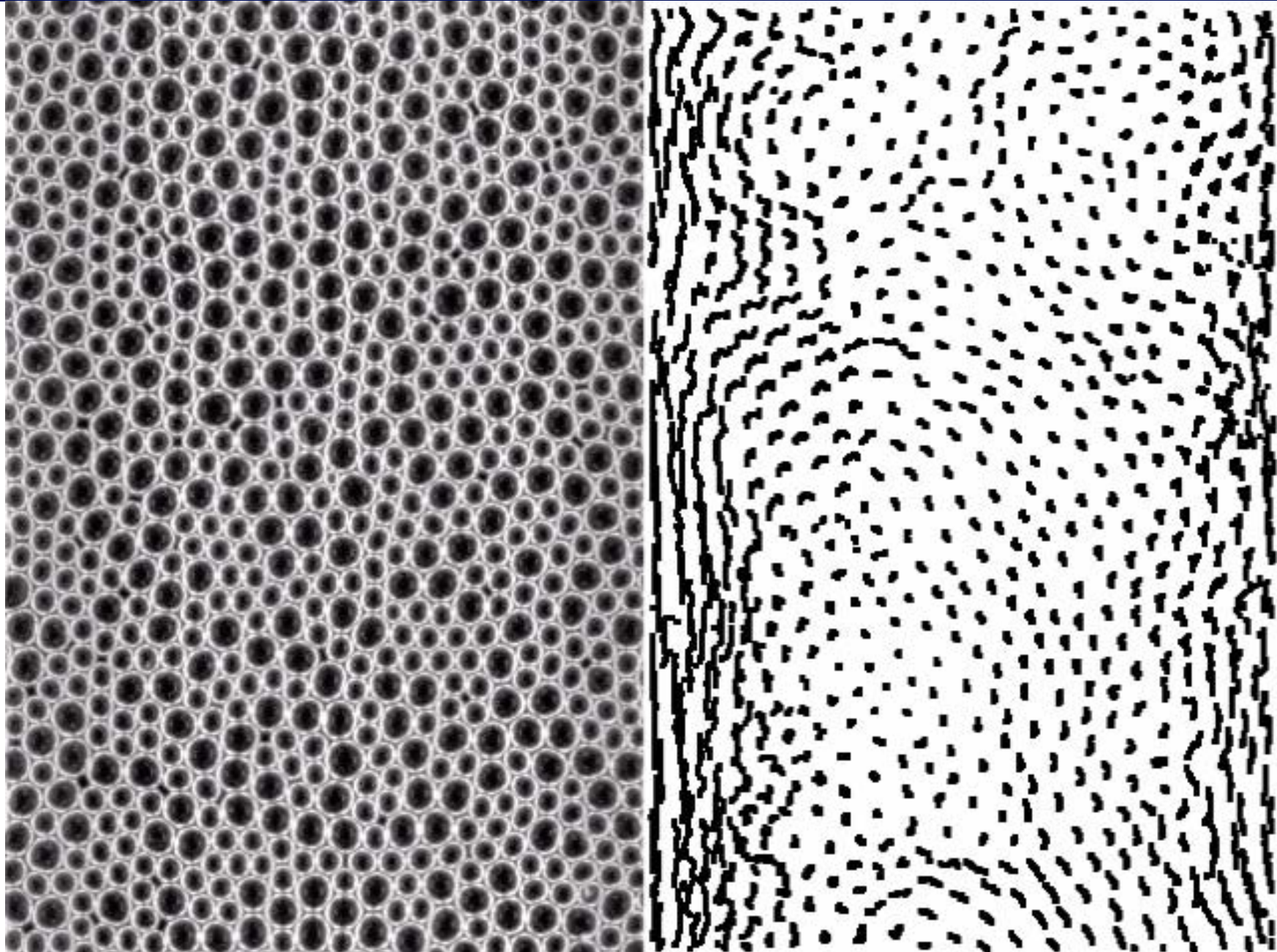


Flow of Foams



Flow of Foams

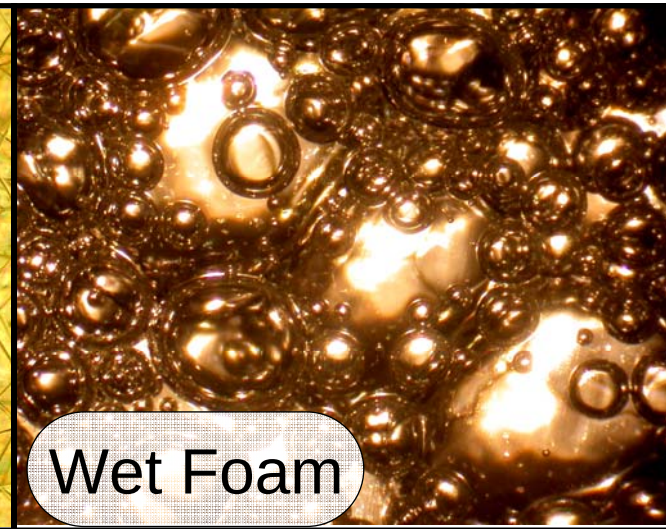
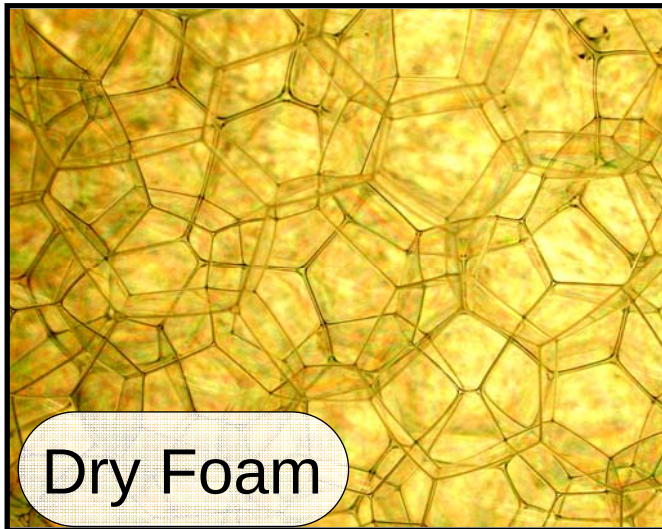
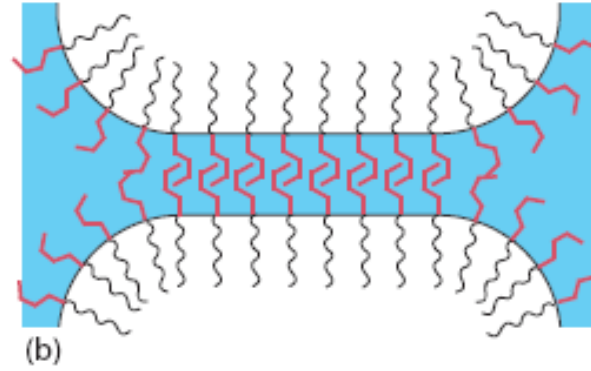
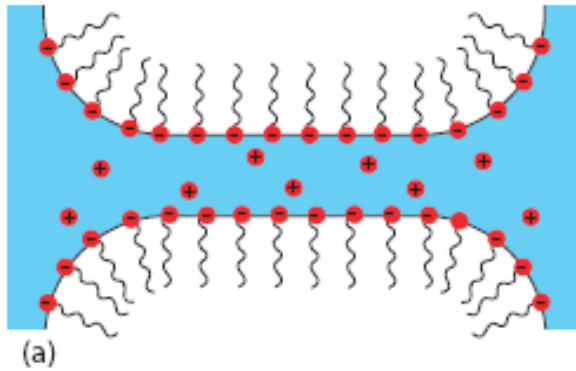
Bubbles vs Grains

