Vibro-fluidized granular matter

clustering and related phenomena

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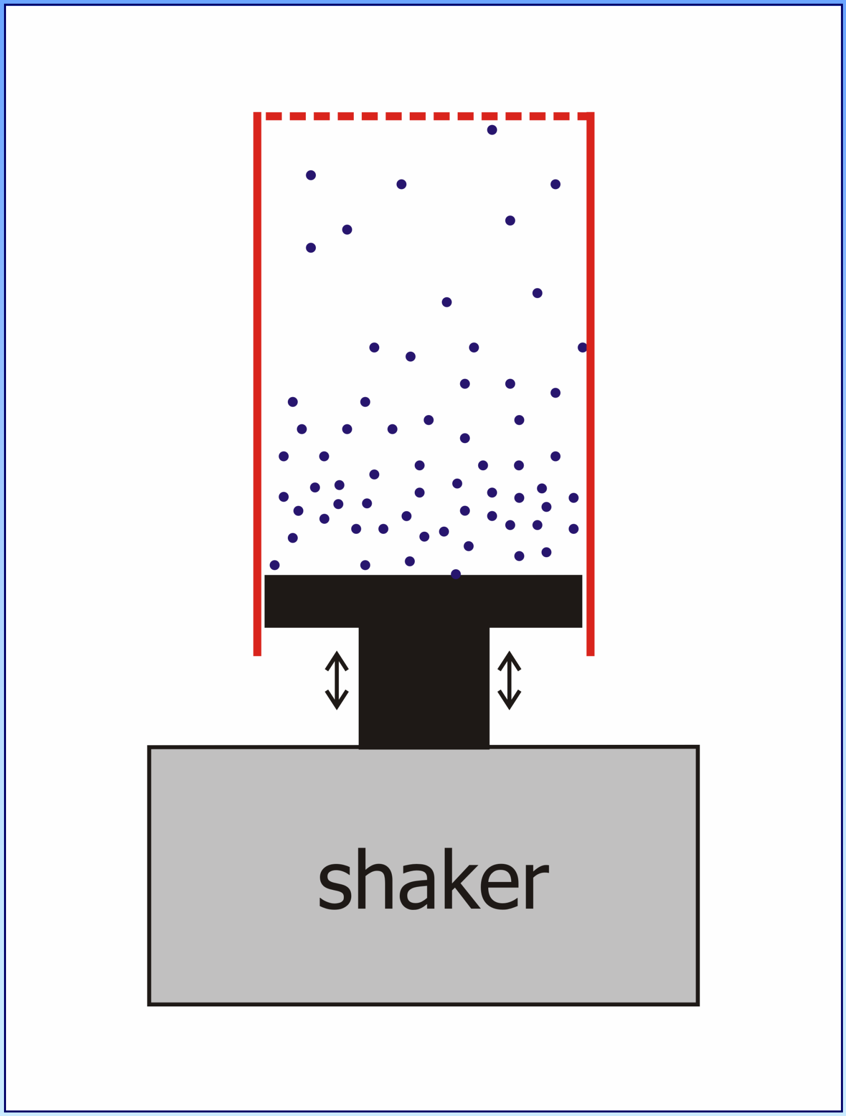
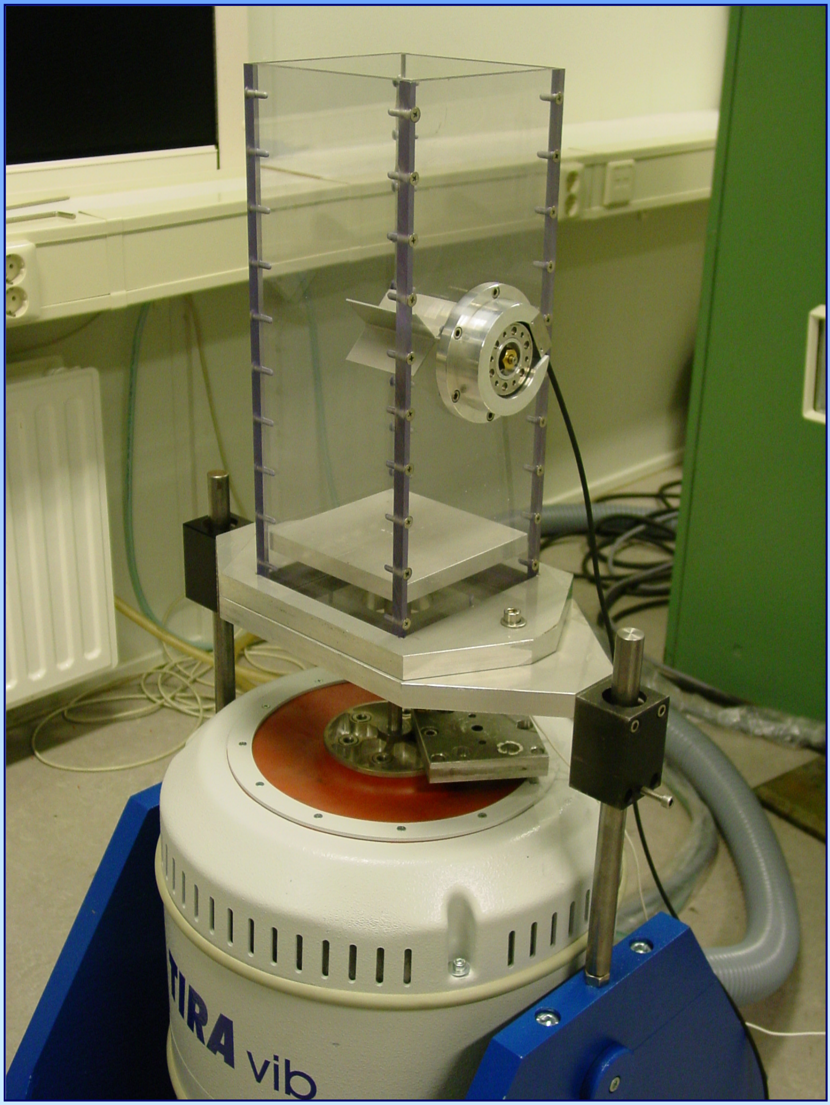
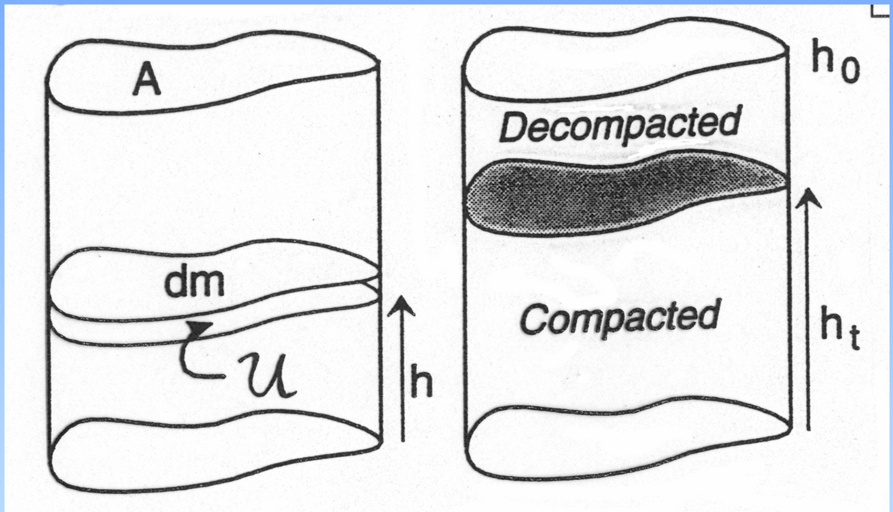
Vibrofluidization = shaking





Decompactification through shaking

Shaking: $a \sin(\omega t)$; dim.less acceleration: $\Gamma = \frac{a\omega^2}{g}$



"Decompacted" means: acceleration overcomes friction

Force balance: acceleration – gravity = (wall) friction:

$$\Gamma\, g\, dm - g\, dm = dF_{friction}$$

Sand moves freely if lhs > rhs !

Decompactification through shaking (threshold calculation)

Total height of stack: h_0

Threshold condition lhs>rhs fullfilled from h_t (<h_0) on.

$$\Gamma = 2 - \exp\left(-\frac{K\mu_s U}{A}(h_0 - h_t)\right)$$

μ_s = static friction coefficient

K = redirection parameter

$\Gamma = 1$ means: $h_t = h_0$; nothing can be fluidized

$\Gamma = 2$ or larger: all can be fluidized

Vertically shaken granular matter: relevant dimensionless parameters

1. $$\frac{\text{bottom energy}}{\text{gravitational energy}} = \frac{a^2\omega^2}{gH}$$ H typical (vertical) lengthscale

Mildly fluidized: particles move with bottom, $H = a$

$$\frac{a^2\omega^2}{ga} = \frac{a\omega^2}{g} = \Gamma$$ dimensionless acceleration

Vigorously fluidized: take intrinsic l.s. $H = R$

$$\frac{a^2\omega^2}{gR} \equiv S$$ dimensionless shaking strength

2. **Dissipation per particle:**

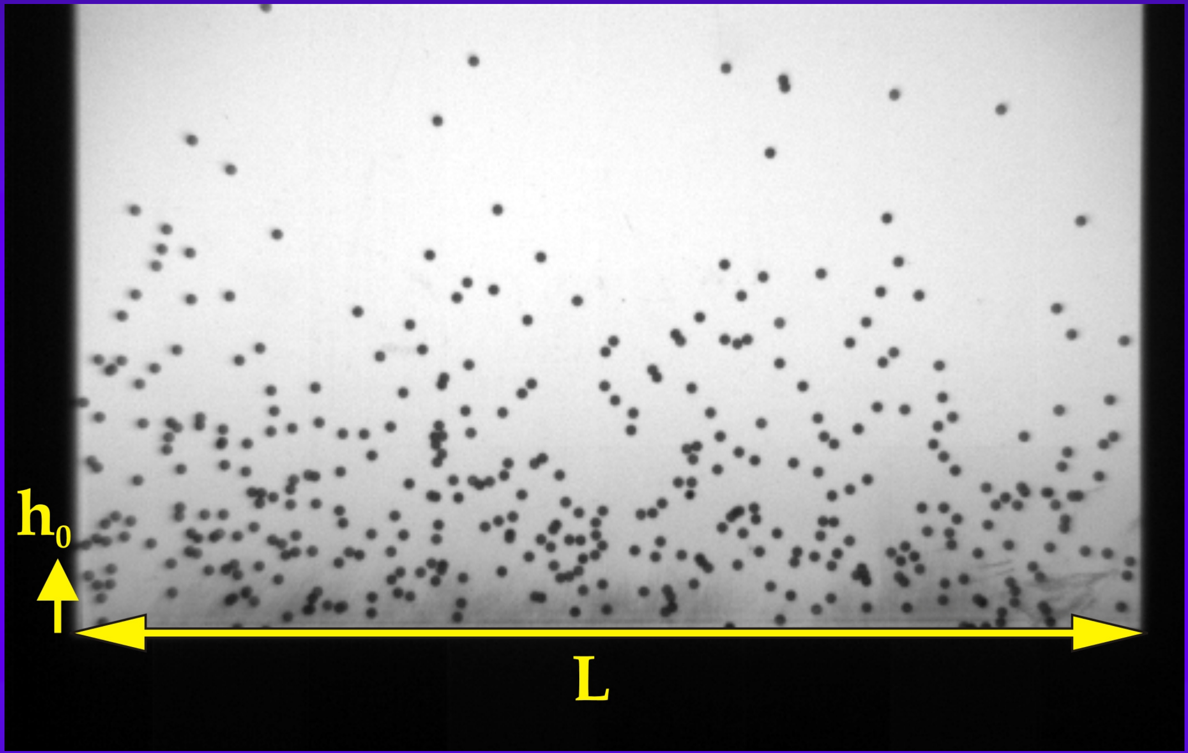
$$\frac{E_{\text{k,bef.}} - E_{\text{k,aft.}}}{E_{\text{k,bef.}}} = \frac{E_{\text{k,bef.}} - e^2 E_{\text{k,bef.}}}{E_{\text{k,bef.}}} = 1 - e^2 \equiv \varepsilon$$ inelasticity

3. **Filling factor: (layers of particles)**

$$\frac{\pi R^2 N}{\Omega} = F$$ filling factor

Quasi 2-D container

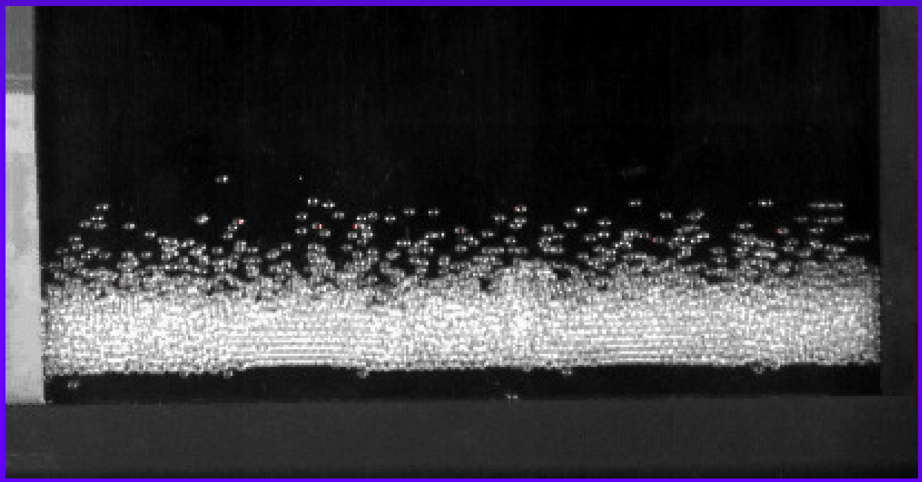
$L \times D \times H = 101 \times 5 \times 150$ mm



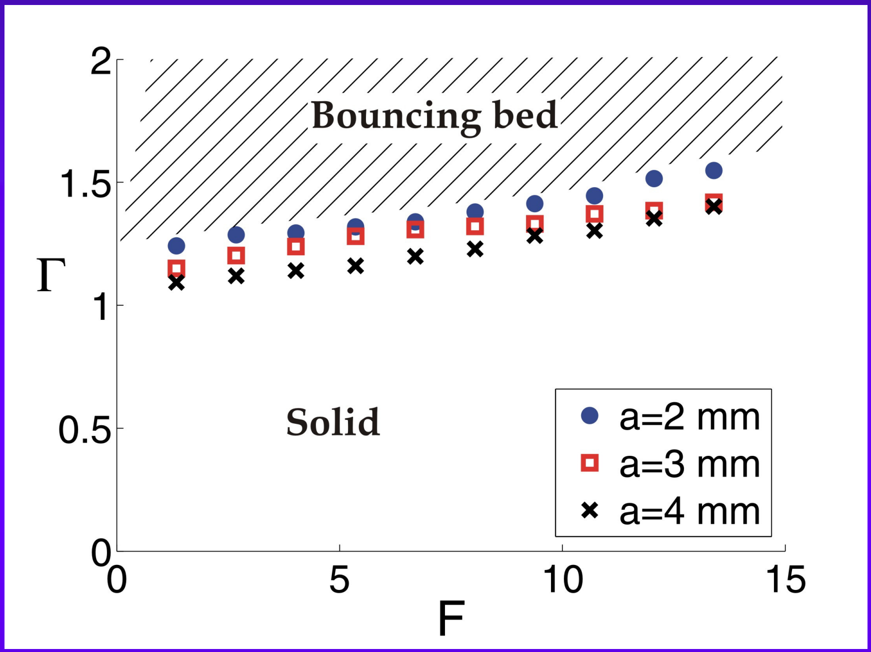
- Glass beads: d =1.0 mm, $e \approx 0.95$
- Frequency f linearly increased, amplitude a fixed

4. Aspect ratio: $\frac{L}{h_0}$ remains large (h$_0$ = bed height at rest)

1. Bouncing bed



F = 8.1 layers, a = 4.0 mm, f = 12 Hz

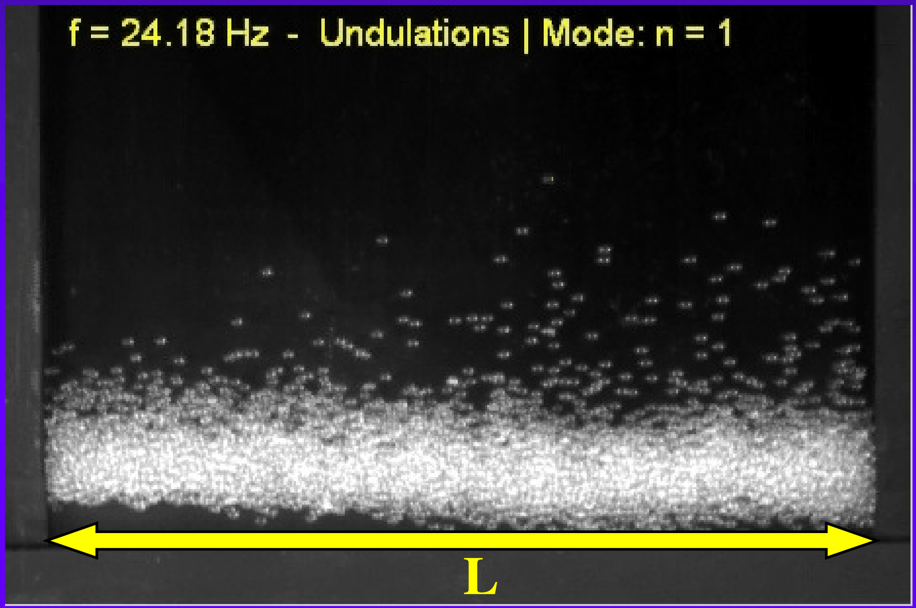


The granular bed bounces as a single body

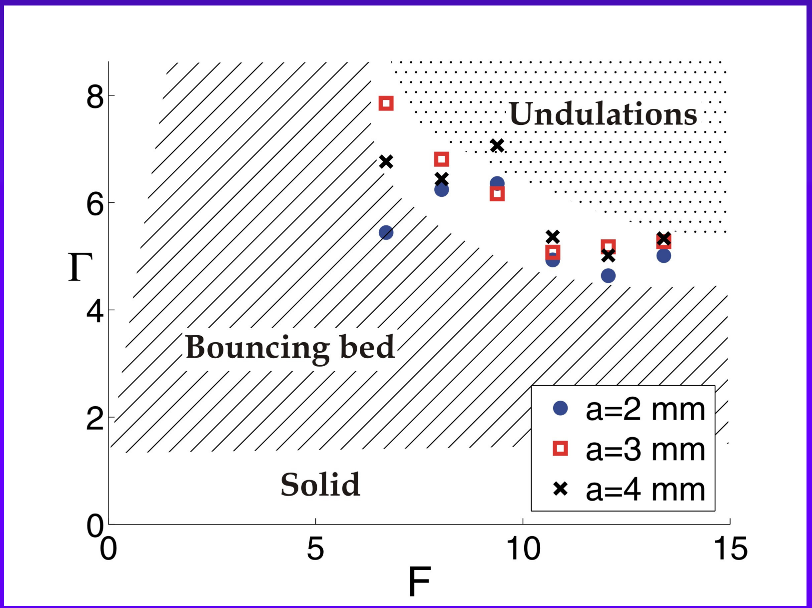
Mild fluidization → Γ

Mehta and Luck, *Phys. Rev. Lett.* **65**, 393 (1990)

2. Undulations



F = 8.1 layers, amplitude *a* = 3.0 mm

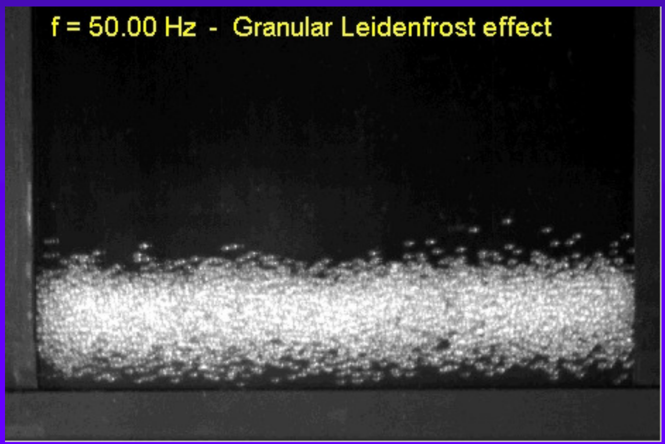


Standing wave pattern oscillating at 2T

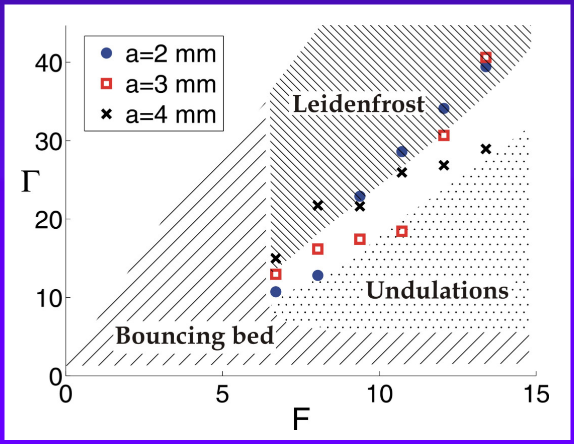
Mild fluidization → Γ

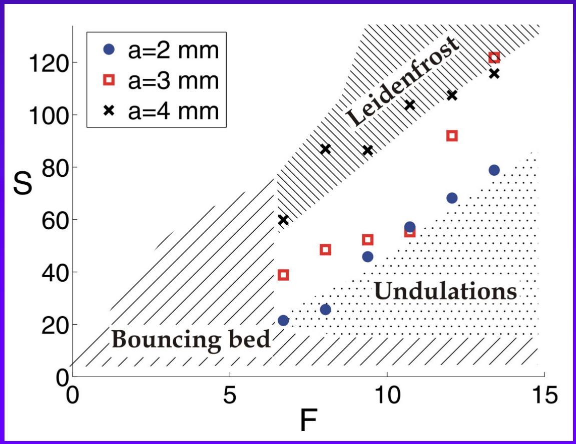
Sano, *Phys. Rev. E* **72**, 051302 (2005)

3. Granular Leidenfrost effect



F = 8.1 layers, a = 3.0 mm



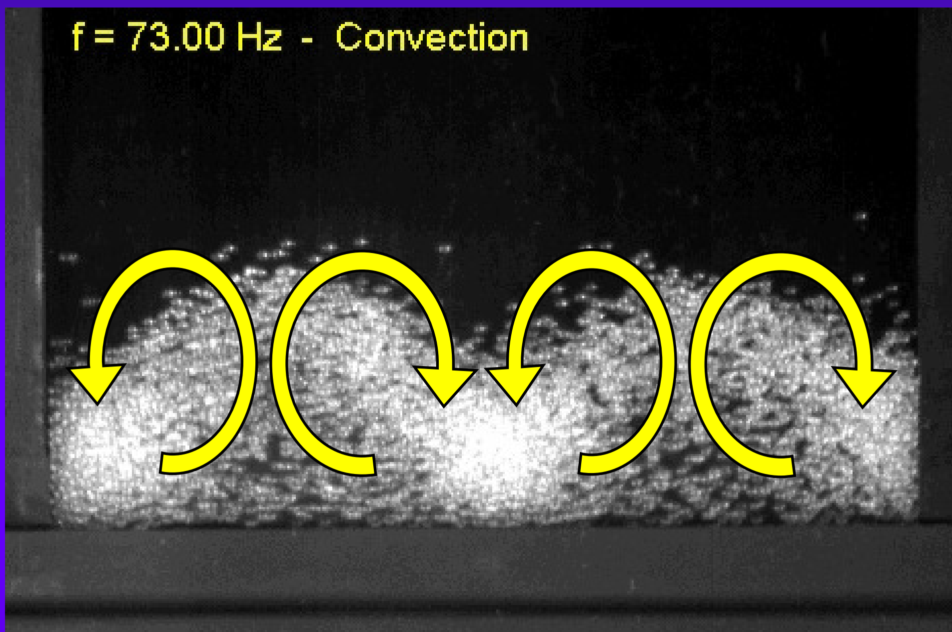


Dense cluster elevated by dilute layer of fast particles

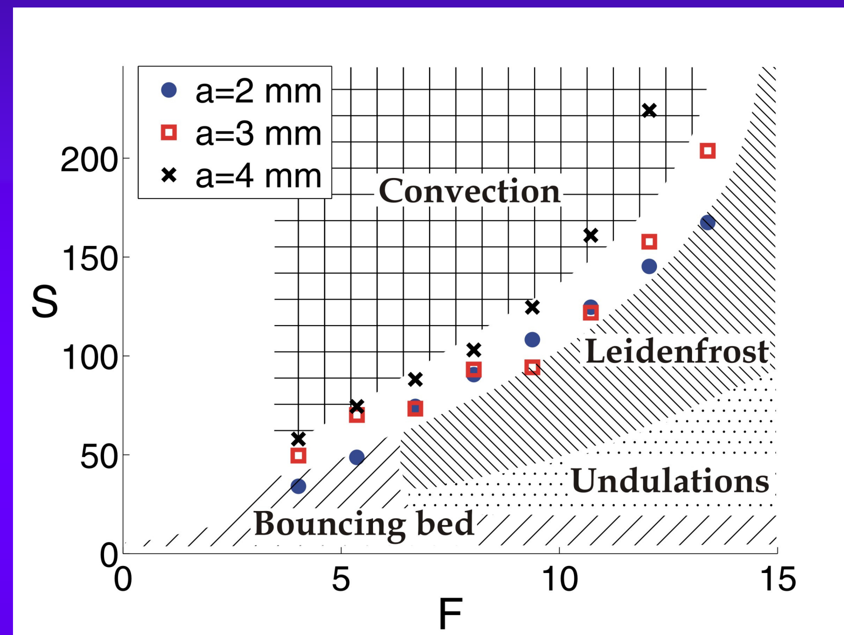
Intermediate fluidization: both T and S → Candidates…

Meerson *et al.*, *Phys. Rev. Lett.* **91**, 024301 (2003)
Eshuis *et al.*, *Phys. Rev. Lett.* **95**, 258001 (2005)

4. Convection



F = 8.1 layers, a = 3.0 mm

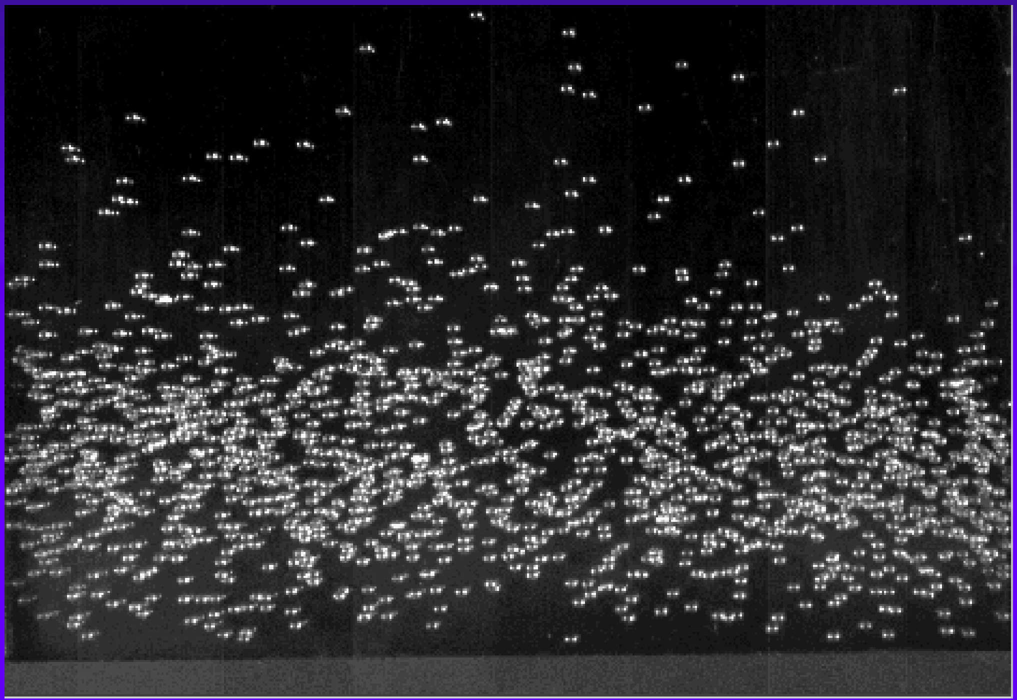


Counter-rotating rolls like Rayleigh-Bénard convection

Strong fluidization → S

Paolotti *et al.*, *Phys. Rev. E* **69**, 061304 (2004)

