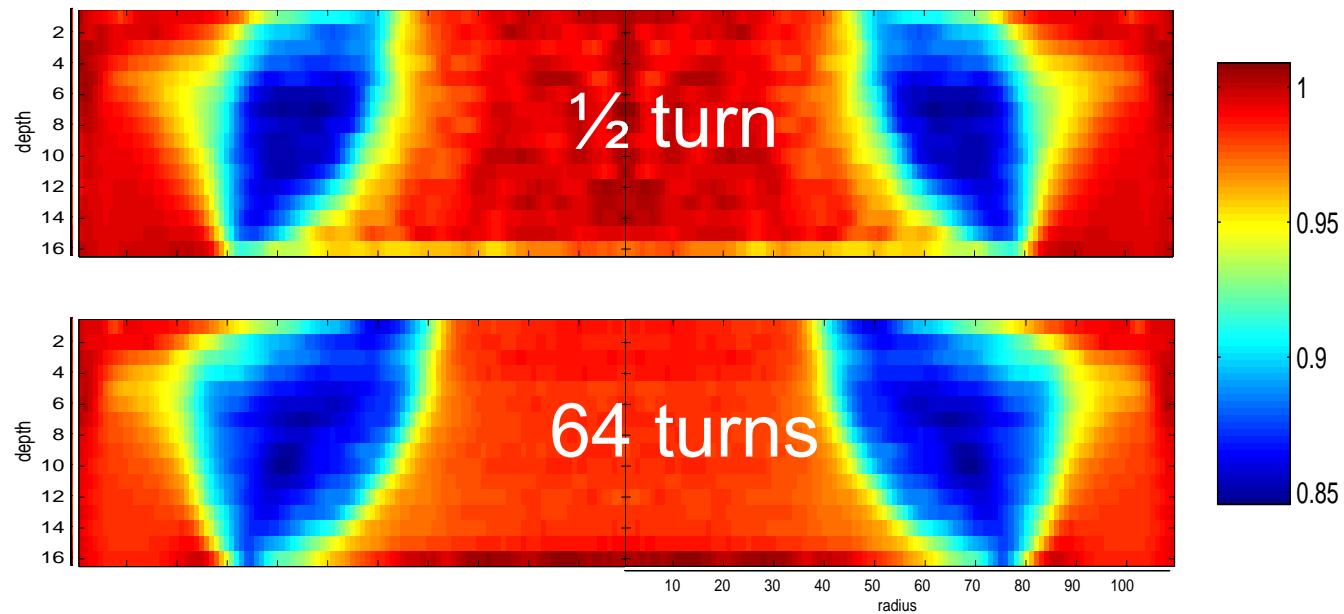
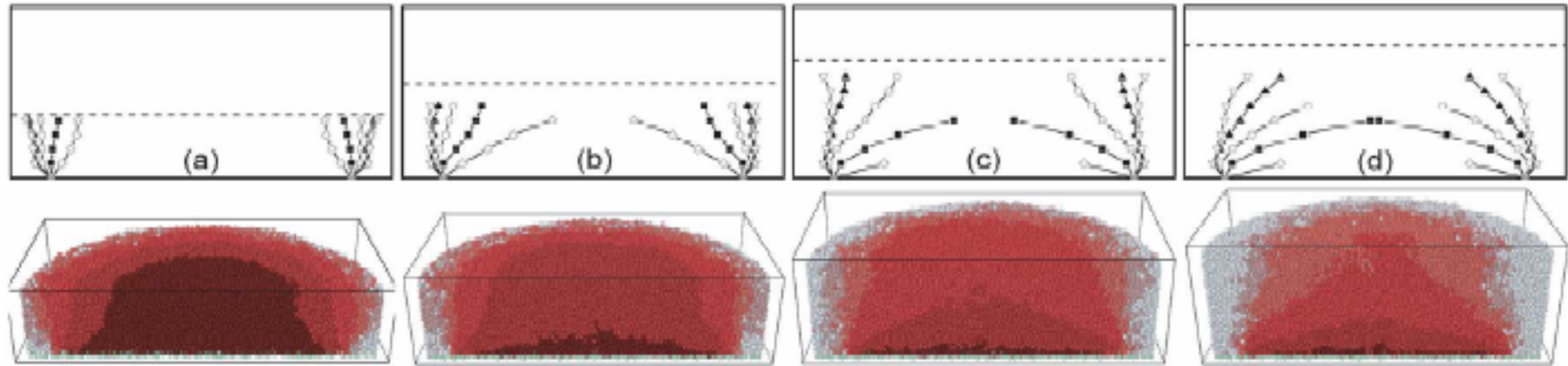
$$(R_s - R_c)/R_s = (H/R_s)^{5/2} : \text{Independent of particles}$$

$$W/d \sim (H/d)^{2/3} : \text{Independent of } R_s$$

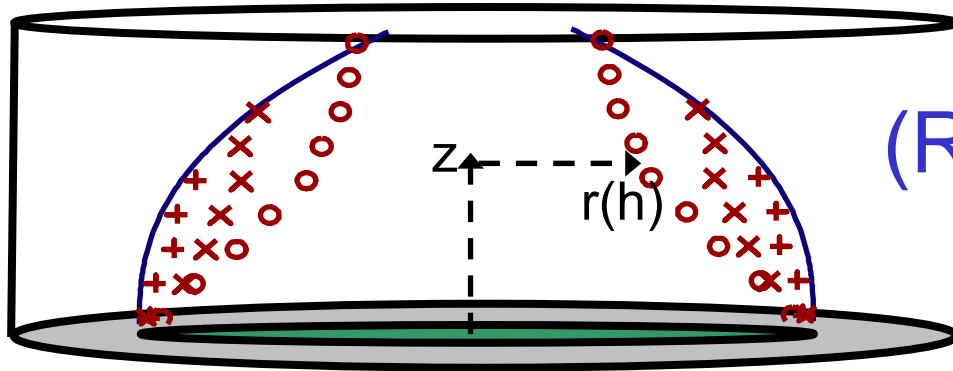
3D Profile: What happens inside?