Experiments: Rheology

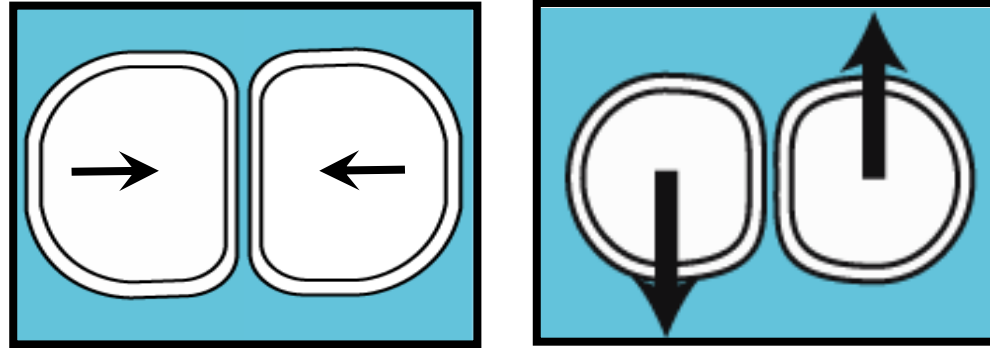
Theory: Flow near Jamming

Foams

Gas (nitrogen) + **Liquid** (water & glycerol) + **Surfactant** (soap)



Bubbles vs Grains

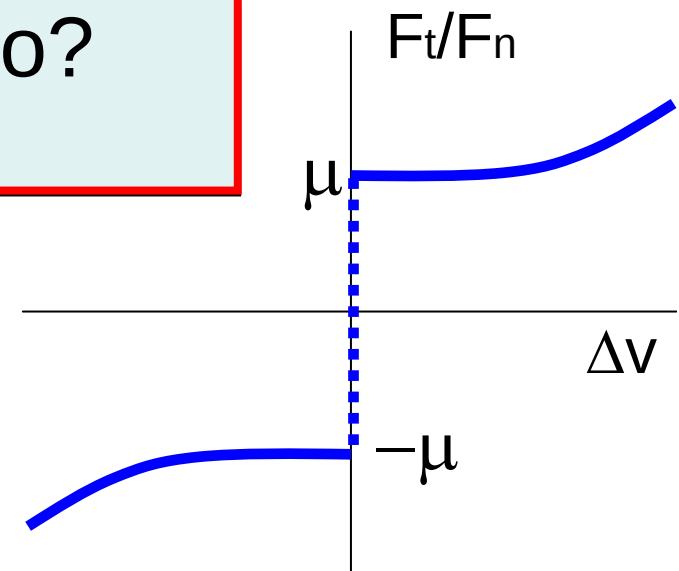


Foam
Bubbles

Grains
Granular Media

Collective Behavior?

Macro from Micro?



Experiments: 2D Foams



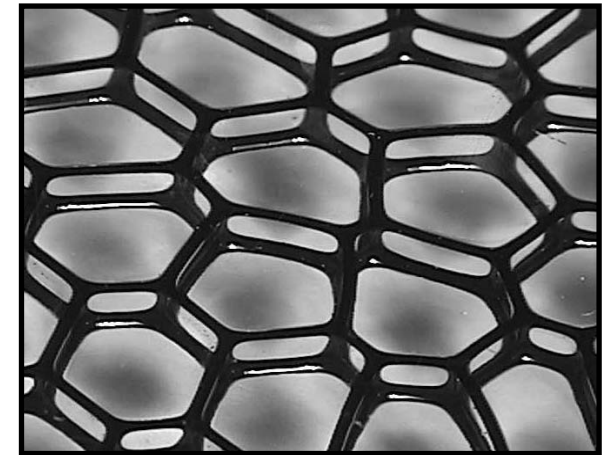
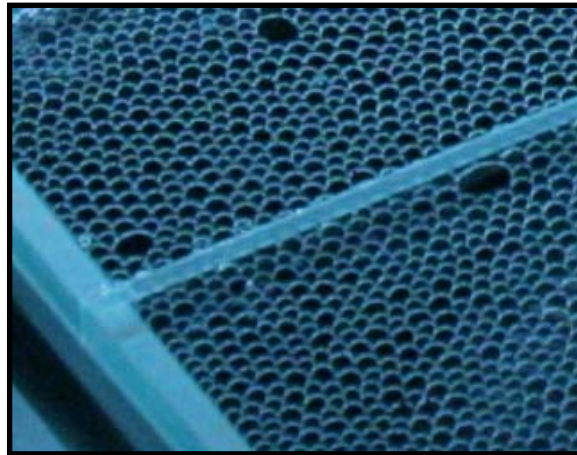
Bubble Raft: **L**iquid-**A**ir



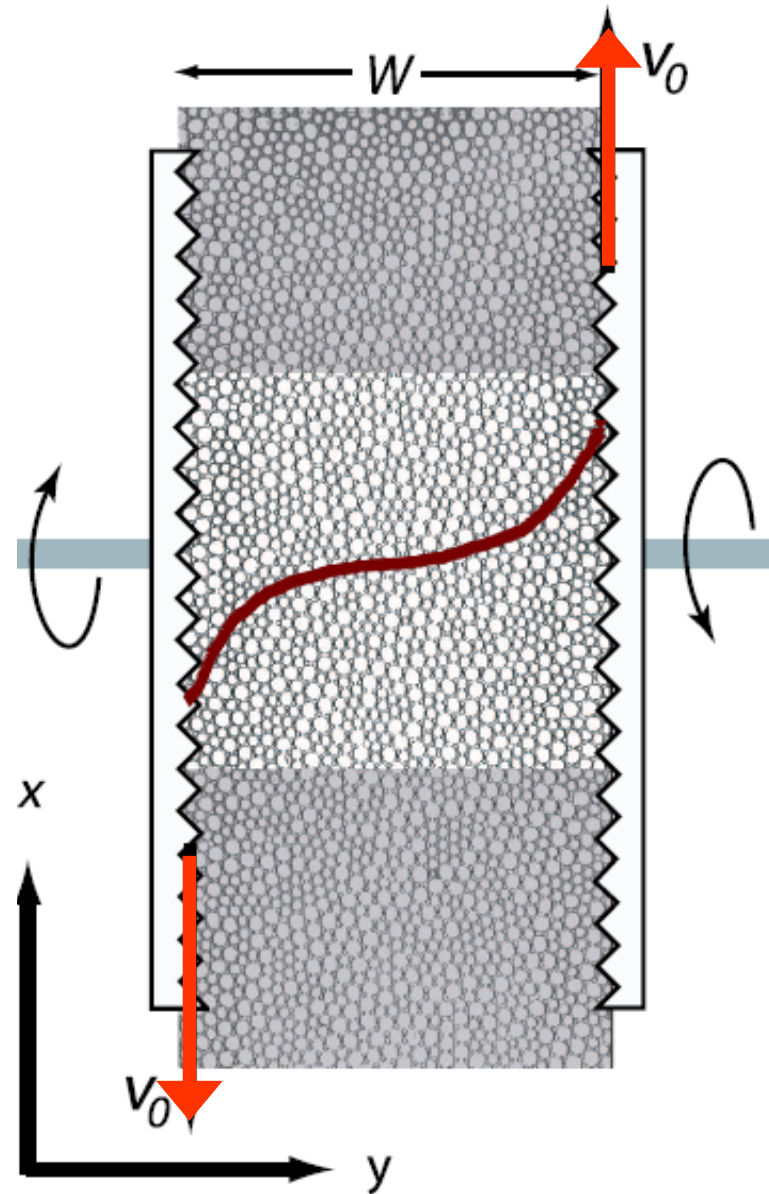
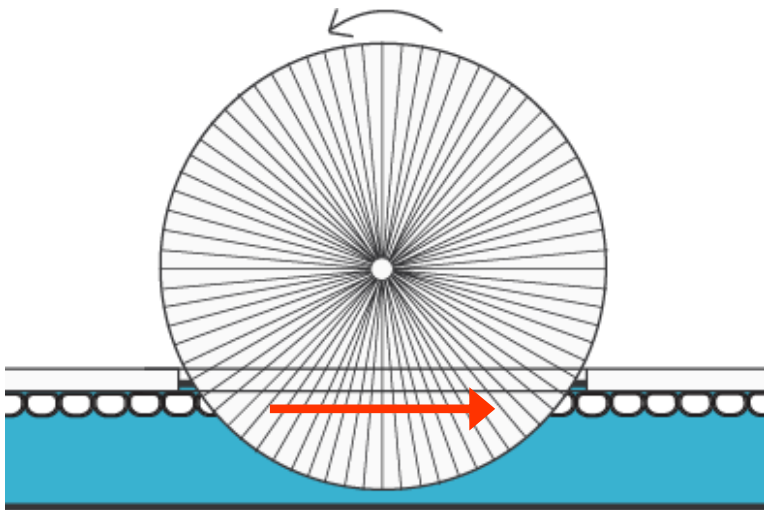
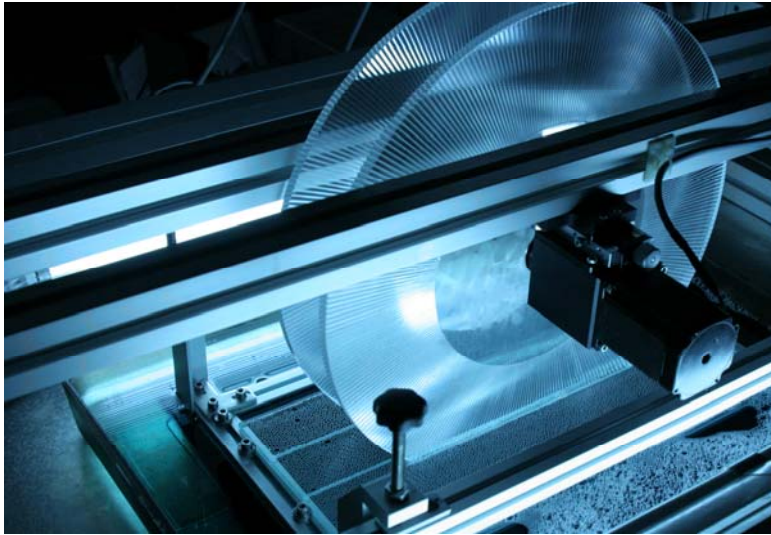
Top Plate: **L**iquid-**G**lass



