- Rare view into iron-ore melting furnace with inflow pellets during inspection cycle (Corus).

Rapid granular flows

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Vreman et al., JFM 2007

Akers & B., Physics of Fluids 2008

Rhebergen & B., Van der Vegt, Comp. Meth. Appl. Mech. Eng. 2009

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Outline

- 1. Introduction
- 2. Experiments: *granular*, *hydraulic* & *slurry*
- 3. Shallow layer theory: intermezzo
- 4. Multiple steady states: 1D theory, granular & hydraulic
- 5. Supercritical flow: 2D theory
- 6. Shallow slurry flows: 2D simulations
- 7. Conclusions
- *References*

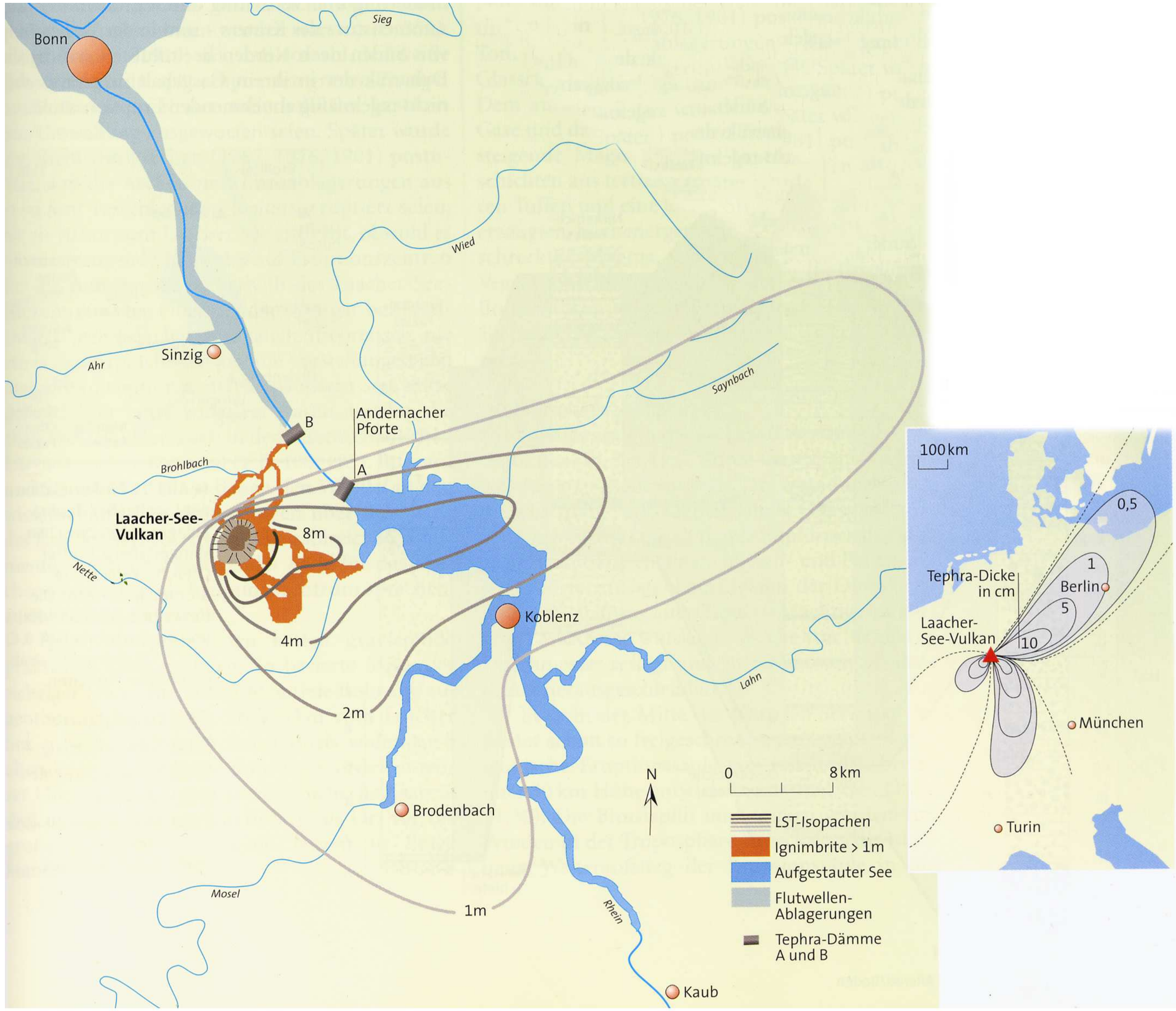
1. Introduction

Once upon a time, . . . a field trip in the *German Eifel* (2002):

- Under guidance of Hans-Ulrich Schmincke.
- Eifel is “young” volcanological field 2.5hrs from Twente.
- Laacher See Volcano exploded in Pleistocene (12.9k aBP):

Laacher See Volcano

- 1–8 meter layer of tephra around volcano causing fluidized tephra flows
- Schmincke 2000



. . . Introduction

Research question:

- Can we make a theory and laboratory experiment in support of this event of dam and lake formation and its collapse?
- Complexity of carrier fluid ("the Rhine") and floating granular material suggests initial simplification:
- ***What flow regimes emerge when dry granular matter flows down an inclined chute with a contraction?***

. . . Introduction

- We will consider shallow ***granular***, ***hydraulic*** & ***slurry*** flows through a contraction, experimentally, analytically and numerically.
- Large variations in particle/water discharges through contracting channels can lead to dramatic changes in the flow state.
- Surges: mud and river flows underneath bridges and in ravines.
- Linear contraction: archetypical contraction geometry.
- E.g. contraction flow with two oblique hydraulic jumps:

. . . Introduction

Fig. 1. H_2O: Oosterschelde storm surge barrier.

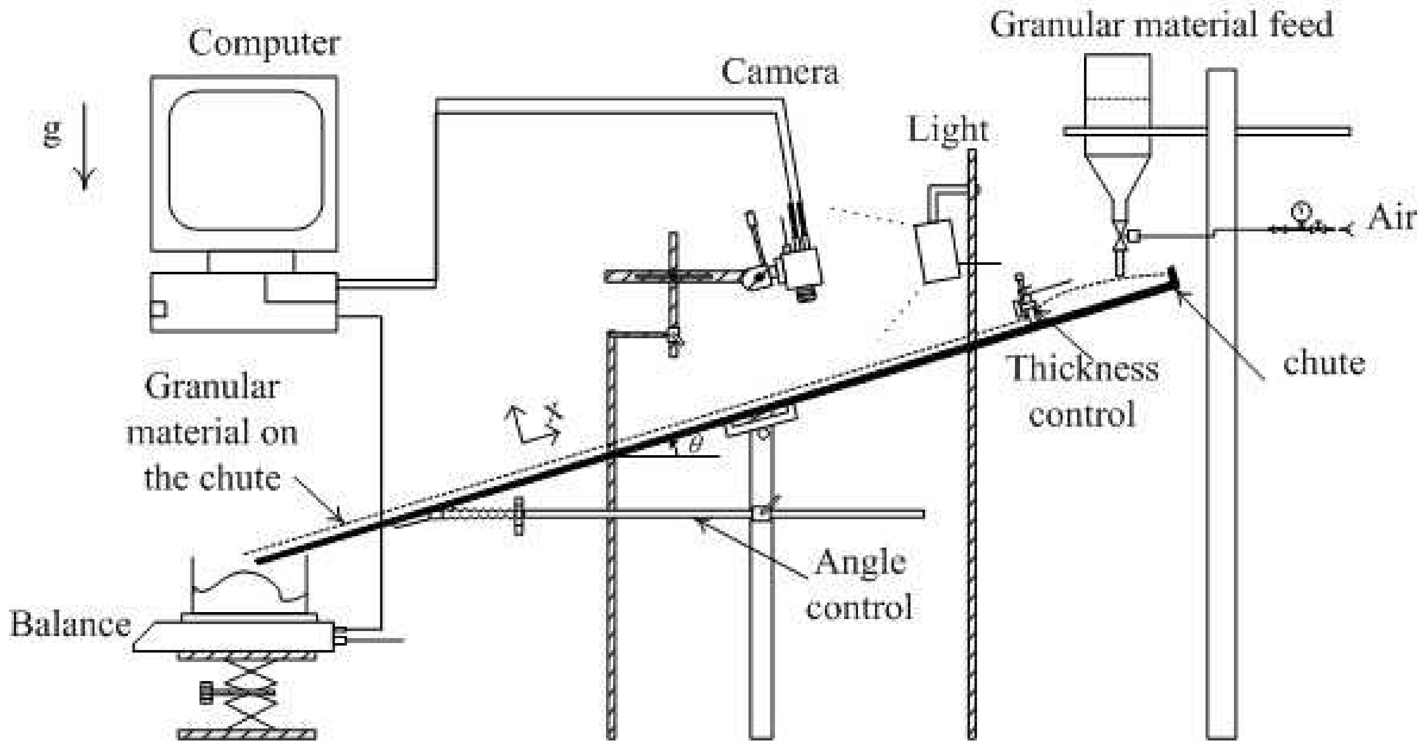
. . . Motivation

- **How do *granular*, *hydraulic* and *slurry* flows through a contraction compare with another, on inclined and flat chutes?**
- **Can we experimentally find three stable steady states for one discharge rate?**
  - Baines & Whitehead (2003) couldn't, for flow over hill.
- **Industrial counterparts: proposal Twente & Eindhoven, rotating granular flows (Corus, BASF, Unilever).**

2. Experiments

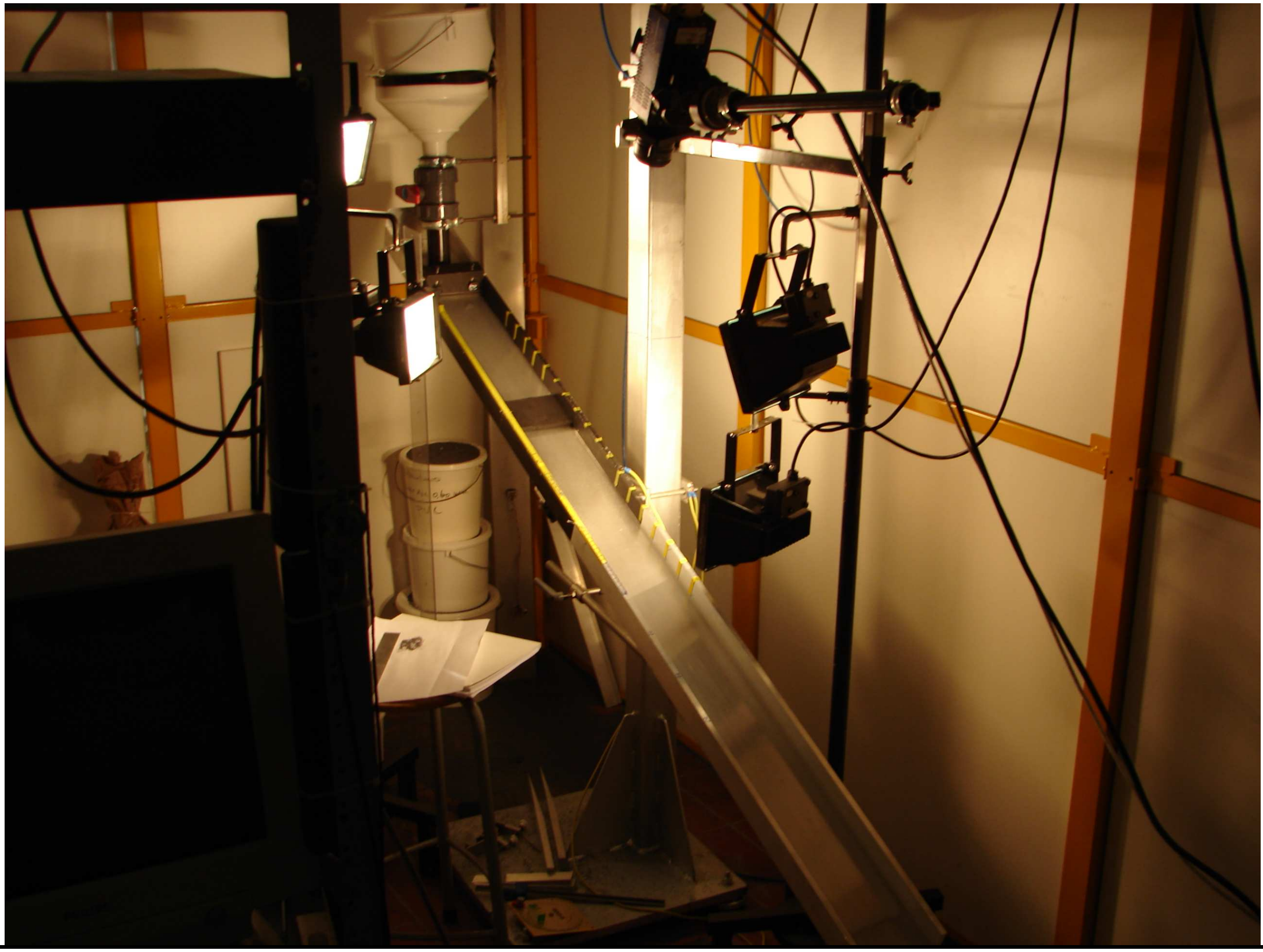
Granular flow (Al-Tarazi, Vreman) Inclined aluminum chute:

- $b_0 = 13$cm wide, 2m long, inclination $\theta, \phi = 15^o$–35^o
- glass beads: inelastic, diameter: S0) $d \in [.5, .6]$mm, S) $[.28, .42]$mm, M) $[.4, .6]$mm, L) $[.75, 1]$mm; density: 2470kg/m^3



. . . Granular experiments

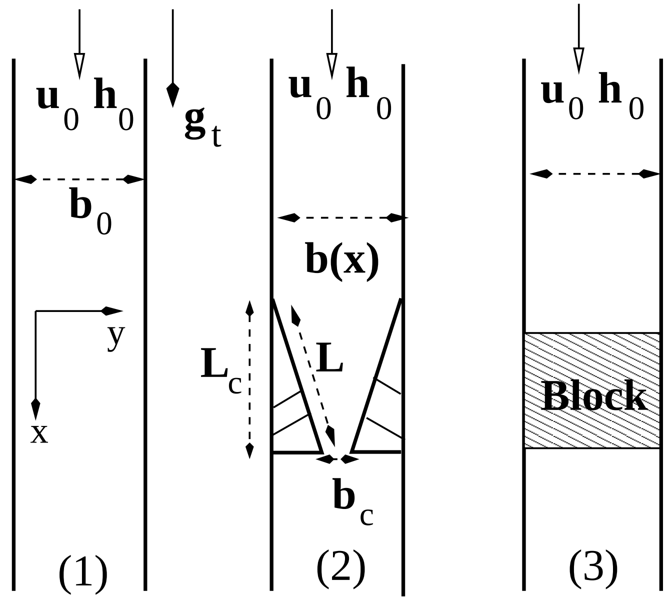
- Granular flow. Set-up: impression

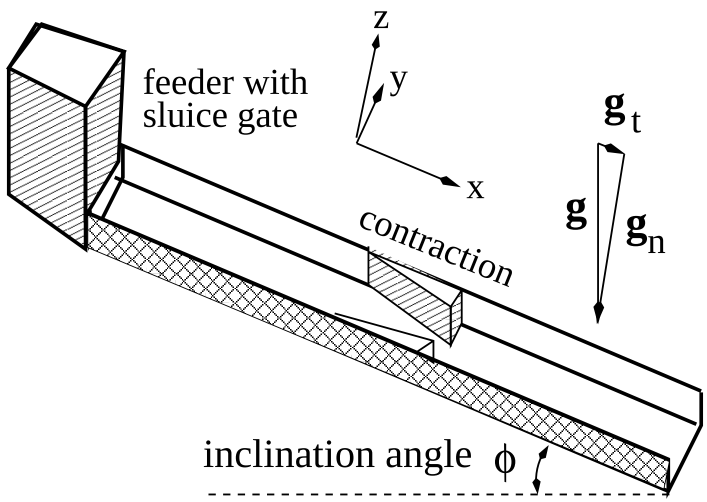


. . . Granular experiments

Top view sketches of granular experiments on **inclined chute:**

- [1] of constant width with no contraction, $b_c = b_0$,
- [2] with a localized contraction, and
- [3] blocked in middle, "$b_c = 0$".

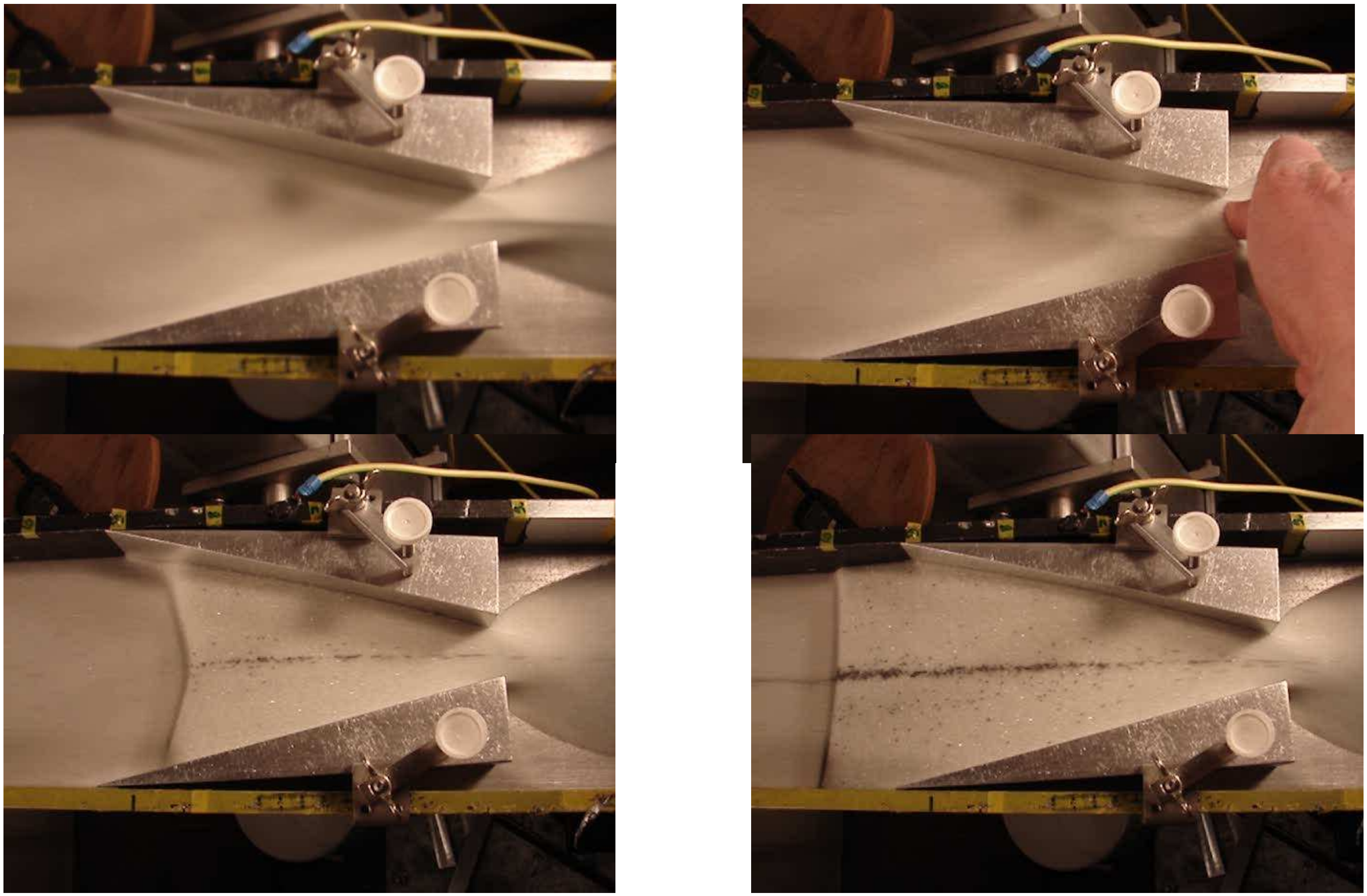




Reason: establish upstream conditions & bores experimentally.

Dry ***granular*** flow on inclined chute:

- Transition from supercritical to granular jump state:



- Steady-state ***granular*** flows: parameter plane upstream Froude # $F_0 = u_0/\sqrt{g_n\, h_0}$ & scaled minimum nozzle width $B_c = b_c/b_0$:

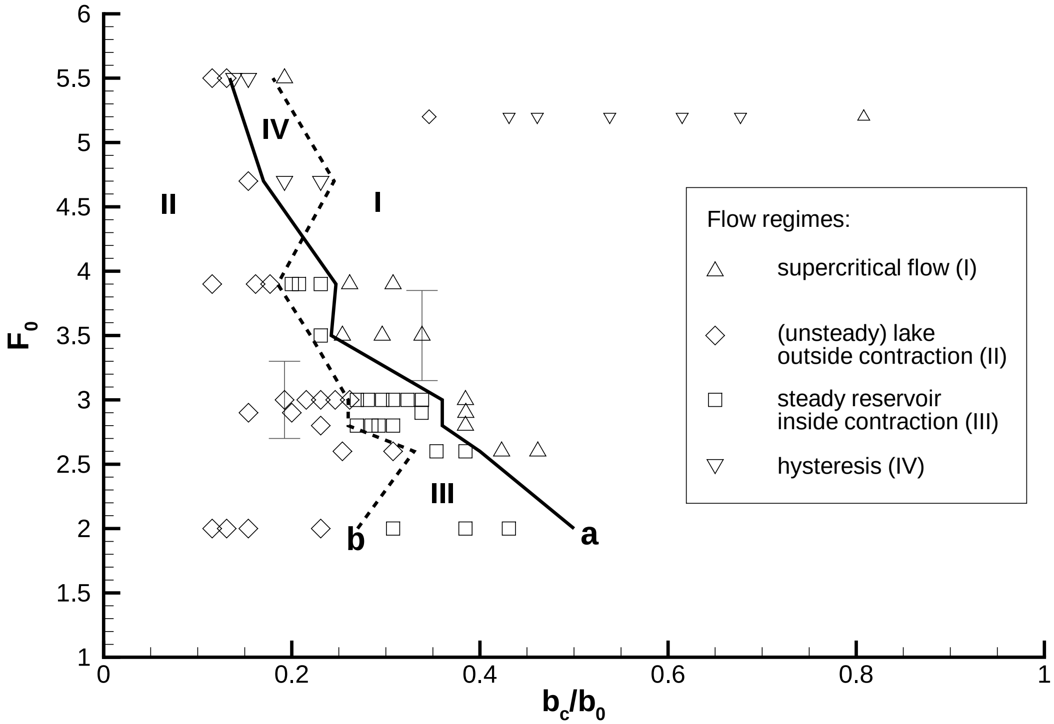


Fig. 2. △*: supercritical oblique jumps;* ▽*: hysteretic oblique jumps;* □*: steady reservoir;* ◇*: lake & upstream bore*

Hydraulic/slurry ...experiments

- Horizontal flume: 110cm long, $b_0 = 20$cm wide, with sluice gate:

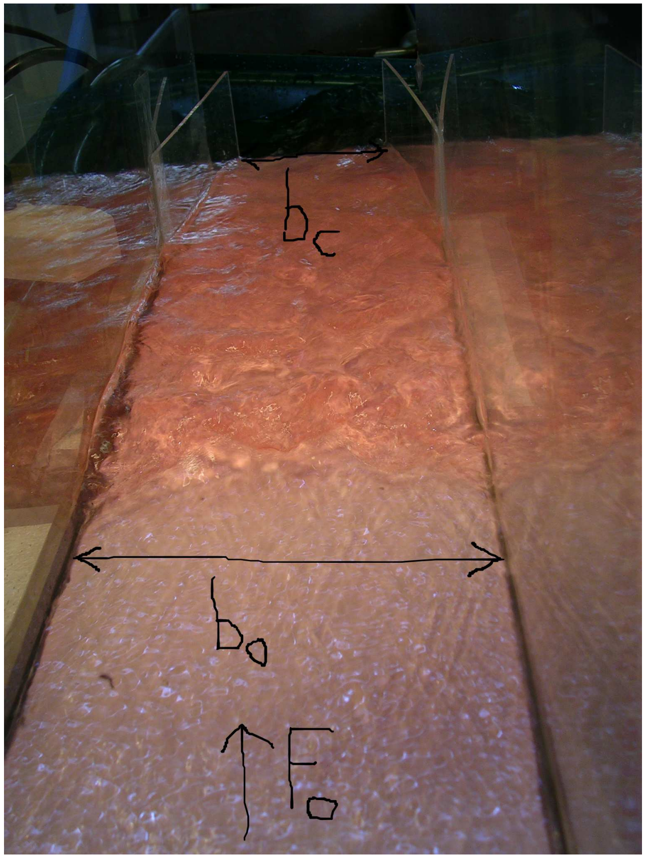


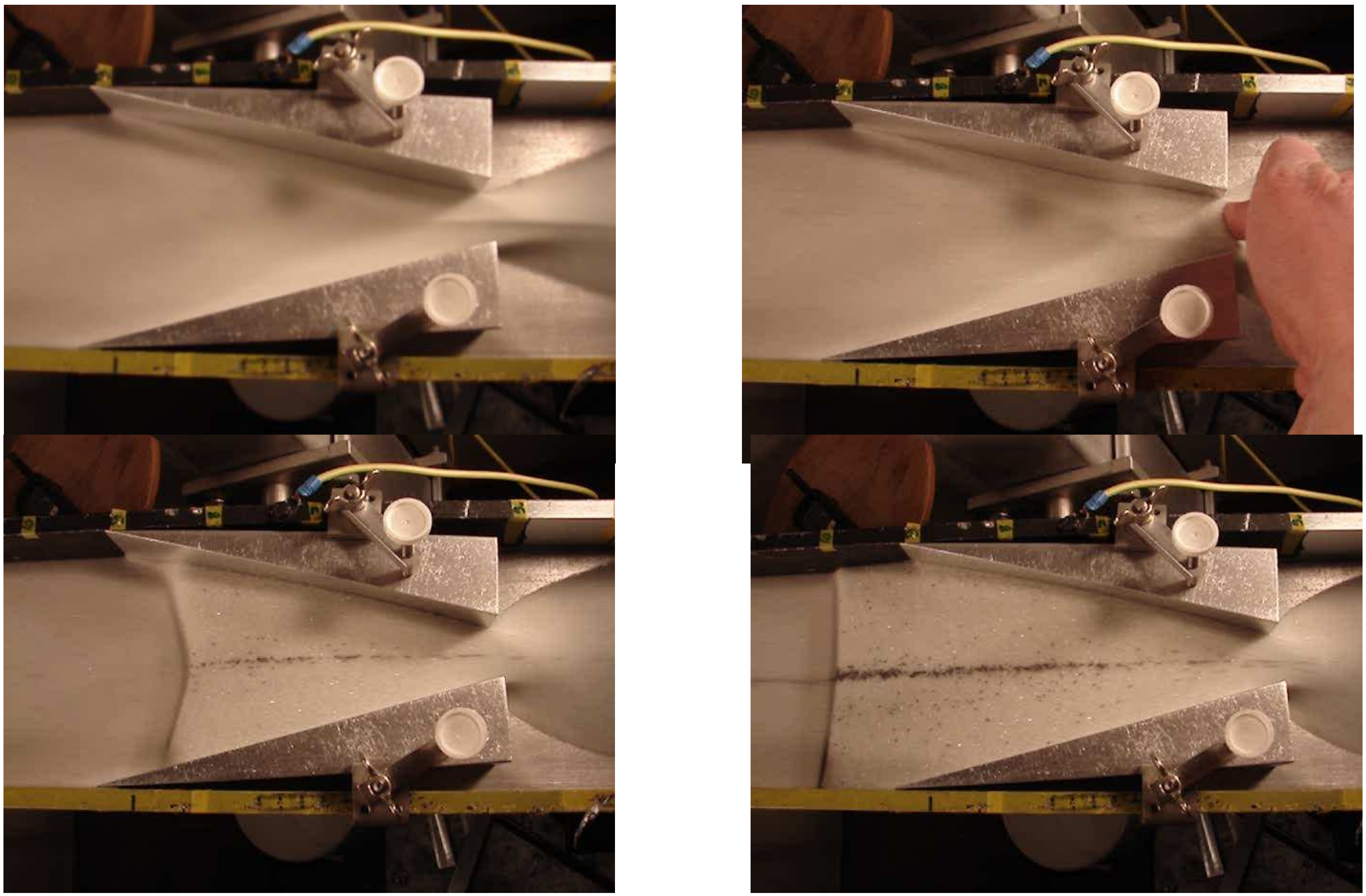
Fig. 3. Looking at the contraction in the flume; $F_0 = u_0/\sqrt{g\,h_0}, B_c = b_c/b_0$.

- Three multiple steady states in regions (i/iii/iv):

Fig. 3. Multiple states: $F_0 = 3.07$ & $B_c = 0.7$. Left to right: upstream steady shock (iii), reservoir state with "Mach stem" (iv), and oblique waves (i). Transitions induced by pushing flow.

Compare with dry granular flow on inclined chute:

- Transition from supercritical to granular jump state:



- Steady-state ***hydraulic*** flows: parameter plane of upstream Froude number F_0 and scaled minimum nozzle width $B_c = b_c/b_0$:

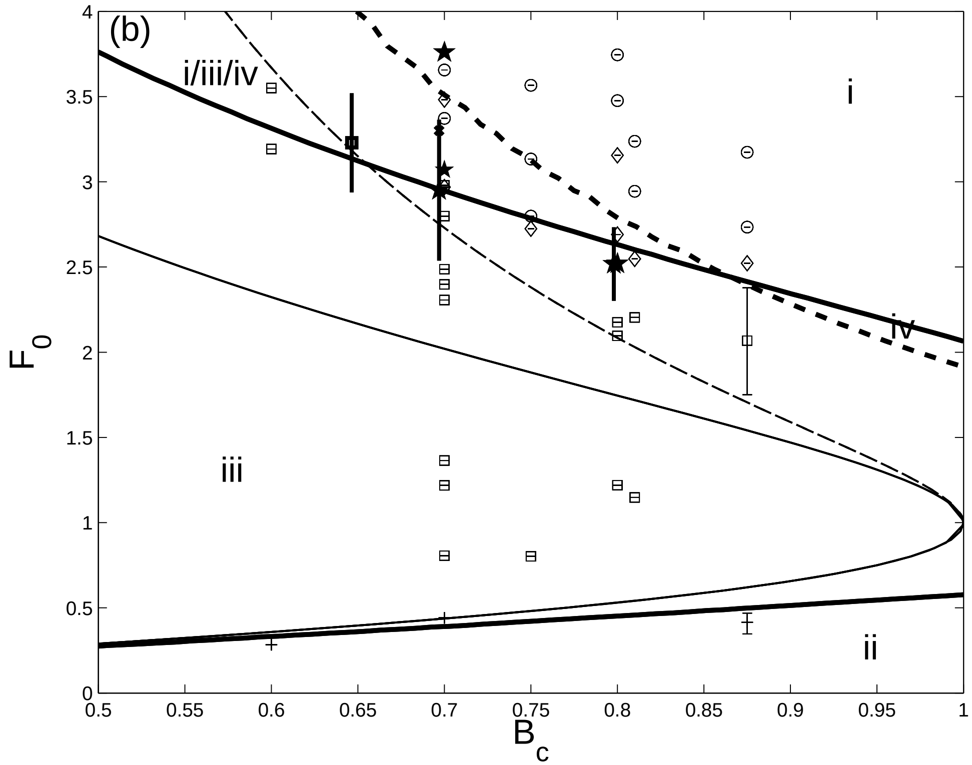


Fig. 4. +: smooth flows (ii); □: upstream moving shocks (iii); ◇: steady shocks (iii); o: oblique waves (i). ⋆: flows with 3 possible states.

- Stars, three multiple steady states in regions (i/iii/iv):

Fig. 4'. Multiple states: $F_0 = 3.07$ *&* $B_c = 0.7$*. Left to right: upstream steady shock (iii), reservoir state with "Mach stem" (iv), and oblique waves (i). Transitions induced by pushing flow.*

- Transition induced upstream ***slurry*** avalanche polystyrene beads:

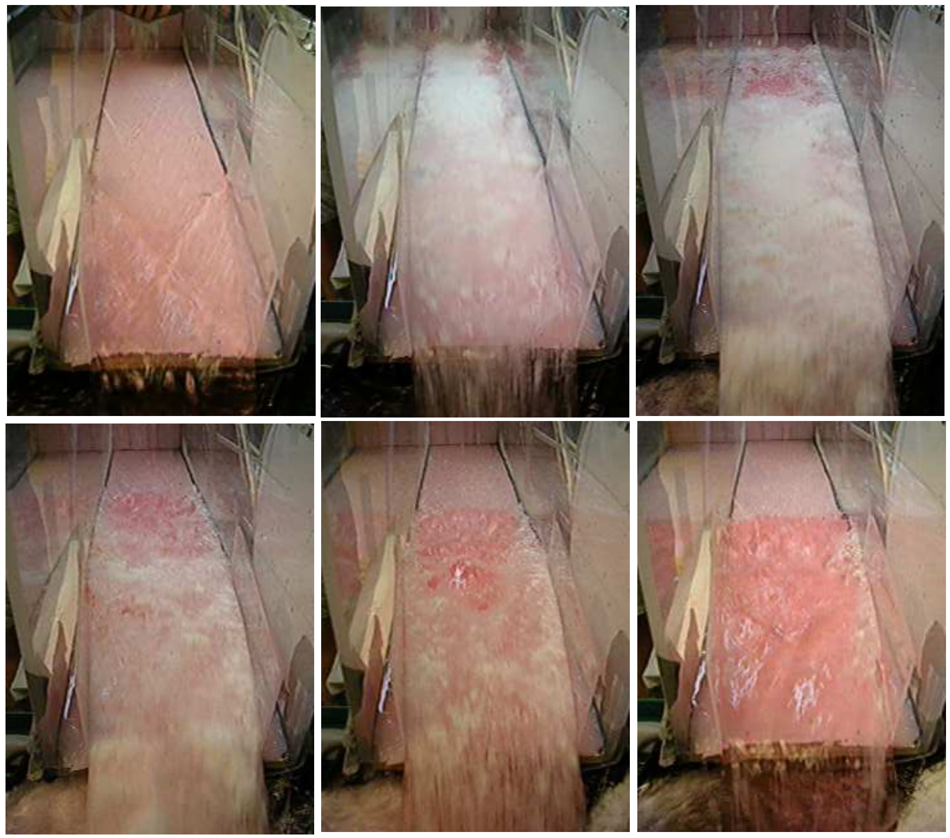


Fig. 5. Oblique wave state (top left) to upstream steady shock state (bottom right). t=1 to 6s. $\sim 900\,kg/m^3$, $F_0 = 3.07$ & $B_c = 0.7$.

3. Shallow layer theory: intermezzo

- Flowing granular materials can act as liquids or gases.
- When flowing granular layers are thin: shallow-layer theory.
- Asymptotic expansion in aspect ratio ϵ plus depth-averaging.
- New derivation with solid fraction: extension of Gray *et al.* (JFM 2003), who did incompressible case.

Shallow layer theory . . .

- Assumption: particles more or less homogenized: no segregation.
- Starting point: "Navier-Stokes" equations for granular flow.
- E.g., Haff (JFM 1983), Lun *et al.* (JFM 1984), Luding's approach.

Shallow layer theory ...

- 2D vertical cross section. Mass, momentum, energy on incline θ:

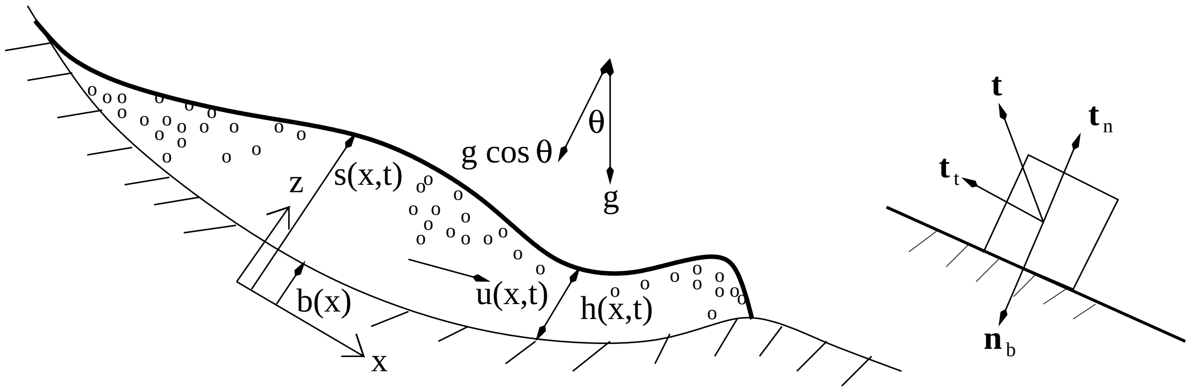
$$
\begin{aligned}
\partial_t\alpha + \partial_x(\alpha u) + \partial_z(\alpha w) &= 0 \qquad (1)\\
\rho_p\alpha(\partial_t u + u\partial_x u + w\partial_z u) &= \rho_p\alpha g\sin\theta + \partial_x\sigma_{xx} + \partial_z\sigma_{xz}\\
\rho_p\alpha(\partial_t w + u\partial_x w + w\partial_z w) &= -\rho_p\alpha g\cos\theta + \partial_x\sigma_{zx} + \partial_z\sigma_{zz}\\
\frac{3}{2}\rho_p\alpha(\partial_t T + u\partial_x T + w\partial_z T) &= \boldsymbol{\sigma} : \boldsymbol{\nabla}\mathbf{u} + \partial_x q_1 + \partial_z q_3 - \gamma
\end{aligned}
$$

- Solid fraction α & granular temperature T.
- Velocity field $\mathbf{u} = (u, v, w)^T$, & pressure p.
- Stress tensor $\boldsymbol{\sigma} = -p(\alpha, T)\mathbf{I} + \boldsymbol{\Sigma}$.
- Acceleration of gravity g; vector $\mathbf{g} = g(\sin\theta, 0, \cos\theta)$.
- Particle density ρ_p & granular damping γ.
- Flux of fluctuation energy $\mathbf{q} = (q_1, q_3)^T$; coordinates x, z, t.

Shallow layer theory ...

Kinematic conditions:

- Sketch



- At free surface $z - s(x,t) = 0$:

$$\partial_t s + u\partial_x s - w = 0 \tag{2}$$

- At bottom $z - b(x) = 0$

$$\partial_t b + u\partial_x b - w = 0 \tag{3}$$

Shallow layer theory ...

Traction conditions:

- Traction free at free surface: $\mathbf{t}^s = \boldsymbol{\sigma}^s \cdot \hat{\mathbf{n}}^s = 0$.
- Coulomn friction model at base. Tangential component proportional to normal component and opposite to velocity:

$$\mathbf{t}_t^b = -\mu |\mathbf{t}_n| \mathbf{u}/|\mathbf{u}|. \tag{4}$$

- The traction at base becomes:

$$\mathbf{t}^b = \boldsymbol{\sigma}^b \hat{\mathbf{n}}^b = \mathbf{t}_t^b + \mathbf{t}_n^b = \mathbf{t}_t^b - \hat{\mathbf{n}}^b |\mathbf{t}_n^b|. \tag{5}$$

- Clearly $|\mathbf{t}_n^b| = -\hat{\mathbf{n}}^b \cdot \mathbf{t}_b = -\hat{\mathbf{n}}^b \cdot \boldsymbol{\sigma}^b \hat{\mathbf{n}}^b$, such that:

$$\boldsymbol{\sigma}^b \hat{\mathbf{n}}^b = \mu(\hat{\mathbf{n}}^b \cdot \boldsymbol{\sigma}^b \hat{\mathbf{n}}^b)\mathbf{u}/|\mathbf{u}| + (\hat{\mathbf{n}}^b \cdot \boldsymbol{\sigma}^b \hat{\mathbf{n}}^b)\hat{\mathbf{n}}^b. \tag{6}$$

- Friction factor μ can depend on the flow variables at the base.