5. Gas

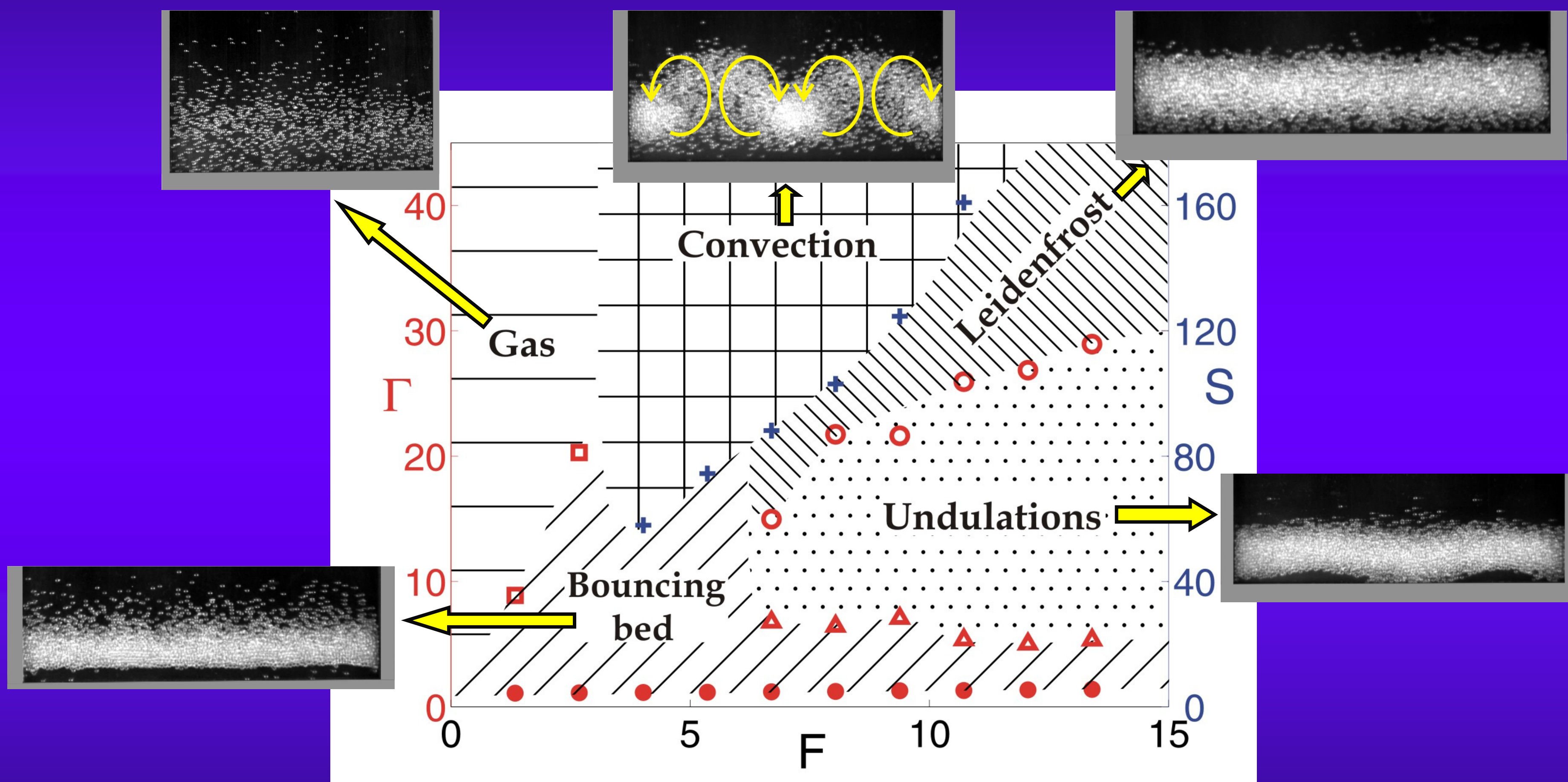


F = 2.7 layers, amplitude *a* = 3.0 mm, frequency *f* = 50 Hz

Shaking parameter → either Γ (from bouncing bed) or S (from convection)

Grossman *et al.*, *Phys. Rev. E* **55**, 4200 (1997)

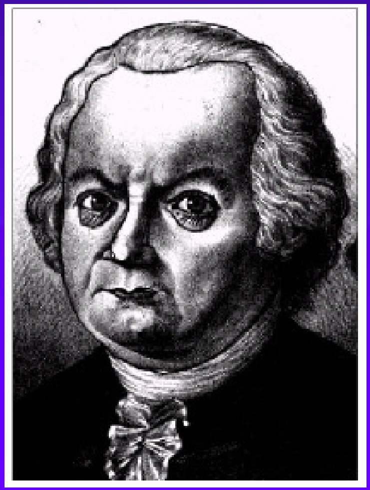
Phase Diagram

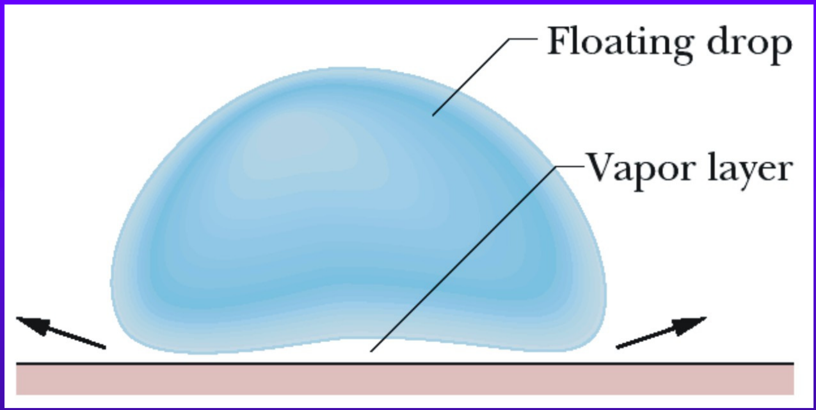


a = 4.0 mm

P. Eshuis, K. van der Weele, D. van der Meer, R. Bos, and D. Lohse, *"Phase Diagram of Vertically Shaken Granular Matter"*, Phys. Fluids **19**, 123301 (2007)

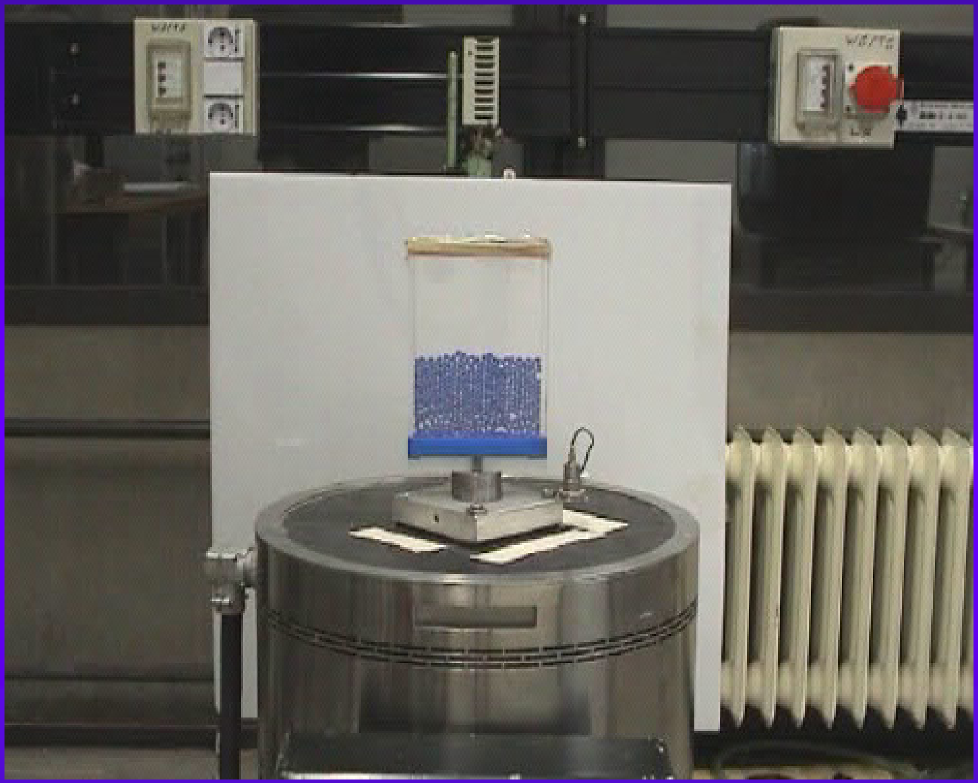
Johann Gottlob Leidenfrost (1756)





Drop of water on a hot plate (≥ 220° C)

Granular version

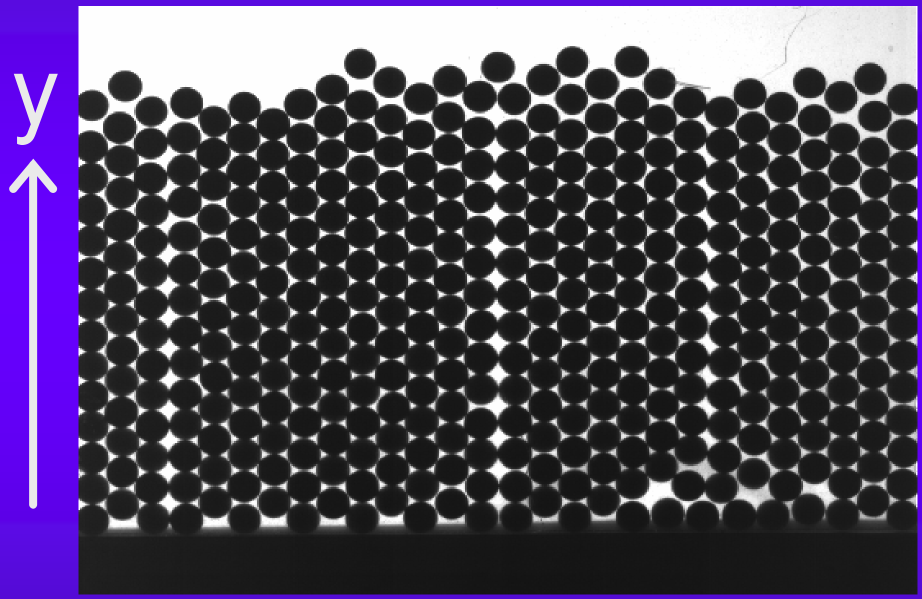


Granular temperature at bottom ~ Shaking strength

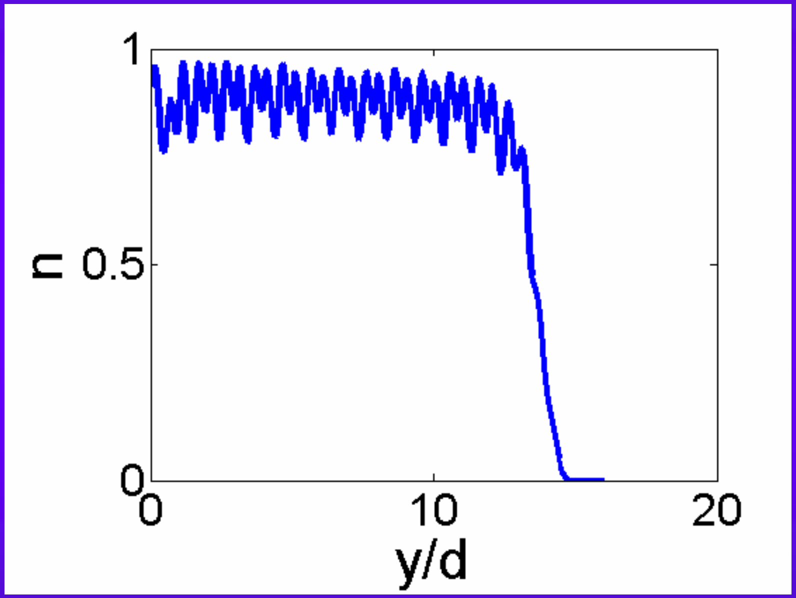
2D container: $10 \times 0.45 \times 14$cm, Glass beads: $d = 4$mm, $\rho = 2.5$g/cm^3, $e \approx 0.9$

Leidenfrost state beyond critical acceleration Γ_c

F=16 layers, f=80Hz



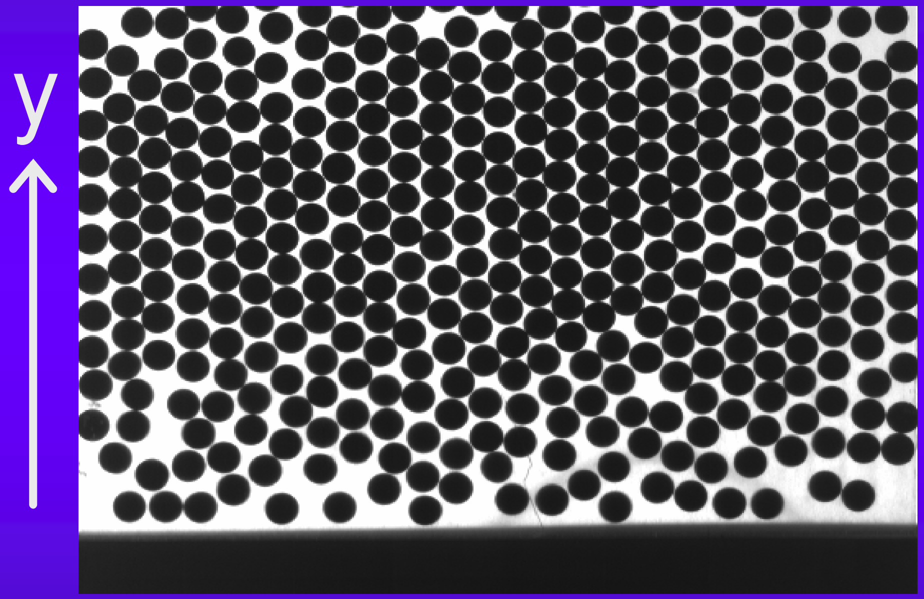
$\Gamma = 7.7$



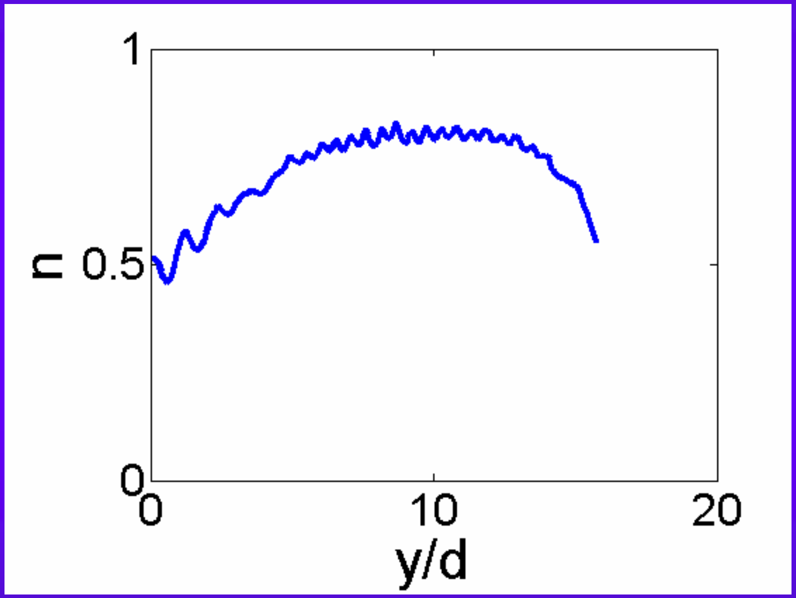
Solid phase

Leidenfrost state beyond critical acceleration Γ_c

F=16 layers, f=80Hz



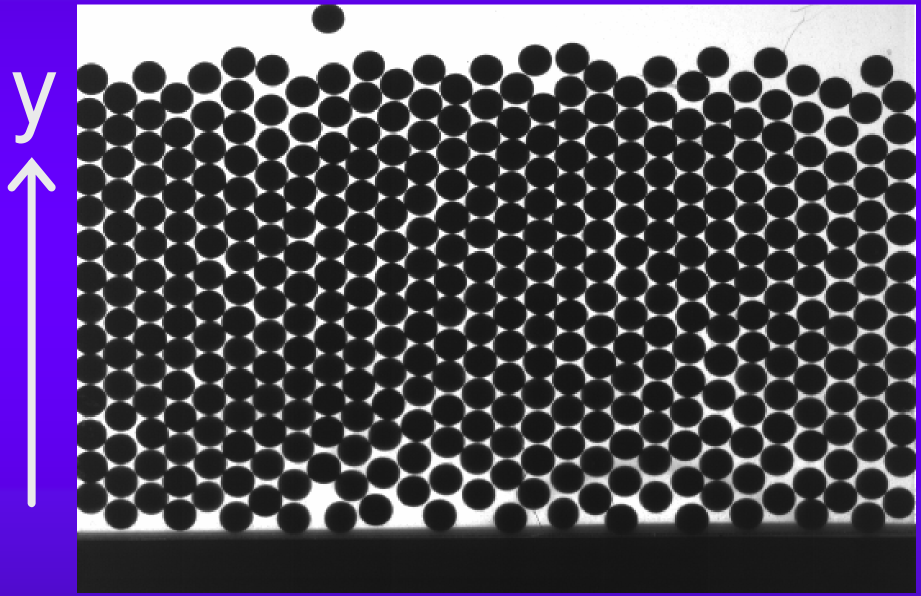
$\Gamma = 51.5$

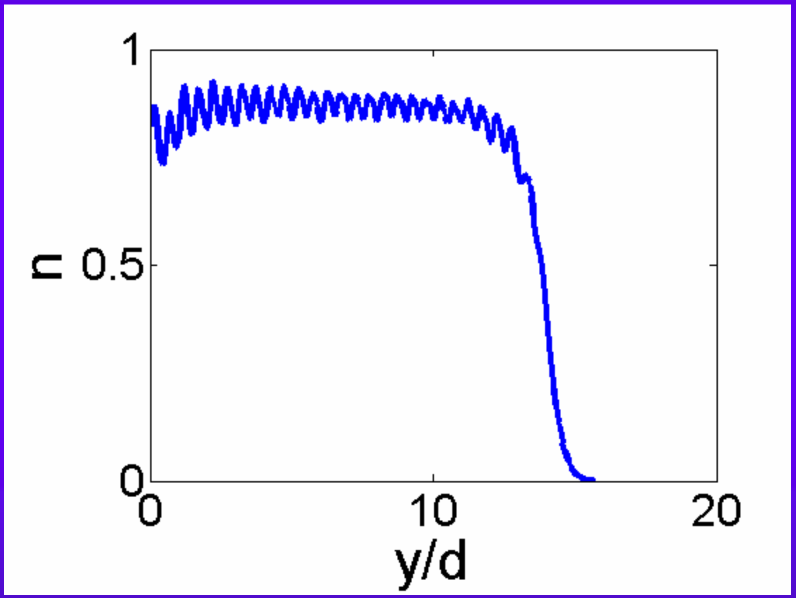


Leidenfrost state

Leidenfrost state beyond critical acceleration Γc

F=16 layers, f=80Hz





$\Gamma = 25.8$

Transition

$\Gamma_c \approx 25$ (for F = 16 layers)

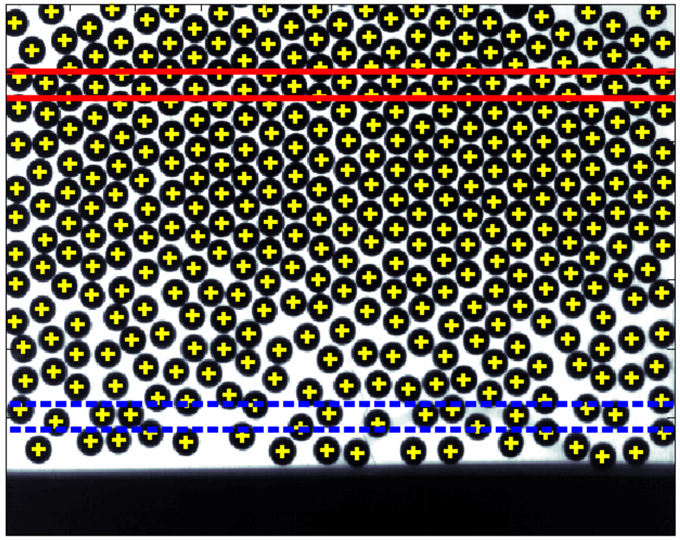
What's a suitable ***order parameter*** to distinguish between the different phases in the Leidenfrost state?

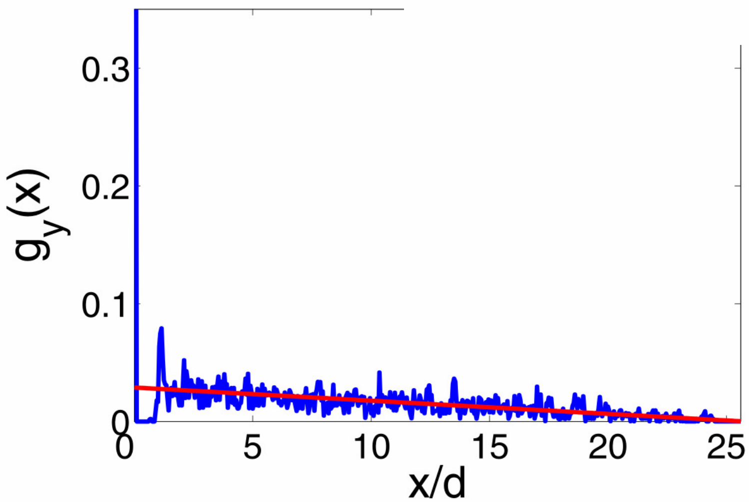
→ Employ the concept of *pair correlations*:

$$g_y(x) = \frac{1}{N} \sum_{i,j \, in \, (y, y+dy)} \sum_{i \neq j} \delta\left(x - \left(x_i - x_j\right)\right)$$