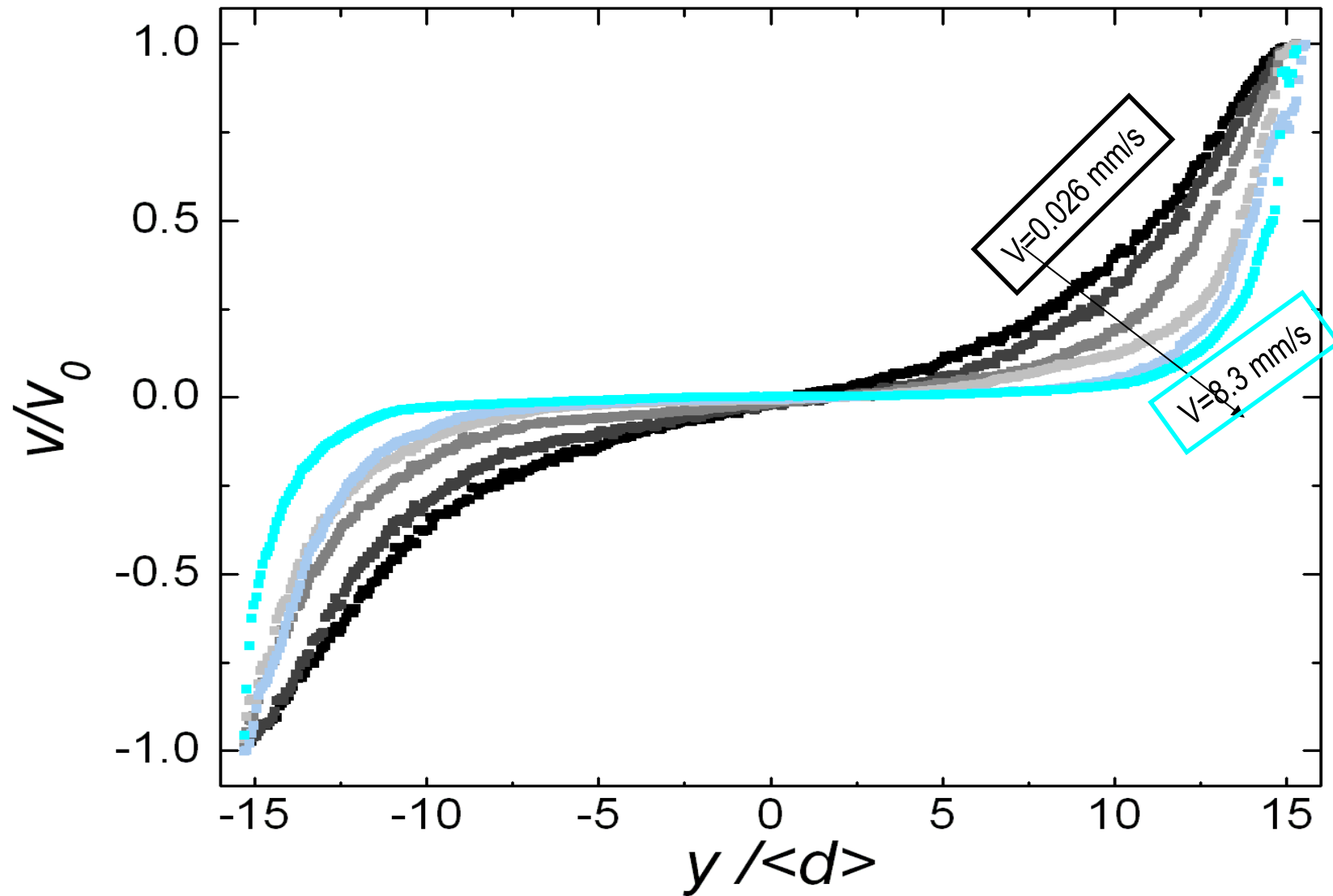
Hele-Shaw: **G**lass-**G**lass



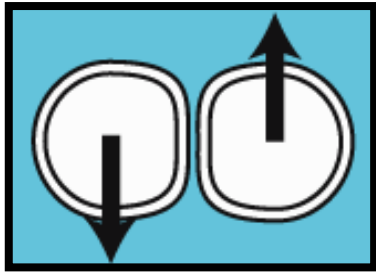
Experiments: 2D Foams



Rate Dependent Shear Bands

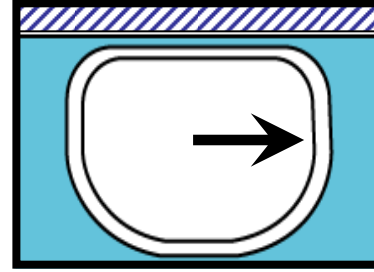


Why Shear Bands?



Bubble-Bubble Drag

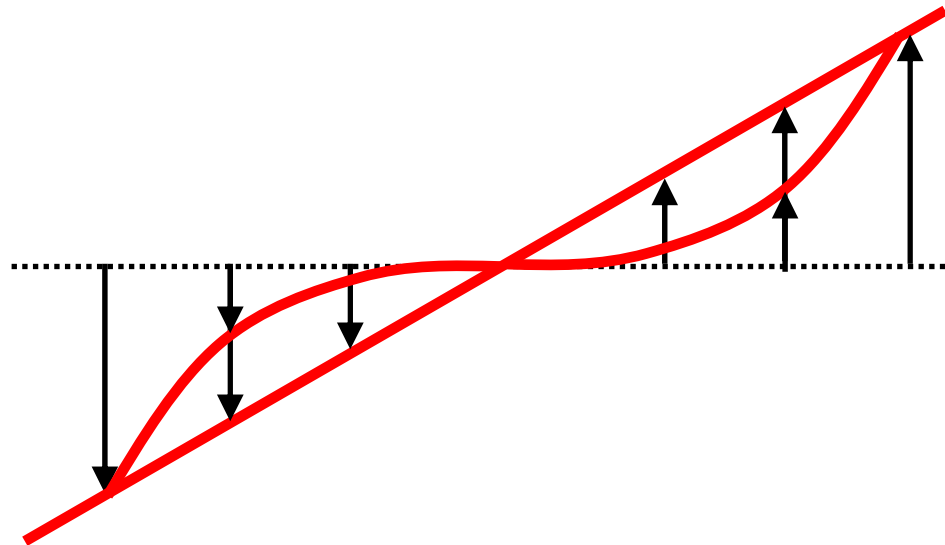
$$F \sim (\eta \Delta v / \sigma)^\alpha$$