More 3D Data



Location in Bulk



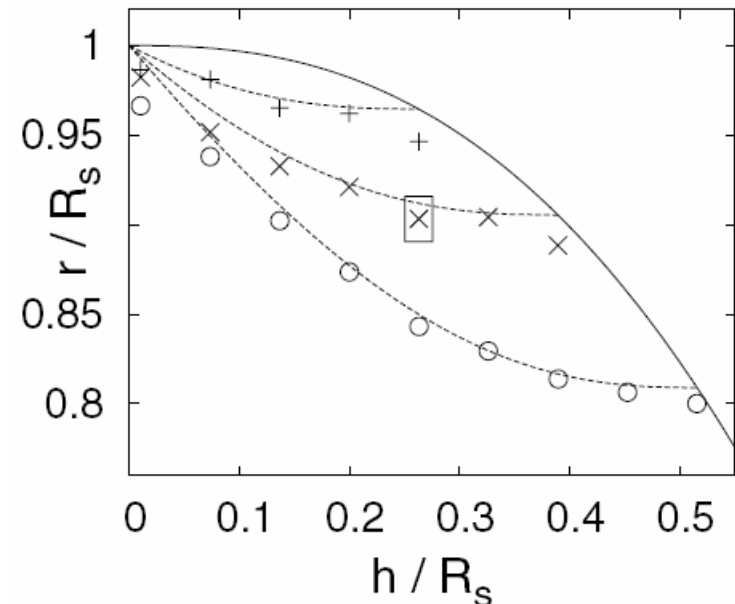
$$(R_s - R_c) / R_s = (H / R_s)^{5/2}$$

R_c in bulk: Scaling argument virtual bottom **OK**

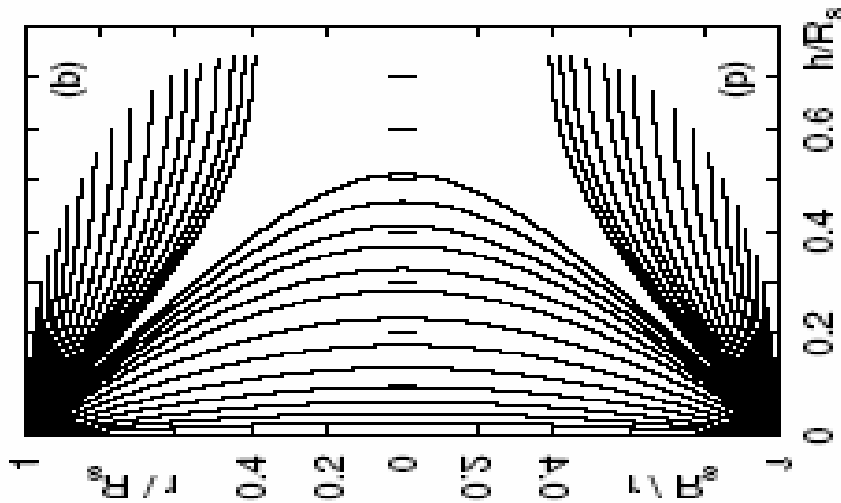
$$R_c(R_s, H) = R_c(r, H - h),$$

$$h = H - r \left[1 - \frac{R_s}{r} \left[1 - (H/R_s)^\alpha \right] \right]^{1/\alpha}$$