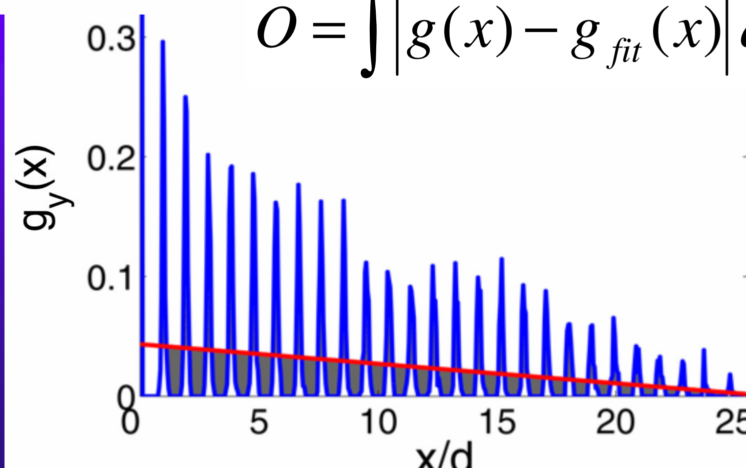
Identifying the order parameter

F=16 layers
Γ=64.4

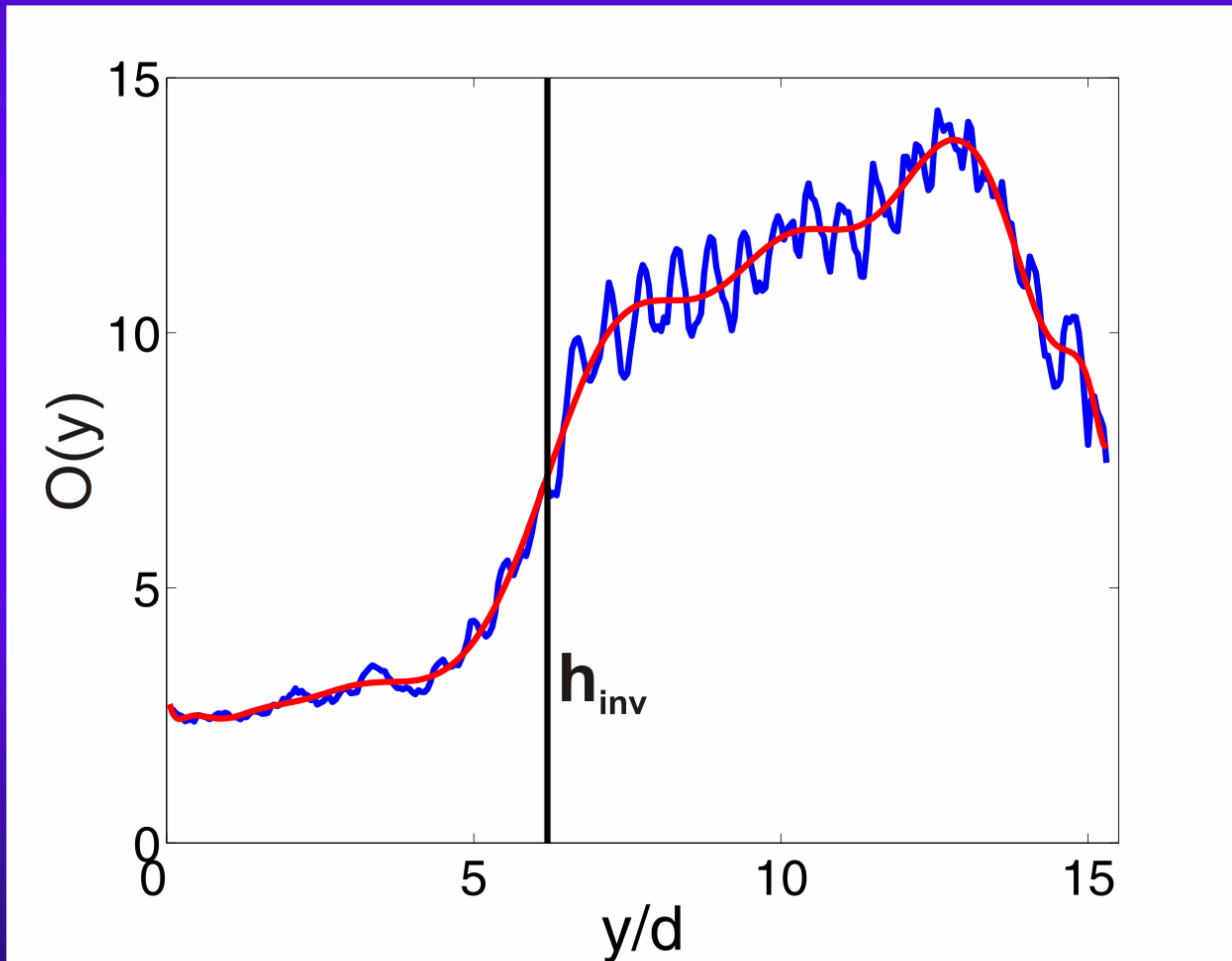




$$O = \int \left| g(x) - g_{fit}(x) \right| dx$$

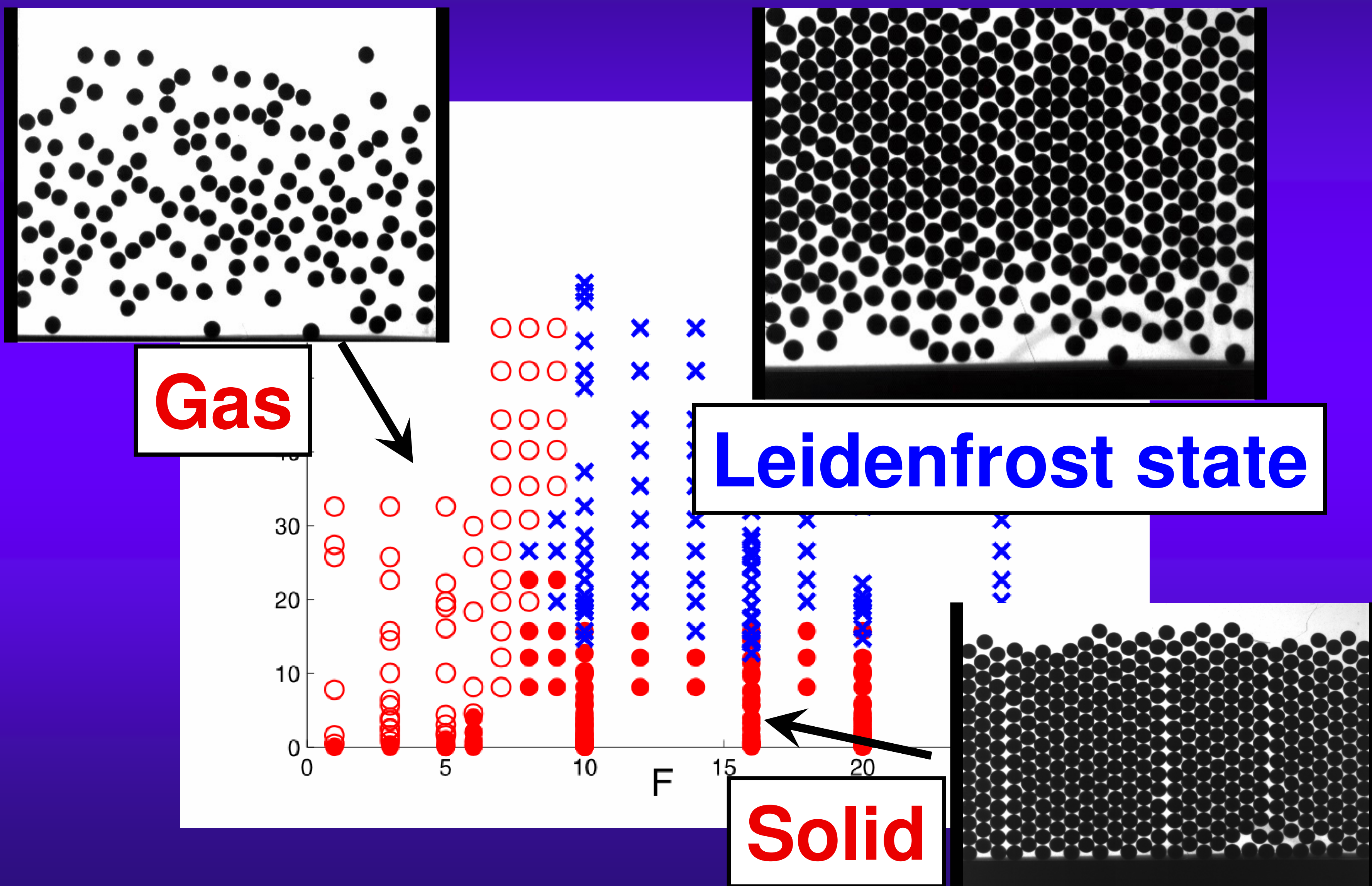


Order parameter *O* determines inversion height:



F=16 layers
Γ=64.4

Phase diagram in S-F plane



Hydrodynamic model

(1) Force balance: $$\frac{dp}{dy} = -mgn$$

(2) Balance between heat flux and dissipation:

$$\frac{d}{dy}\left\{\kappa \frac{dT}{dy}\right\} = \frac{\mu}{\gamma l} \varepsilon n T^{3/2}$$

(3) Equation of state: $$p = nT \frac{n_{cp} + n}{n_{cp} - n}$$

cf. Meerson *et al.*, PRL **91** (2003)

3 Boundary conditions

- **Prescribed granular temperature at bottom:**

$$T_0 \propto (af)^2$$

- **Zero heat flux at top:**

$$\lim_{y\to\infty}\left(\kappa(y)\frac{dT}{dy}\right)=0$$

- **Conservation of total number of particles:**

$$\int_0^\infty n(y)dy = F\, n_{cp} d$$

Dimensionless control parameters

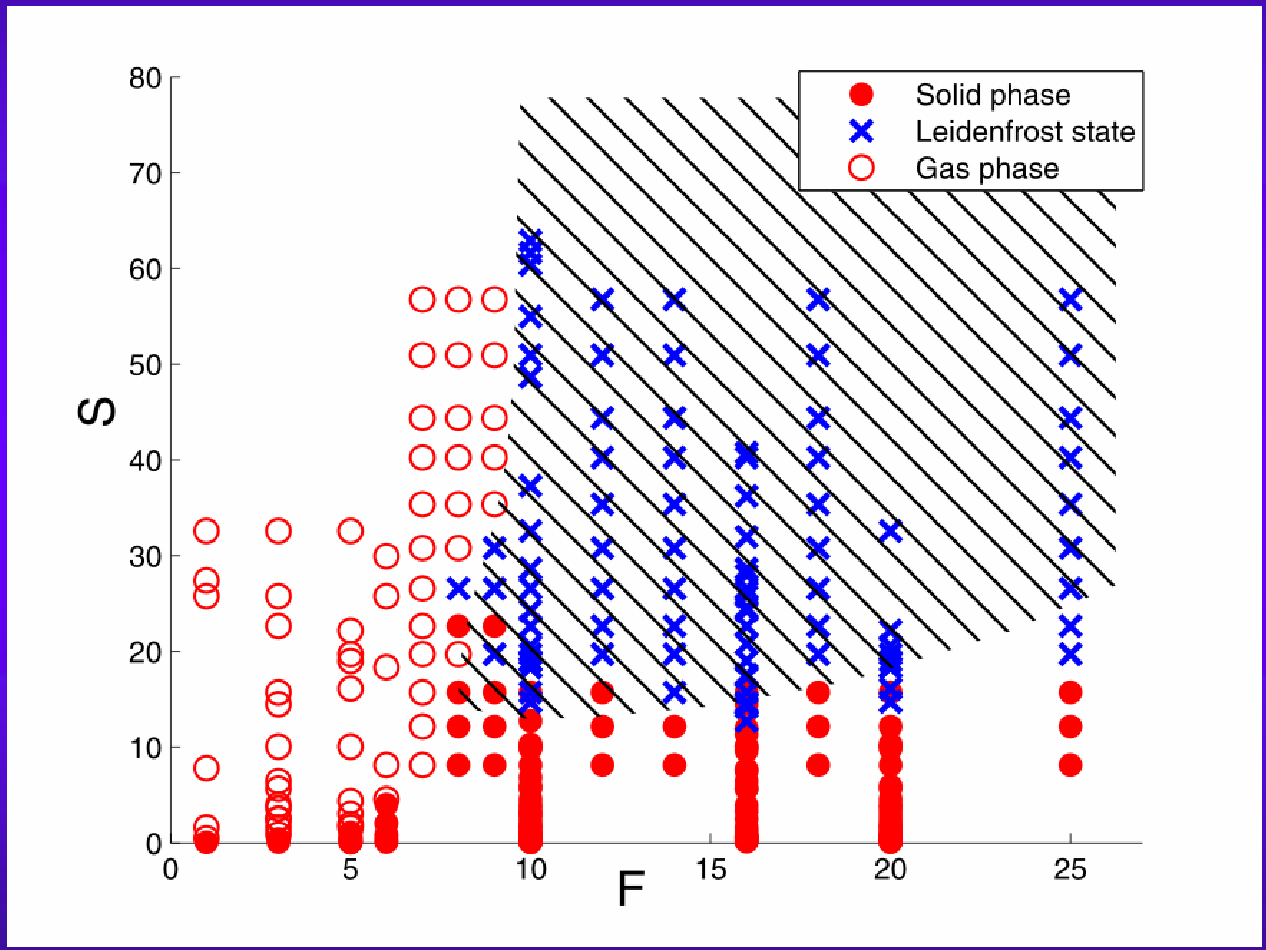
Energy input: $S = \dfrac{4\pi^2 (af)^2}{gd}$

Inelasticity: $\varepsilon = (1 - e^2)$

Number of layers: F

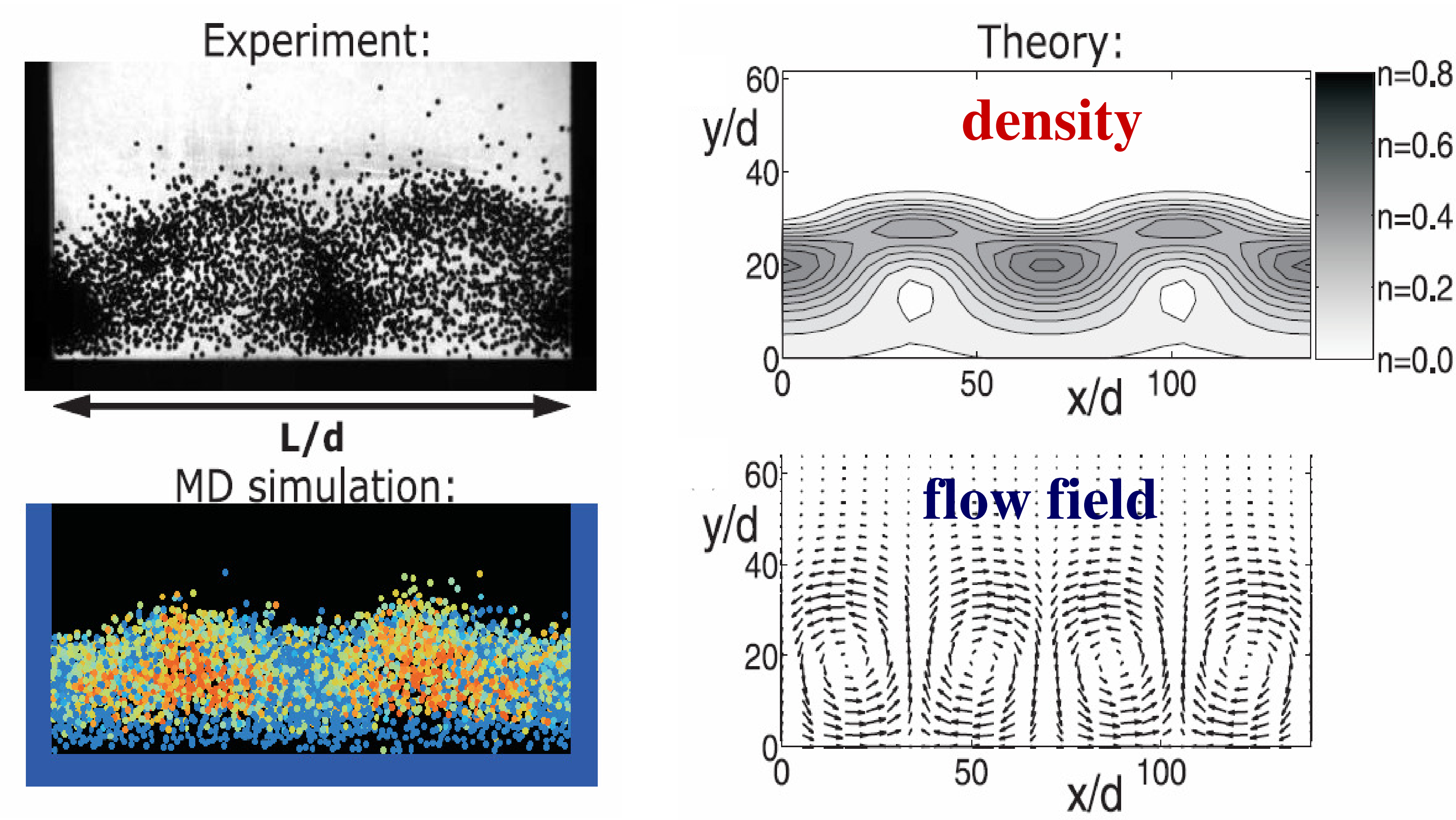
Just as in experiment, the relevant shaking parameter is $S \equiv \Gamma A$ (not Γ)

Experimental phase diagram and theoretical!



P. Eshuis, K. van der Weele, D. van der Meer, D. Lohse, *Granular Leidenfrost effect: Experiment and theory of floating particle clusters*, Phys. Rev. Lett. **95, 258001 (2005)**

Granular convection



How did we obtain the theoretical result ?

Starting point:

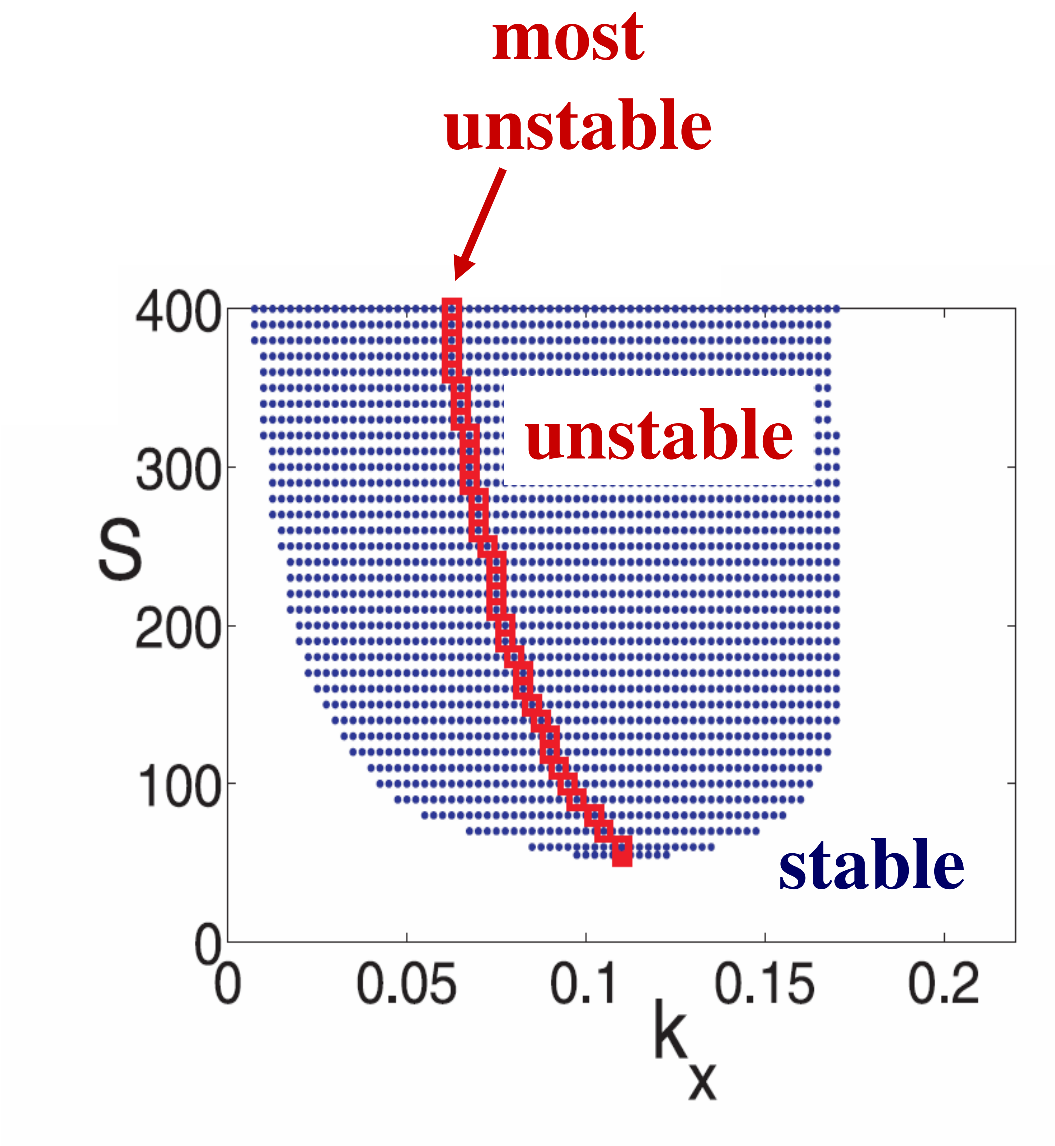
Leidenfrost solution $n_L(y)$, $T_L(y)$

Perform linear stability analysis of the full granular hydrodynamic equations

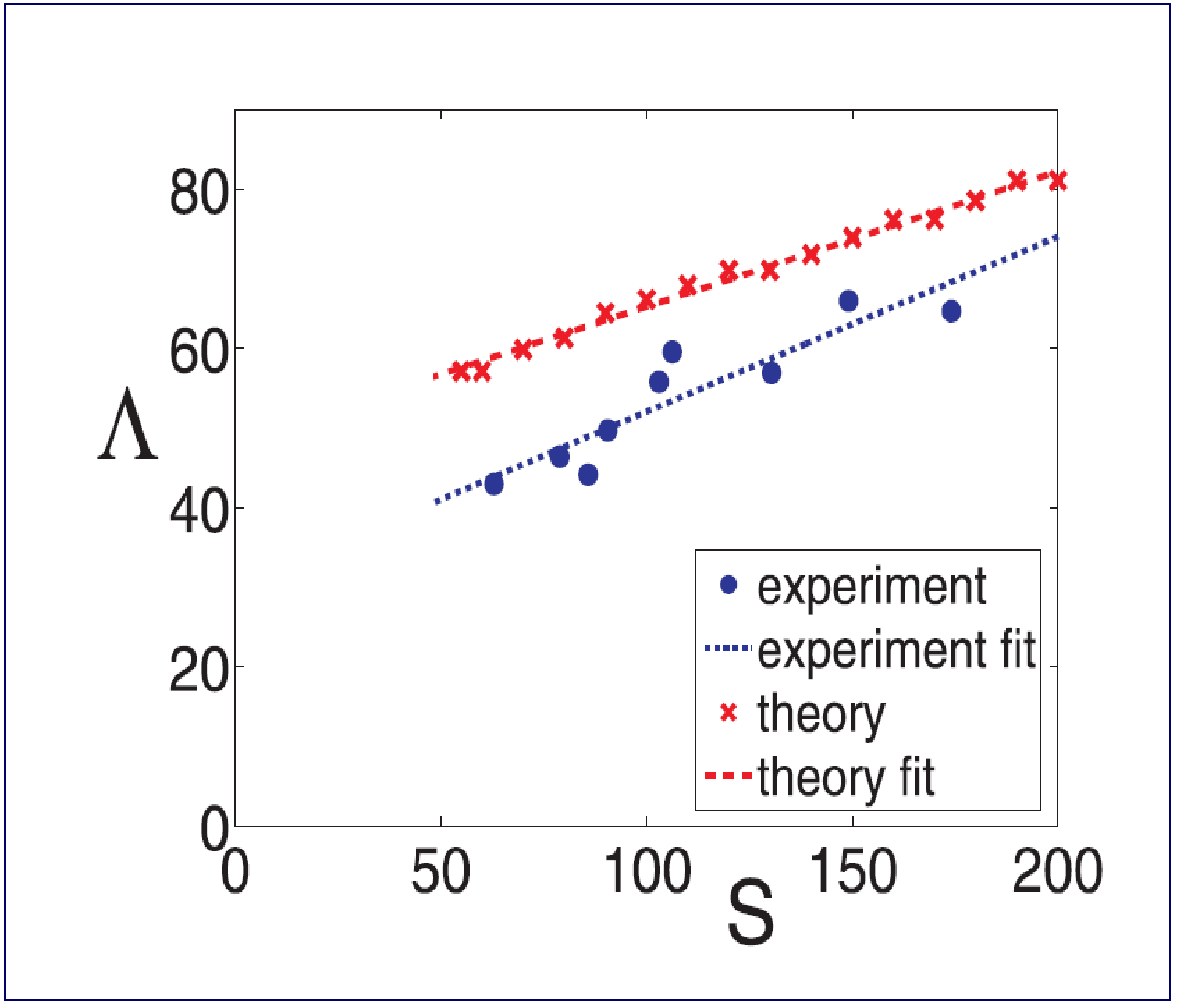
Determine the most unstable wavelength

$\rightarrow$ length of convection roll

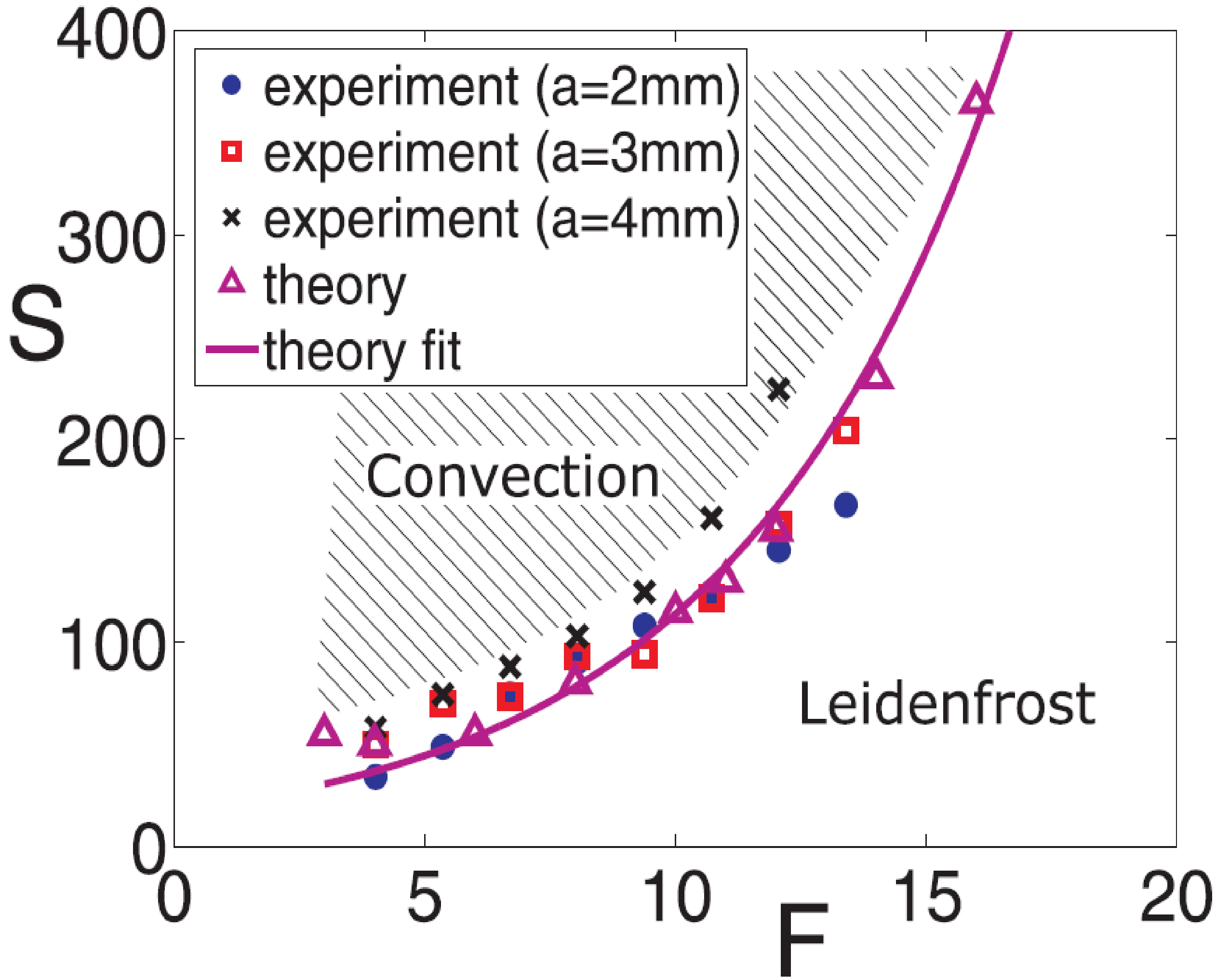
Granular convection



Granular convection



Again, the phase diagram



Granular gases

Vertically vibrated granular gas

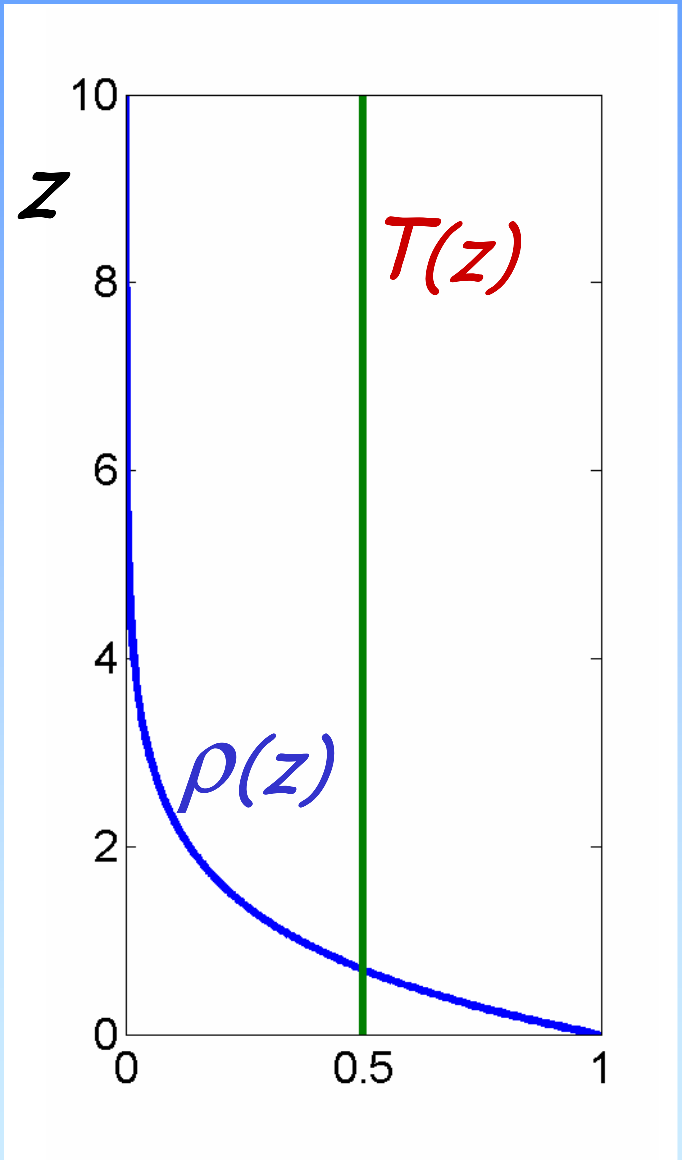
* Granular temperature:

$$T \equiv \langle v^2 \rangle$$

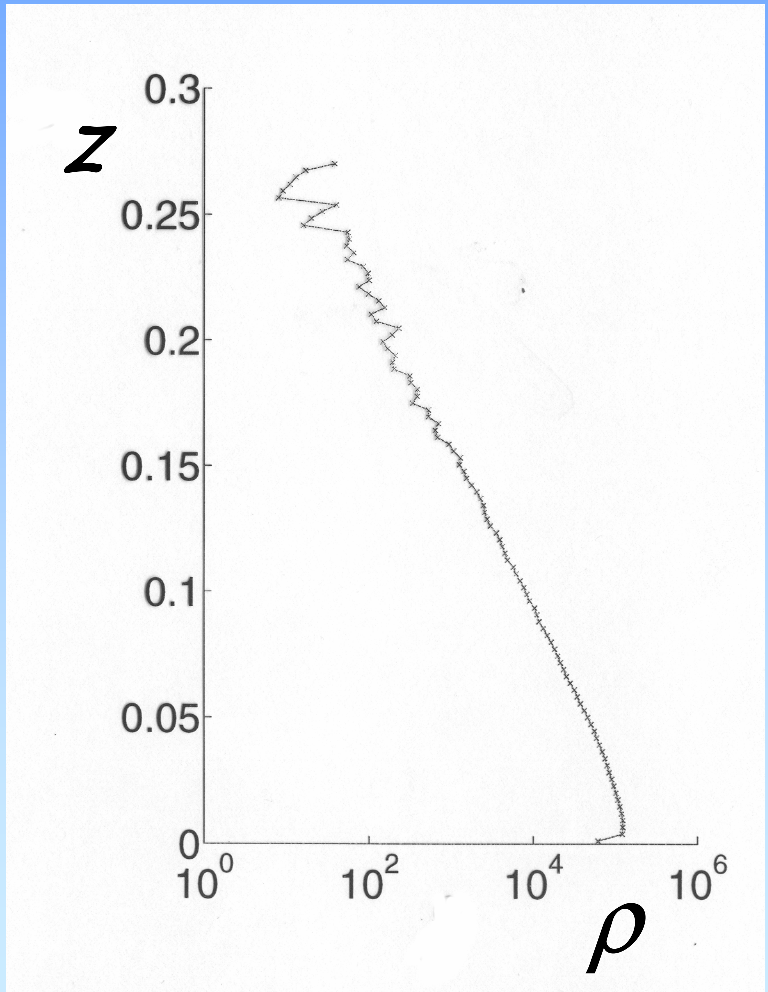
* For dilute system:
  T roughly independent of z