Shallow layer theory ...

Scale equations:

- Scaling with aspect ratio $\epsilon = H/L$:

$$
\begin{aligned}
x &= Lx', z = Hz', t = \sqrt{L/g}t', u = \sqrt{Lg}u', w = \epsilon\sqrt{Lg}w', \\
\sigma_{xx} &= \rho_p g H \sigma'_{xx}, \sigma_{xz} = \mu \rho_p g H \sigma'_{xz}, \sigma_{zz} = \rho_p g H \sigma'_{zz}, \qquad (7)
\end{aligned}
$$

- Apply to system & drop primes:

$$
\begin{aligned}
\partial_t \alpha + \partial_x(\alpha u) + \partial_z(\alpha w) &= 0 \qquad (8) \\
\alpha(\partial_t u + u\partial_x u + w\partial_z u) &= \alpha \sin\theta + \epsilon\partial_x \sigma_{xx} + \partial_z(\mu\sigma_{xz}) \\
\alpha\epsilon(\partial_t w + u\partial_x w + w\partial_z w) &= -\alpha\cos\theta + \epsilon\partial_x(\mu\sigma_{zx}) + \partial_z\sigma_{zz}
\end{aligned}
$$

- Leading order z–momentum equation ($\alpha = \bar{\alpha} + \hat{\alpha}$):

$$
\begin{aligned}
\sigma_{zz} &= \cos\theta \int_{b+h}^{z} \alpha(x,\tilde{z},t)\mathrm{d}\tilde{z} = (z-s)\bar{\alpha}\cos\theta + \cos\theta\int_{b+h}^{z}\hat{\alpha}(x,\tilde{z},t)\mathrm{d}\tilde{z} \\
\Rightarrow \sigma_{zz}^{b} &= -h\bar{\alpha}\cos\theta.
\end{aligned}
$$

Shallow layer theory . . .

Scale tractions:

- With $\Delta^b = \sqrt{1 + (\epsilon \partial_x b)^2}$:

$$\epsilon \sigma^b_{xx} \partial_x b - \mu \sigma^b_{xz} = (\hat{\mathbf{n}}^b \cdot \boldsymbol{\sigma}^b \hat{\mathbf{n}}^b)(\Delta^b \mu \frac{u}{|\mathbf{u}|} + \epsilon \partial_x b) \quad (9)$$

$$\epsilon \mu \sigma_{xz} \partial_x b - \sigma_{zz} = (\hat{\mathbf{n}}^b \cdot \boldsymbol{\sigma}^b \hat{\mathbf{n}}^b)(\Delta^b \mu \epsilon \frac{w}{|\mathbf{u}|} - 1) \quad (10)$$

- Using $\hat{\mathbf{n}}^b = (\epsilon \partial_x b, -1)^T / \Delta^b$, one finds that $\hat{\mathbf{n}}^b \cdot \boldsymbol{\sigma}^b \hat{\mathbf{n}}^b = \sigma^b_{zz} + O(\epsilon) \approx -h \bar{\alpha} \cos \theta$.
- Also $h \overline{\sigma}_{zz} \approx -\frac{1}{2} h^2 \bar{\alpha} \cos \theta$.
- Mohr-Coulomb theory for material in yield gives $\sigma_{xx} = K \sigma_{zz}$ with $K = K(\phi_i, \phi)$ depending on the internal angle of friction ϕ_i and ϕ the basal friction angle.

Shallow layer theory ...

Keep leading order terms in ϵ & $\mu = O(\epsilon^{\varphi})$ with $0 < \varphi < 1$:

- depth-average, $\int_{b(x)}^{h(x,t)+b(x)} \mathrm{d}z$, continuity equation using kinematic conditions:

$$\partial_t(h\bar{\alpha}) + \partial_x(h\overline{\alpha u}) = 0. \tag{11}$$

- depth-average x–momentum equation using ...:

$$\partial_t(h\overline{\alpha u}) + \partial_x\left(h\overline{\alpha u^2}\right) = h\bar{\alpha}\sin\theta + \epsilon\partial_x(h\bar{\sigma}_{xx}) - \mu\sigma_{xz}\big|^b$$

gives

$$\partial_t(h\overline{\alpha u}) + \partial_x\left(h\overline{\alpha u^2} + \frac{1}{2}\epsilon\bar{\alpha}K\cos\theta h^2\right) = h\bar{\alpha}\sin\theta - h\bar{\alpha}\cos\theta\mu\frac{u}{|u|}. \tag{12}$$

Shallow layer theory ...

Final leading-order shallow layer equations:

- after dropping overbars (flat/linear profiles, e.g., GDR MIDI 2004):

$$\begin{aligned} \partial_t(h\alpha) + \partial_x(h\alpha u) &= 0 \\ \partial_t(h\alpha u) + \partial_x\big(h\alpha u^2 + \frac{1}{2}\epsilon\alpha K \cos\theta h^2\big) &= h\alpha\sin\theta - h\alpha\cos\theta\mu\frac{u}{|u|}. \end{aligned} \quad (13)$$

- Not closed ... yet! Relation between α and h required from granular temperature equation?

- "Standard incompressible" case emerges for constant α.

Shallow layer theory ...

Granular jump and bore relations, aka shock relations:

- Say discontinuity located at position $x = x_b(t)$, then integration continuity equation around $x_b(t)$ yields

$$\lim_{\epsilon \to 0} \int_{x_b(t)-\epsilon}^{x_b(t)+\epsilon} \partial_t(h\alpha) + \partial_x(h\alpha u)\mathrm{d}x = 0 \qquad (14)$$

$$\lim_{\epsilon \to 0} \frac{\mathrm{d}}{\mathrm{d}t} \int_{x_b(t)-\epsilon}^{x_b(t)+\epsilon} h\alpha \mathrm{d}x - S[h\alpha]_-^+ + [h\alpha u]_-^+ = 0 \qquad (15)$$

$$-S[h\alpha]_-^+ + [h\alpha u]_-^+ = 0 \qquad (16)$$

with shock speed $S = \mathrm{d}x_b/\mathrm{d}t$ and jump $[\cdot]_-^+$ in quantity $(\cdot)$ across discontinuity.

Shallow layer theory ...

- Similarly for momentum equation; this yields the bore relations (two relations, five unknowns):

$$-S[h\alpha]+[h\alpha u]=0 \qquad -S[h\alpha u]+[h\alpha u^2+\epsilon K\cos\theta\alpha h^2]=0. \quad (17)$$

- Relevance: 1D blocked granular flow.
- I.e., fast 1D, thin ($h/d \approx 2, 8$), airy granular layer running into moving shock with still thick matter thereafter.

Shallow layer theory ...

Determination of friction μ:

- Supercritical flows on "smooth" chutes: $K = 1$.
- Straight and oblique shocks/jumps: Gray *et al.* (2003), Hákonardóttir & Hogg (2005).
- Granular slumping: Kerswell (2005); constant K and μ combined in one fitting parameter.
- Rough chutes: $\mu = \mu(F, h; \ldots)$, Pouliquen-Jenkins law (Pouliquen and Fortere 2002, Jenkins 2006, Börzsönyi & Ecke 2007).
- $\ldots \mu$ depends on flow for subcritical flows $F < 1$ and flows around criticality $F \approx 1$.

Shallow layer theory ...

- Similarly, derive 2D compressible shallow granular flow equations:

$$\partial_t(h\alpha) + \partial_x(h\alpha u) = 0 \quad (18)$$

$$\partial_t(h\alpha u) + \partial_x\Big(h\alpha u^2 + \frac{1}{2}\epsilon\alpha K\cos\theta h^2\Big) + \partial_y(h\alpha v u) = h\alpha\sin\theta - h\alpha\cos\theta\mu\frac{u}{|\mathbf{u}|} \quad (19)$$

$$\partial_t(h\alpha v) + \partial_x(h\alpha u v) + \partial_y\Big(h\alpha u^2 + \frac{1}{2}\epsilon\alpha K\cos\theta h^2\Big) = -h\alpha\cos\theta\mu\frac{v}{|\mathbf{u}|} \quad (20)$$

  with lateral direction y and velocity v, and here $\mathbf{u} = (u, v)^T$.

- Consider chute of constant width b_0 up- and downstream of a contraction where width is $b = b(x)$.
- Width-average 2D shallow granular flow equations, to obtain ....

4. Multiple steady states: 1D theory

- Steady-state and "shock" solutions of dimensionless 1D cross-sectionally averaged shallow water equations:

$$u_t + u\,u_x + h_x/F_l^2 = \begin{cases} \sim (\tan\phi - \mu(F)) & \text{granular} \\ -C_d\,u^2/h & H_2O \end{cases} \quad (21)$$

$$(bh)_t + (buh)_x = 0 \quad (22)$$

- with velocity $u = u(x,t)$, depth $h = h(x,t)$, width $b = b(x)$,
- Froude number $F_l = F_0$ upstream or $F_l = F_m$ at contraction,
- granular: angle ϕ, $\mu(F > 1) \leq \tan\phi$, $\mu_{sub}(F < 1) < \tan\phi$ acceleration integral: $Z_{1,2} = \int_{x_0}^{x_c} (\tan\phi - \mu_{sub}) \mathrm{d}x/h_0$.
- H_2O: demarcation lines friction (thin $C_d = 0$, thick: $C_d > 0$).

4. Multiple steady states: 1D theory

- Steady-state and "shock" solutions of dimensionless 1D cross-sectionally averaged shallow water equations:

$$\frac{\mathrm{d}}{\mathrm{d}x}(u^2/2 + h/F_l^2) = \begin{cases} \sim (\tan\phi - \mu(F)) & \text{granular} \\ -C_d\, u^2/h & H_2O \end{cases} \quad (23)$$

$$buh = Q \quad (24)$$

- with velocity $u = u(x,t)$, depth $h = h(x,t)$, width $b = b(x)$,
- Froude number $F_l = F_0$ upstream or $F_l = F_m$ at contraction,
- granular: angle ϕ, $\mu(F > 1) \leq \tan\phi$, $\mu_{sub}(F < 1) < \tan\phi$ acceleration integral: $Z_{1,2} = \int_{x_0}^{x_c} (\tan\phi - \mu_{sub})\mathrm{d}x/h_0$.
- H_2O: demarcation lines friction (thin $C_d = 0$, thick: $C_d > 0$).

...

- Frictionless case, say $F_l = F_0$:

$$Q = 1 = buh, \quad \frac{\mathrm{d}}{\mathrm{d}x}(\frac{1}{2}u^2 + h/F_0^2) = 0, \quad F = \frac{F_0 u}{\sqrt{h}} = \frac{F_0}{(bh^{3/2})}. \quad (25)$$

- Substitute $u = 1/(bh)$ in ODE and rewrite:

$$(1 - F^2)\frac{\mathrm{d}h}{\mathrm{d}x} = \frac{F_0^2}{b^3 h^2}\frac{\mathrm{d}b}{\mathrm{d}x} \quad (26)$$

- Hence, when $F < 1$ subcritical $\mathrm{d}b/\mathrm{d}x$ & $\mathrm{d}h/\mathrm{d}x$ in phase, while out of phase when $F > 1$ supercritical.

- Rewrite all in terms of Froude number $F = F(x)$, to get:

$$\frac{2(F^2 - 1)}{(2 + F^2)F}\frac{\mathrm{d}F}{\mathrm{d}x} = \frac{\mathrm{d}\ln b}{\mathrm{d}x} \quad \frac{F_0}{F}\Big(\frac{2 + F^2}{2 + F_0^2}\Big)^{3/2} = \frac{b}{b_0}. \quad (27)$$

...