$\alpha = 5/2$



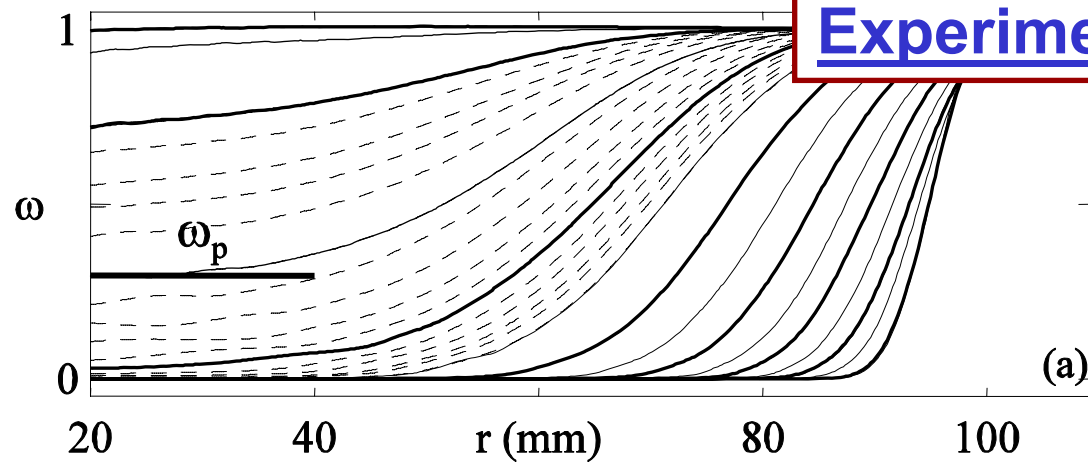
Model: Friction in Continua



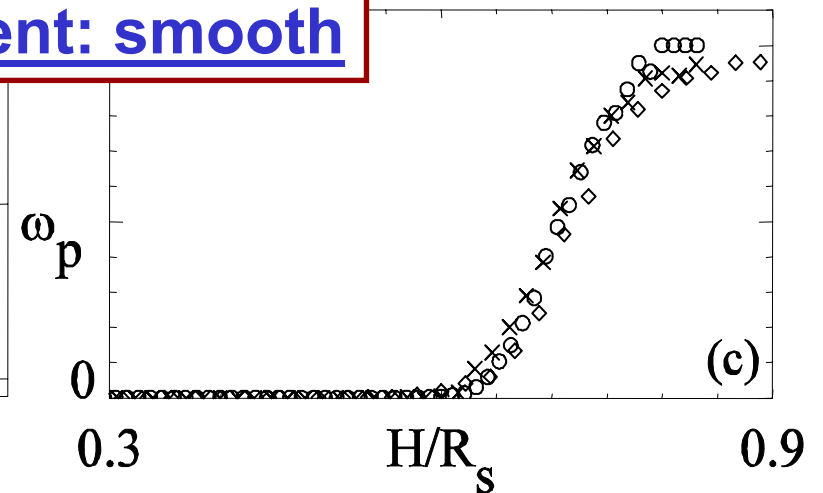
Constant Friction
Torque minimization

$$\int dz r^2(z) \sqrt{r'^2 + 1} (H - z)$$

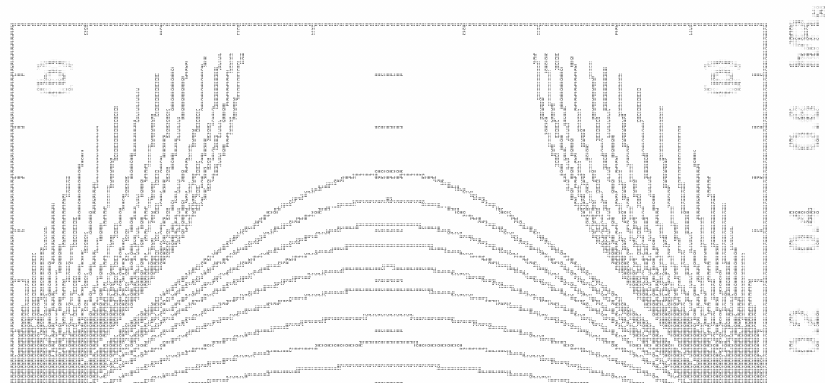
Theory: first order transition



Experiment: smooth



Model: Friction in Continua

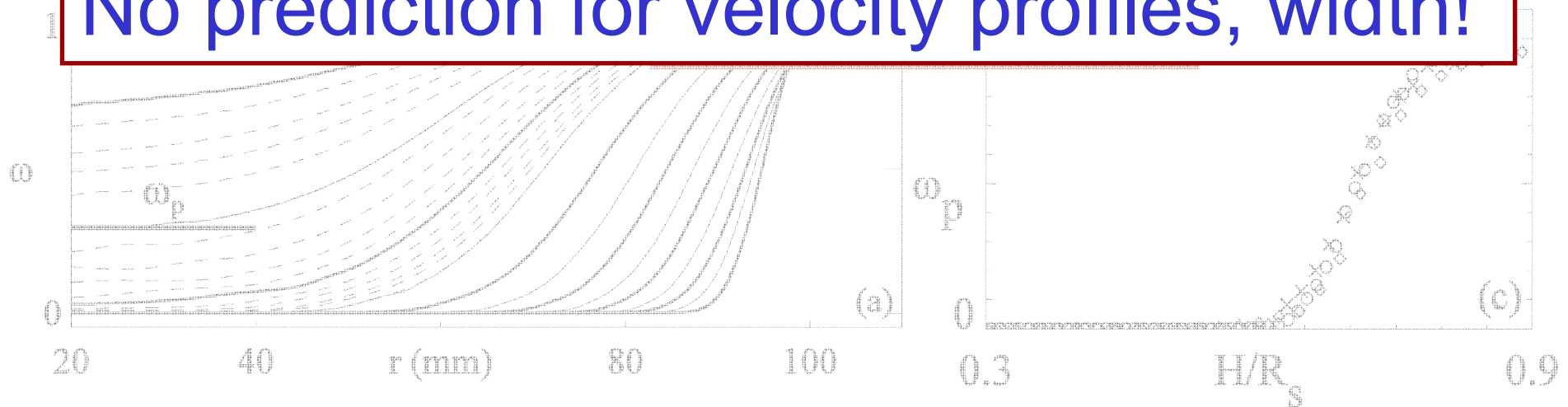


Constant Friction
Torque minimalization

$$\int dz r^2(z) \sqrt{r'^2 + 1} (H - z)$$

Onset of precession: roughly ok, but is smooth in experiment
3D shearband phenomenology: position ok.

No prediction for velocity profiles, width!



Break

Not Always Narrow Shearbands

No Theory for Wide Shear Zones

What do these flows imply for **Stress & Rigidity**

Stresses in Smooth, Slow, Dense Granular Flows

Rate Independence: Stresses independent of driving rate.
 Velocity profiles independent of driving rate

Wide Shear Zones

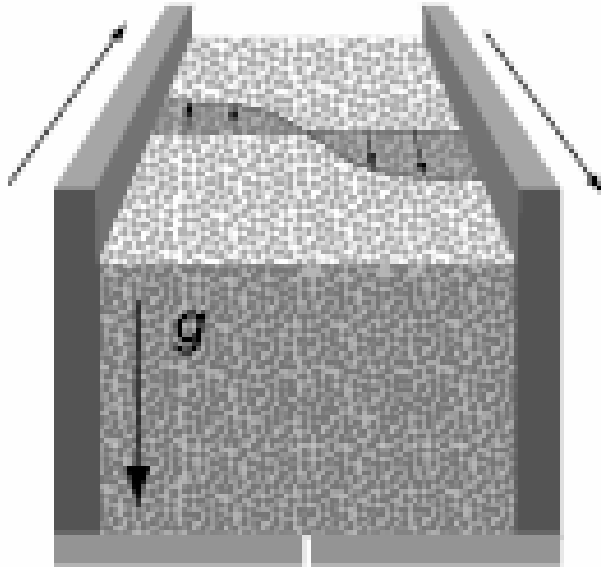
Stress fluctuations relax fast :

stresses are dynamically generated
(no elastic stresses)

M. Depken, W. van Saarloos and MvH., Phys. Rev. E 73, 031302 (2006).

M. Depken, J. B. Lechman, MvH, W. van Saarloos and G. S. Grest, EPL 78, 58001 (2007).

Linear Geometry: Shear Free Sheets



Rate Independence:
Wide Shear Zones

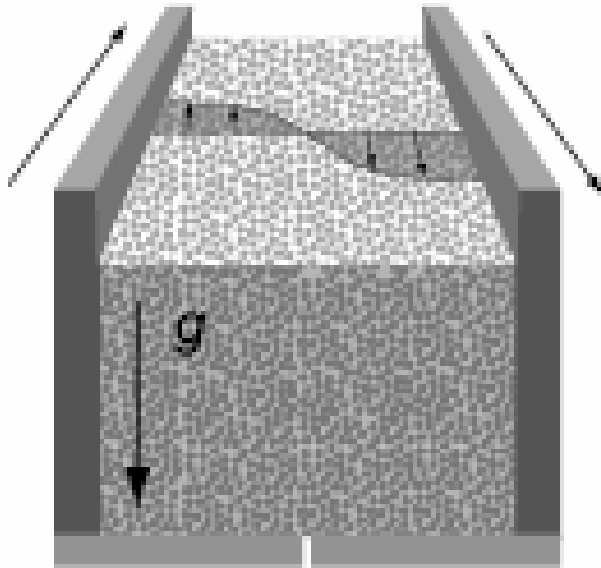
Stresses independent of driving rate.

Stress fluctuations relax fast : No shearflow, no shear stress

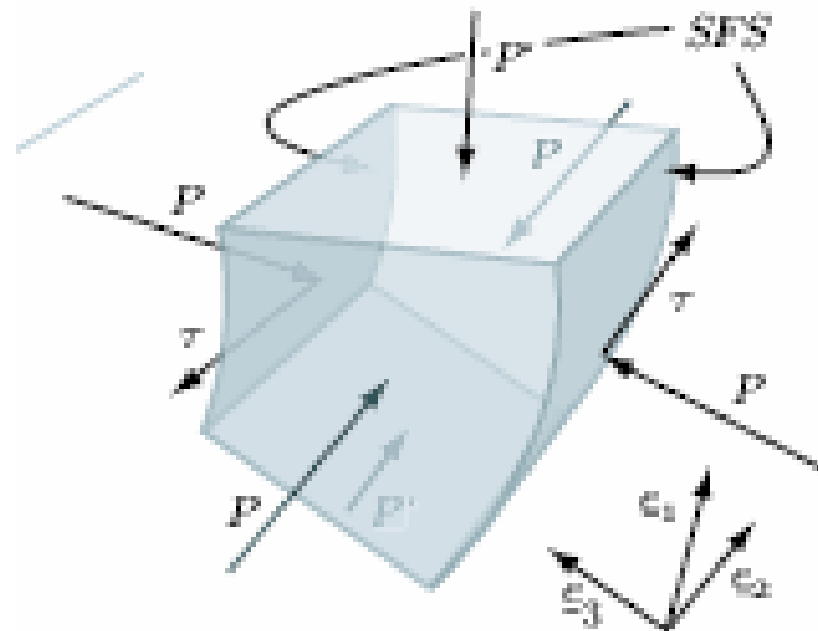
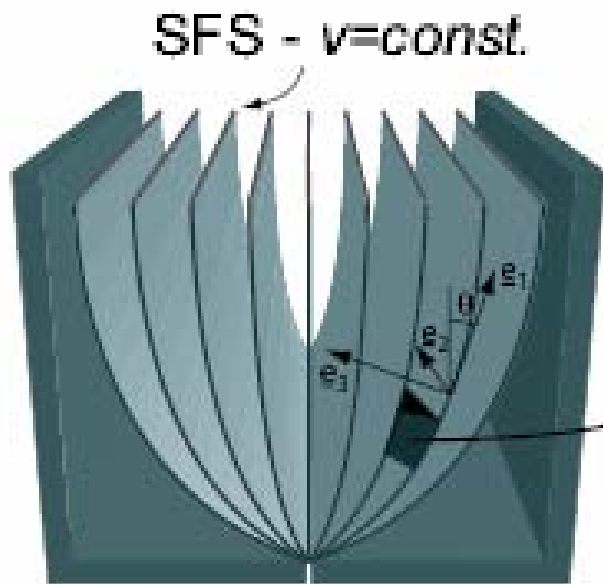
M. Depken, W. van Saarloos and MvH., Phys. Rev. E 73, 031302 (2006).