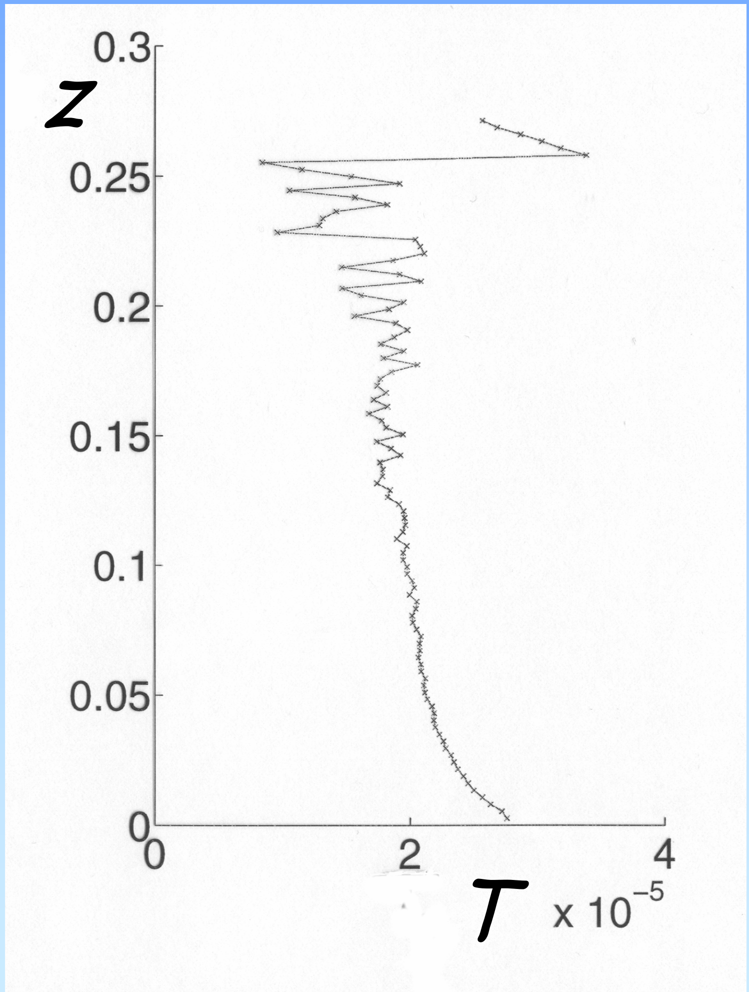
* Barometric height distribution:

$$\rho(z) \equiv \frac{gN}{T} \exp\left(-\frac{gz}{T}\right)$$



Density and temperature in MD simulations





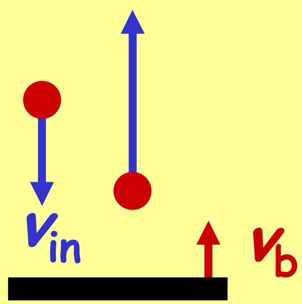
T follows from energy balance

Energy input at bottom:

$$E_{in} \sim \rho(0)\sqrt{T}\,\Delta E_k \sim \frac{gN}{T}\sqrt{T}\;\boxed{v_b\sqrt{T}}$$

For sawtooth driving:

v_{out} = $-v_{in} + 2v_b$



take: $v_{in} = \sqrt{T}$

$$\begin{aligned}\Delta E_k &= \tfrac{1}{2} m v_{out}^2 - \tfrac{1}{2} m v_{in}^2 \\ &= \tfrac{1}{2} m(\sqrt{T} + 2v_b)^2 - \tfrac{1}{2} m T \\ &\approx 2 m v_b \sqrt{T}\end{aligned}$$

T follows from energy balance

Energy input at bottom:

$$E_{in} \sim \rho(0)\sqrt{T}\,\Delta E_k \sim \frac{gN}{T}\sqrt{T}\,v_b\sqrt{T}$$

Energy dissipation in the system

$$E_{diss} \sim \int_{z=0}^{\infty} \rho(z)^2 \sqrt{T}\;\varepsilon T\,dz \sim \varepsilon T^{3/2}\,\frac{gN^2}{T}$$

Integral gives:

$$\int_{z=0}^{\infty} \rho(z)^2\,dz = \left(\frac{gN}{T}\right)^2 \int_{z=0}^{\infty} \exp\left(-\frac{2gz}{T}\right) dz = \frac{gN^2}{2T}$$

T follows from energy balance

Energy input at bottom:

$$E_{in} \sim \rho(0)\sqrt{T}\,\Delta E_k \sim \frac{gN}{T}\sqrt{T}\,v_b\sqrt{T}$$

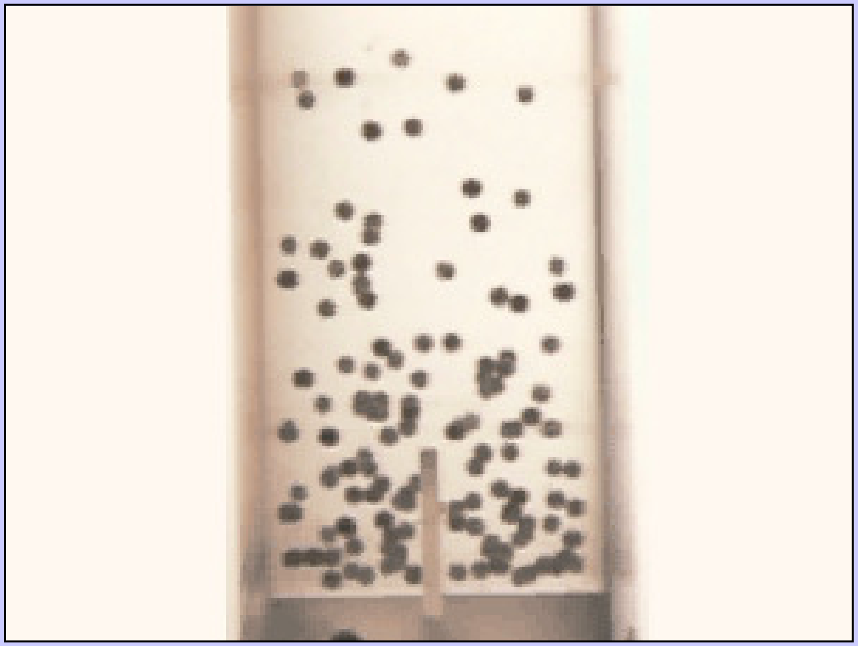
Energy dissipation in the system

$$E_{diss} \sim \int_{z=0}^{\infty} \rho(z)^2\sqrt{T}\;\varepsilon T\,dz \sim \varepsilon T^{3/2}\frac{gN^2}{T}$$

Equating energy input and dissipation gives:

$$gNv_b \sim g\varepsilon N^2\sqrt{T} \quad \Longrightarrow \quad T \sim \frac{v_b^2}{\varepsilon^2 N^2}$$

Compartmentalized granular gases



Shaking strength: high low

Reason clustering:

Inelastic collisions !

Flux function

Flux through the hole is:

F= density * velocity * area hole

$$F \sim \rho_1(h)\sqrt{T_1}\,S$$

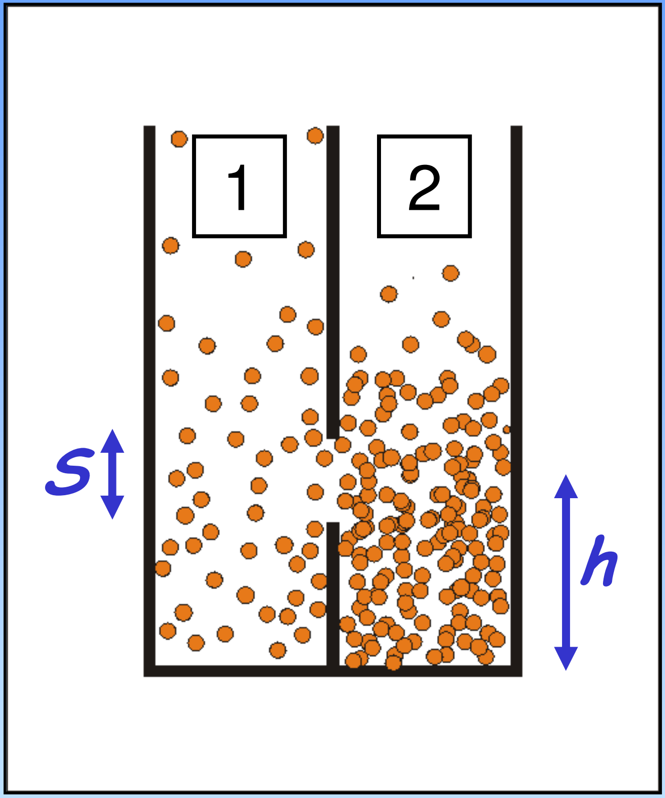
$$= \frac{gN_1S}{\sqrt{T_1}}\exp\left(-\frac{gh}{T_1}\right)$$

Use: $T \sim \dfrac{v_b^2}{\varepsilon^2 N^2}$

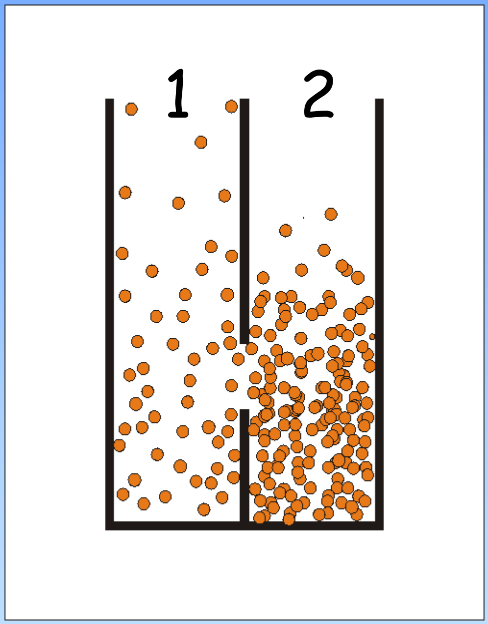
$$F(N_1) \sim N_1^2 \exp\left(-BN_1^2\right)$$

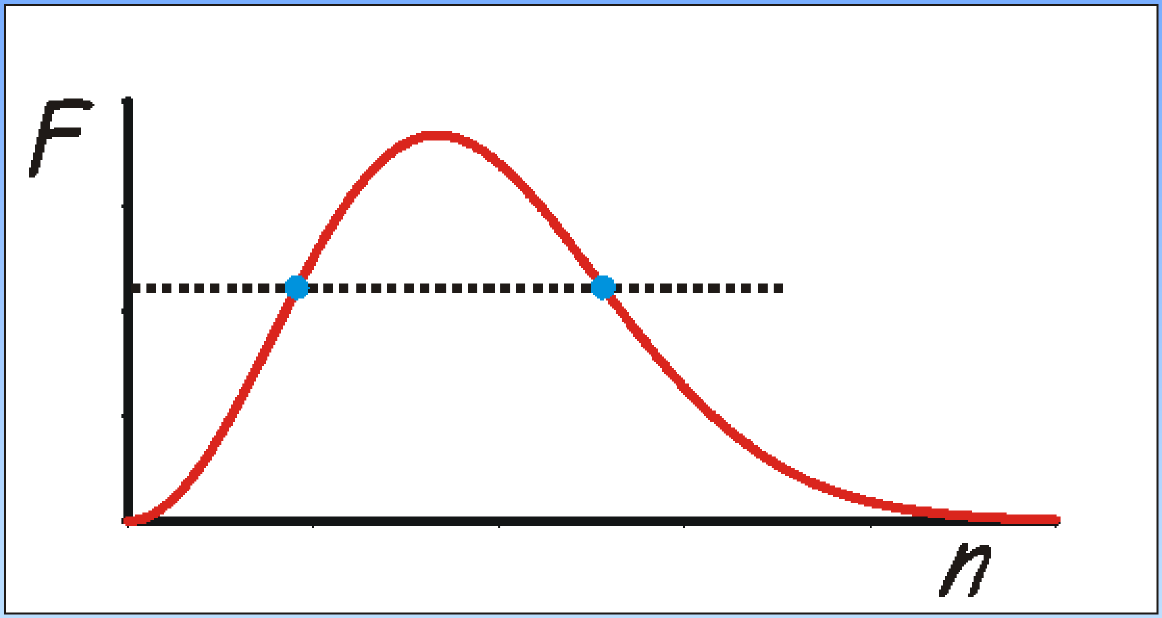
with:

$$B \propto (1-e^2)^2 \frac{gh}{a^2\omega^2}$$



Flux explains the clustering:





$$F_{1\rightarrow 2} = F_{2\rightarrow 1}$$

Stability analysis 2 box system

$$n_1 = \frac{N_1}{N_1 + N_2}; \quad n_2 = 1 - n_1$$

$$\frac{d}{dt} n_1 = F(n_2) - F(n_1)$$
$$= F(1 - n_1) - F(n_1)$$

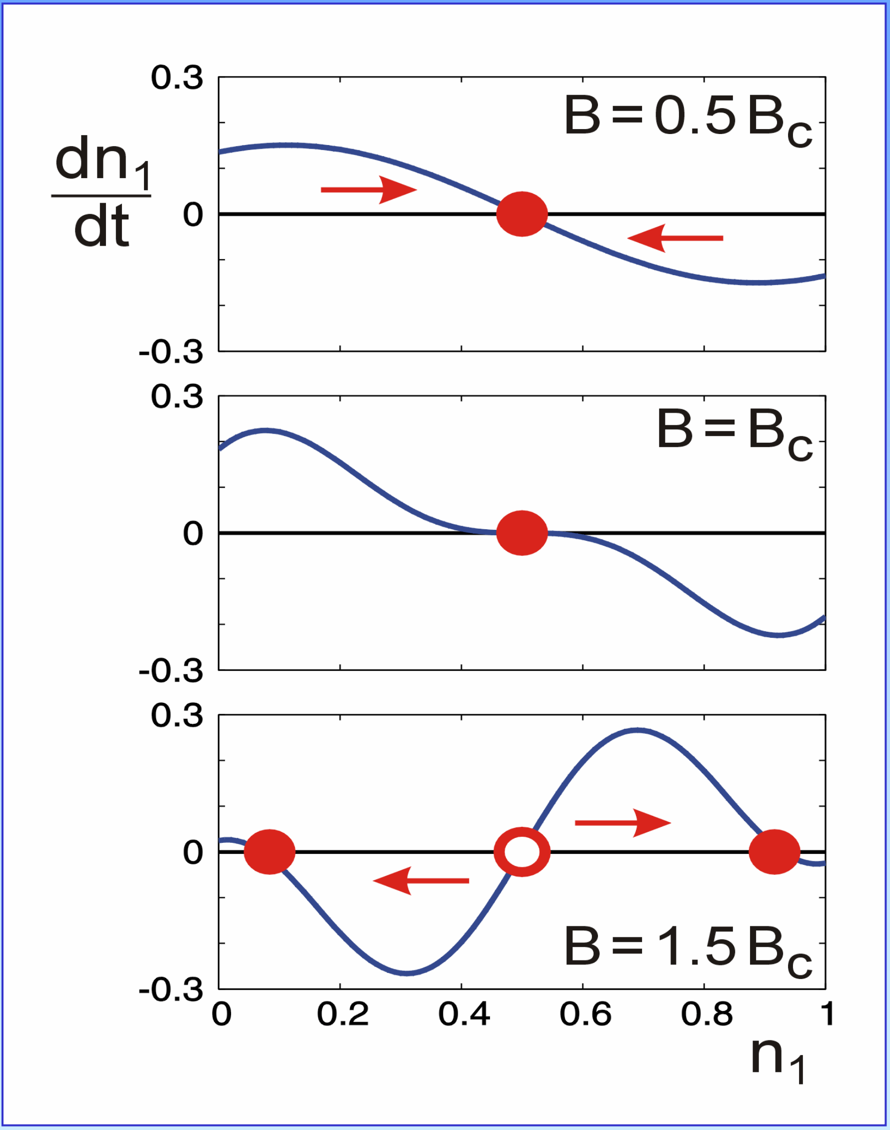
Around n_1=1/2:

$$n_1 = \tfrac{1}{2} + \delta n_1$$

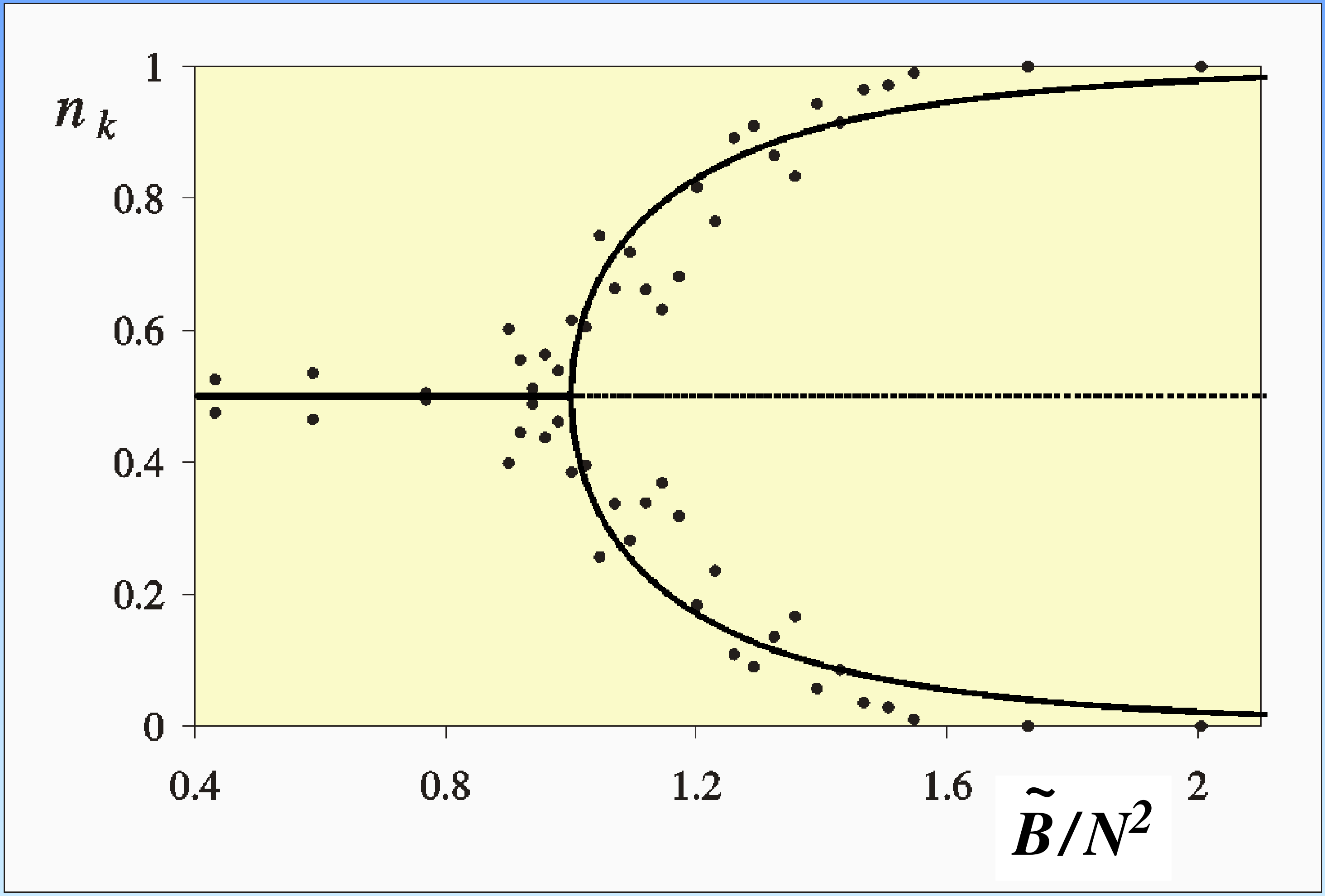
$$\frac{d}{dt} \delta n_1 = -2F'(n_1)\, \delta n_1$$
$$\propto -2e^{-B}(B_c - B)\, \delta n_1$$