- This smooth solution exist in limit part of parameter space. Threshold when $F = 1$ at nozzle $b = b_c$, so width $b = b_0$ one finds threshold curve (Akers & B. 2008 PoF):

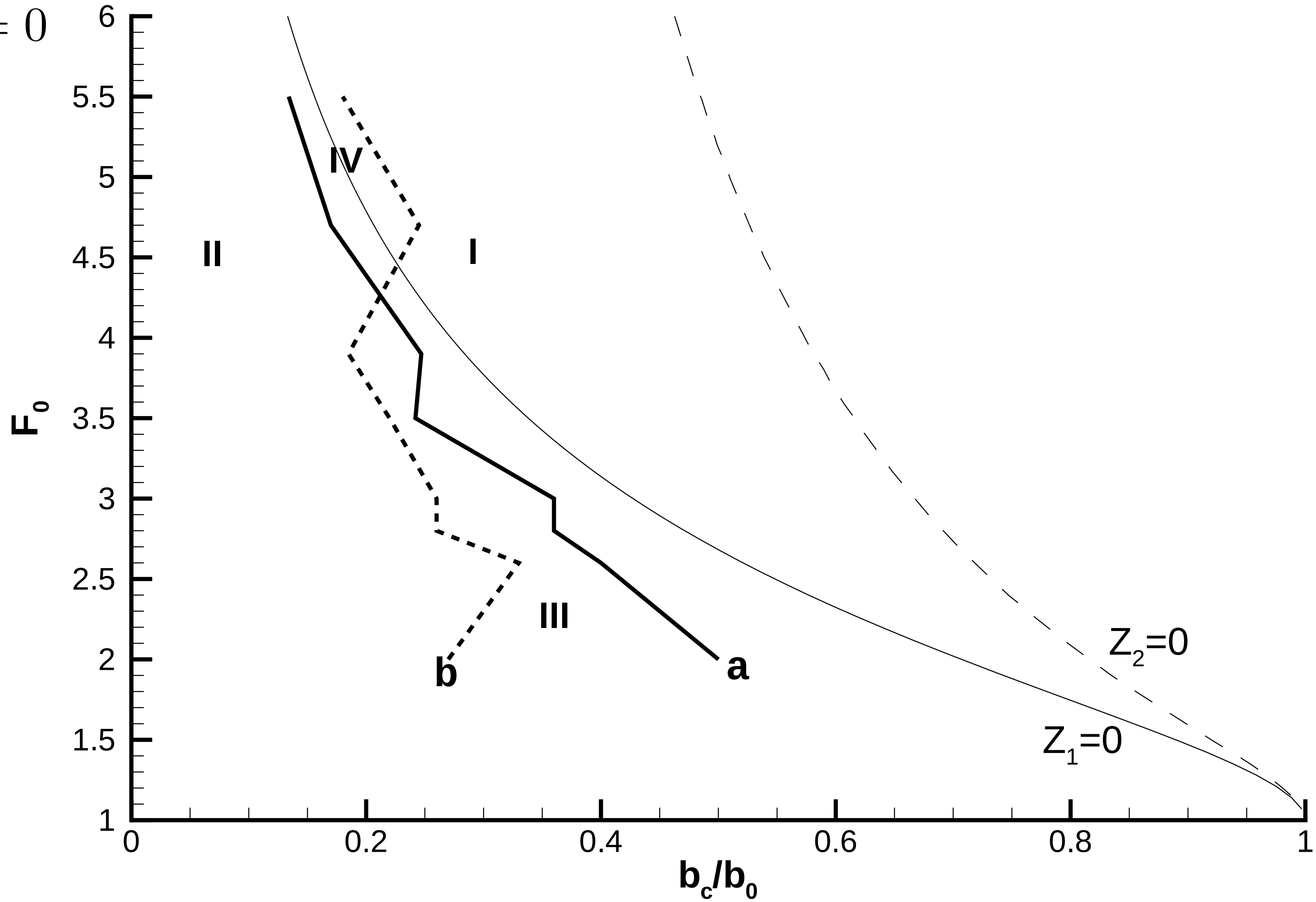
$$F_0 \left(\frac{3}{2 + F_0^2}\right)^{3/2} = B_c. \tag{28}$$

- For frictional case: solve ODE numerically (Akers & B. 2008), or apply simplification (Vreman et al. 2007 JFM).

- Shock curve, borderline case: place stationary shock/jump with $S = 0$ at contraction entrance and construct supercritical solution before and subcritical one after this granular jump, with criticality at nozzle

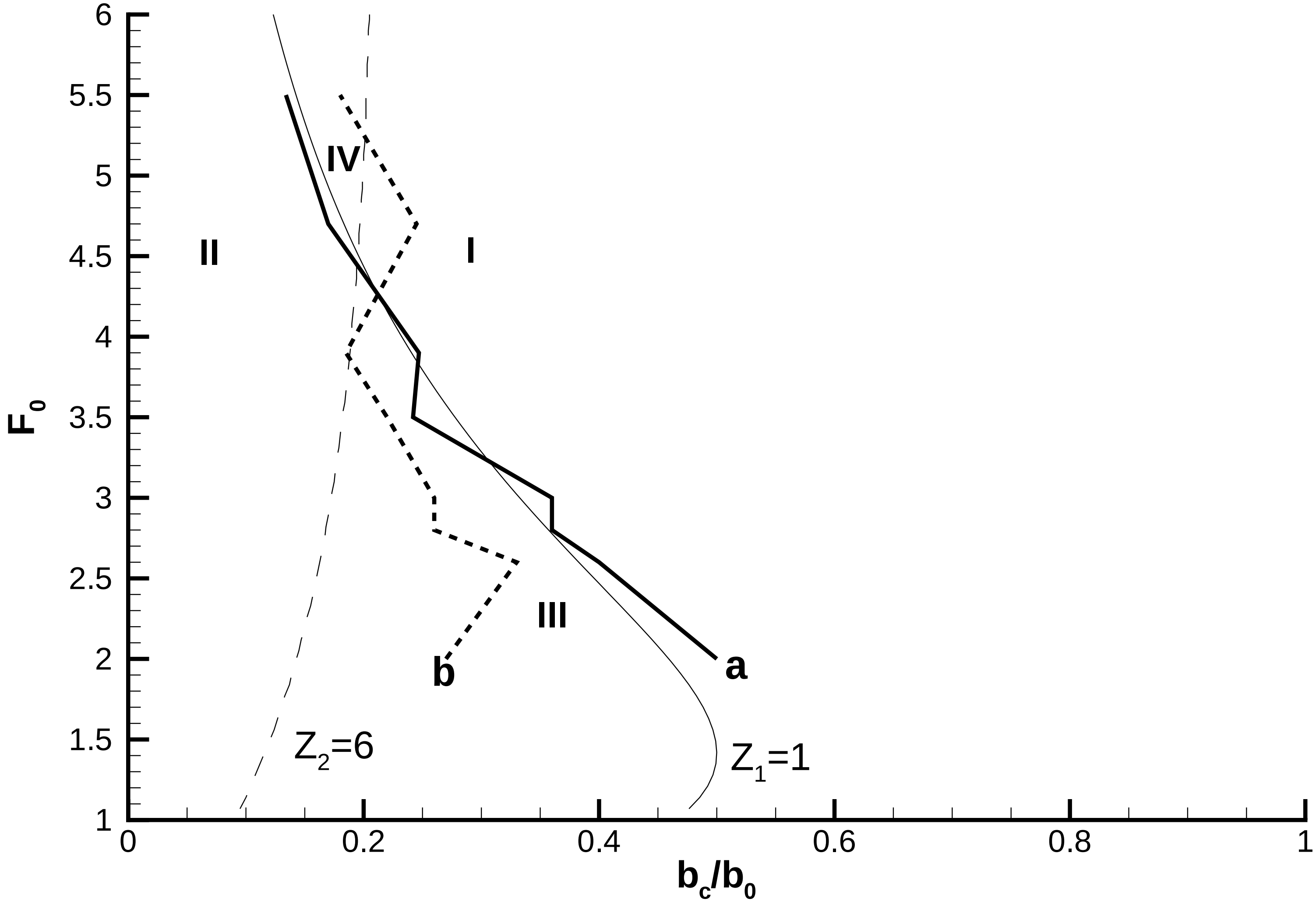
Granular: effectively inviscid F_0, b_c–parameter plane

- Isolines of $Z = 0$
- Solid: critical curve & dashed: shock curve



Granular: modified F_0, b_c–parameter plane best fit Z_1, Z_2

- Isolines of Z
- Solid critical curve & dashed shock curve



- H_2O: steady-state flows: parameter plane F_0 versus $B_c = b_c/b_0$:

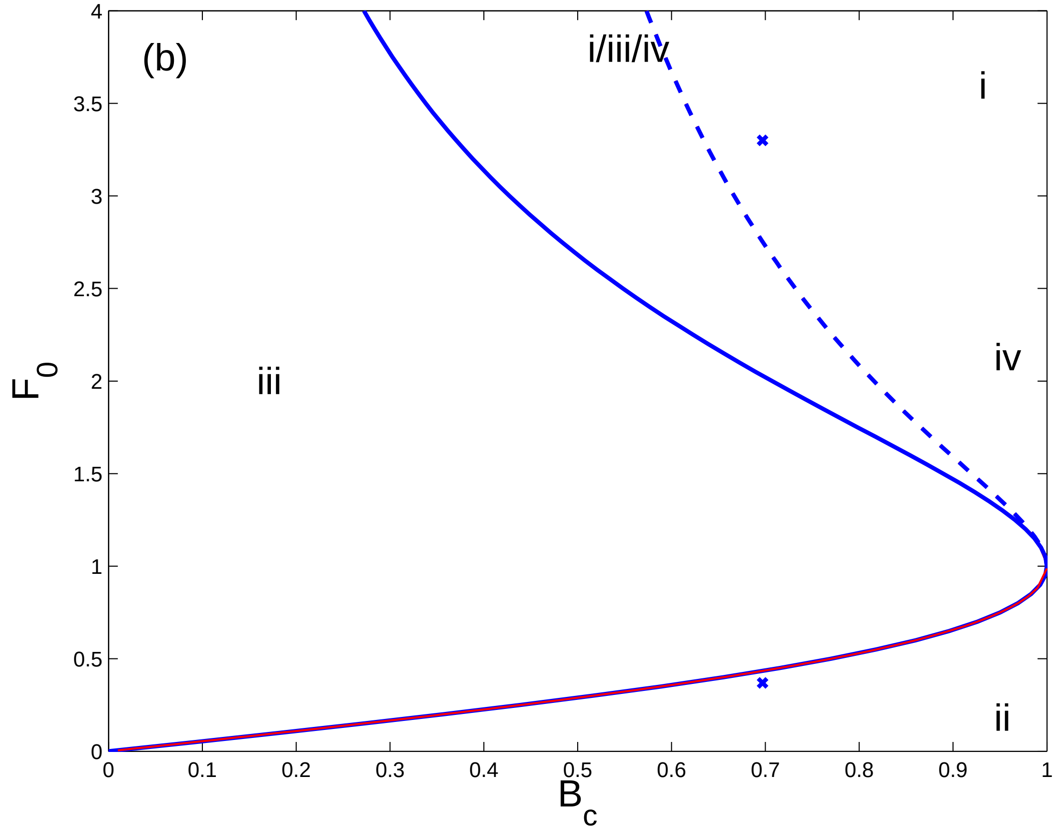


Fig. 4'. From 3 (inviscid) . . . .

- Steady-state flows: parameter plane F_0 versus $B_c = b_c/b_0$:

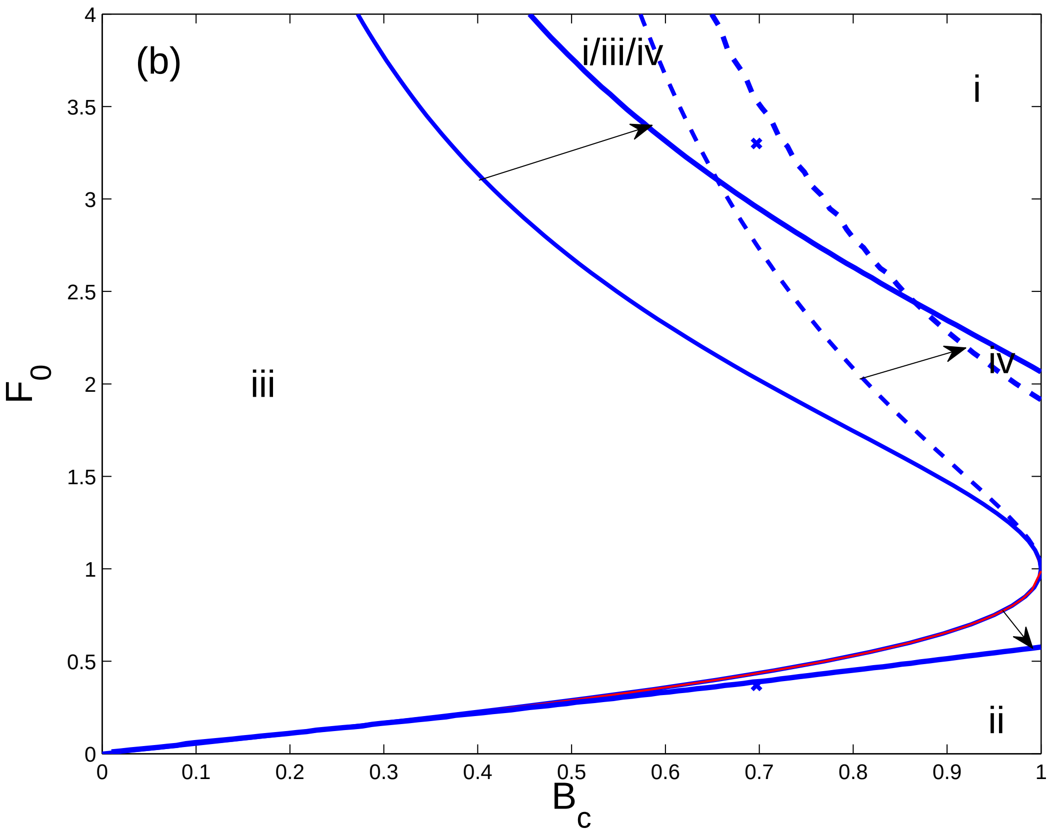
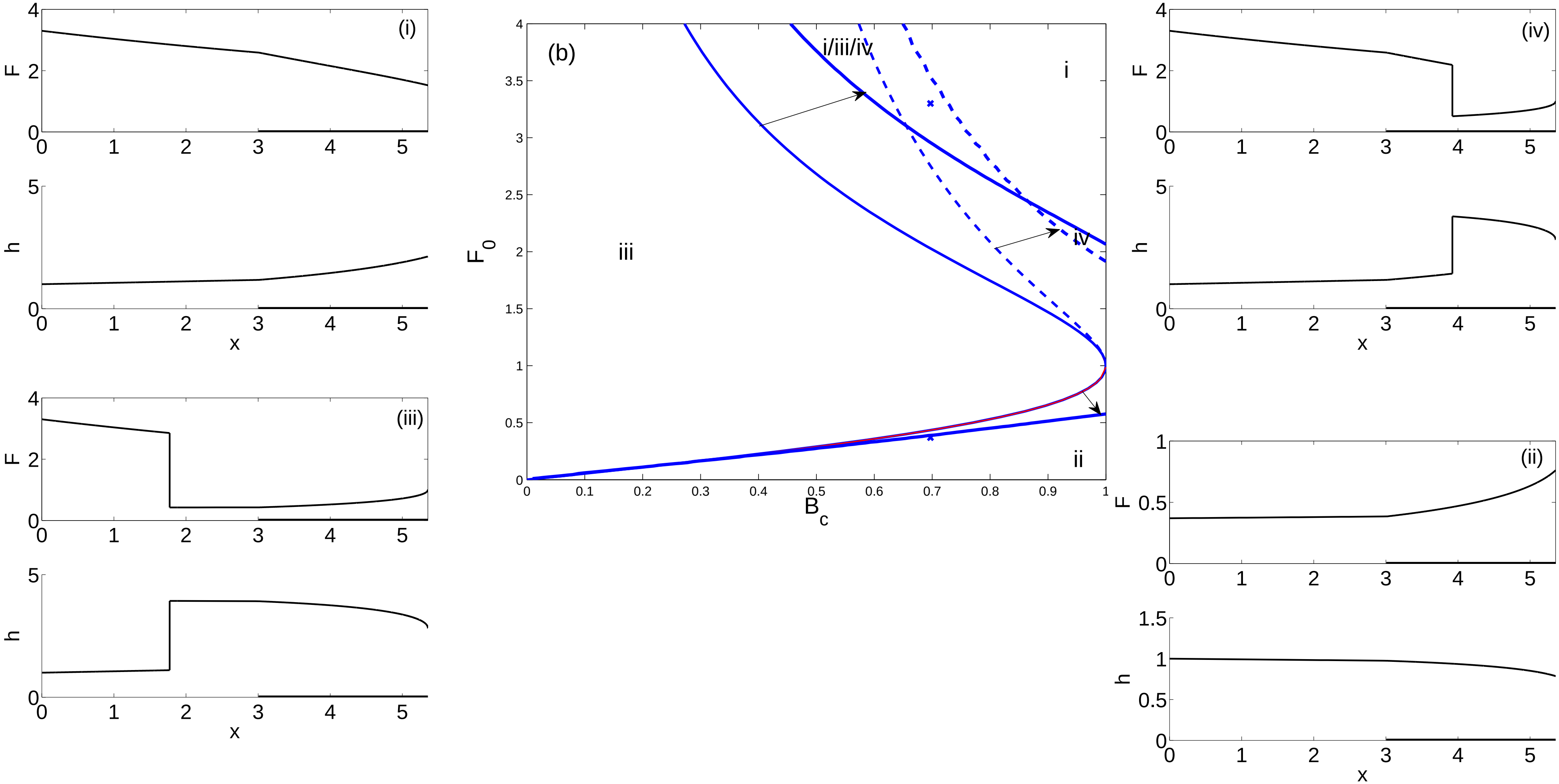


Fig. 4". . . . 3 (inviscid) to 4 (friction) flow regimes: bifurcation.

- Steady-state flows: parameter plane F_0 versus $B_c = b_c/b_0$:



- Three multiple steady states in regions (i/iii/iv):

- Steady-state flows: parameter plane F_0 versus $B_c = b_c/b_0$:

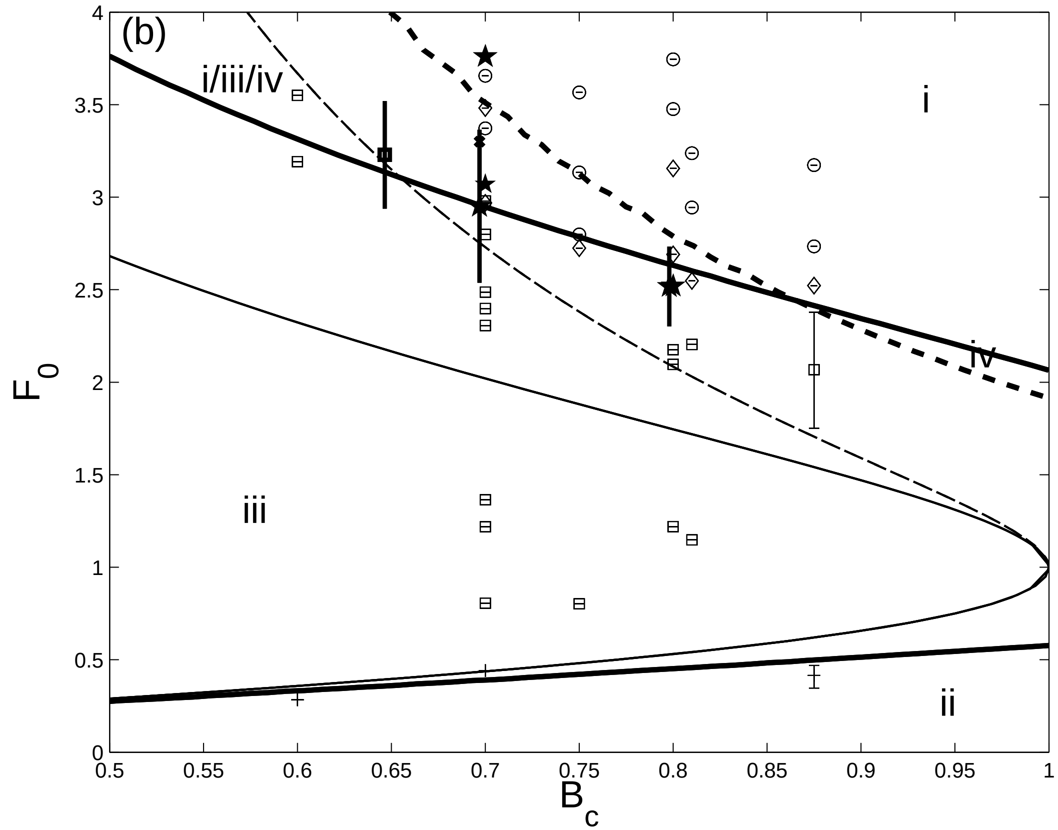
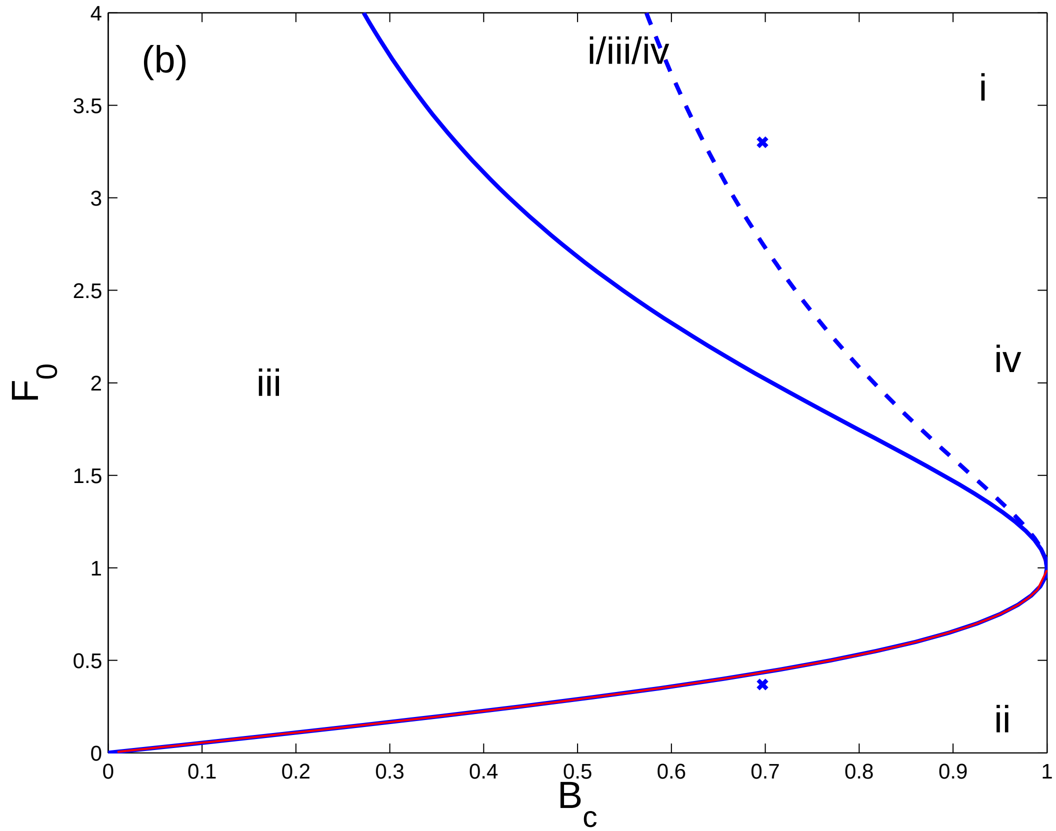


Fig. 4"'. +: smooth flows (ii); □: upstream moving shocks (iii); ◇: steady shocks (iii); ○: oblique waves (i). ⋆: flows with 3 possible states.

5. Supercritical flows: 2D theory

- Some people did not believe 1D theory.



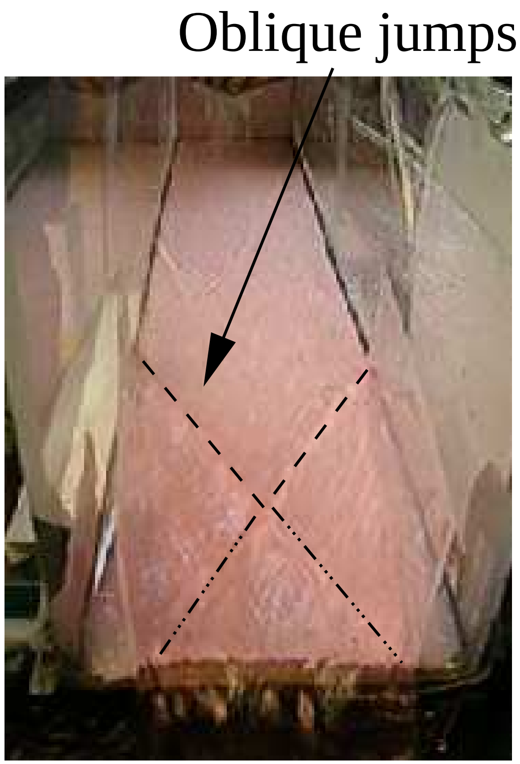


Fig. 4'. . . . inviscid case.

...2D theory

- Existence 2D oblique granular/hydraulic jumps no net friction:

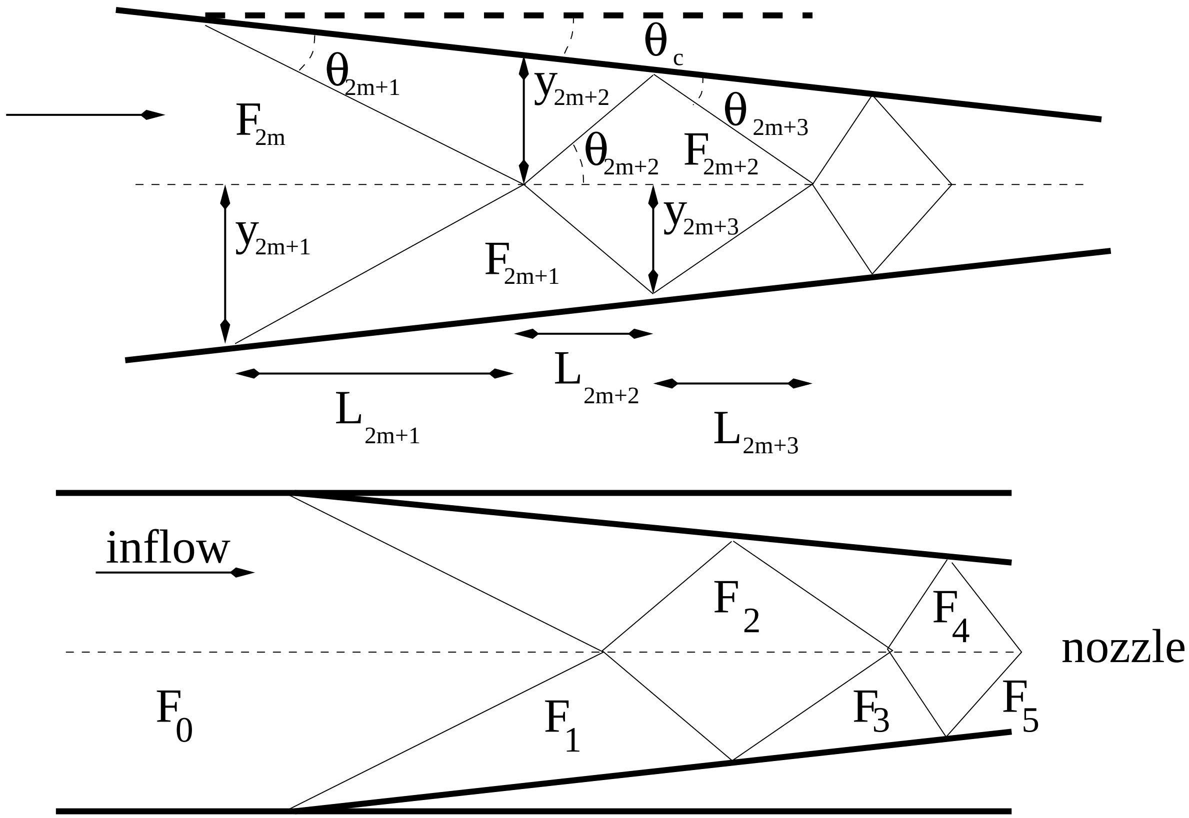
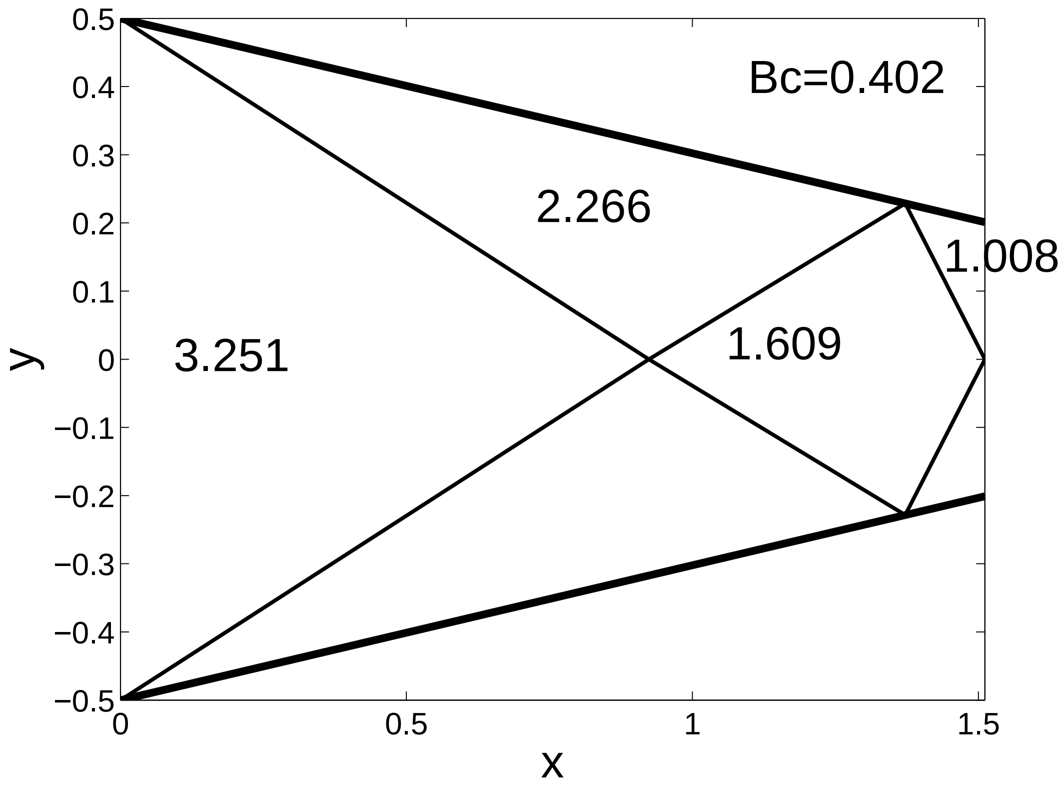


Fig. 6. Sketch of angles θ and Froude numbers F_{2m}, F_{2m+1}.

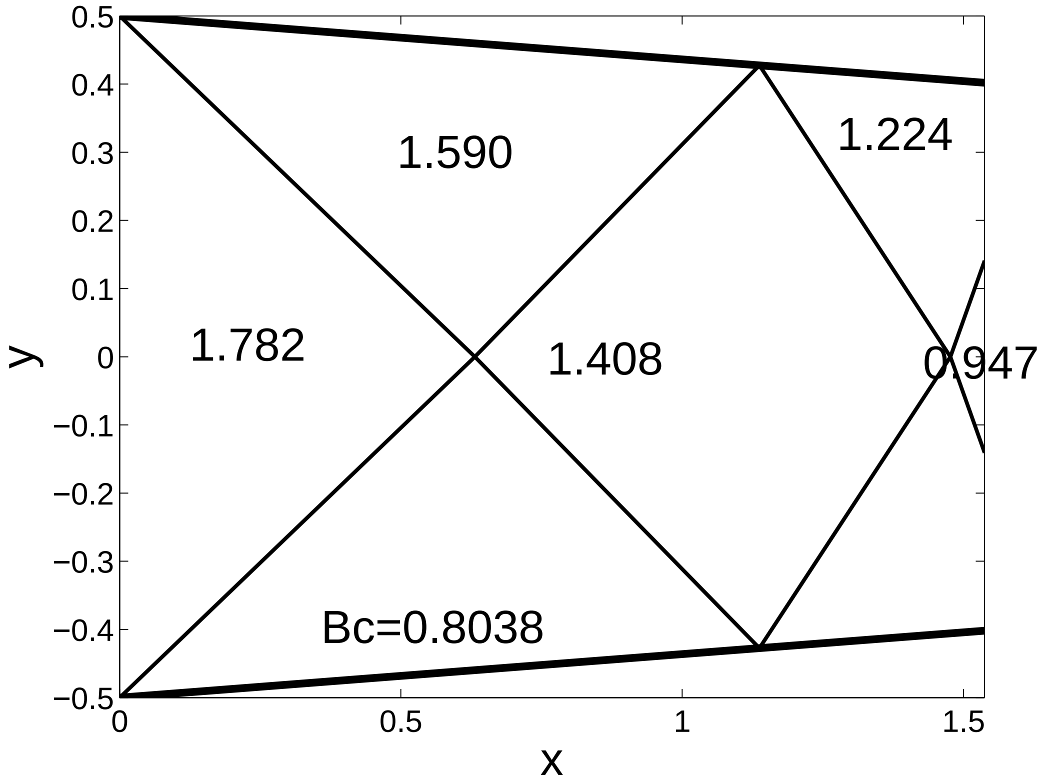
...2D theory

- when the downstream angles θ_{2m+1} or θ_{2m+2} can be calculated for Froude numbers above a critical F_0, or



. . . 2D theory

- Froude number of last polygon entirely fitting within contraction lies above unity for certain critical F_0 at inflow:



- Demarcation: supercritical solutions and upstream moving jumps determined with 1D/2D hydraulic theory, and simulations.

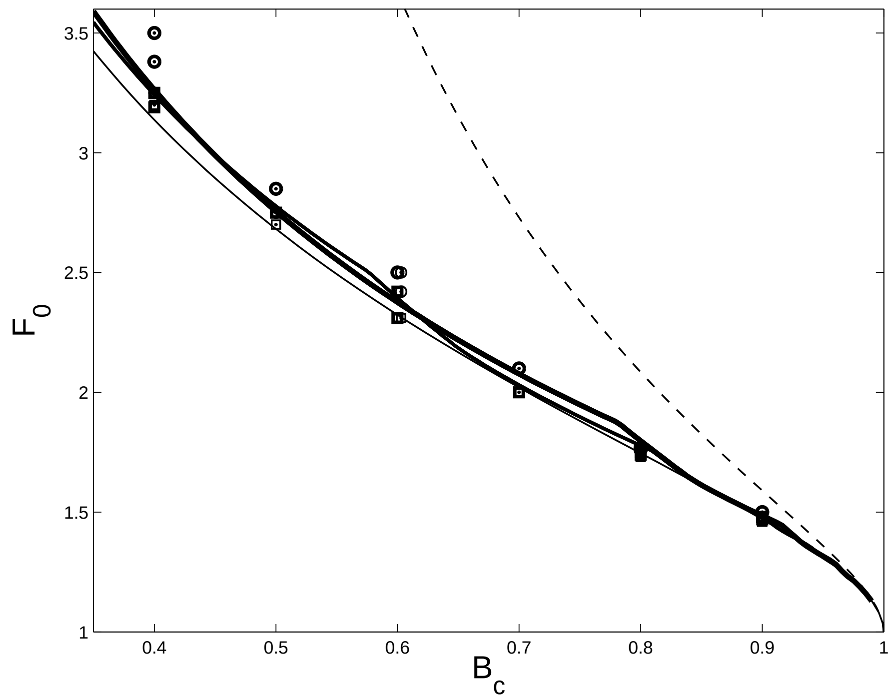
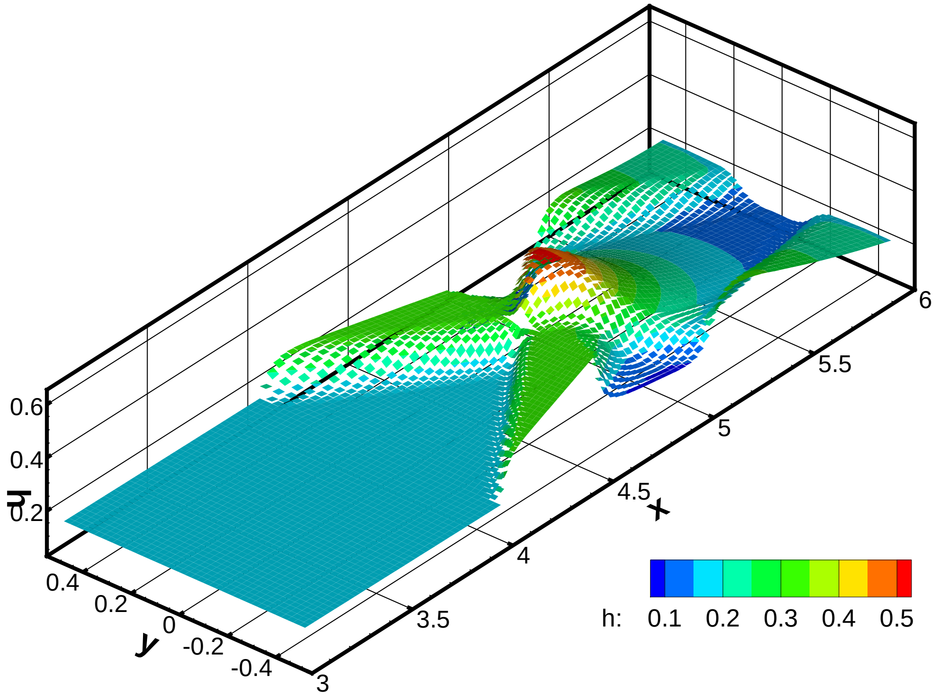
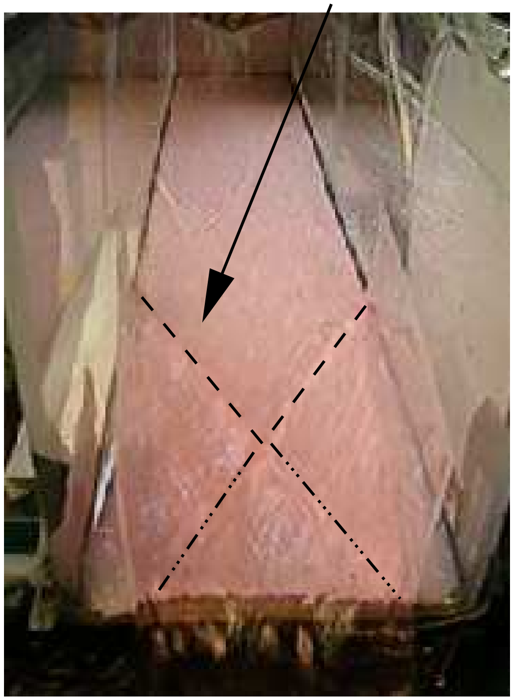


Fig. 7. 1D theory (thin solid); 2D theory: paddle lengths $L = 0.305m$ (thicker) and $L = 0.465m$ (thickest); and, simulations $L = 0.305m$ ($\circ/\Box$) and $L = 0.465m$ ($\circ/\Box$ with dot in center).