M. Depken, J. B. Lechman, MvH, W. van Saarloos and G. S. Grest, EPL 78, 58001 (2007).

Linear Geometry: Shear Free Sheets



$$\underline{(\underline{\sigma})}_{\text{SFS}} = \begin{pmatrix} P' & 0 & 0 \\ 0 & P & \tau \\ 0 & \tau & P \end{pmatrix}$$



Test Stress Tensor

$$\begin{pmatrix} P' & 0 & 0 \\ 0 & P & \tau \\ 0 & \tau & P \end{pmatrix}$$

$$\sigma_{11}/\sigma_{33} = ?$$



$$\sigma_{12}/\sigma_{33} = 0$$

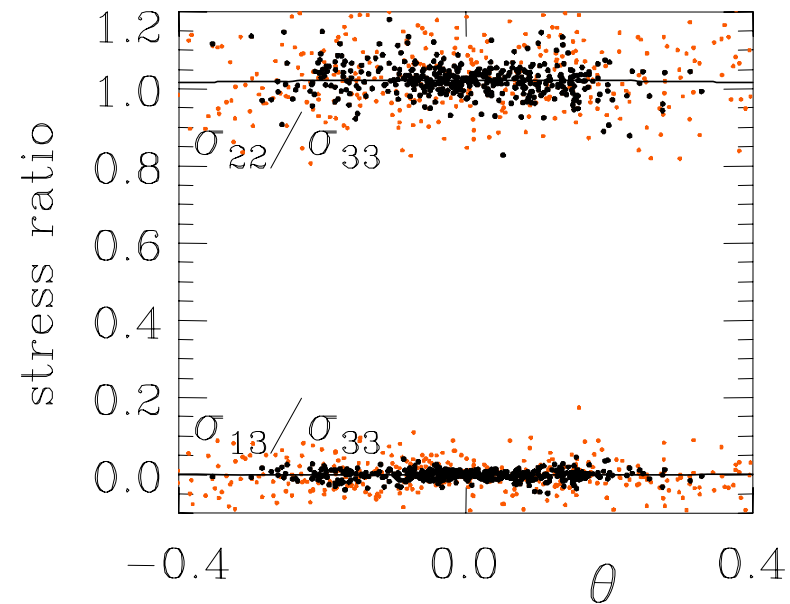
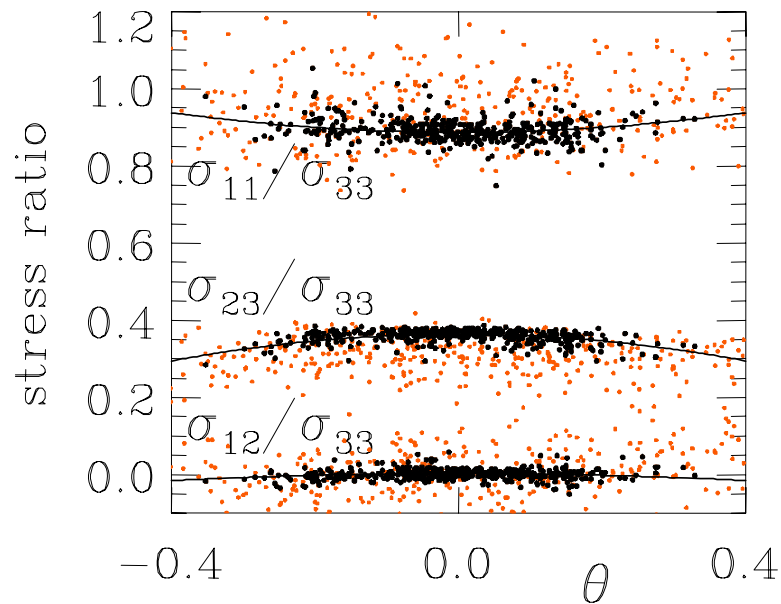


$$\sigma_{13}/\sigma_{33} = 0$$

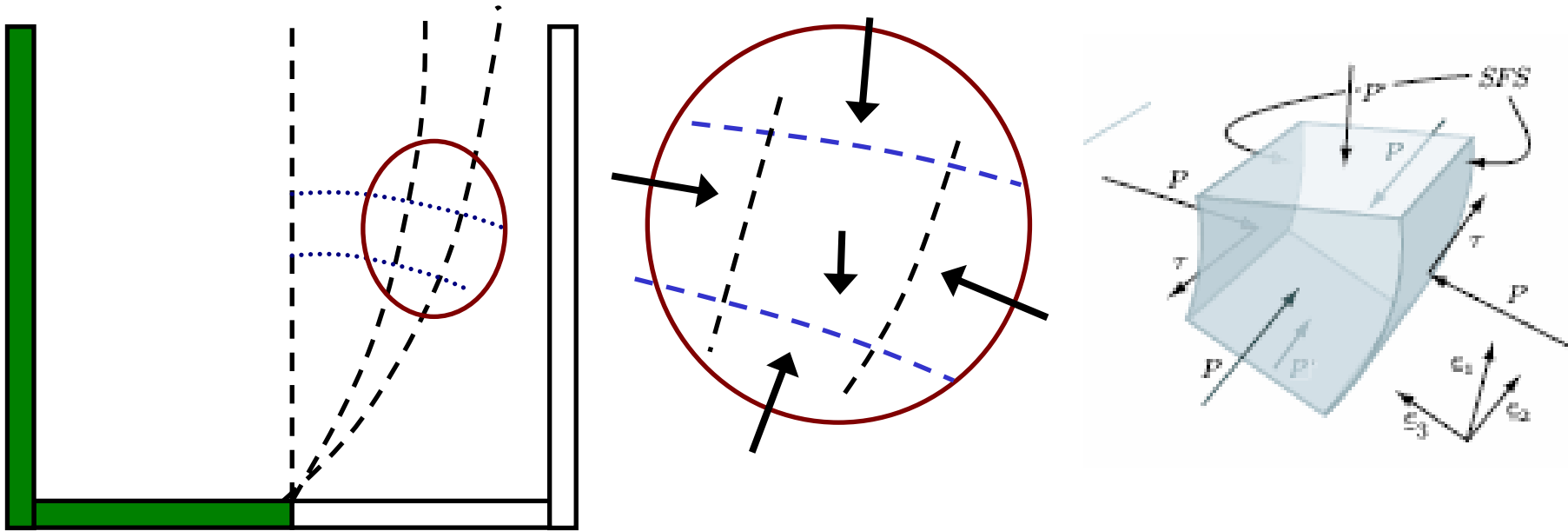
$$\sigma_{22}/\sigma_{33} = 1$$



$$\sigma_{23}/\sigma_{33} = \mu = ?$$



Effective friction cannot be constant!