$B < B_c$: $\frac{d}{dt} \delta n_1 < 0$ ***stable***

$B > B_c$: $\frac{d}{dt} \delta n_1 > 0$ ***unstable***



Bifurcation diagram



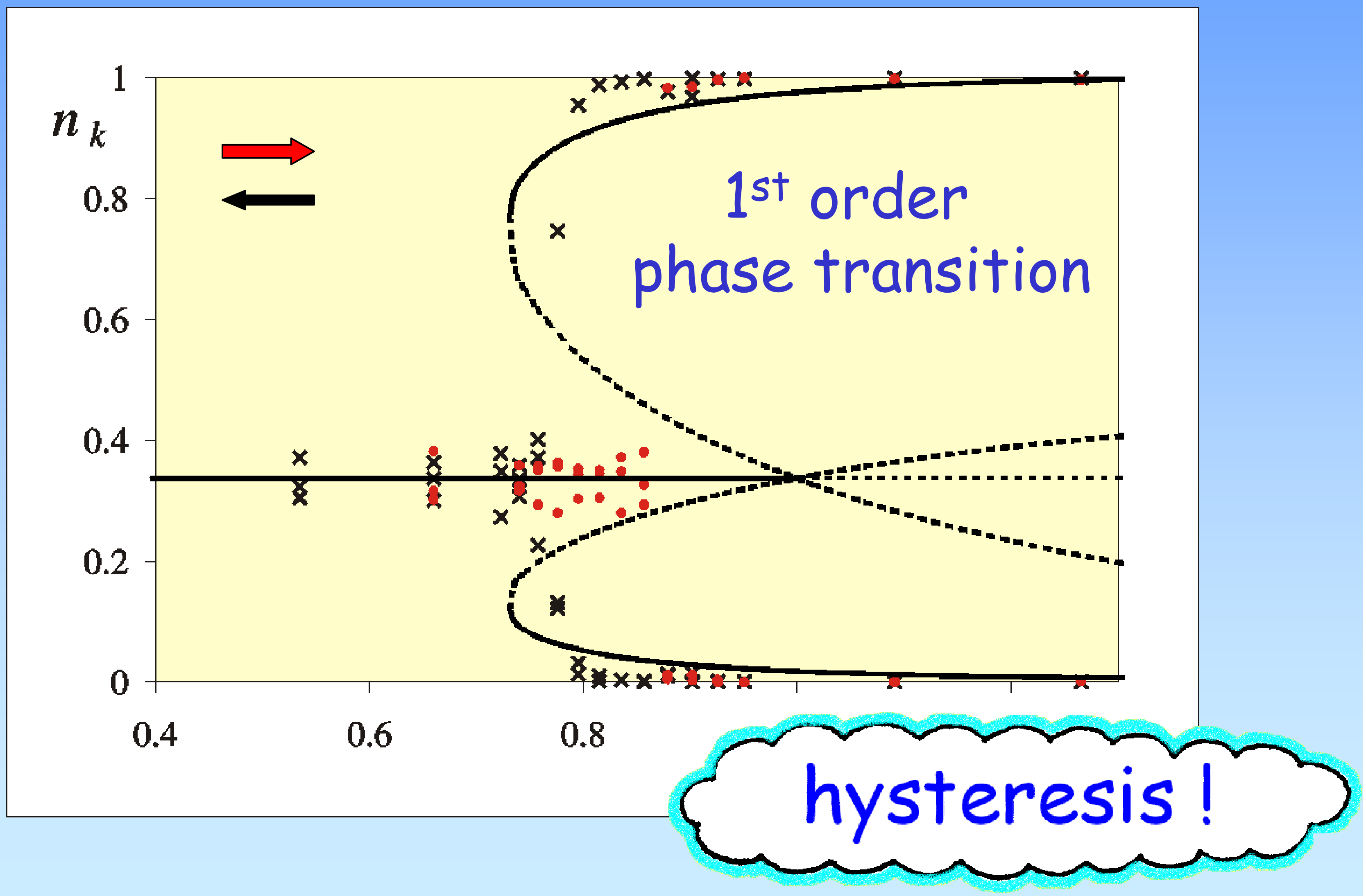
Dynamics for N-box system

$$\dot{n}_k = F(n_{k+1}) + F(n_{k-1}) - 2F(n_k) + \xi_k$$

$$\sum_{k=1}^{N} n_k = 1$$

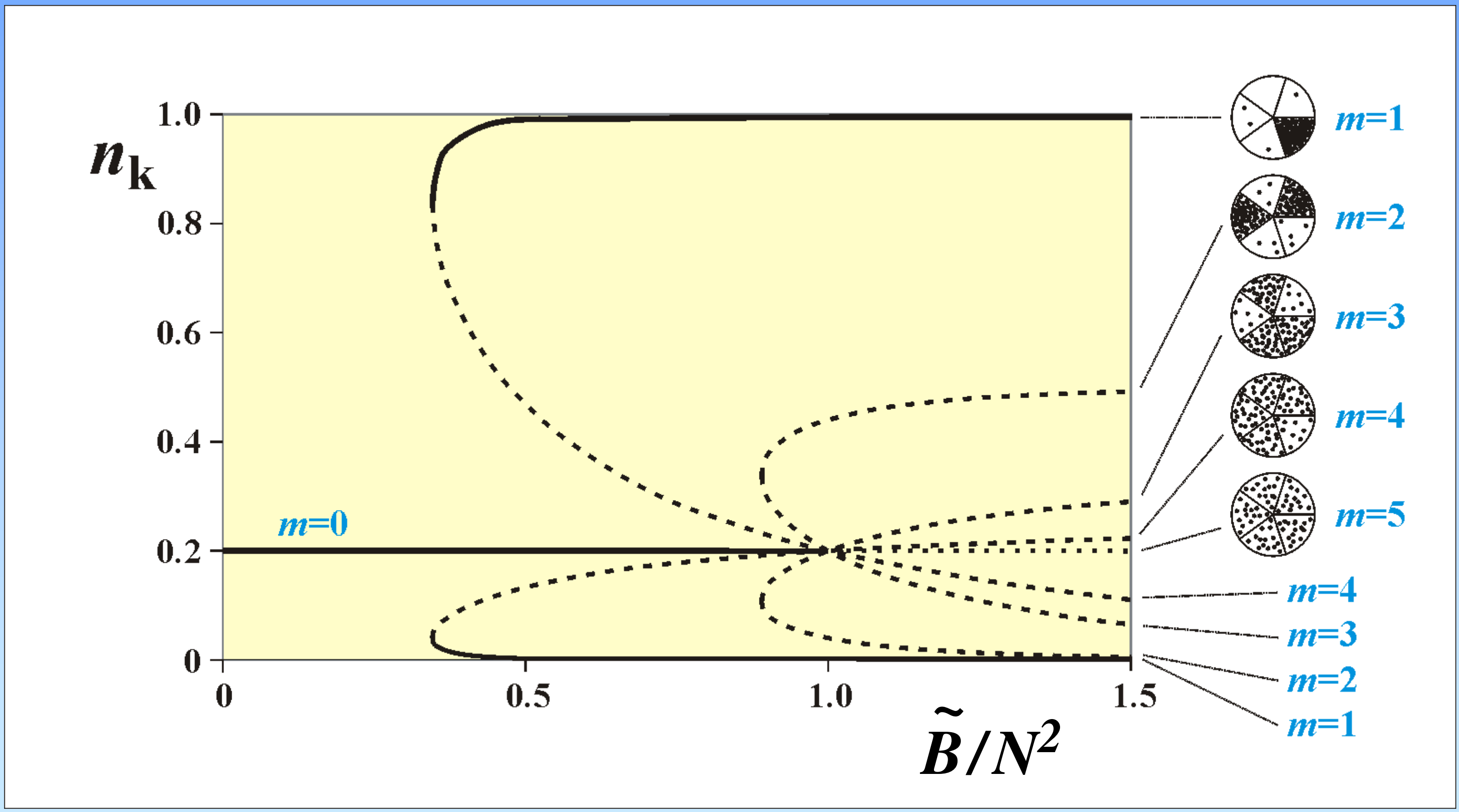
noise term

Bifurcation diagram for $N = 3$

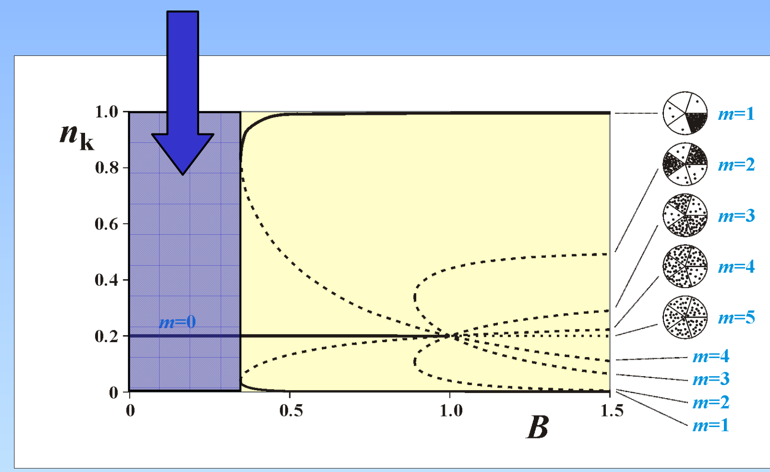


5 boxes in experiment

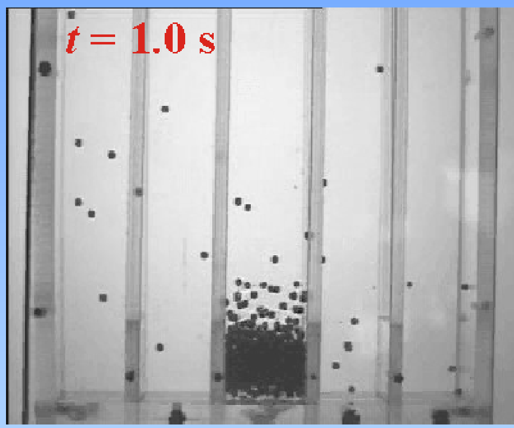
Bifurcation diagram for N=5

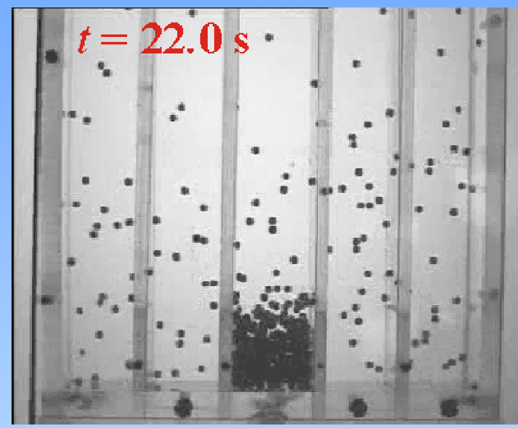


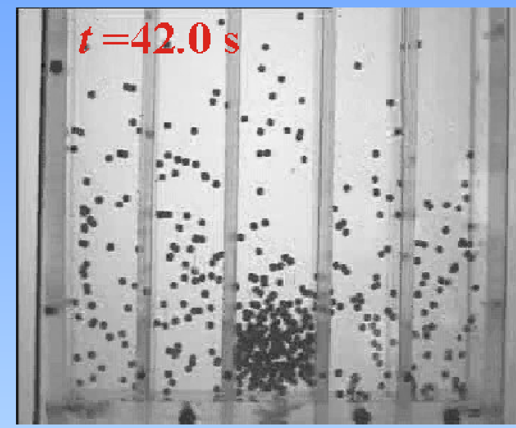
Declustering:

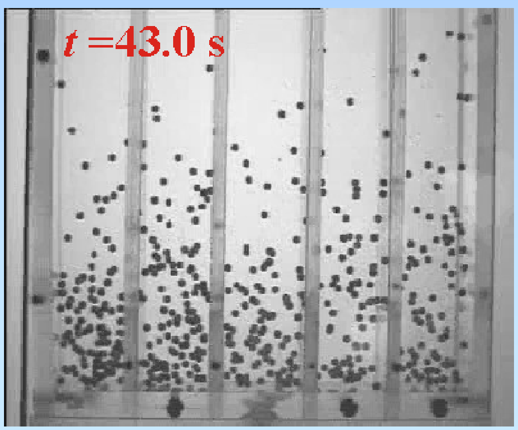


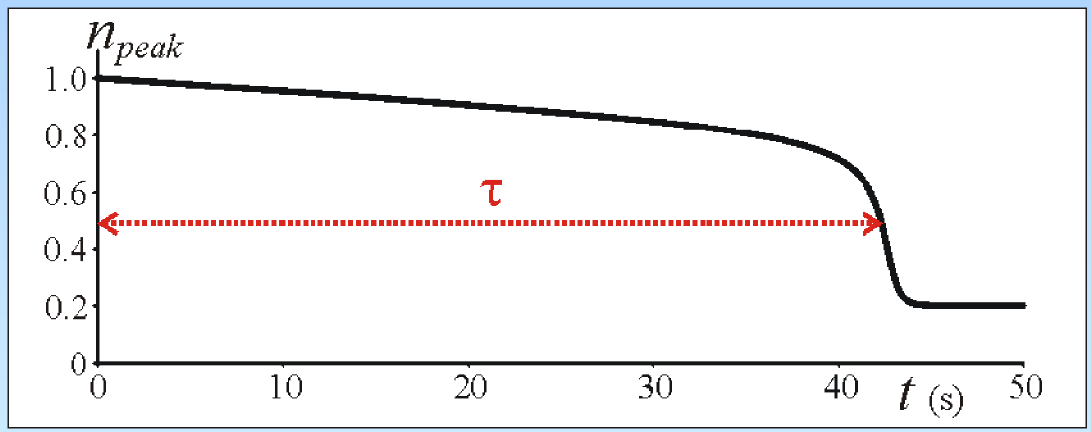
Lifetime of a cluster



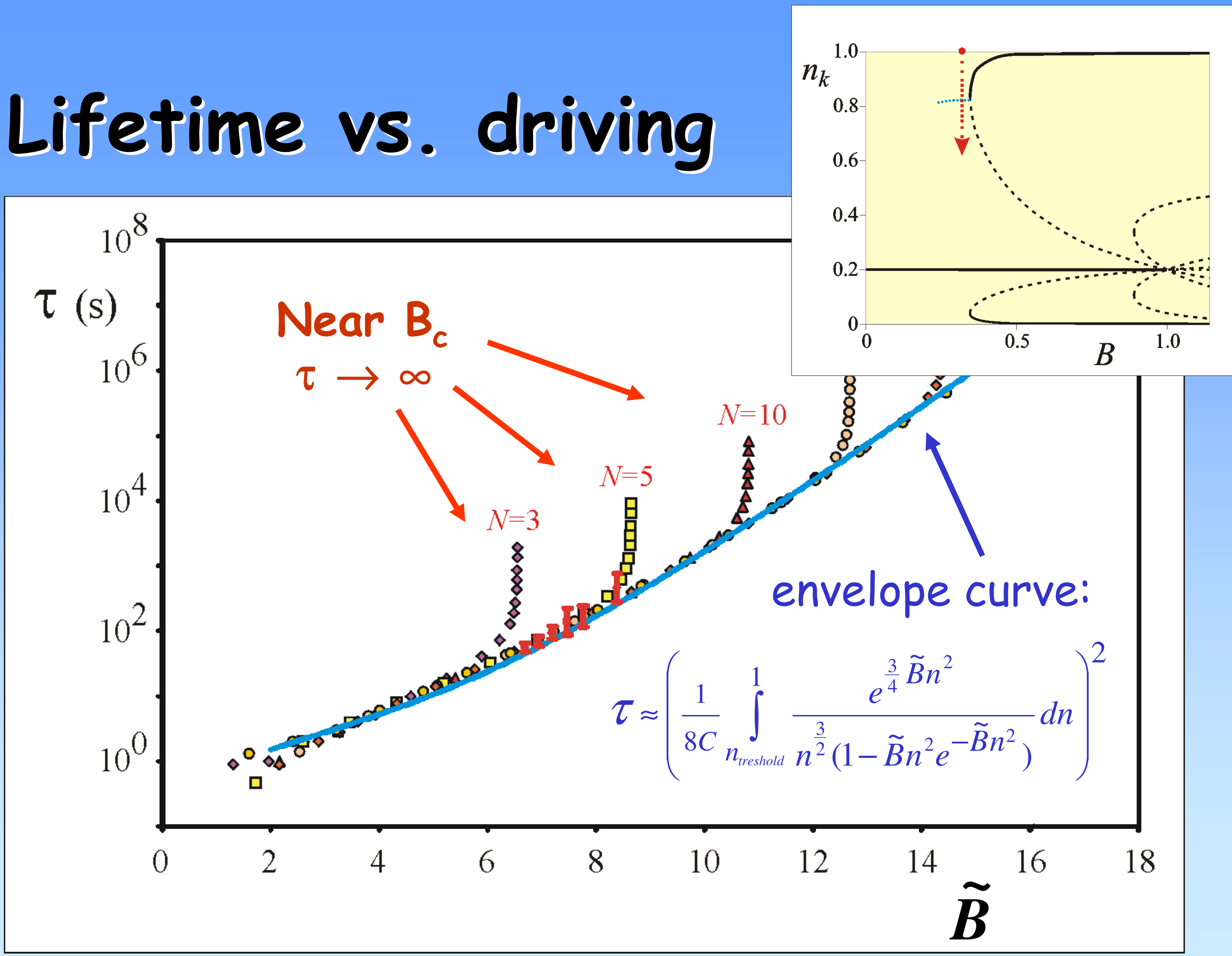




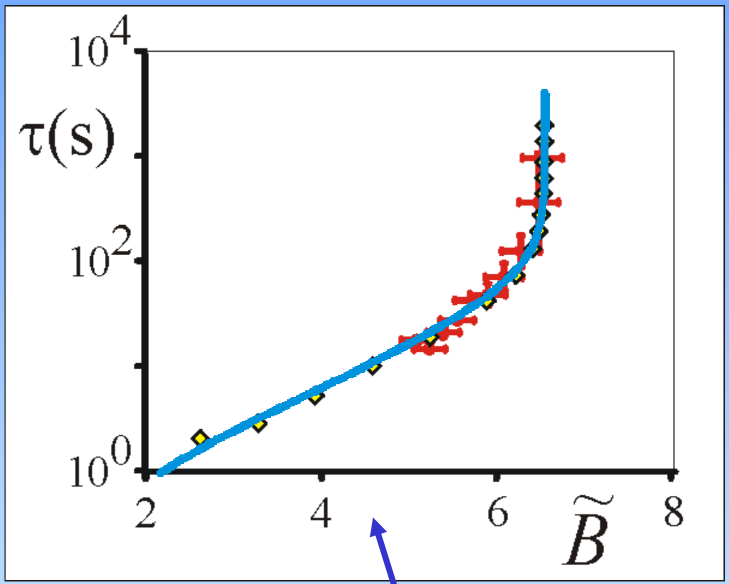




Lifetime vs. driving



Exact solution for N=3



cluster:

$$n_2 = n$$

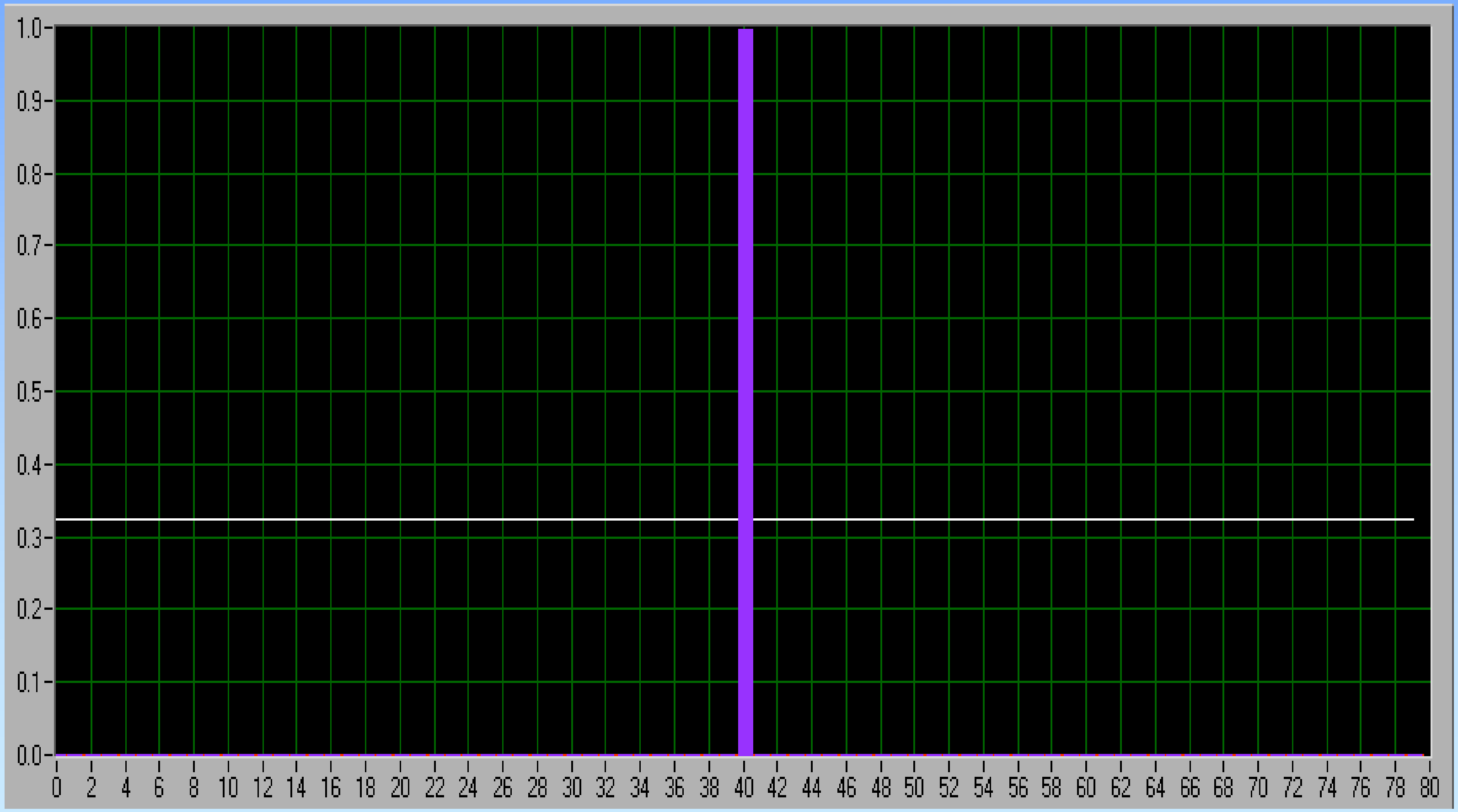
symmetry:

$$n_1 = n_3 = \tfrac{1}{2}(1-n)$$

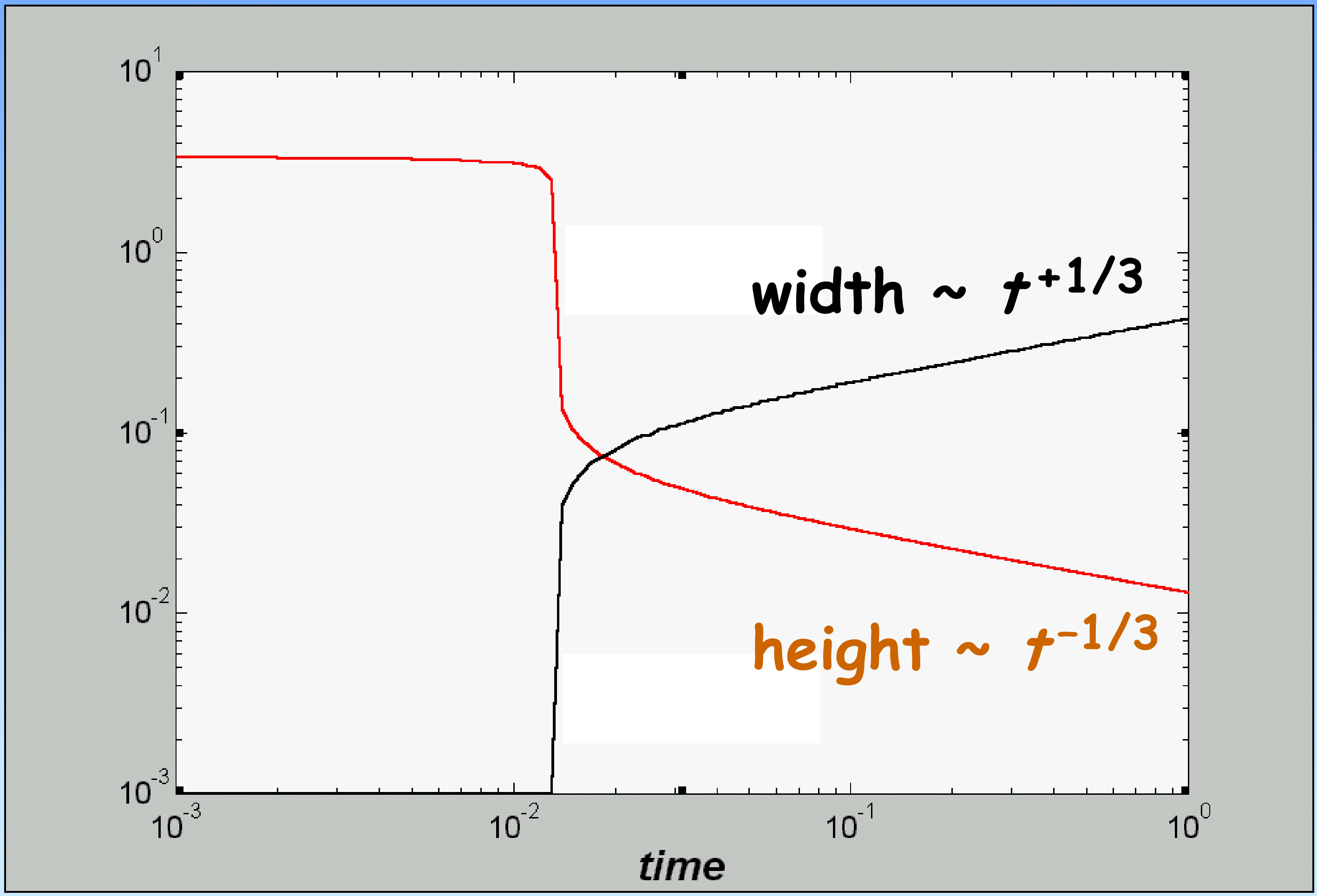
$$\frac{dn}{dt} = 2F(\tfrac{1}{2}(1-n)) - 2F(n)$$

$$\tau = \tfrac{1}{2}\int_{n_{threshold}}^{1} \frac{dn}{F(n) - F((1-n)/2)}$$

In 80 compartments



Time evolution of the cluster



Continuum limit

$$\frac{\partial n(x,t)}{\partial t} = \frac{\partial^2 F(n(x,t))}{\partial x^2} + \xi$$

$$= C(n)\left(\frac{\partial n}{\partial x}\right)^2 + D(n)\frac{\partial^2 n}{\partial x^2} + \xi$$

generalized KPZ equation

Discrete system:

$$\dot{n}_k = F(n_{k+1}) + F(n_{k-1}) - 2F(n_k) + \xi$$

Shape of the decaying cluster

$$\frac{\partial n}{\partial t} = 2A\left(\frac{\partial n}{\partial x}\right)^2 + 2An\frac{\partial^2 n}{\partial x^2}$$

(limit $\tilde{B}$=0)

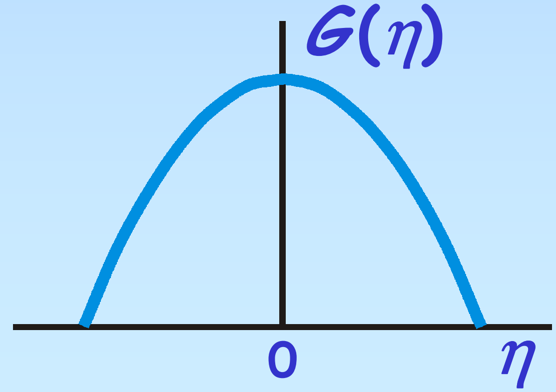
$$n(x,t) = \frac{1}{A^{1/3}t^{1/3}}G(\eta) \quad \text{with} \quad \eta \equiv \frac{x}{A^{1/3}t^{1/3}}$$

$$G + \eta G' + 6(G')^2 + 6GG'' = 0$$

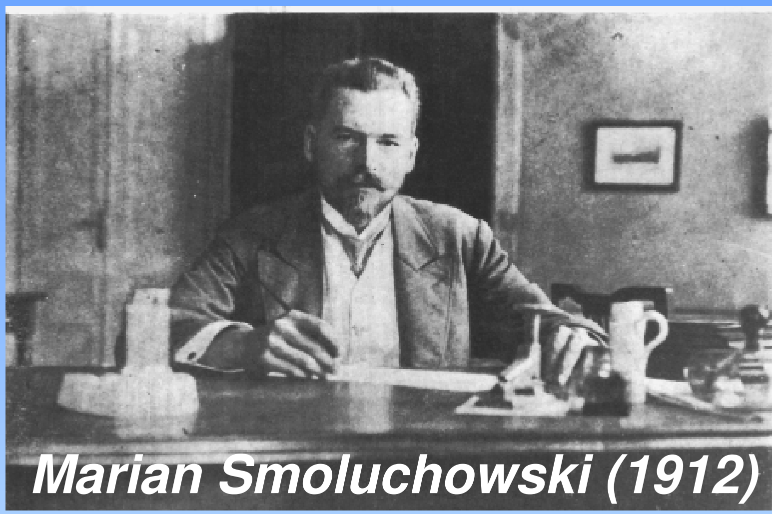
SELF-SIMILARITY !

solution:

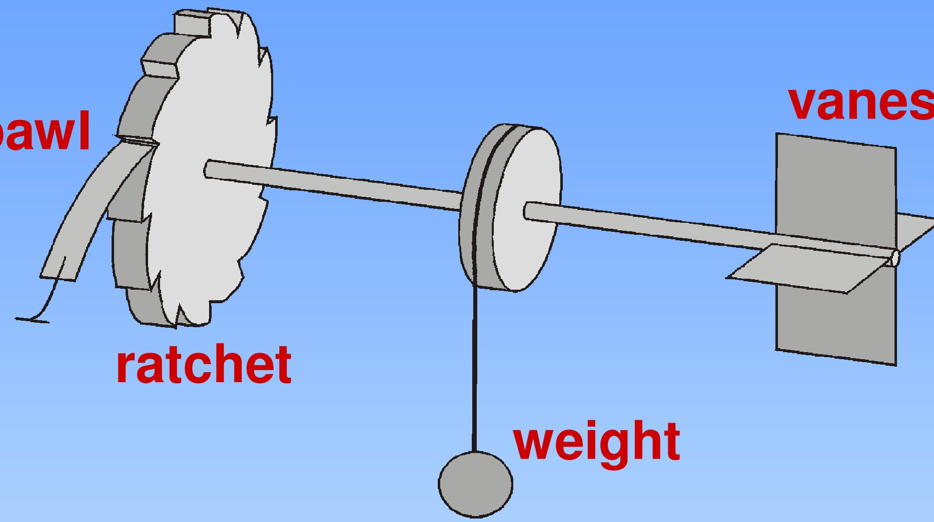
$$G(\eta) = C - \tfrac{1}{12}\eta^2$$

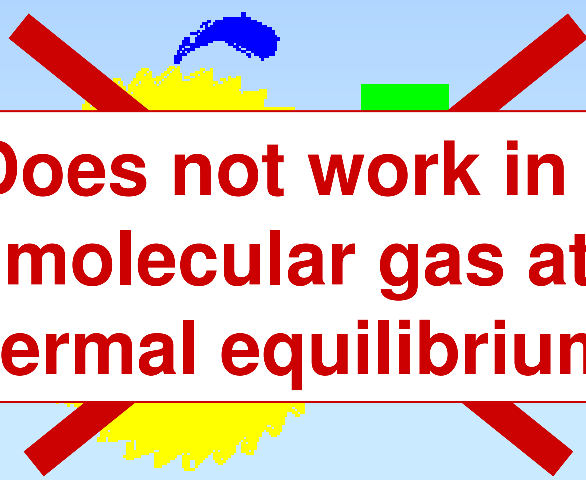


Smoluchowski-Feynman ratchet

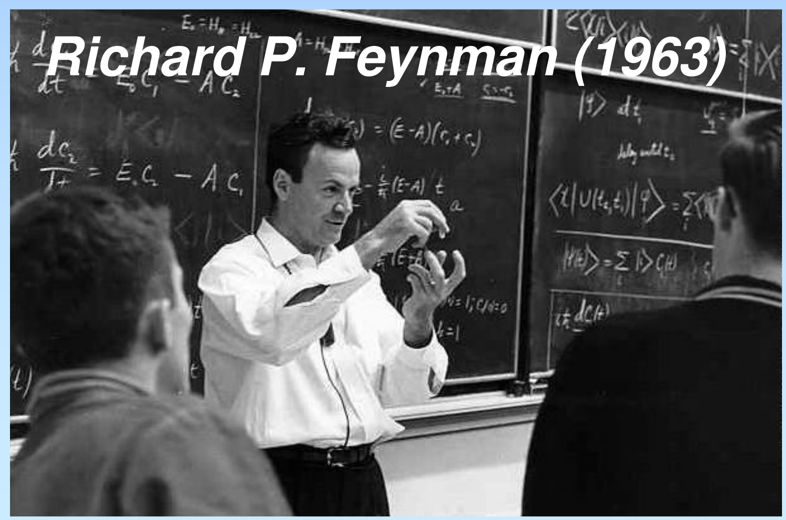


Marian Smoluchowski (1912)





Does not work in a molecular gas at thermal equilibrium !