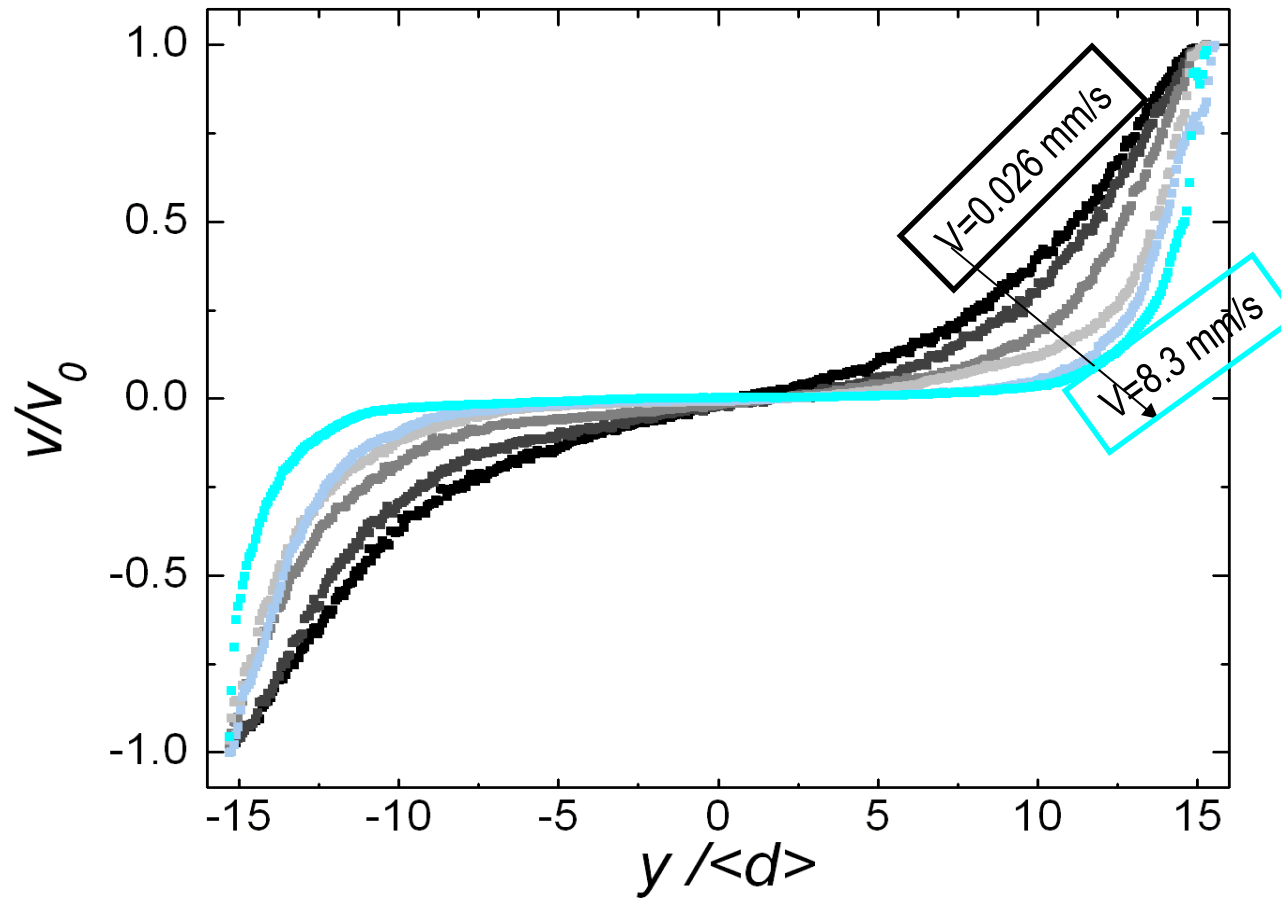
Bubble-Wall Drag

$$F \sim (\eta v / \sigma)^{2/3}$$

Linear Profile Impossible

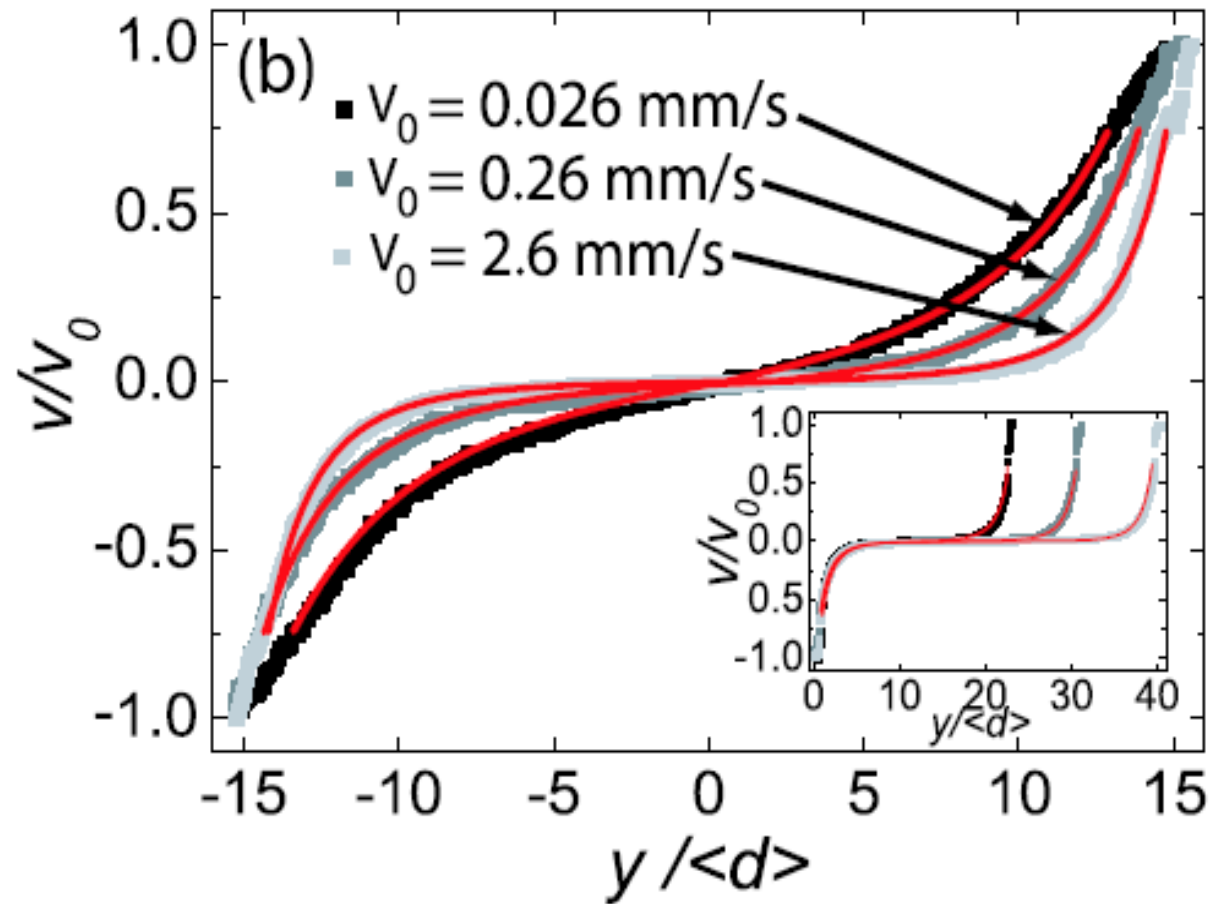


Rate Dependence



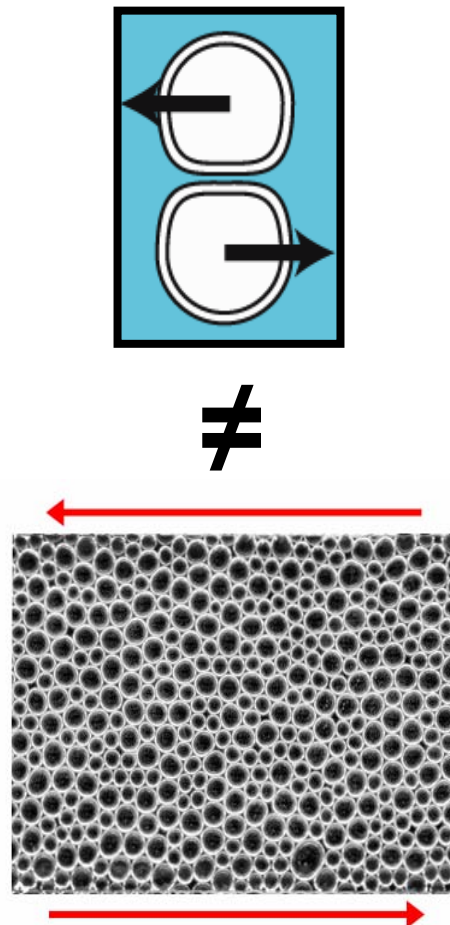
Foam Drag and Wall Drag Scale Differently