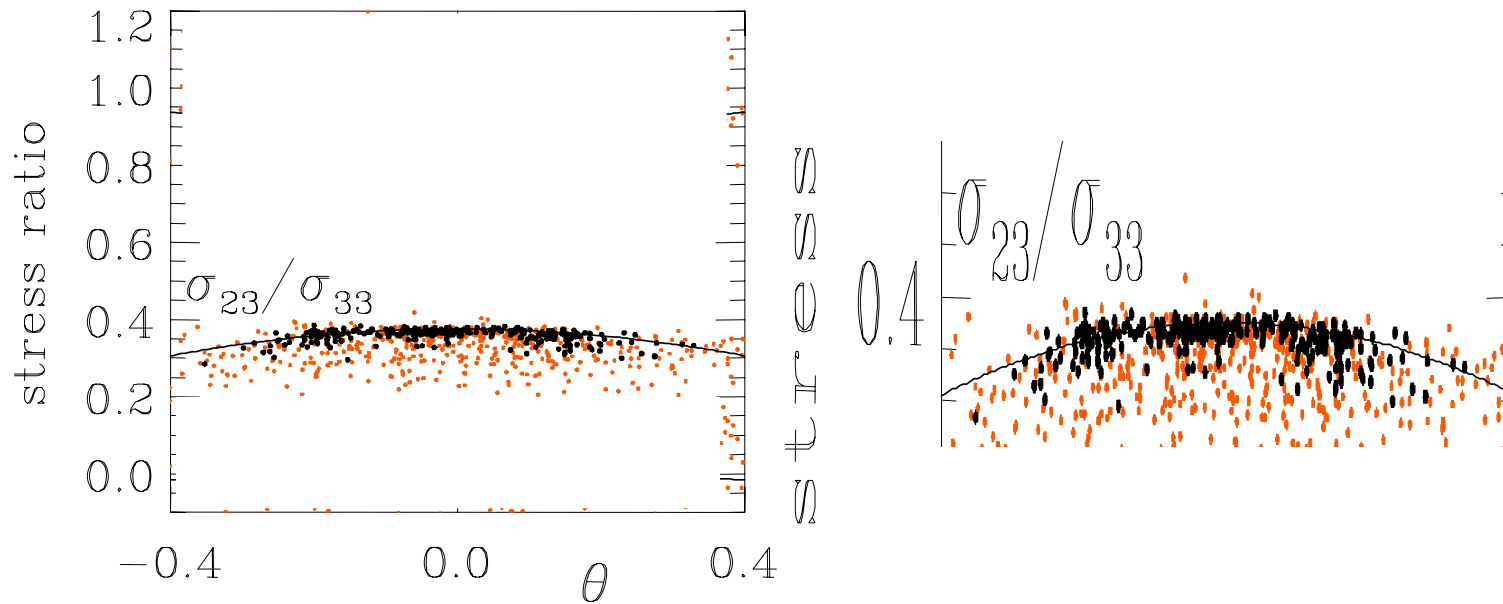


μ constant: NO Force balance



μ has to decrease away from vertical SFS

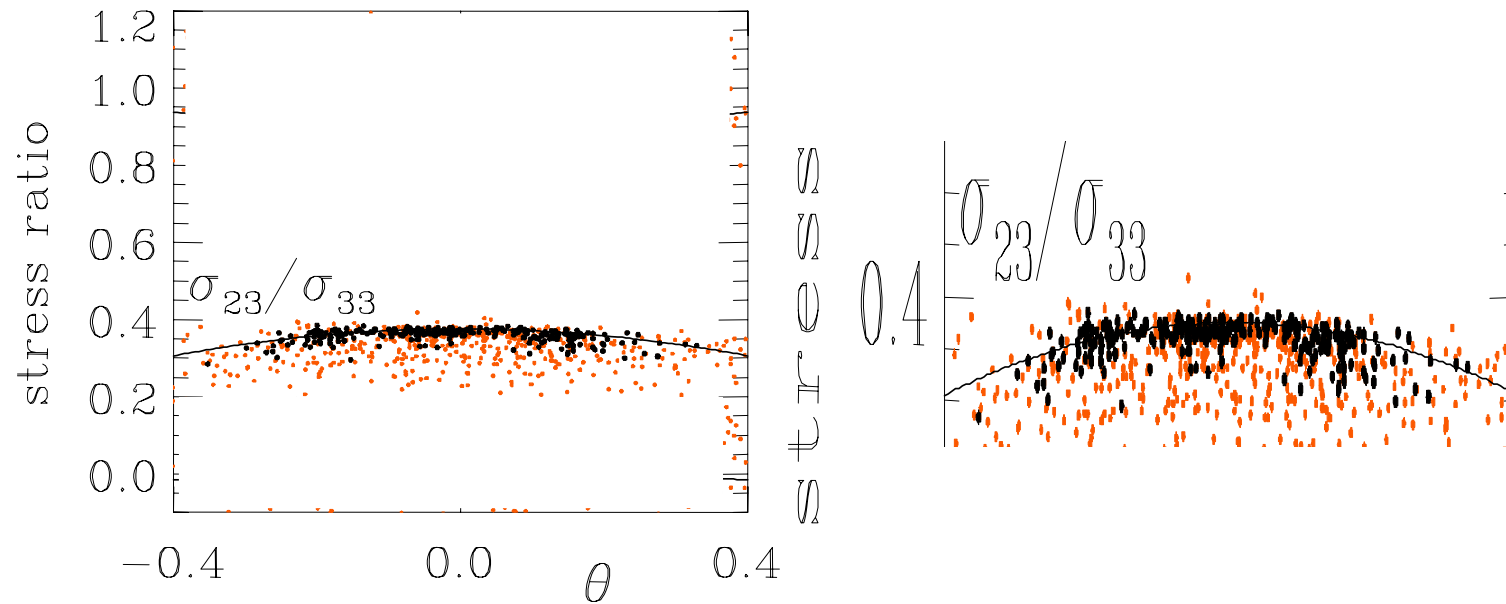
Effective friction cannot be constant!



μ constant: NO Force balance

μ has to decrease away from vertical SFS

Effective friction cannot be constant!