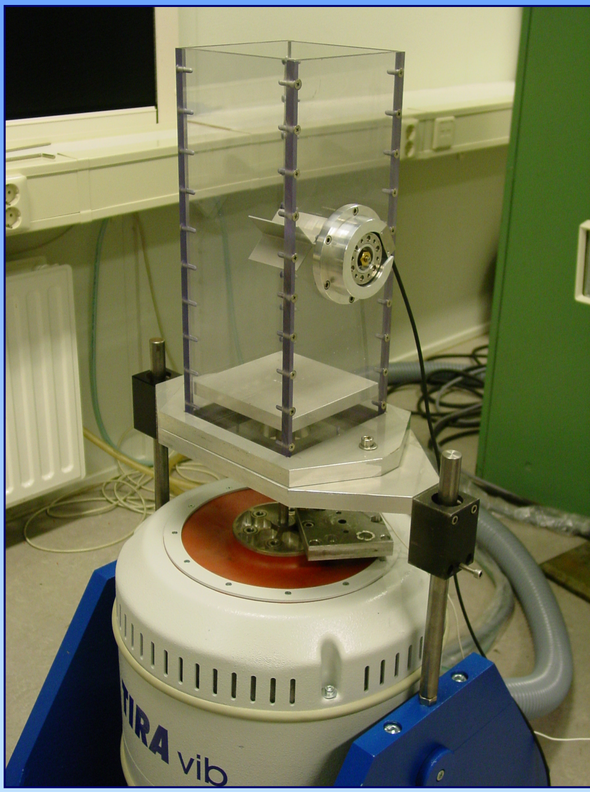
Richard P. Feynman (1963)

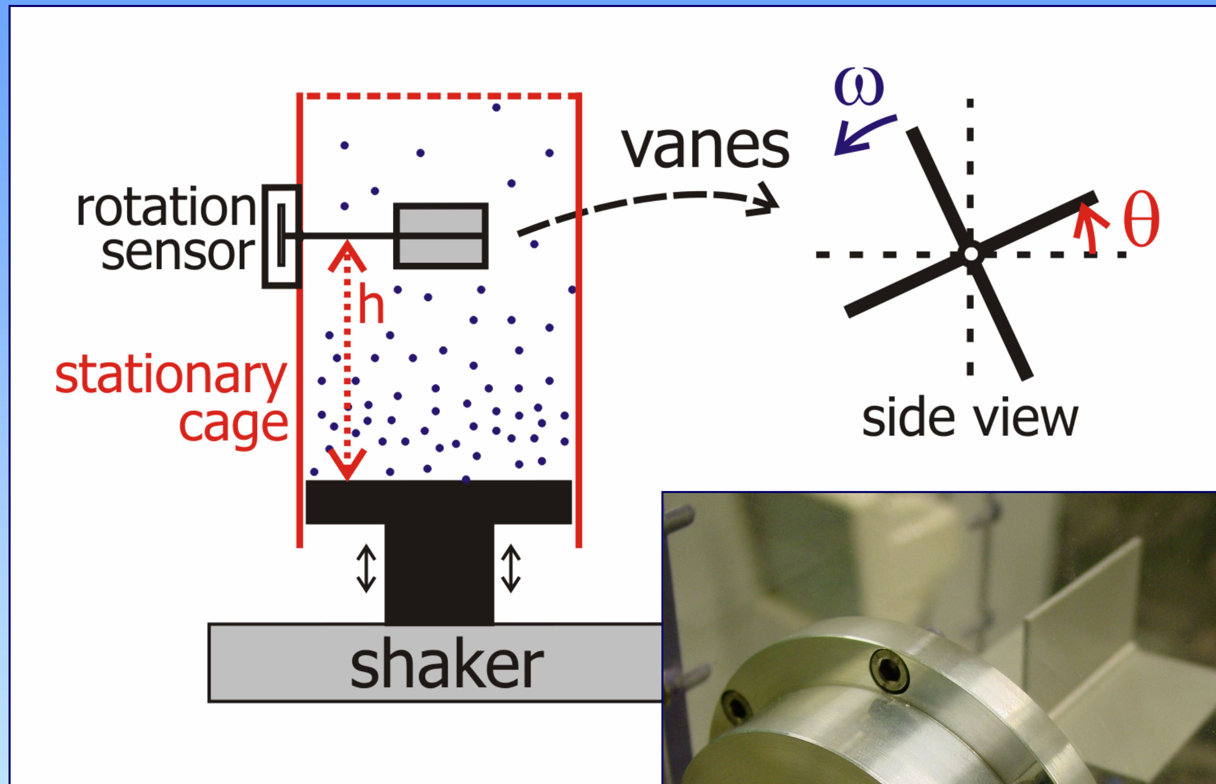
But in a granular gas …

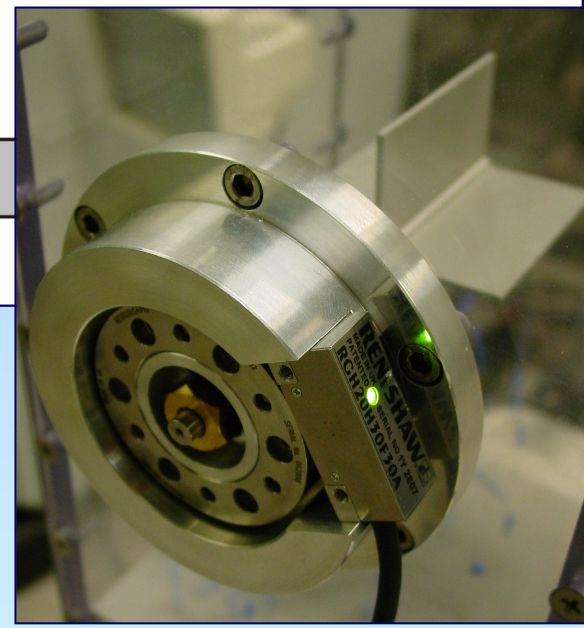
… the ratchet works !

Freshmen's physics project, University of Twente

Experimental setup



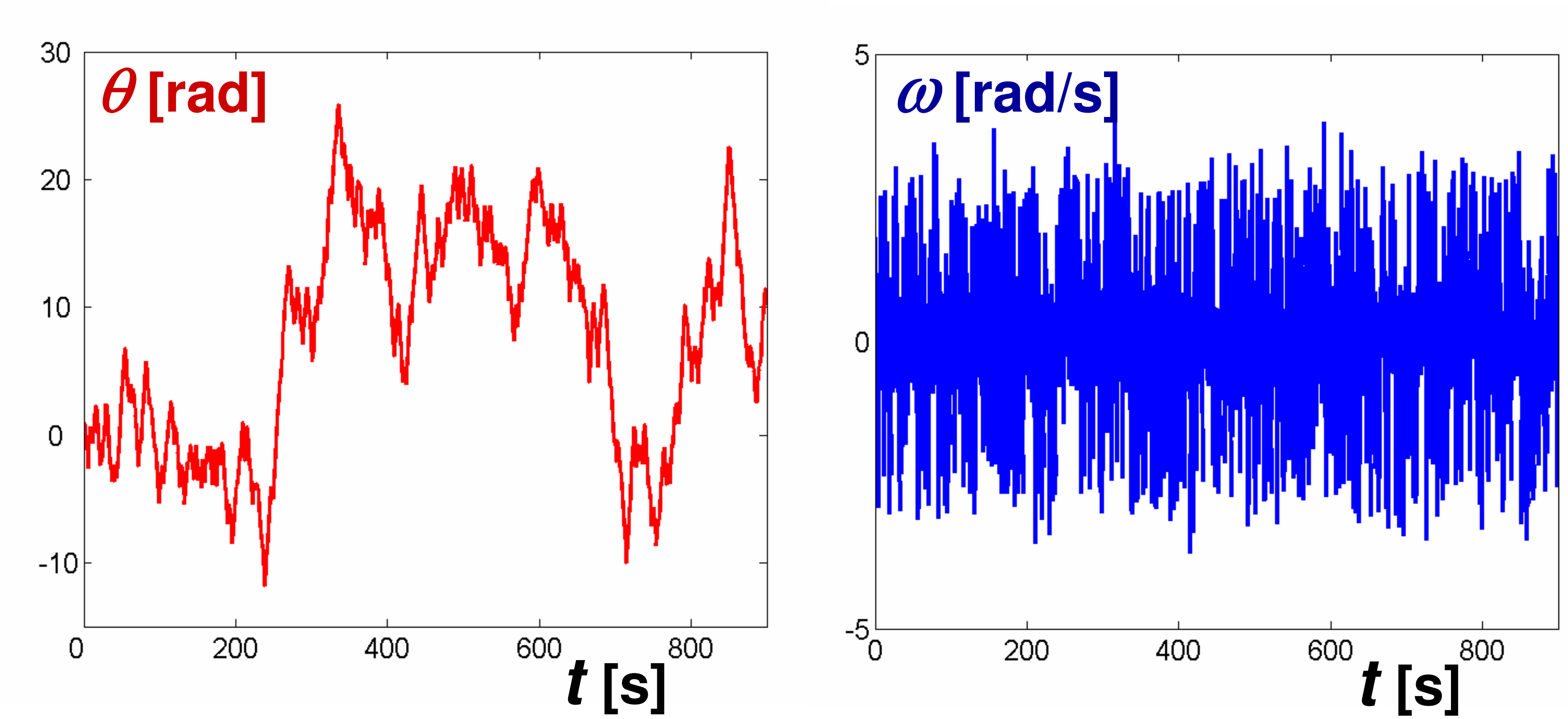



Rotational position sensor

Governing parameter:

$$S \equiv \frac{4\pi^2 a^2 f^2}{gh} \sim \frac{\Delta U_k \text{ at bottom}}{\Delta U_p \text{ at vane}}$$

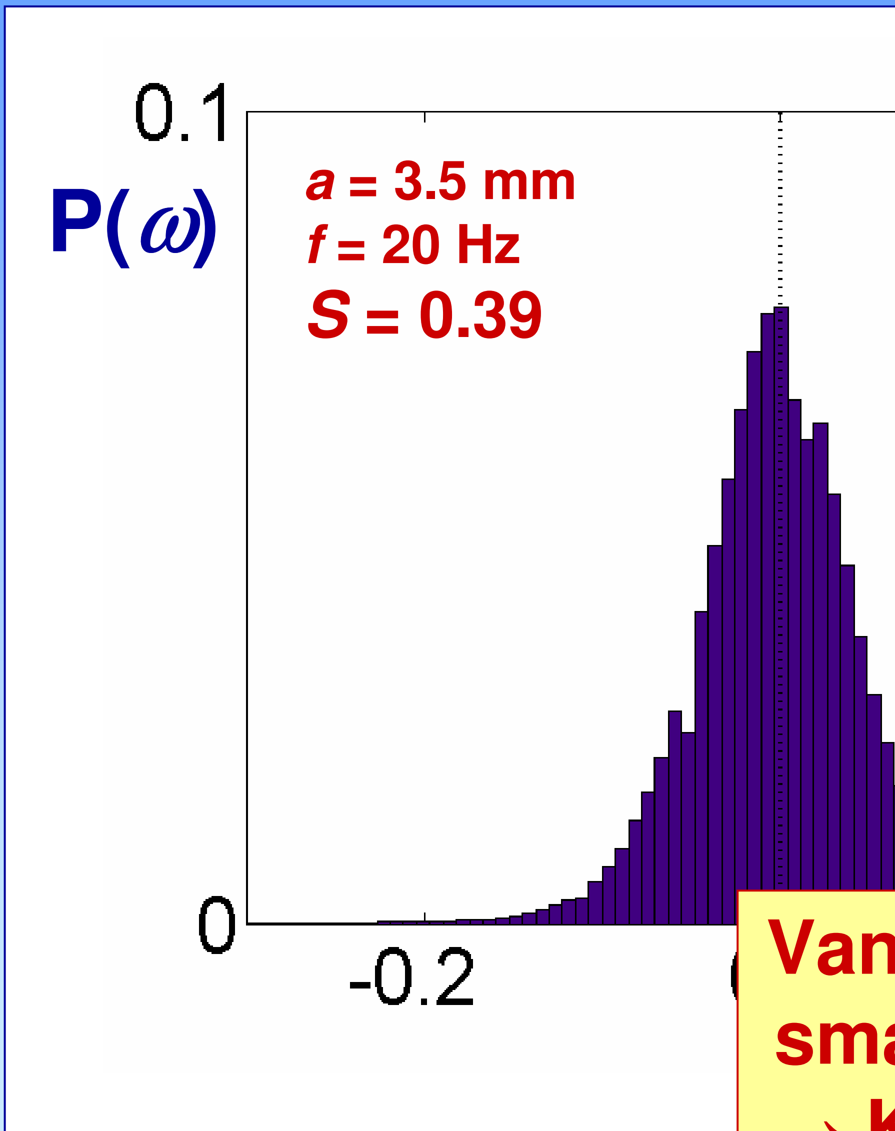
Granular mill: typical time series

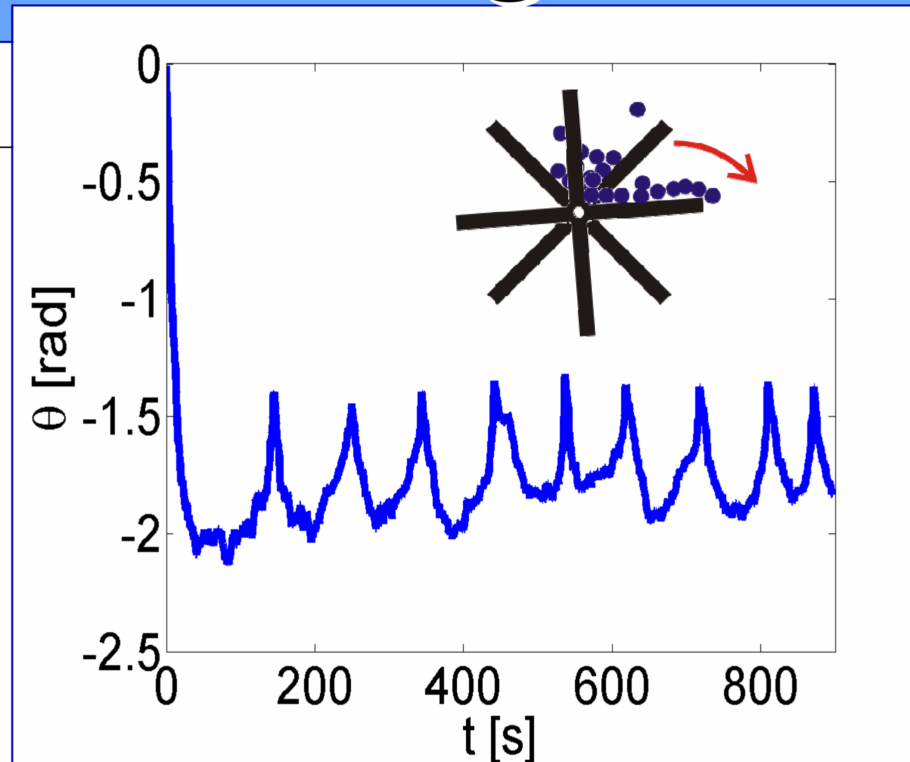


(N = 2000 particles, h = 51 mm, a = 1.5 mm, f = 110 Hz, S = 2.15)

Look at probability distribution function of ω and θ

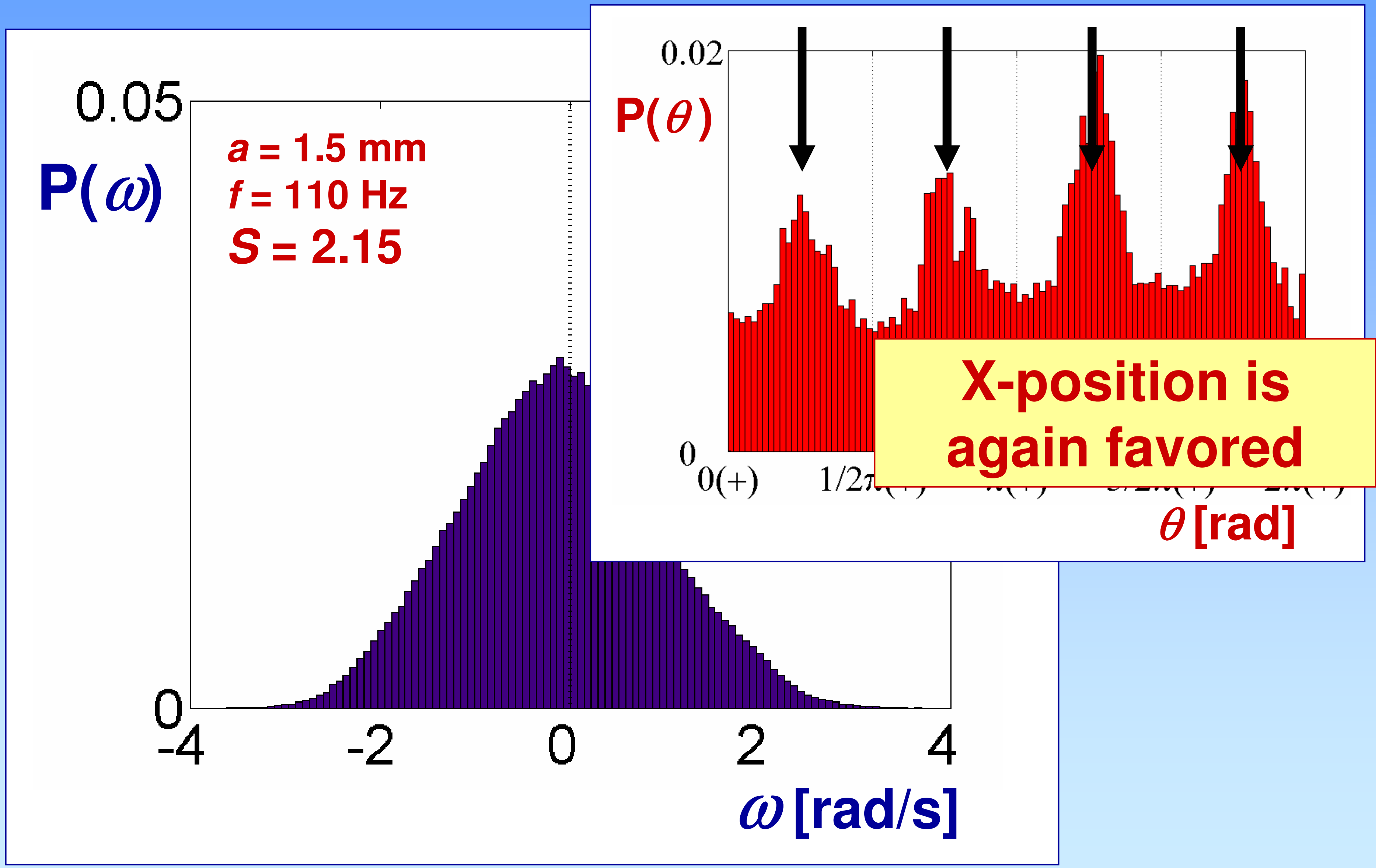
Mill: mild shaking



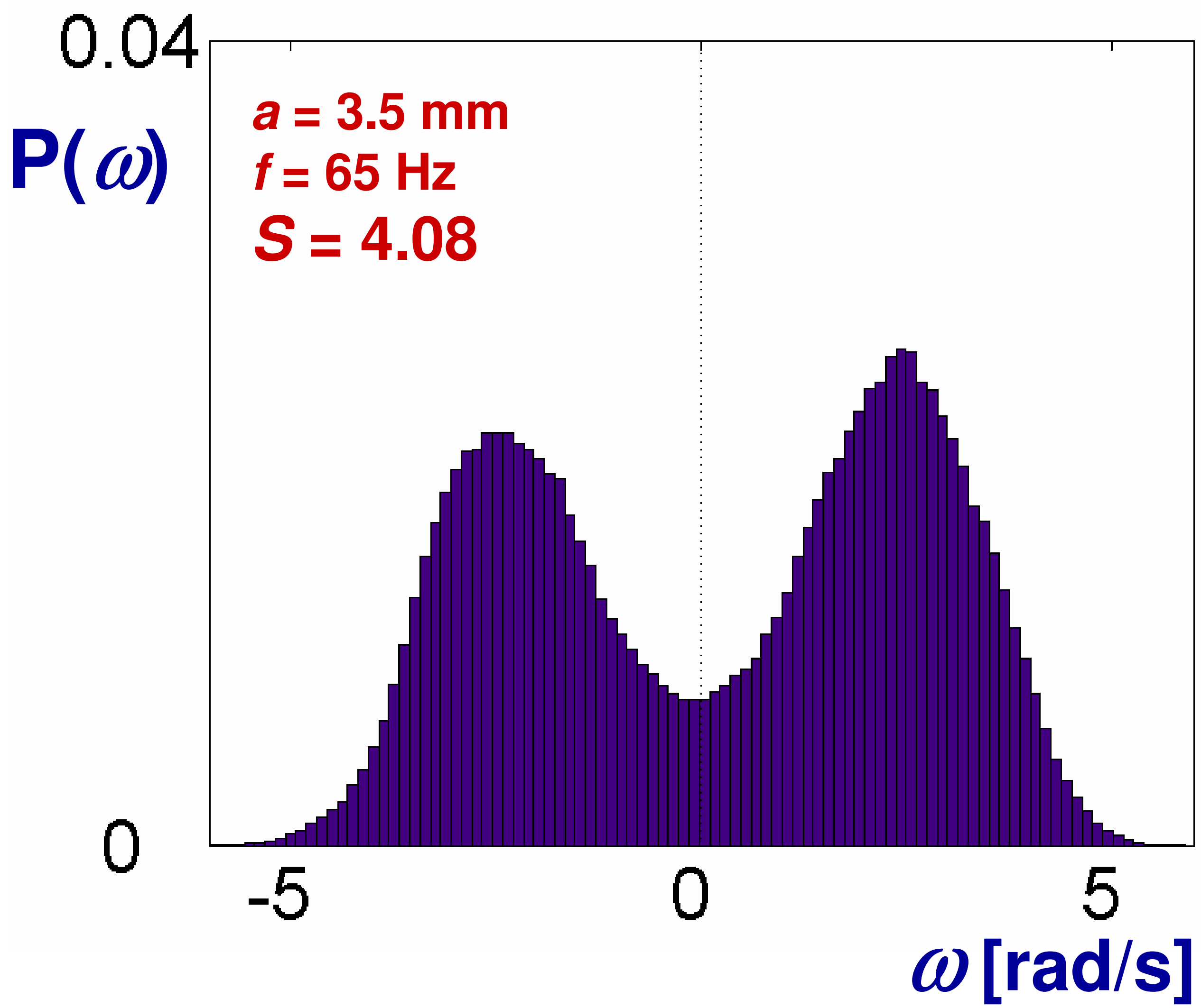


Vane motion is confined to a small region of phase space → Kramers' escape problem