Rate Dependence



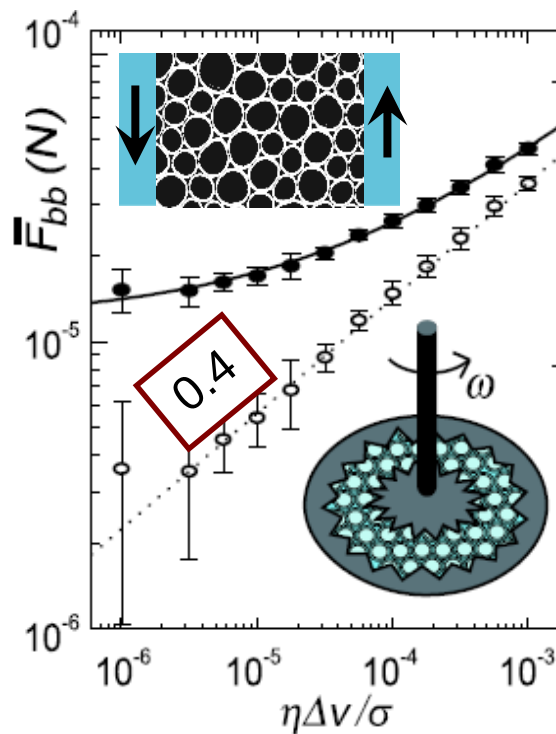
Foam Drag and Wall Drag Scale Differently

Disorder



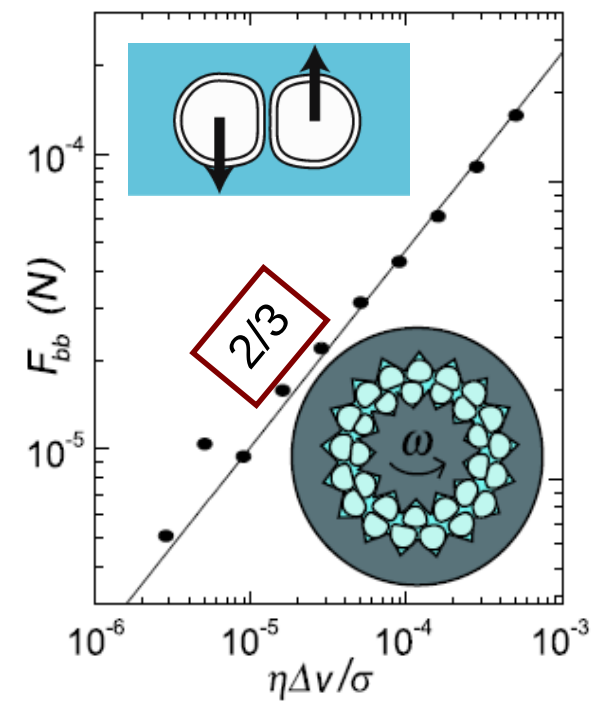
Coarse Grained

$$\bar{F}_{bb} \sim F_y + \Delta v^{0.4}$$

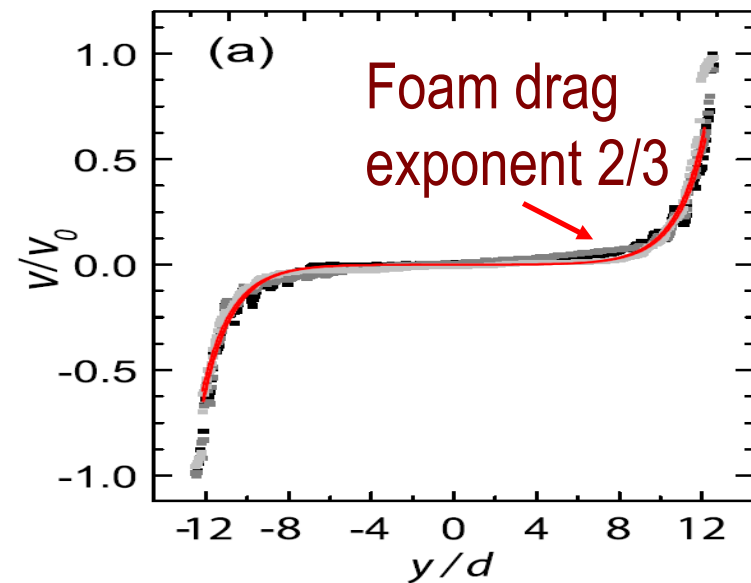
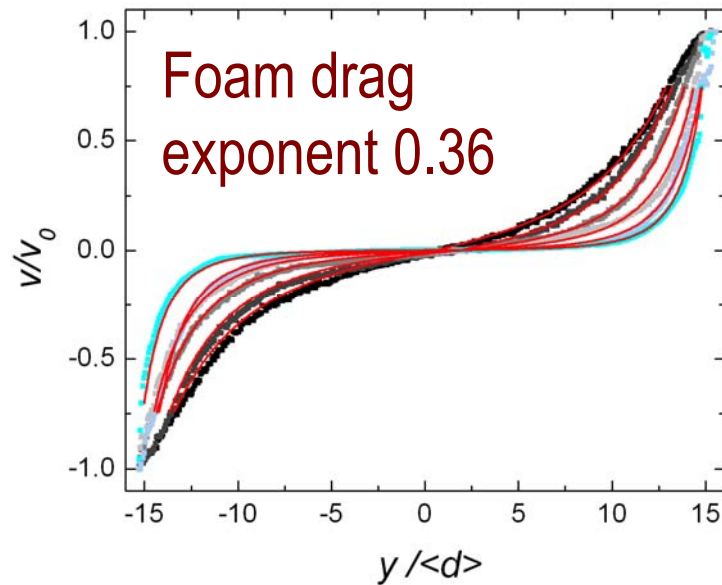
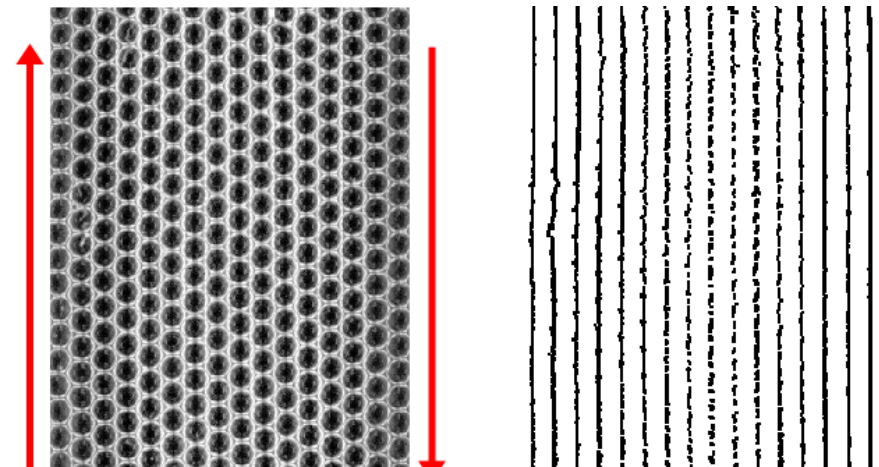
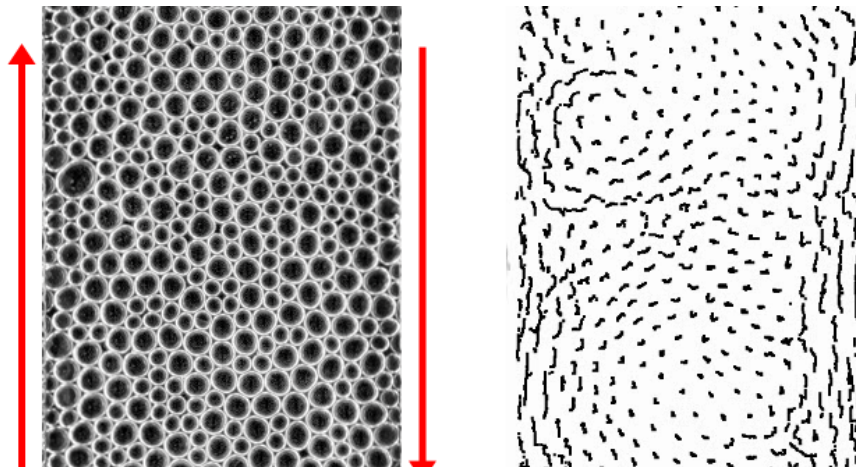