Subtle, but necessary

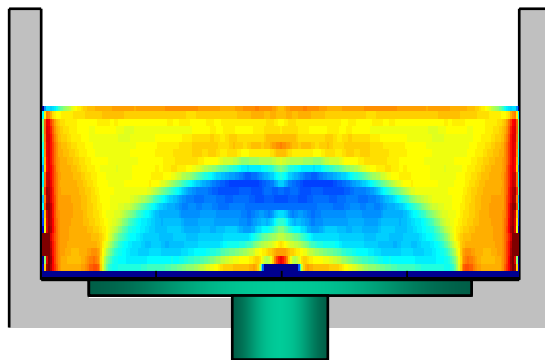
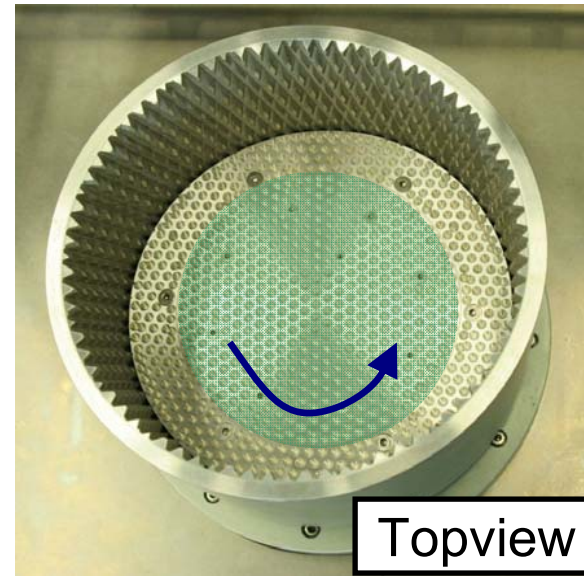
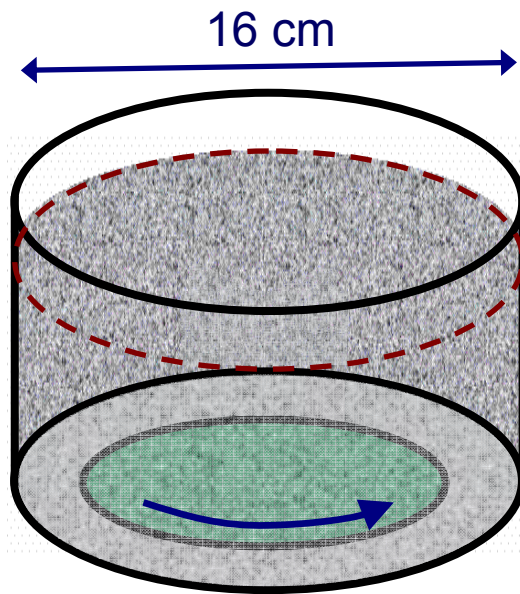
Consistent with width shear zones

Unger et al see similar variation in μ in zero gravity simulations (PRE 2007)

Classical constant friction picture fails

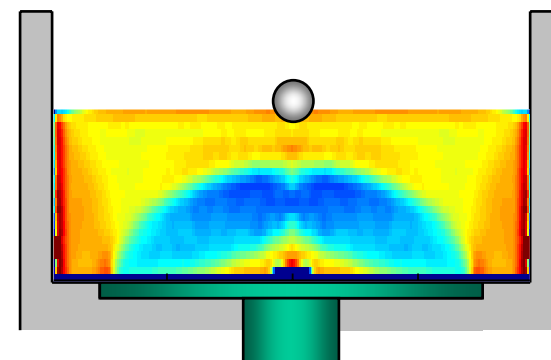
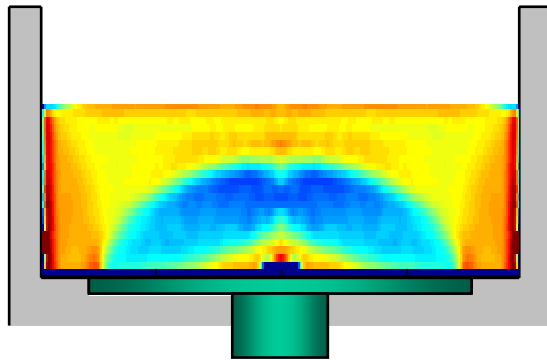
... what about this stress relaxation?

Setup



Yield Criterion in
presence of flow?

Yield stress is lowered!!!



A Stationary Granular Fluid

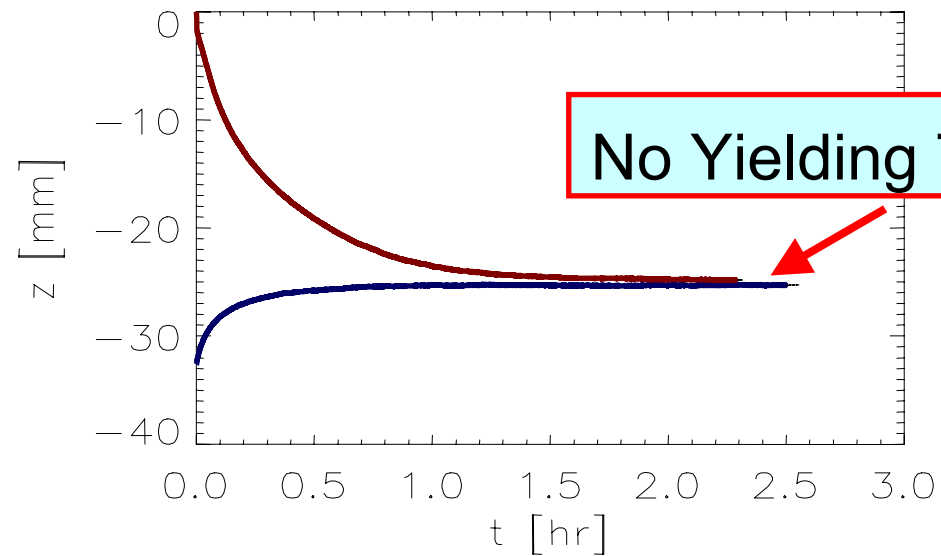
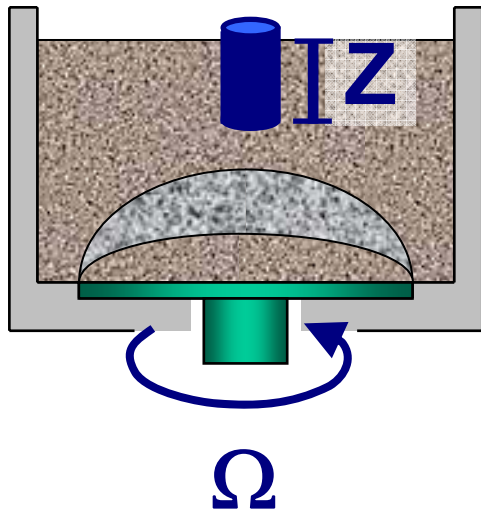
What happened to yield stress?