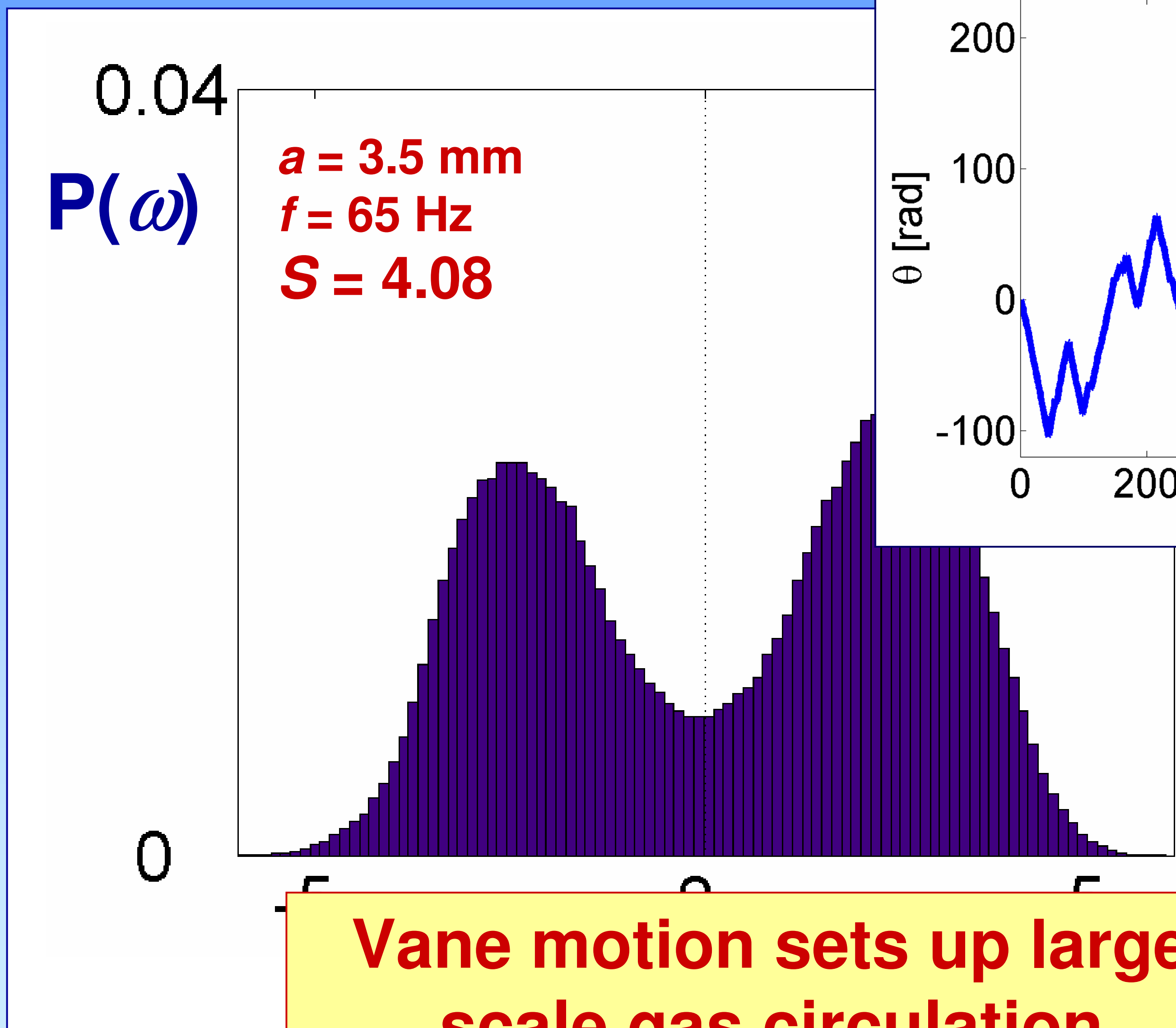
Mill: moderate shaking

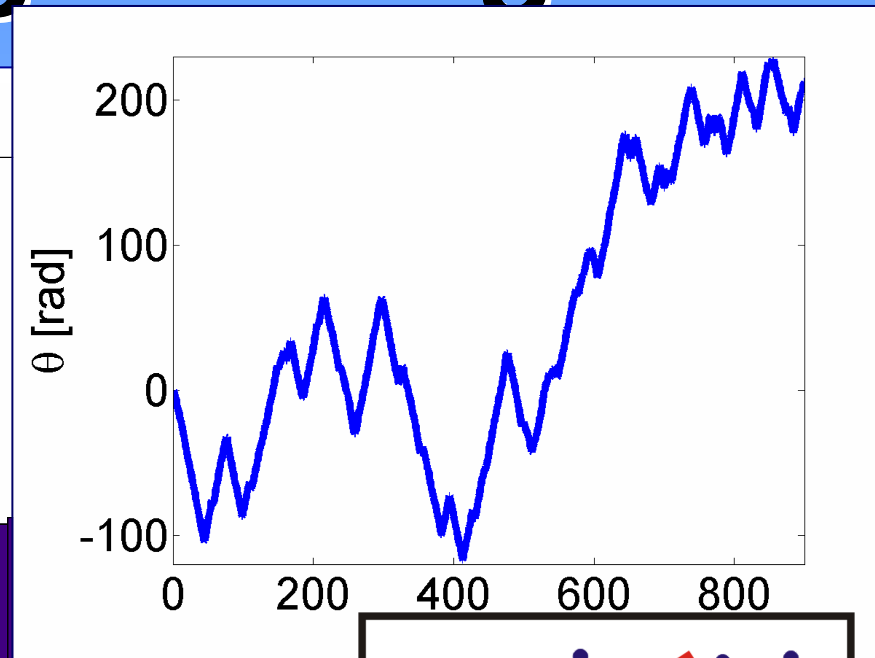


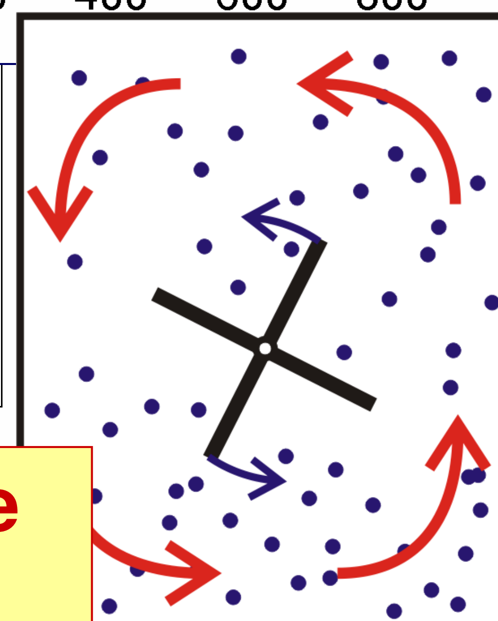
Mill: strong shaking



Mill: strong shaking







Vane motion sets up large scale gas circulation

Breaking the symmetry

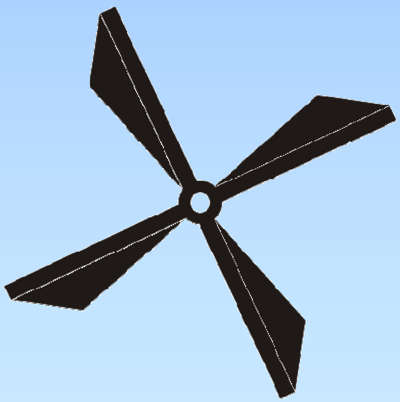
Possibility 1:

Introduce ratchet and pawl on axis

Disadvantage: **granular gas properties may vary within the container**

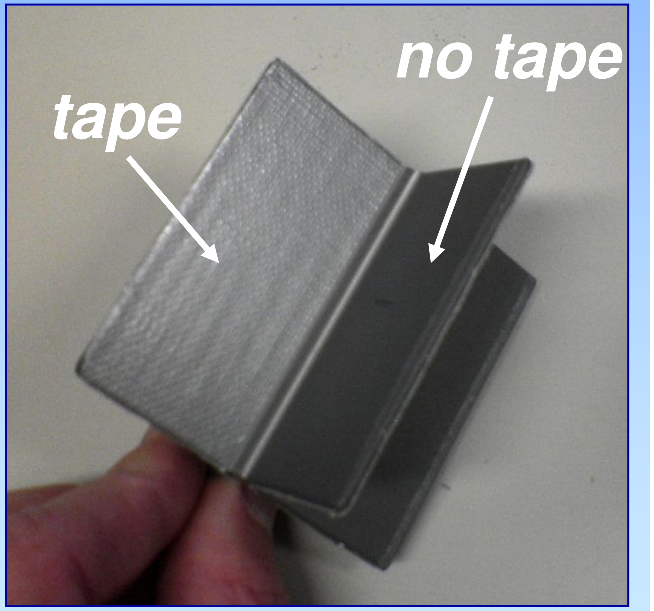
Possibility 2:

Break symmetry at the vanes:

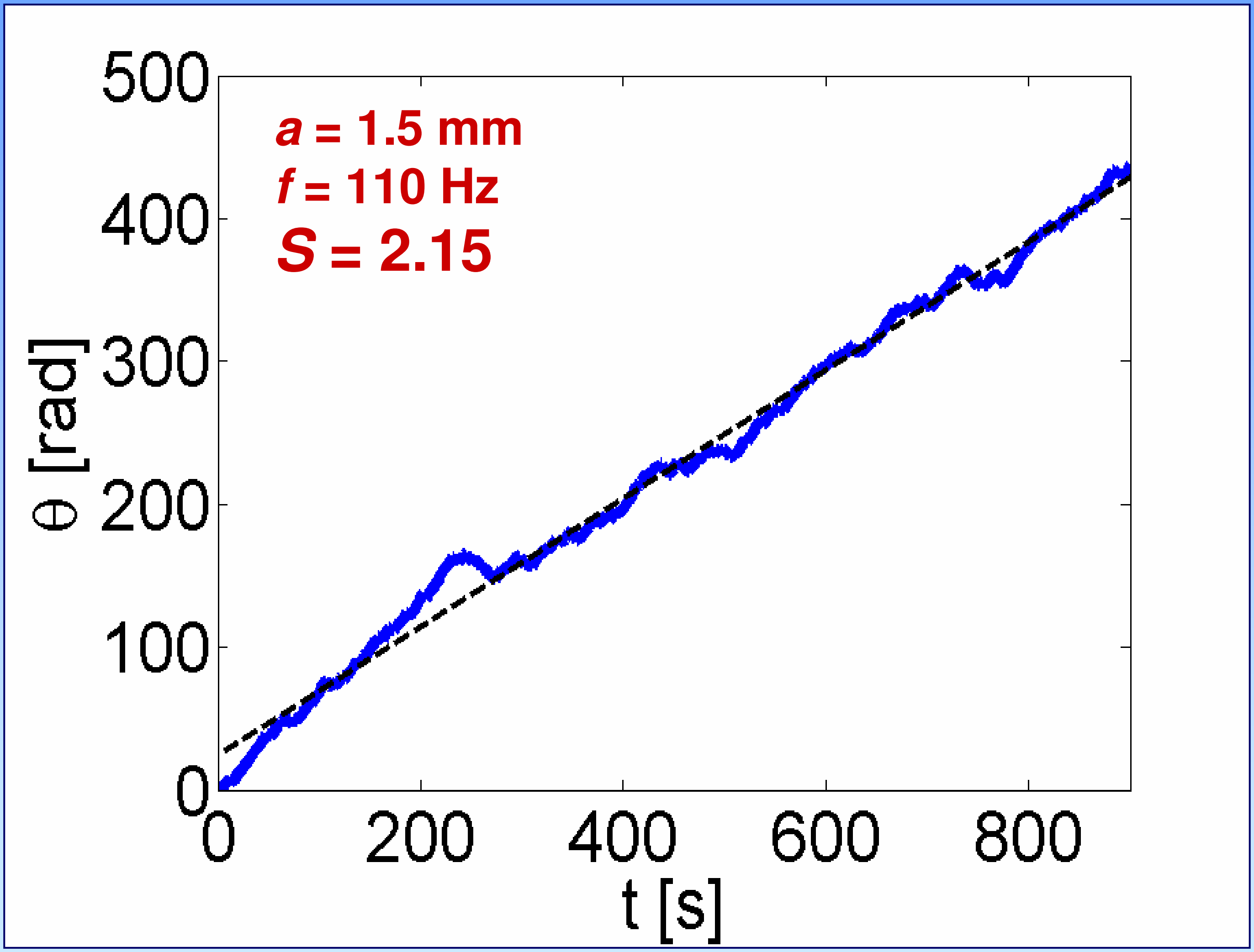


geometrically

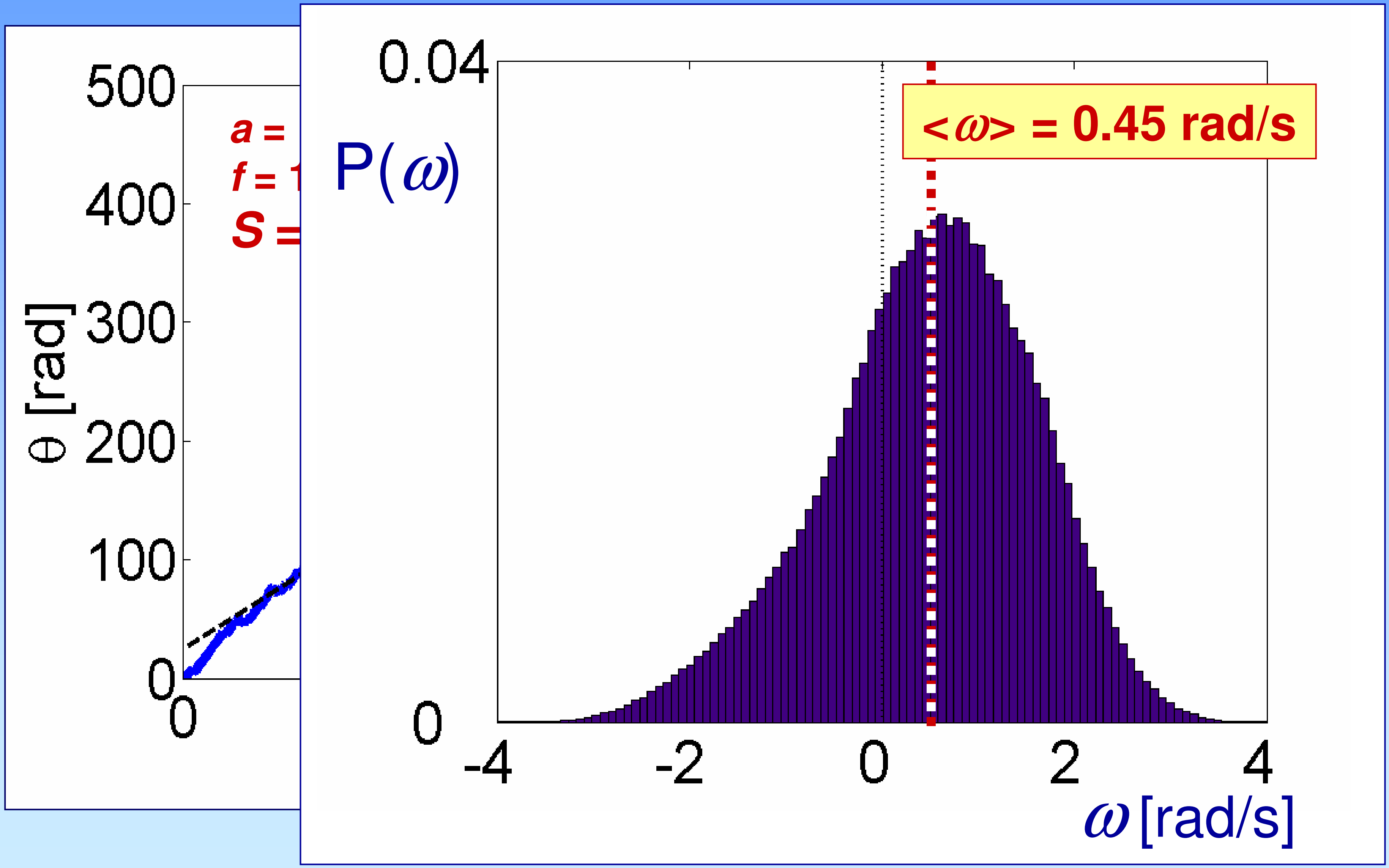
by changing the collisional properties



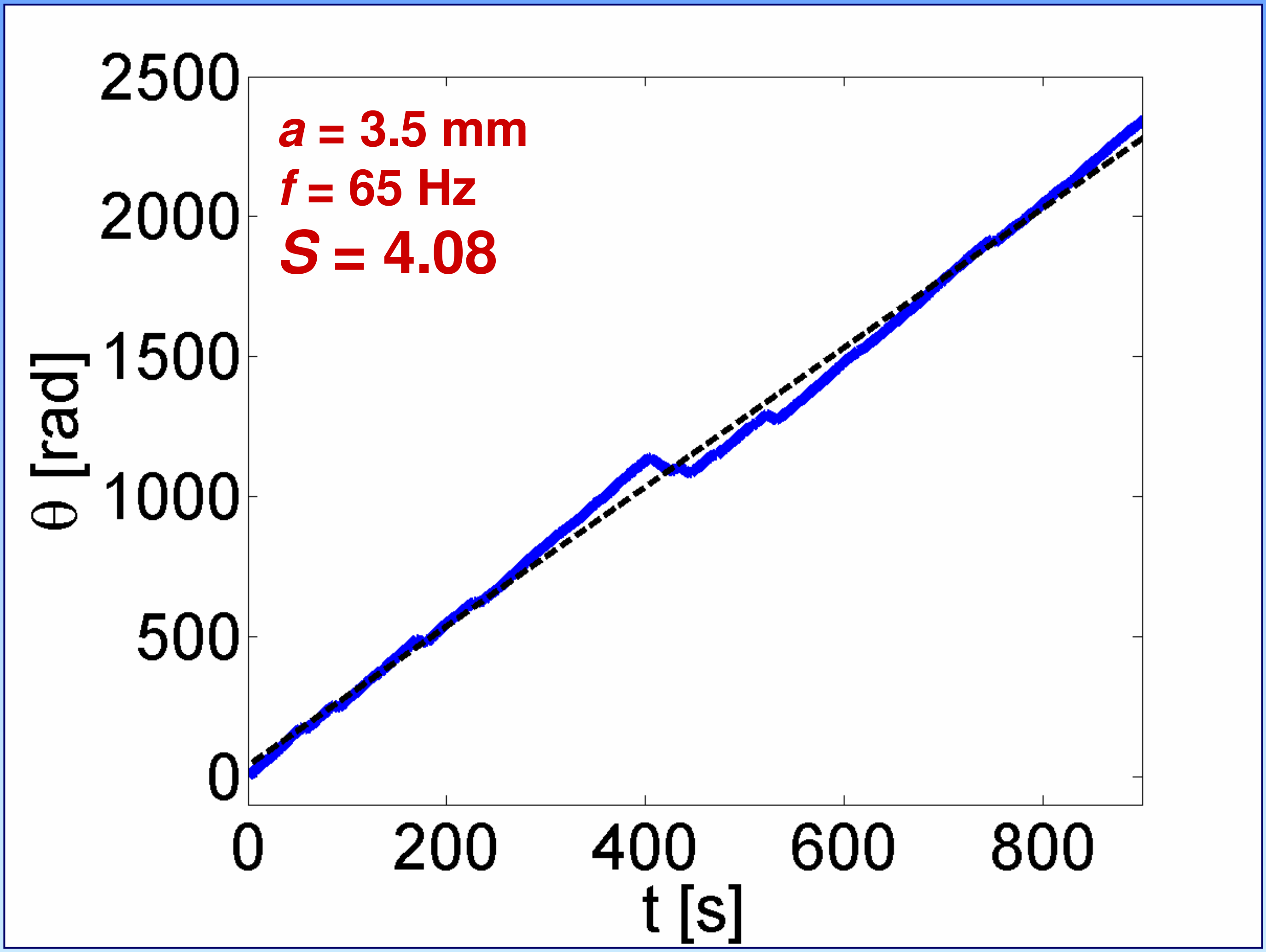
Ratchet: moderate shaking



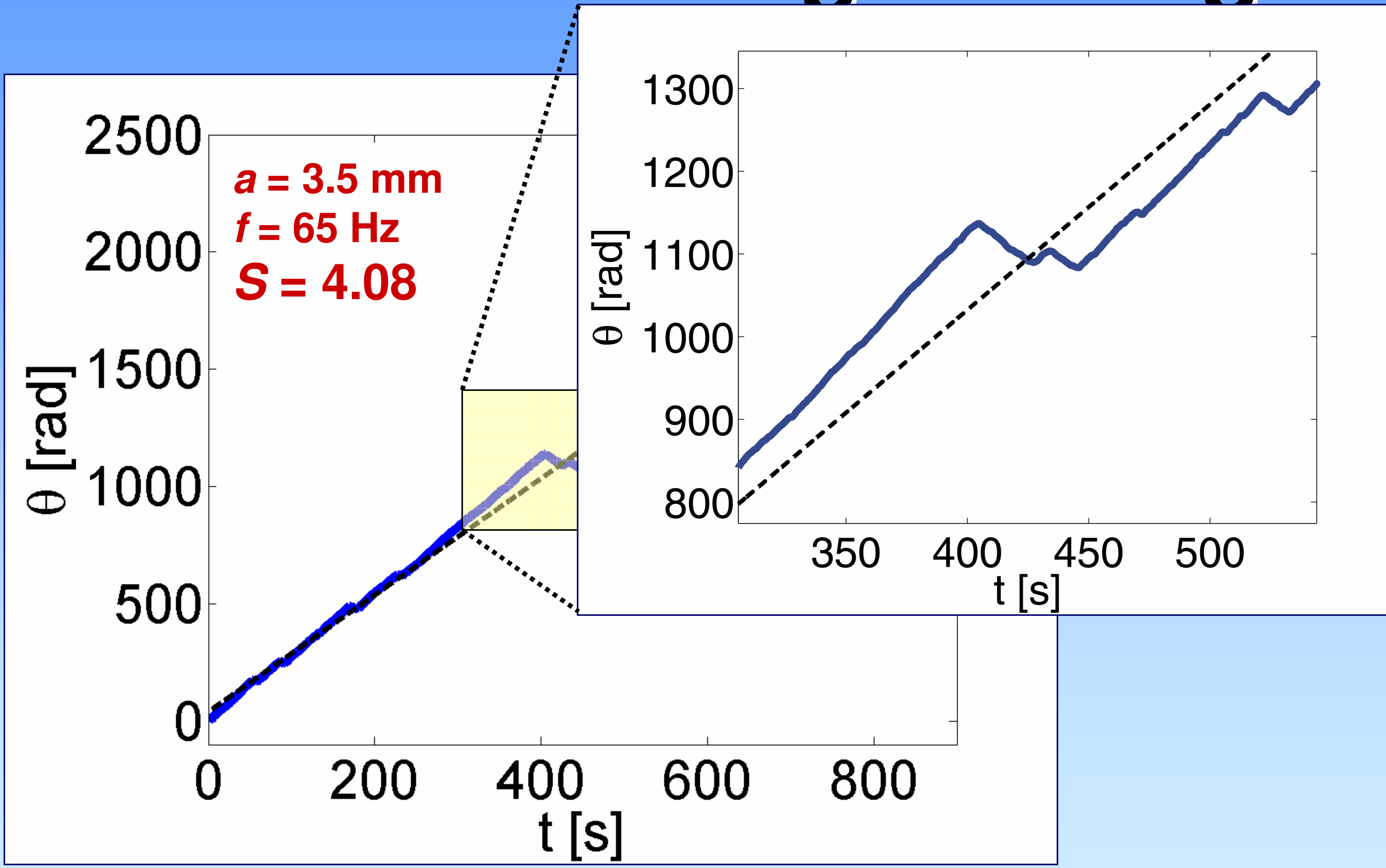
Ratchet: moderate shaking



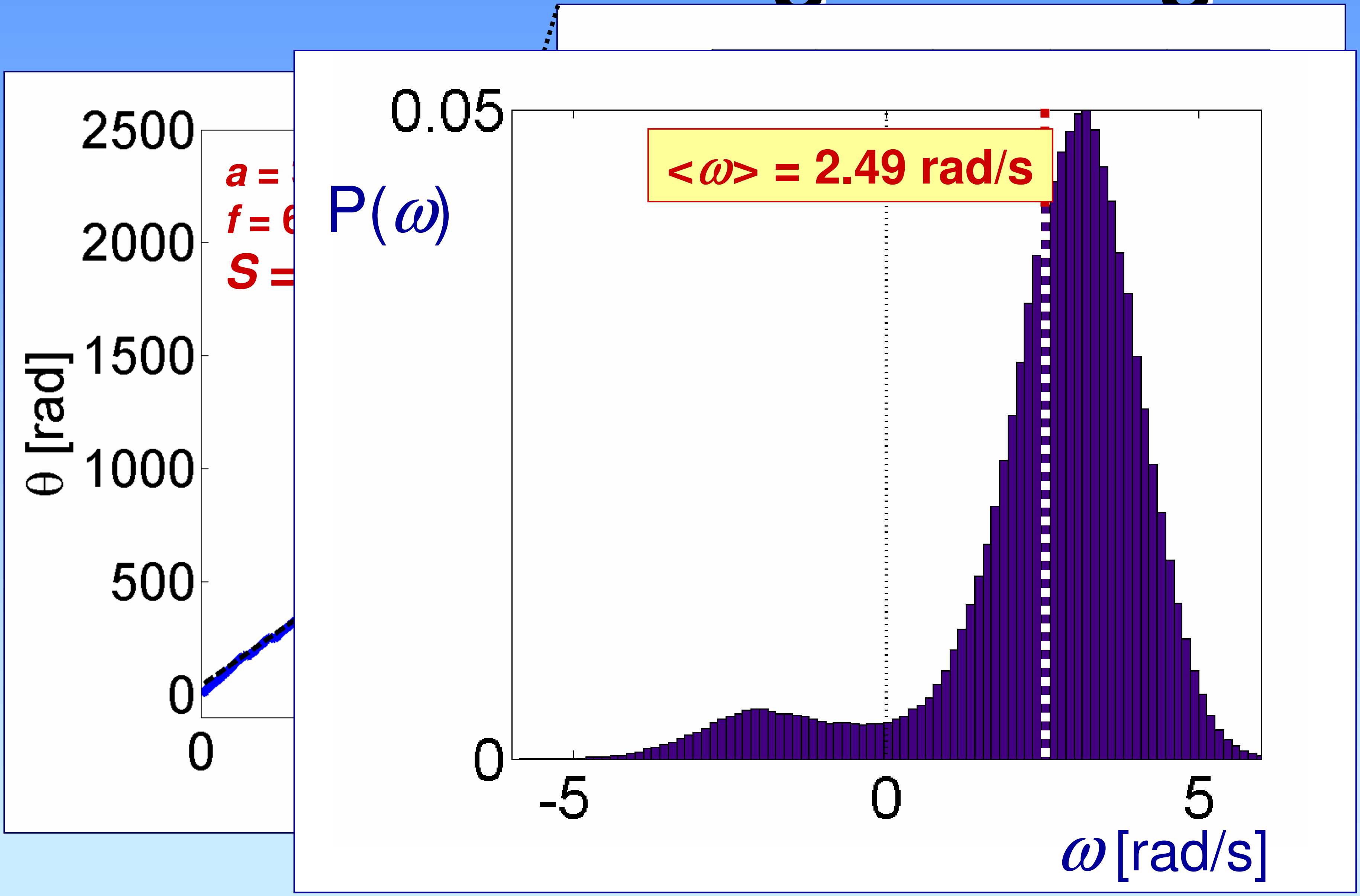
Ratchet: strong shaking



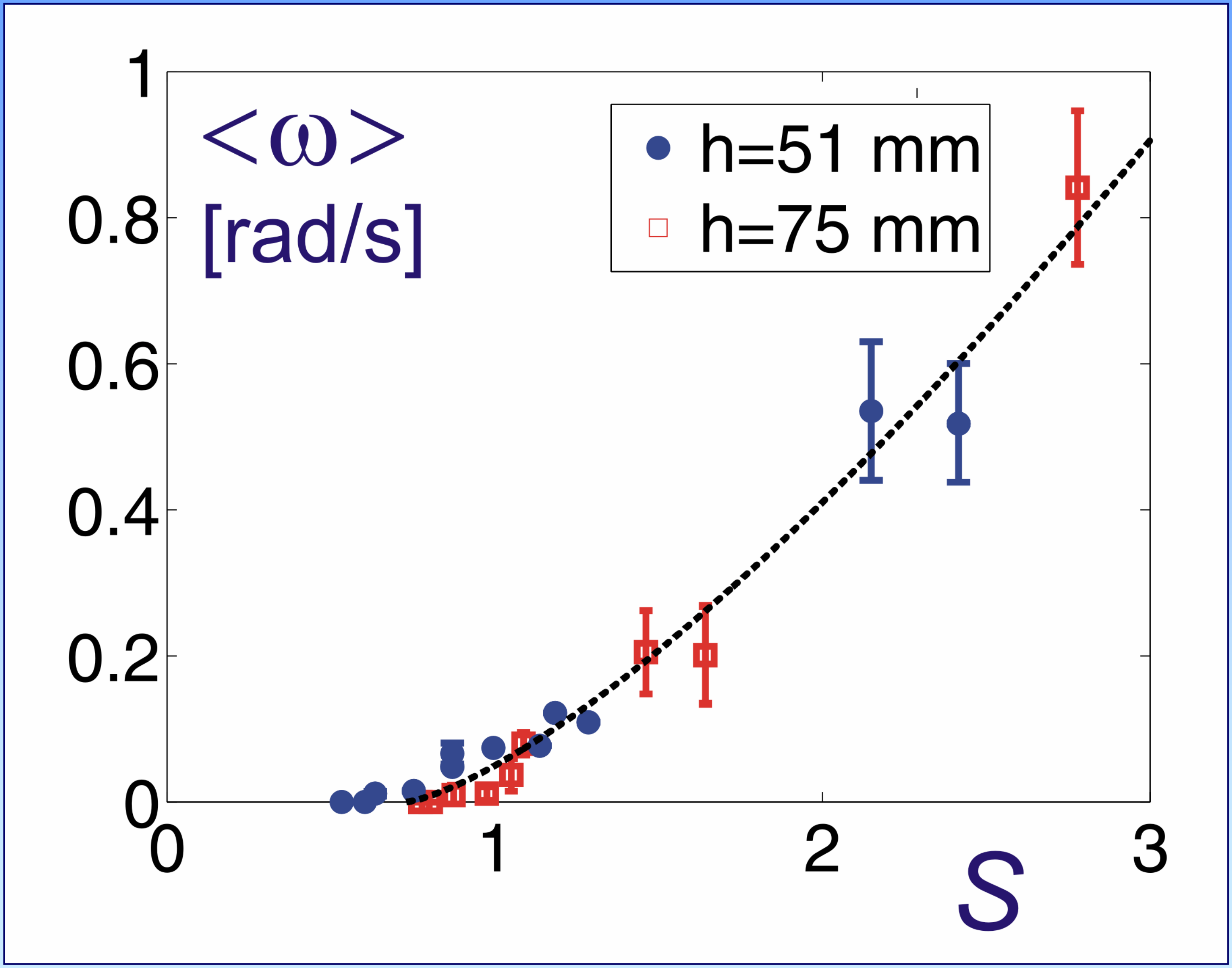
Ratchet: strong shaking



Ratchet: strong shaking



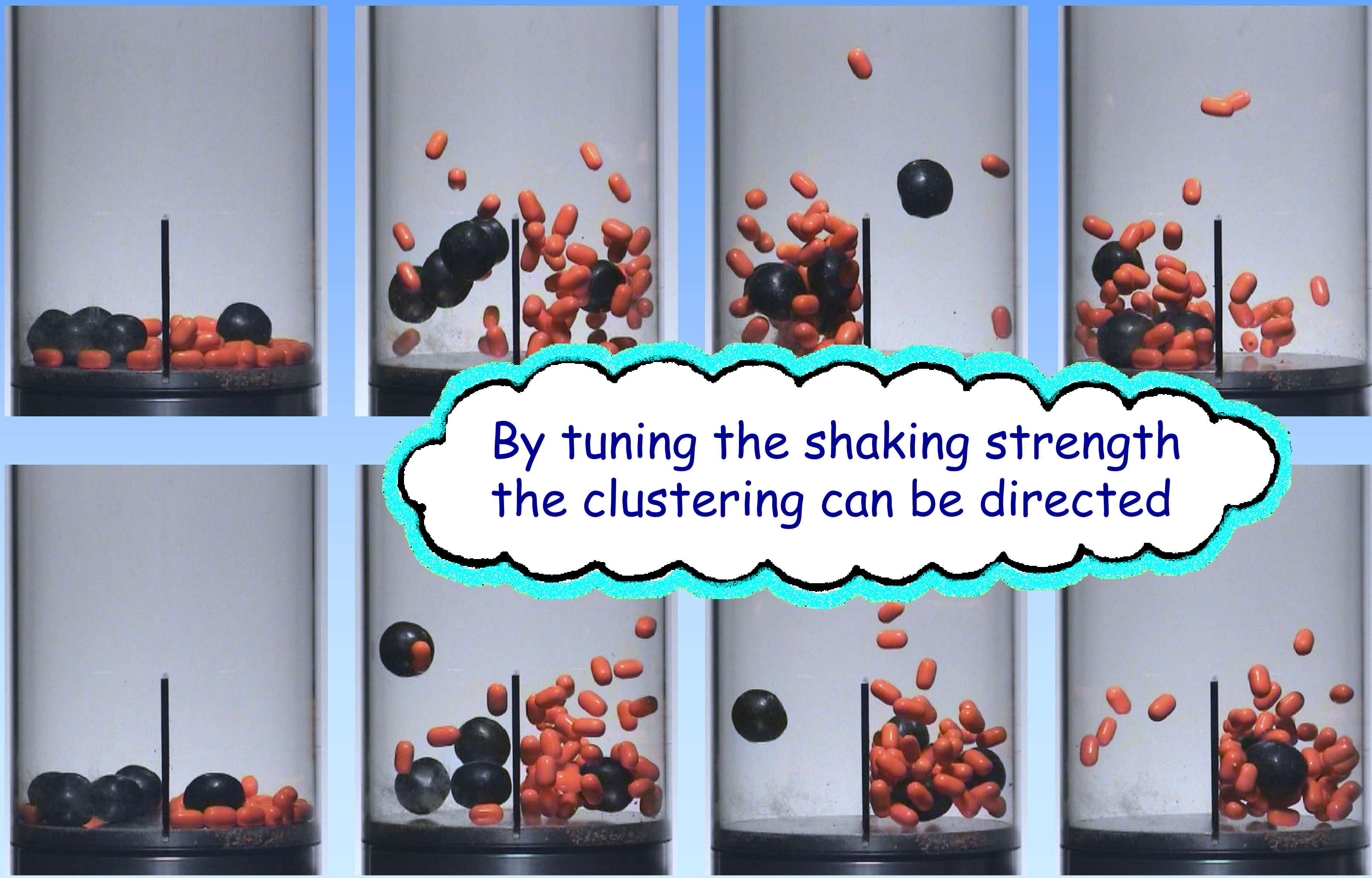
Ratchet: ω vs. S



There is more !

oscillons
Umbanhowar, Melo, and Swinney,
Nature **382** (1996)

Bidisperse systems



By tuning the shaking strength
the clustering can be directed

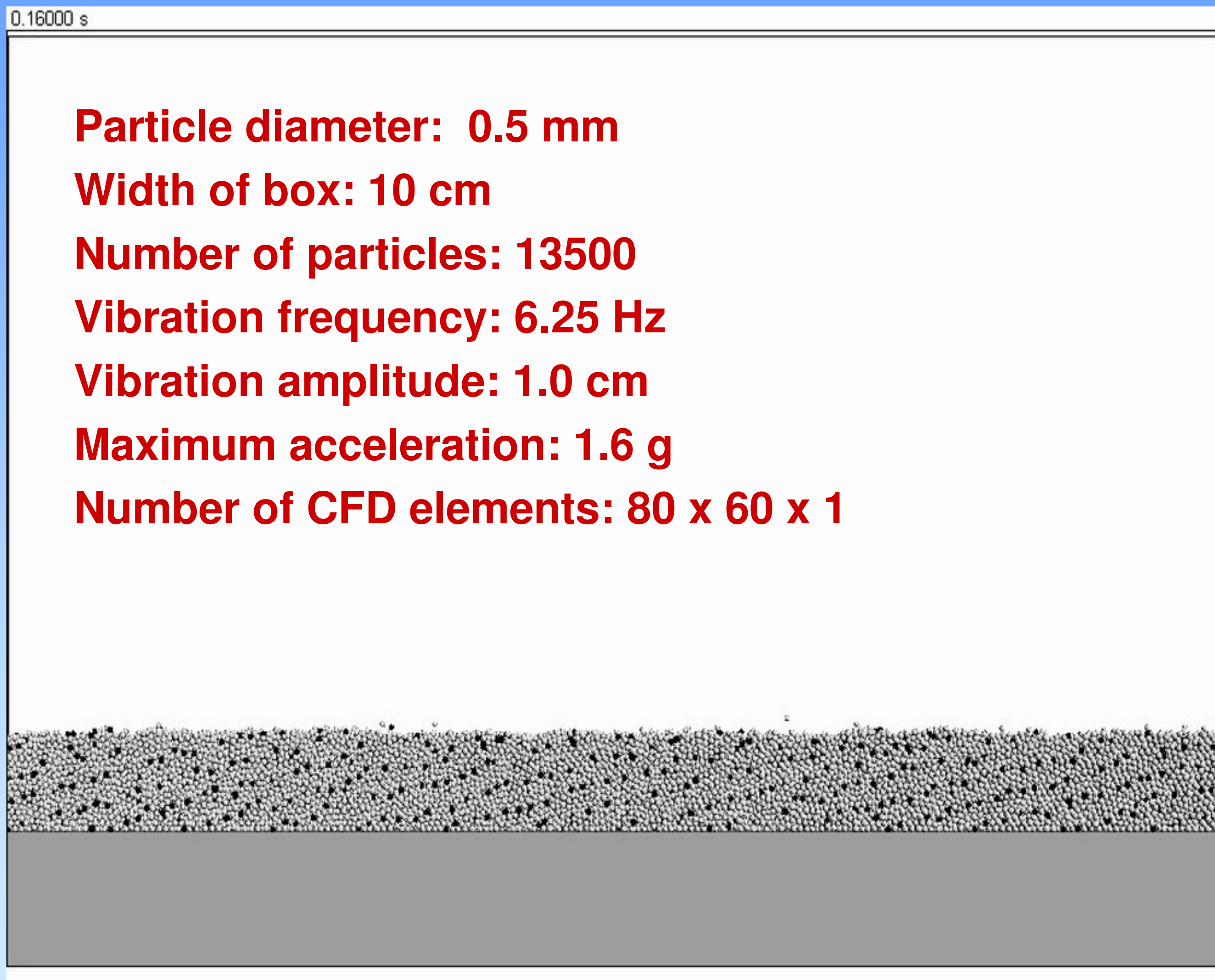
The role of air

@ Γ = 1.6

van Gerner, van der Hoef, van der Meer, van der Weele,
Phys. Rev. E 76, 051305 (2007)

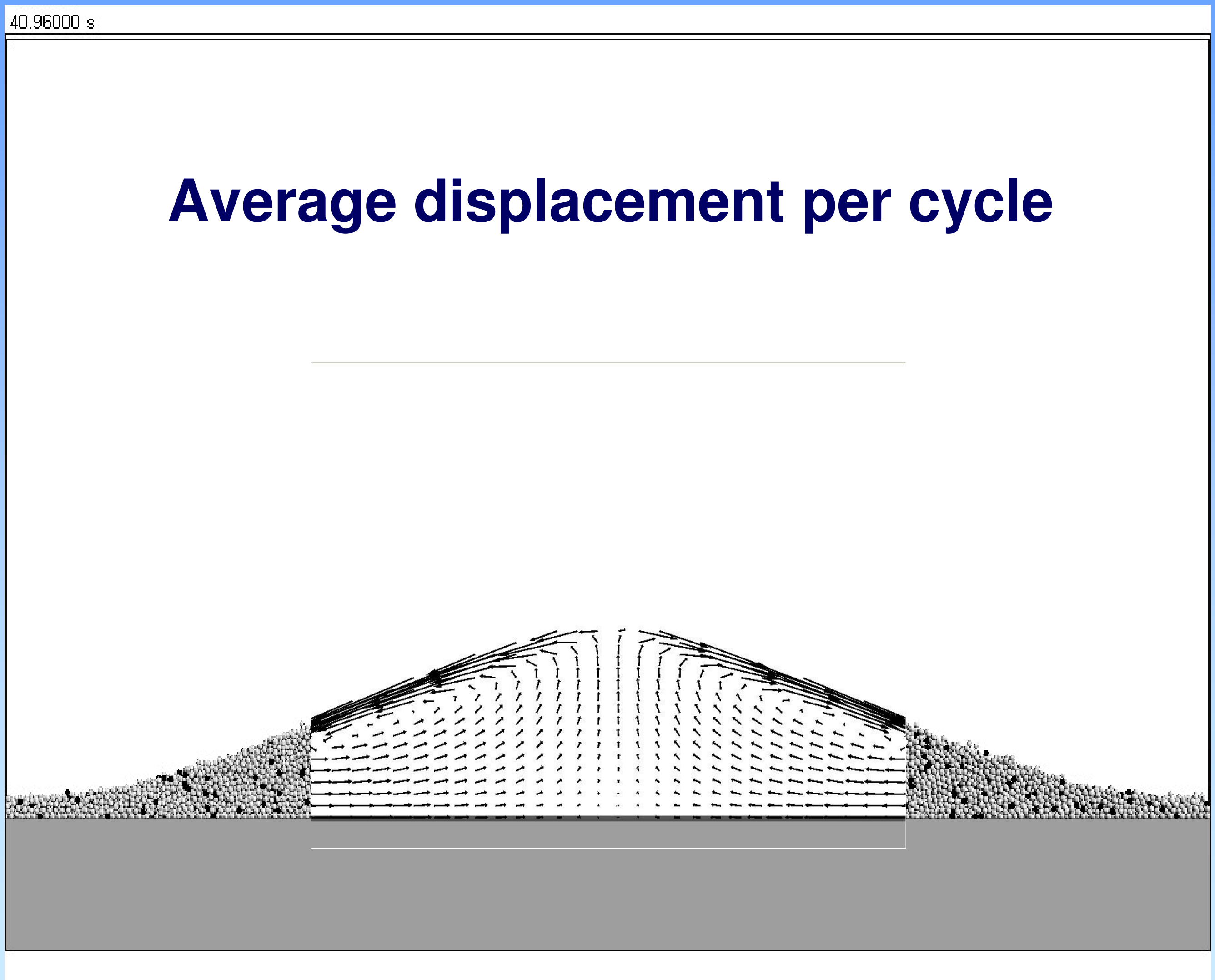
Faraday heaping

Particle diameter: 0.5 mm
Width of box: 10 cm
Number of particles: 13500
Vibration frequency: 6.25 Hz
Vibration amplitude: 1.0 cm
Maximum acceleration: 1.6 g
Number of CFD elements: 80 x 60 x 1



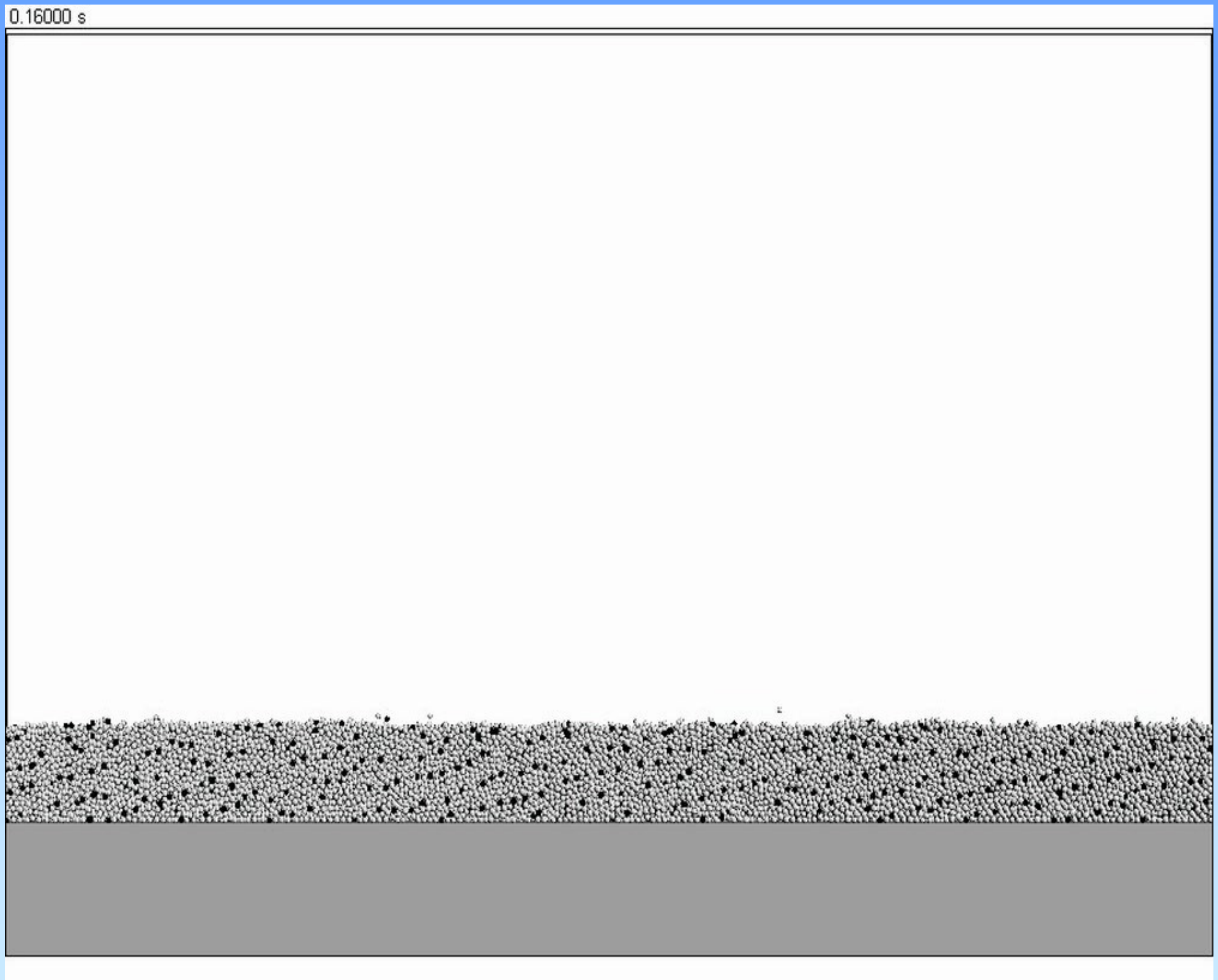
Numerical simulation of heaping with a hybrid GD-CFD code

Faraday heaping



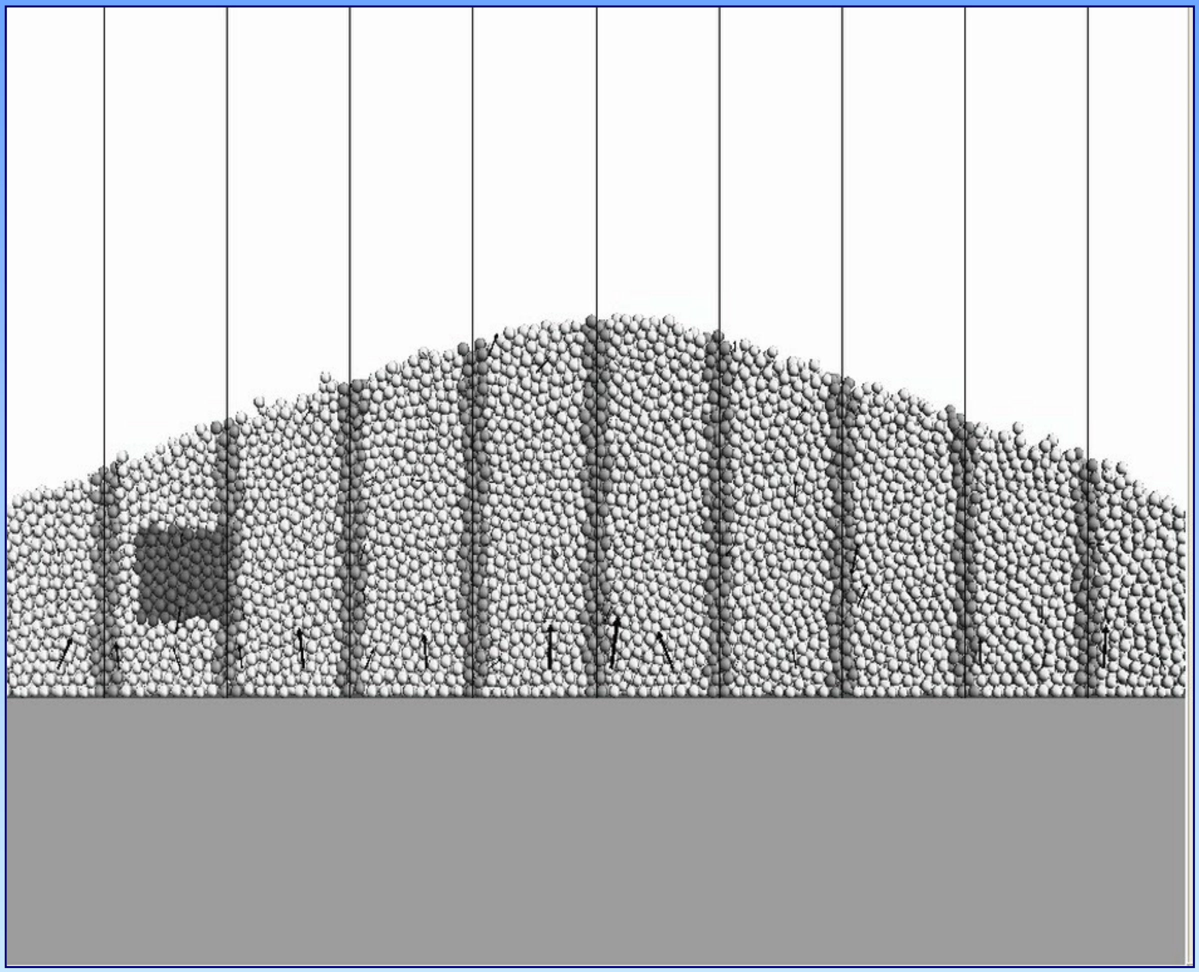
Numerical simulation of heaping with a hybrid GD-CFD code

Without air…



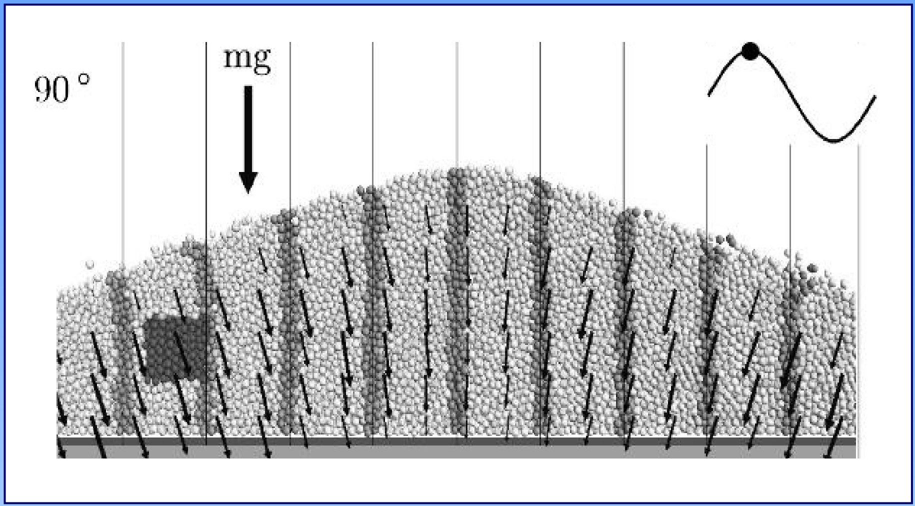
… there is no heap !

Steady state

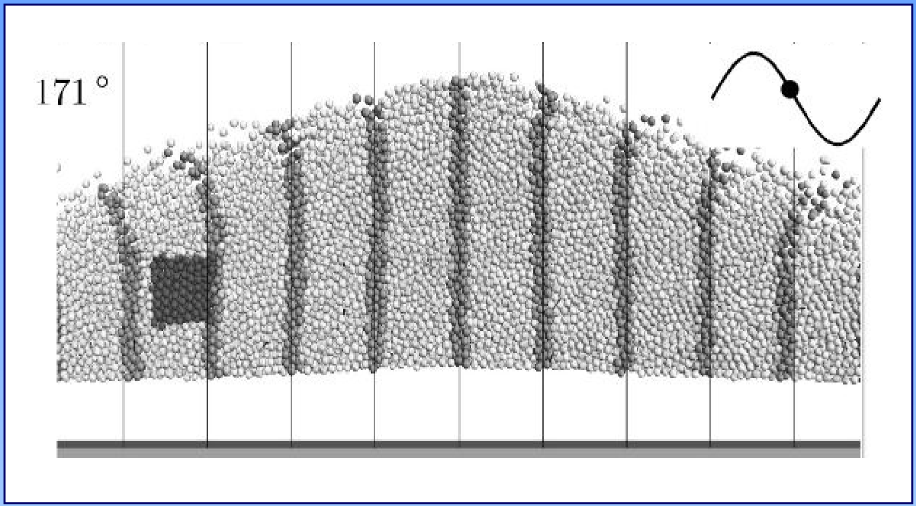


But why does the bulk only move inwards ?

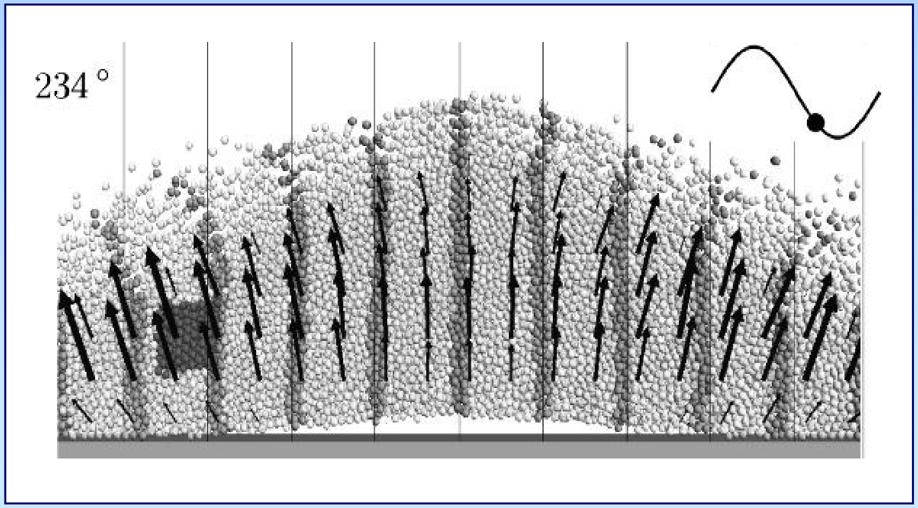
4 snapshots



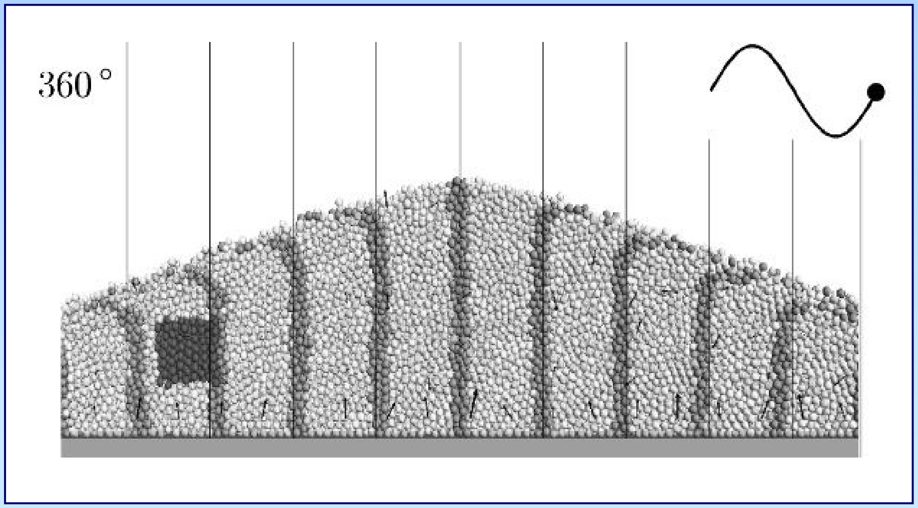
inward drag, loose packing



no drag, loose packing



outward drag, dense packing



no drag, dense packing

Thanks for

your attention !