Bubble-Bubble

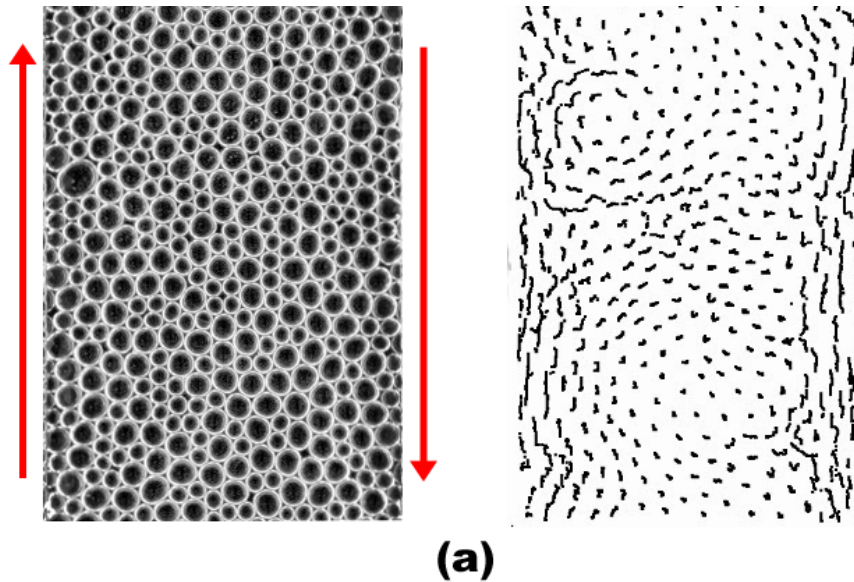
$$F_{bb} \sim \Delta v^{2/3}$$



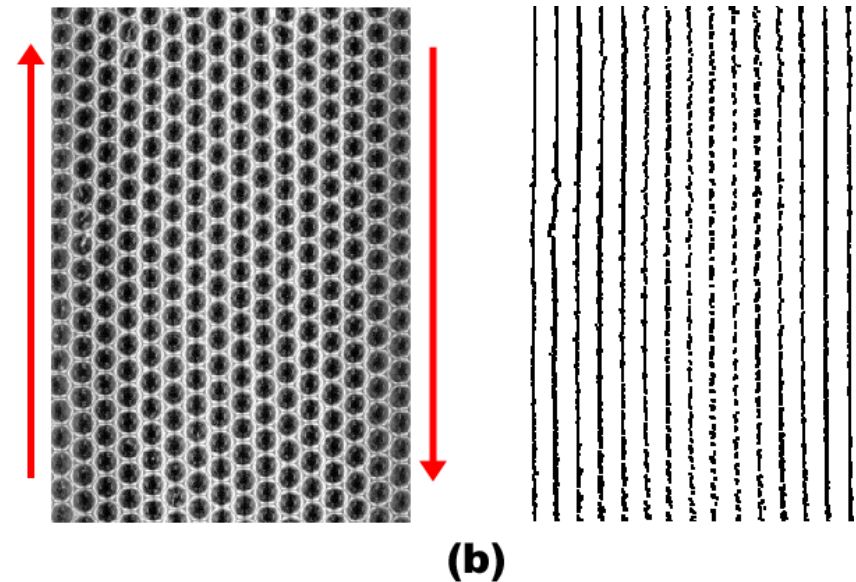
Role of Disorder



Role of Disorder



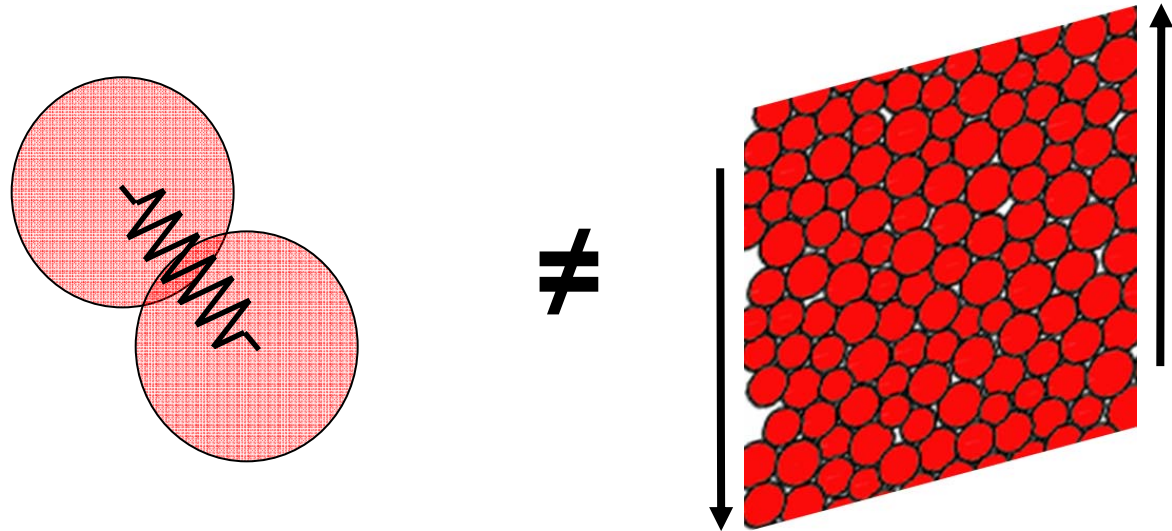
Global Drag Different from Local Drag
Large Fluctuations
Fluctuations depend on Flow