Yield stress vanishes - Archimedes Law

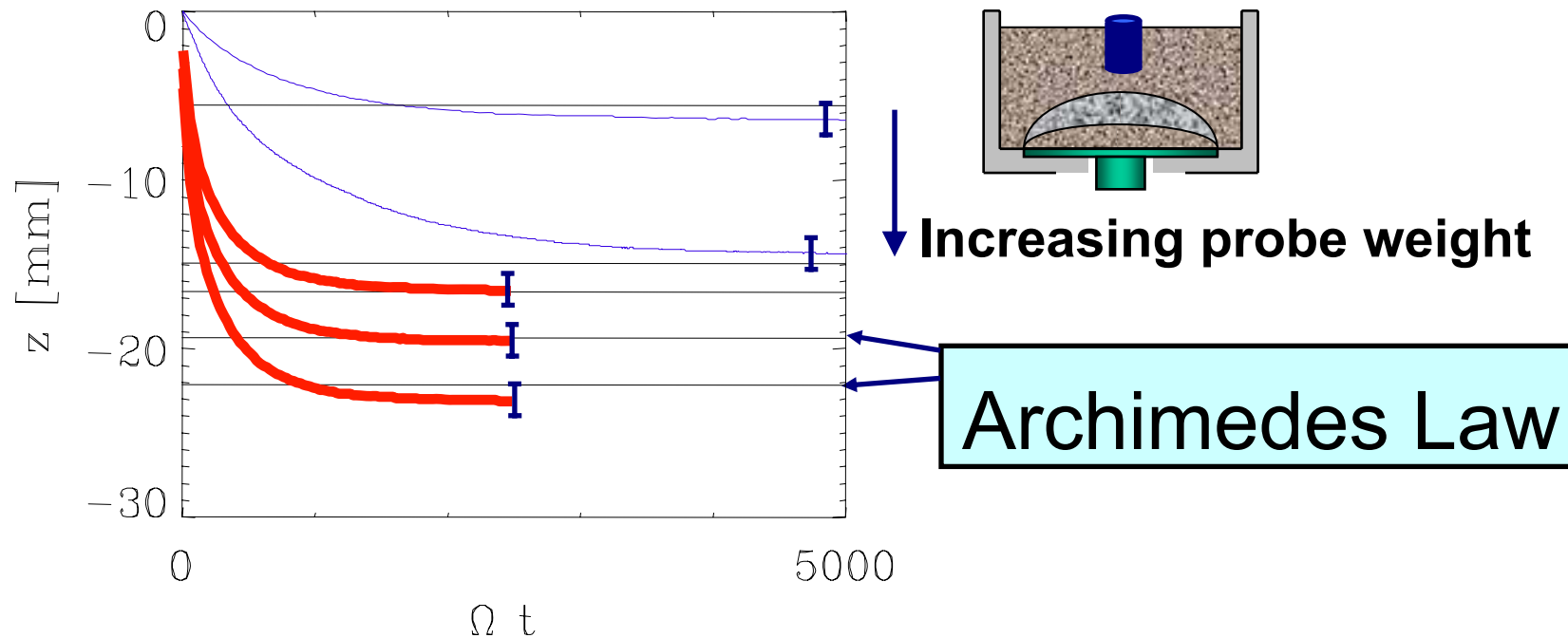
Mechanism?

Agitations - Nonlocal

Floating in Sand

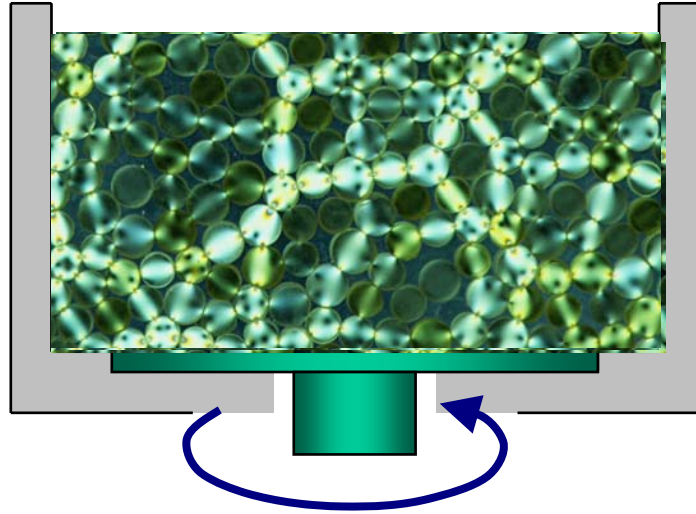


Floating in Sand



No yielding threshold in presence of flow

A Stationary Granular Fluid: Agitations



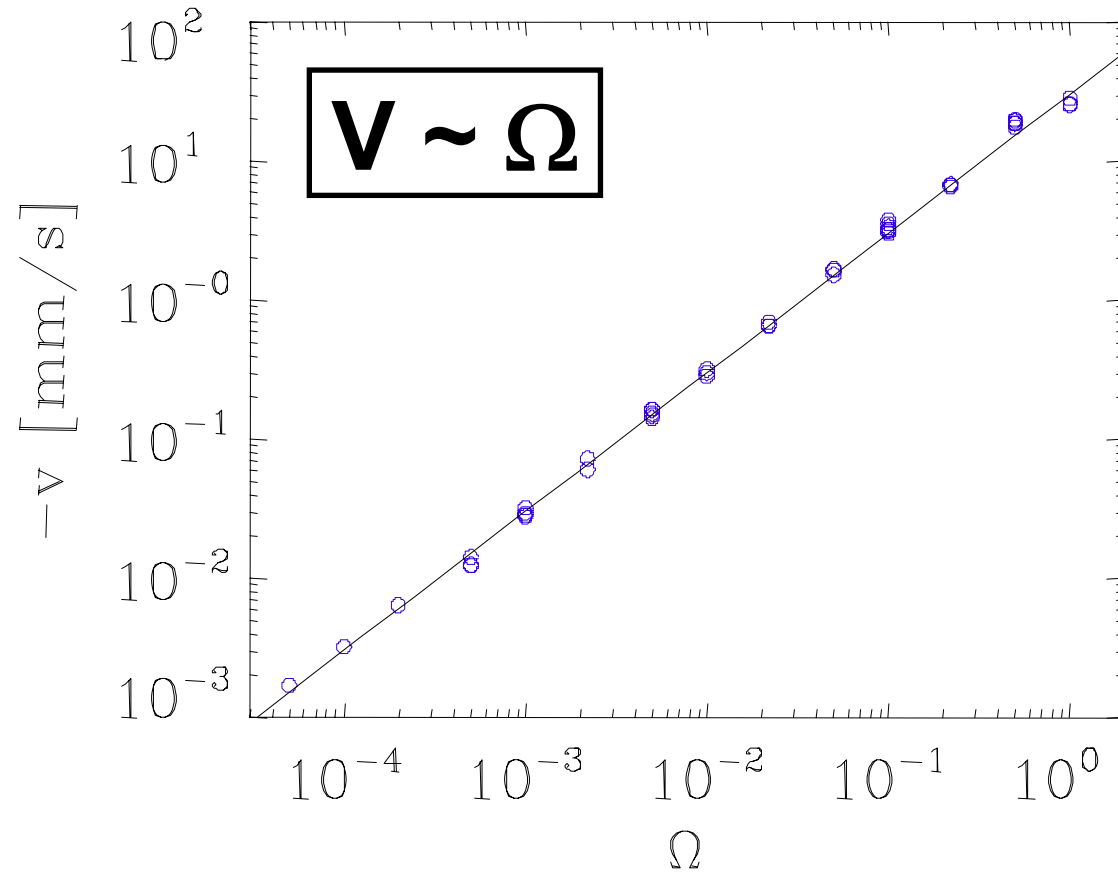
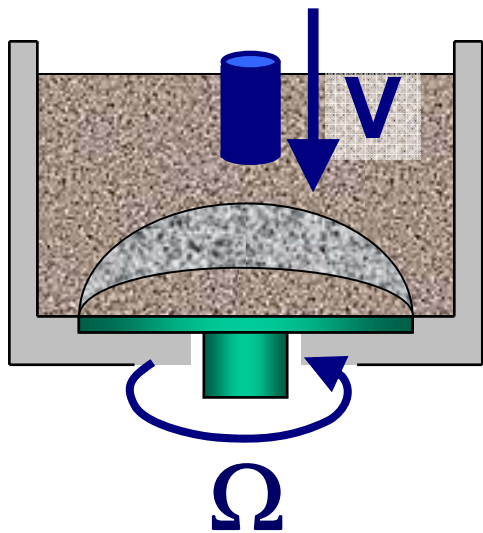
Contact Forces + Flow

Fragile (heating 1 grain by 1C....)