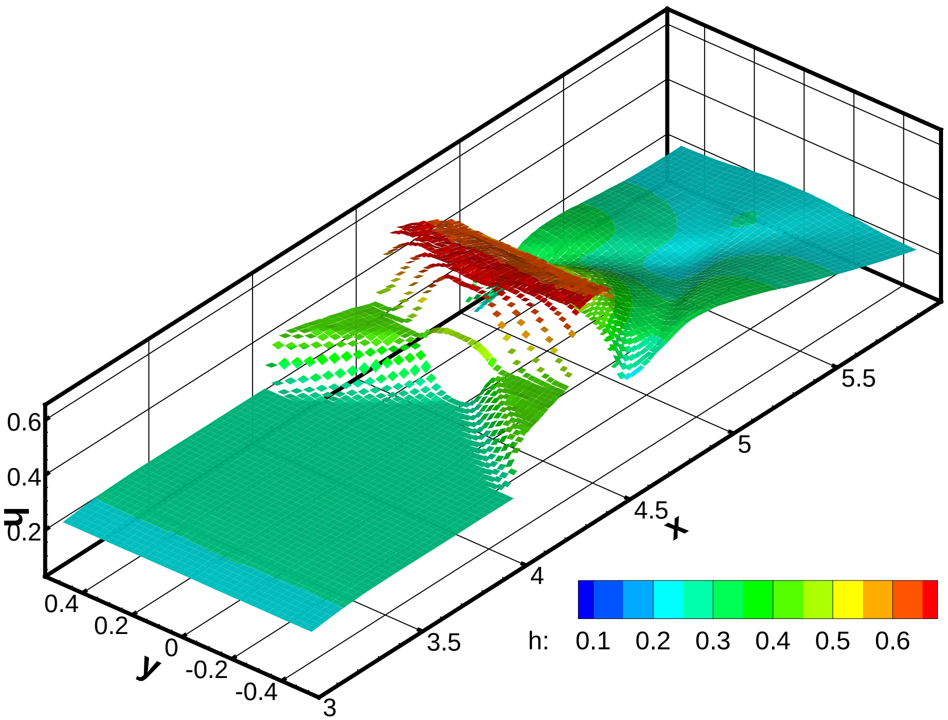
6. Slurry Flows



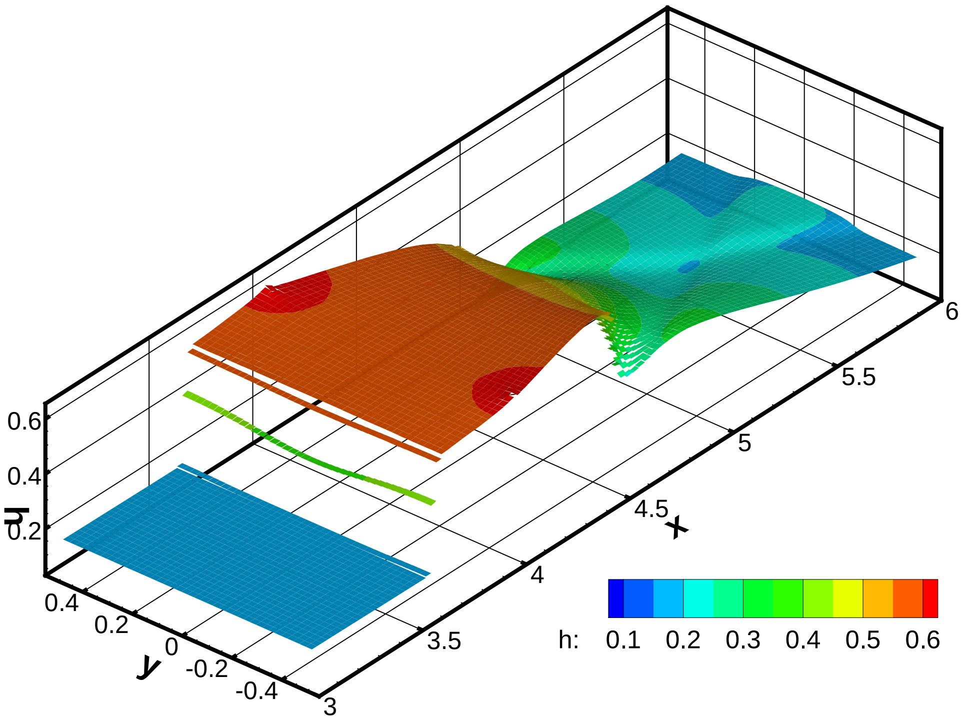
Oblique jumps

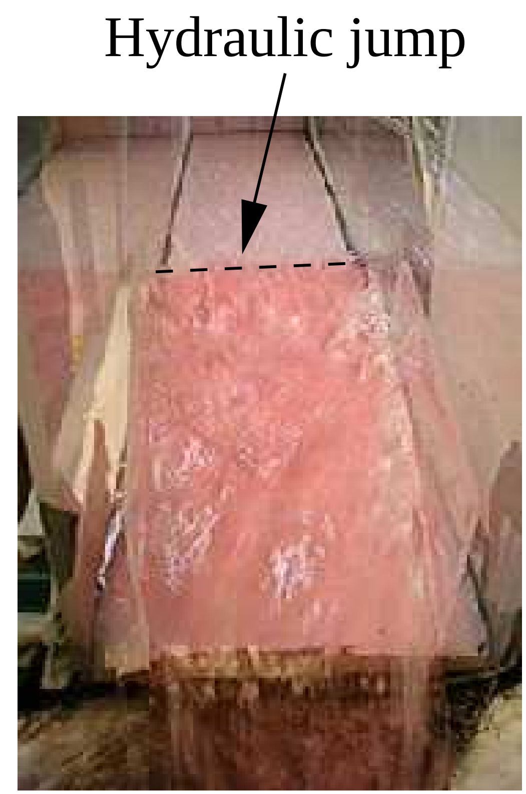


Depth-averaged 2-phase model



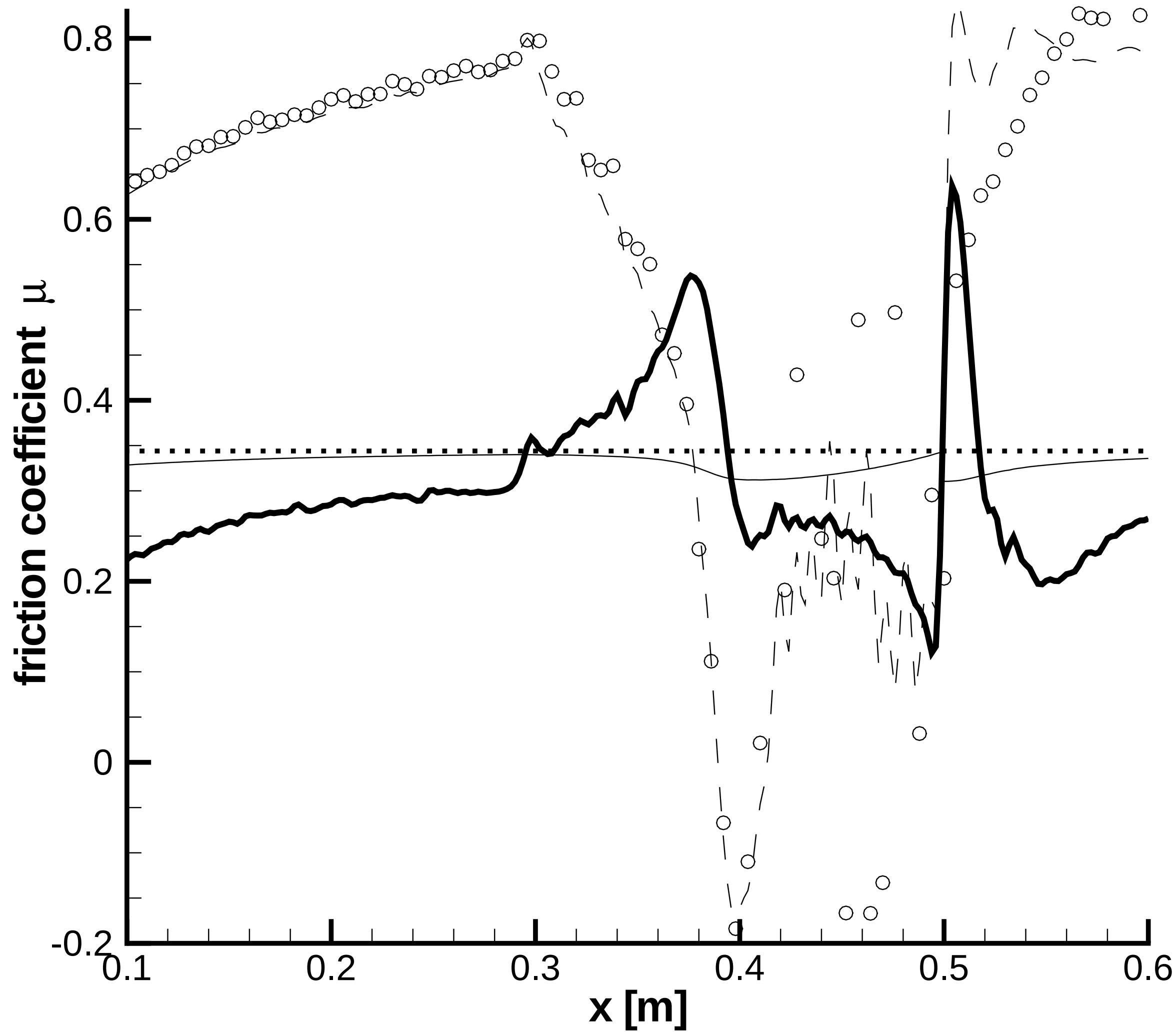
Discontinuous Galerkin FEM





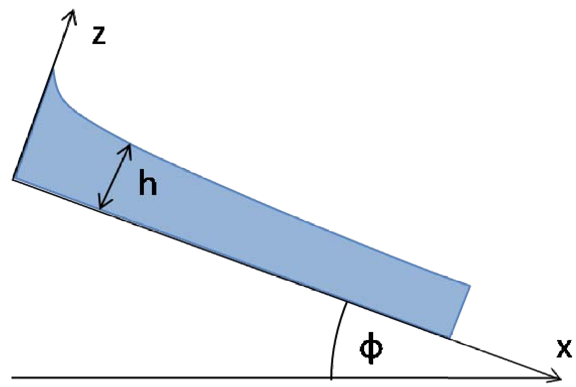
Heterogeneous Multiscale Modeling: granular flows

- Why?
  Rheology on smooth chutes incorrect: actual μ (thick solid); Cst. Coulomb friction (dotted); Lun et al. (1984) 1D (dashed) & 2D (circles); Savage & Hutter (thin solid).



Heterogeneous Multiscale Modeling

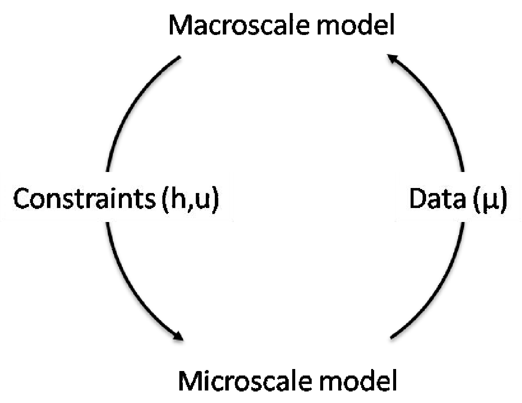
Granular chute flow



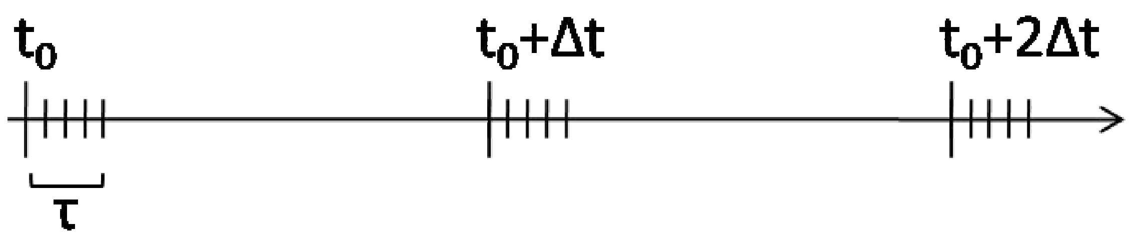
- Macro-model: Discontinuous Galerkin discretization of Shallow Water Equations with unknown basal friction coefficient μ.
- Micro-model: Molecular Dynamics simulation of granular flow around quadrature points constrained by macro-model.
- Test case: constitutive equation $\mu(h, \vec{u})$ known for flow over rough chutes (Pouliquen flow rule).

Heterogeneous Multiscale Modeling

- Coupling: Fix macro-variables while evolving micro model short time, then use data micro model to advance macro-model.



- Time-scale separation: Micro-variables relax in short time.



7. Conclusions

- ***Granular*** and ***hydraulic*** experiments and 1D theory with turbulent friction ***match quite well***.
- 1D theory demarcates 2D supercritical flow boundary surprisingly well, for inviscid bulk flow.
- Discrete particle model simulates granular flows well: analysis.
- ***Depth-averaged two-phase slurry flow simulation*** recovers first experiments.
- Extension to 2D hydraulic and morphodynamic theory of oblique jumps in progress.
- Advanced experiments ***fluid-particle slurry flows*** in progress.
- Hetereogeneuous Multiscale Modeling (Thornton, Weinhart)
- Current interest: ....

References

- Vreman, Al-Tarazi, Kuipers, Van Sint Annaland, & B. 2007: Supercritical shallow granular flow through a contraction: experiment, theory and simulation. *J. Fluid Mech.* 578.
- Akers & B., 2008: Hydraulic flow through a channel contraction: multiple steady states. *Phys. Fluids* 20.
- Tassi, Rhebergen, Vionnet, & B. 2008: A discontinuous Galerkin finite element model for morphological evolution under shallow flows. *Comp. Meth. Appl. Mech. Eng.* 197.
- Rhebergen, B. and Van der Vegt 2008: Discontinuous Galerkin finite element methods for hyperbolic nonconservative partial differential equations. *J. Comp. Phys.* 227.
- Pesch, Bell, Sollie, Ambati, B., & Van der Vegt 2007: hpGEM- A software framework for Discontinuous Galerkin finite element methods, *ACM Trans. Soft.* 33.
- www.math.utwente.nl/∼bokhoveo/ & http://eprints.eemcs.utwente.nl/