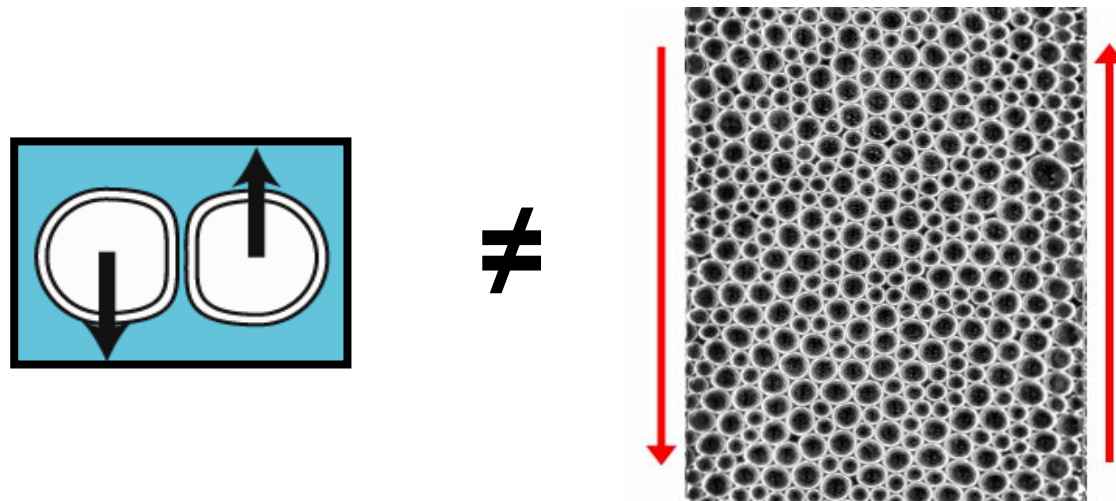
Global = Local
Boring

Jamming and Rheology: Commonalities

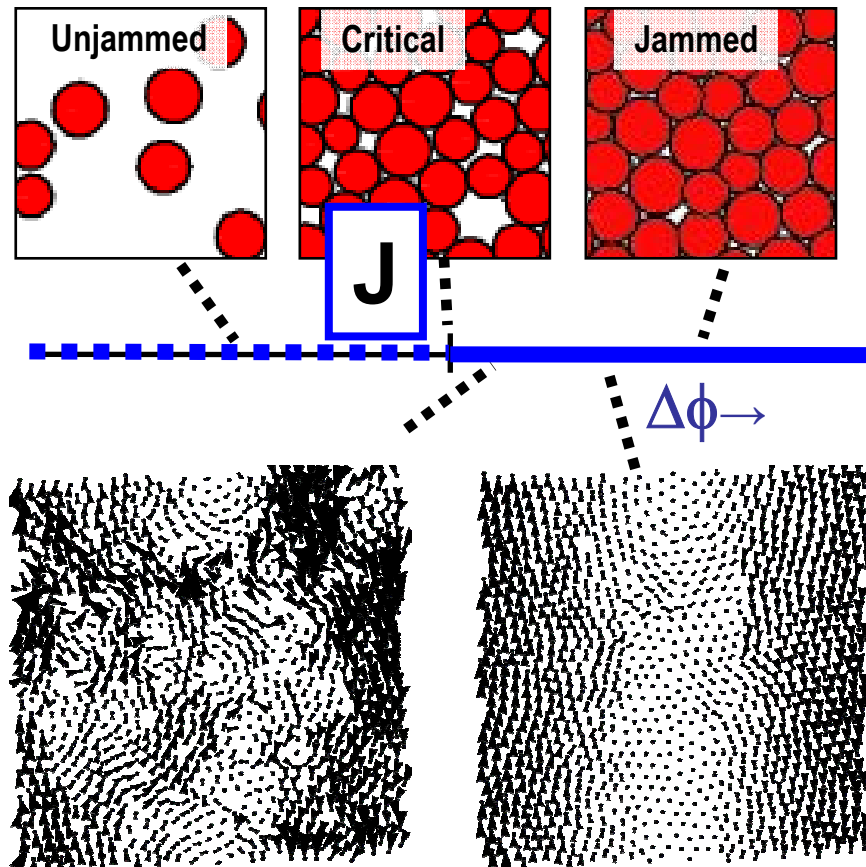
Jamming



Rheology



Role of Disorder: Elastic Simulation



Singularly Strong Non-affine:

$$\mathbf{G}/\mathbf{K} \sim \Delta\phi^{1/2}$$

$$\sim \mathbf{P}^{1/2} \text{ (harmonic)}$$

O'Hern, Silbert, Liu, Nagel, **PRE** 68 011306 (2003)

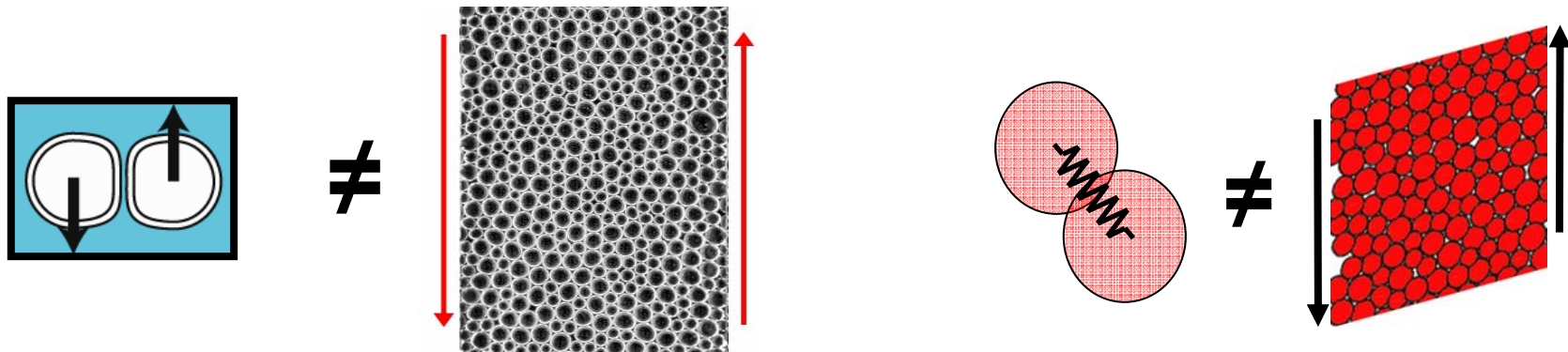
Ellenbroek, Zeravcic, van Saarloos and MvH, **EPL** 87 34004 (2009)

Jamming and Rheology: Commonalities

Strong Disorder Changes Elastic Scaling Near Jamming

..... So What Causes Anomalous Rheology?

Anomalous Strong Fluctuations!



Rheology Near Jamming

Microscopic Model

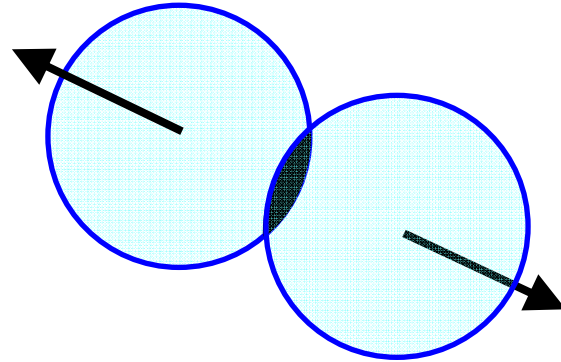
Energy Balance: Anomalous Fluctuations

Scaling Model for Rheology Near Jamming

Microscopic Model

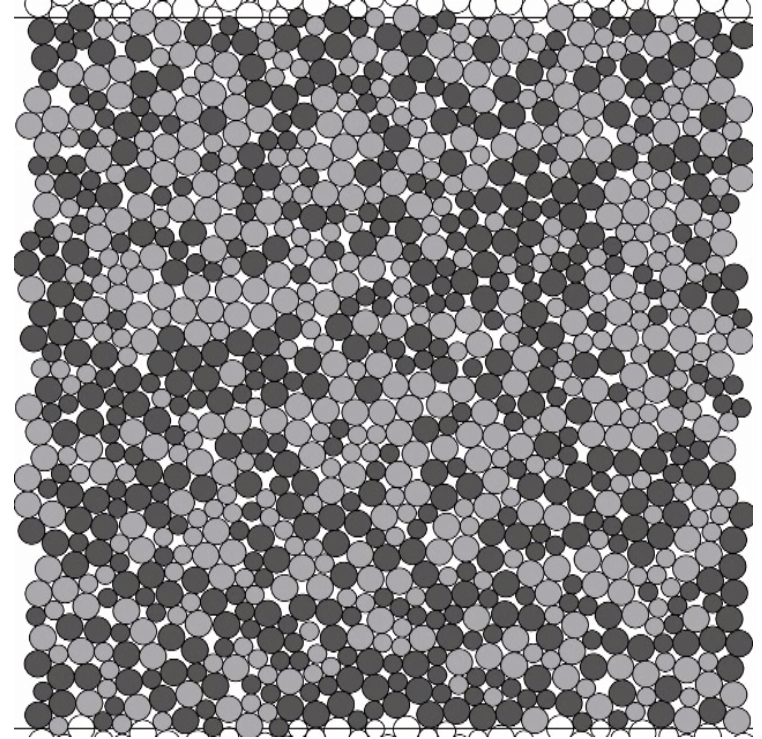
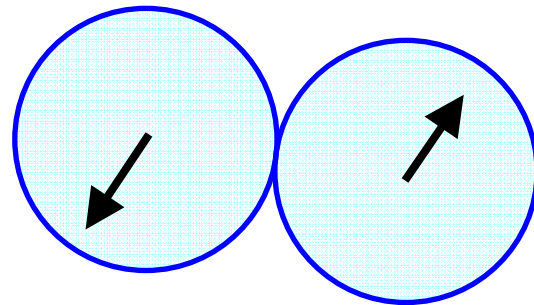
Elastic