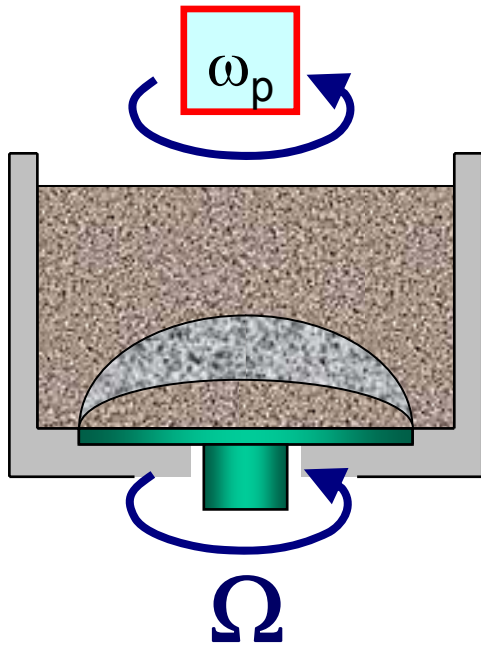
Hard Grains (mm/nm)

Rotation rate Ω sets probe speed?

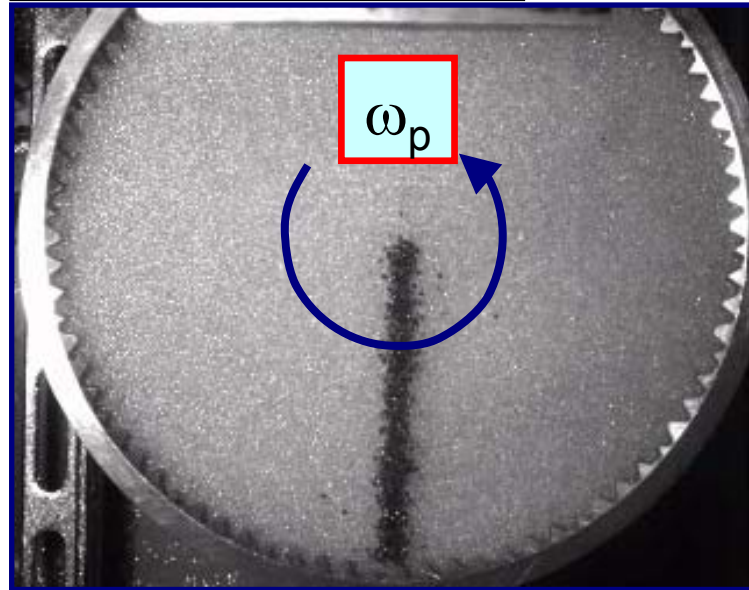
A Stationary Granular Fluid: Mechanism



A Stationary Granular Fluid: Mechanism



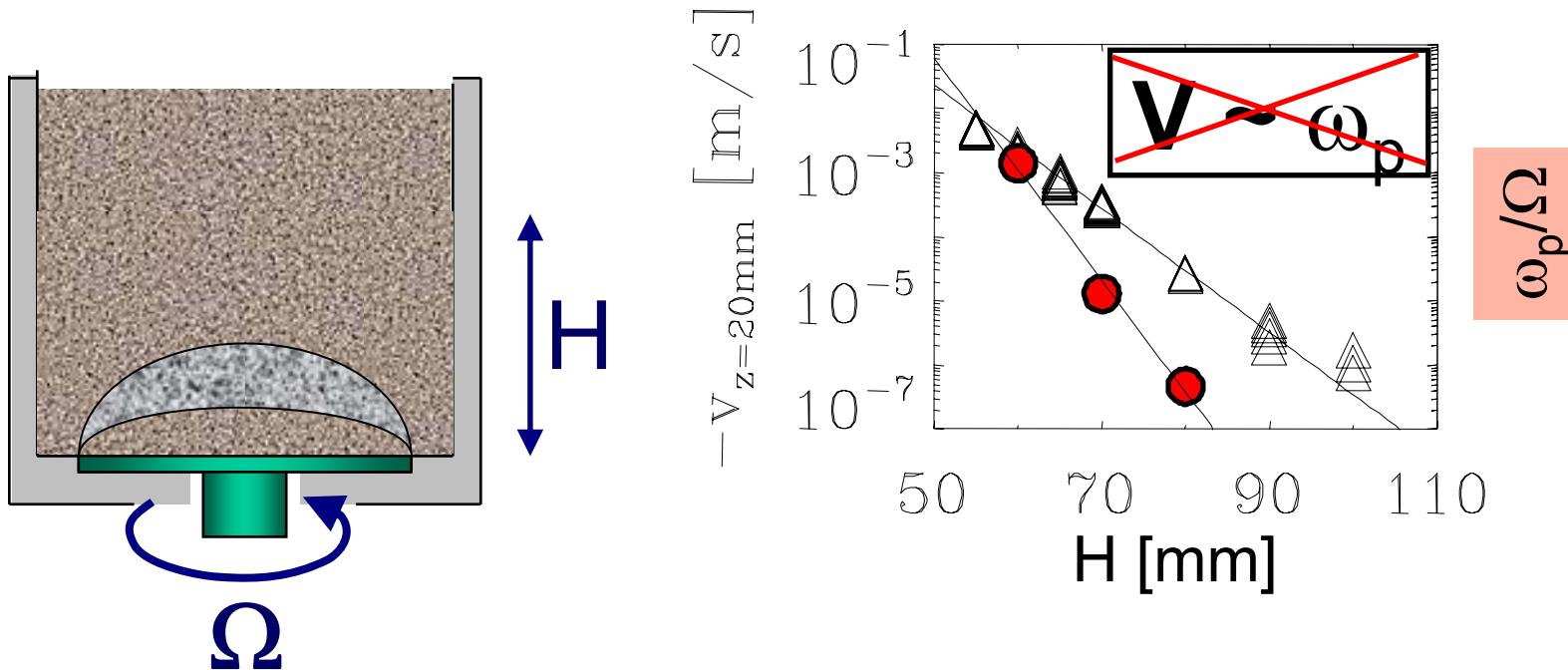
1 frame / 5 rotations



$$\omega_p \sim \Omega \text{ \& \ } V \sim \Omega$$

$$V \sim \omega_p ?$$

A Stationary Granular Fluid: Mechanism



NONLOCAL:

Flowrate in A, determines flowrate in B

Slow Granular Flows

Not Always Shear Bands

Classical Picture based on Friction **Fails**

New Framework: **Shear Free Sheets**

Experiments: **Melting** Sand by Stirring

Outlook: Agitation Field

Relations with fast and wet flows

Openings for grad students and postdocs

www.physics.leidenuniv.nl/sections/cm/grm