$$F_e \sim \delta$$



Viscous

$$F_v \sim (\Delta v)^\alpha$$

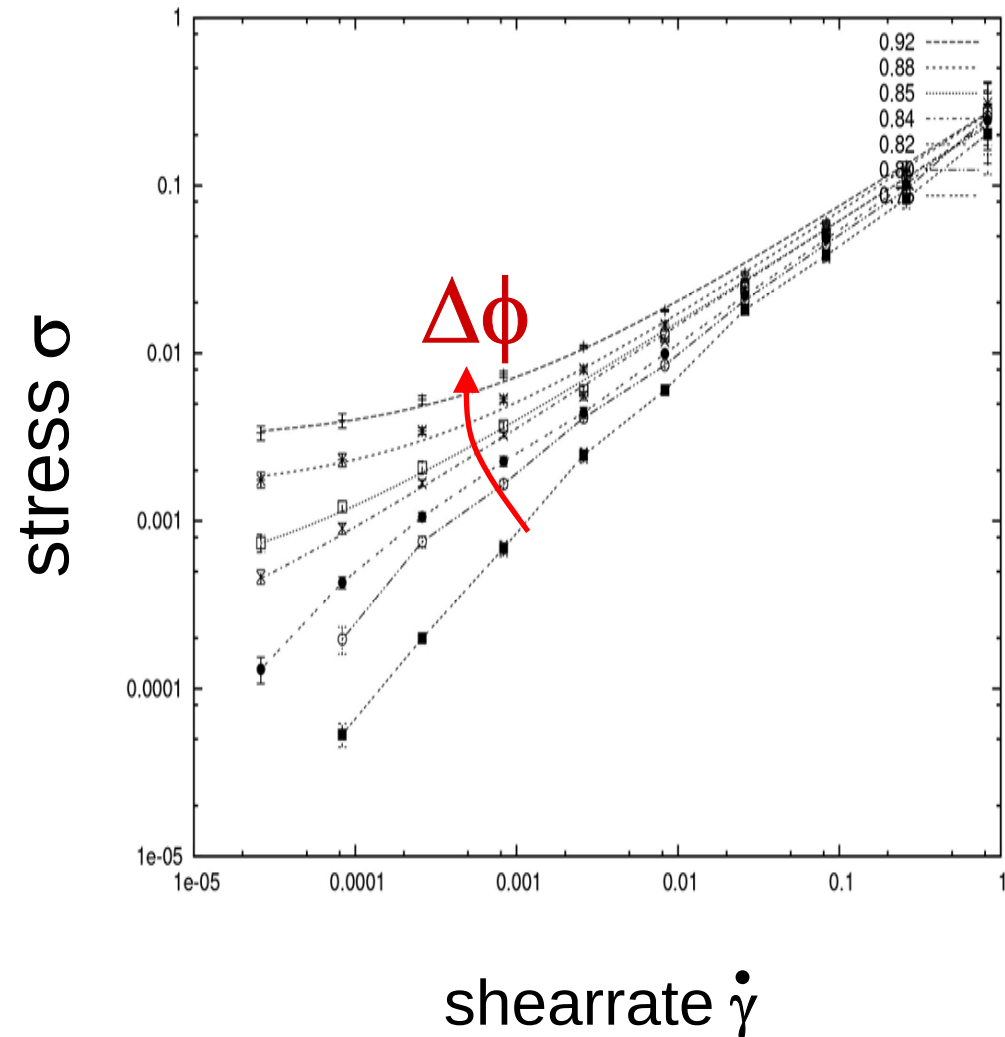


No inertia, force balance at all times

Microscopic Model

Control $\Delta\phi, \dot{\gamma}$

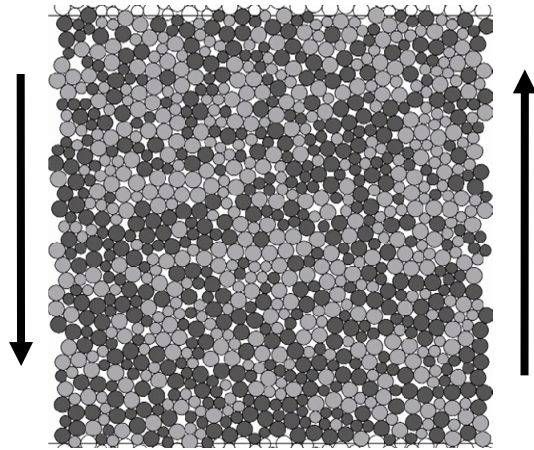
Measure σ



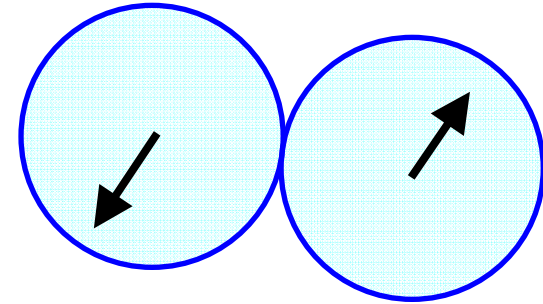
Doug Durian 1995 (Bubble Model)

Energy Balance: Anomalous Fluctuations

Power in $\sim \sigma \dot{\gamma}$



Power out $\sim \langle \Delta \mathbf{v} \Delta \mathbf{v}^\alpha \rangle$



From now on: $\alpha=1$

$$\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$$

Energy Balance: Anomalous Fluctuations

$$\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$$

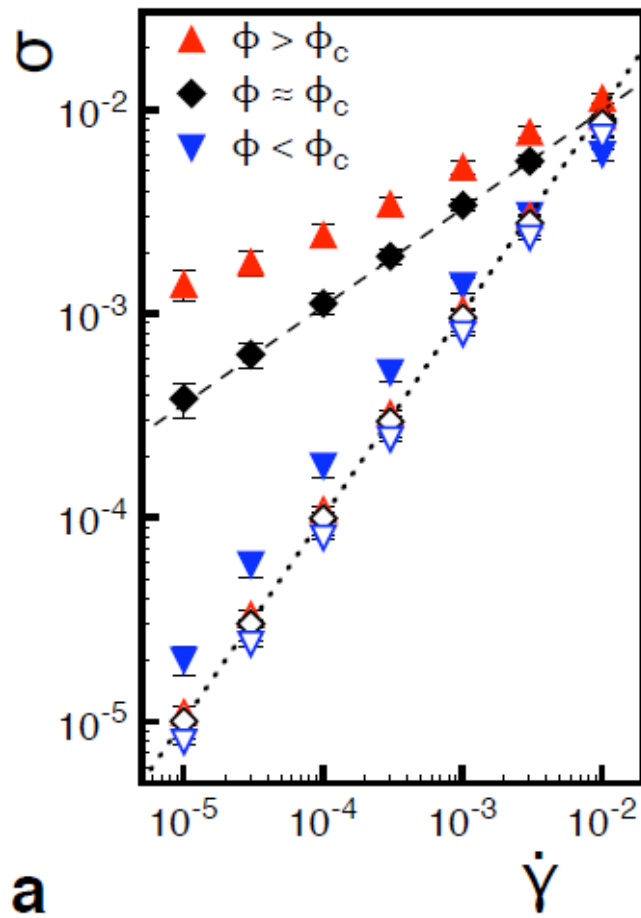
If rheology and fluctuations trivial

$$\sigma \sim \dot{\gamma} \quad \text{and} \quad \langle \Delta v^2 \rangle \sim \dot{\gamma}^2$$

BUT...

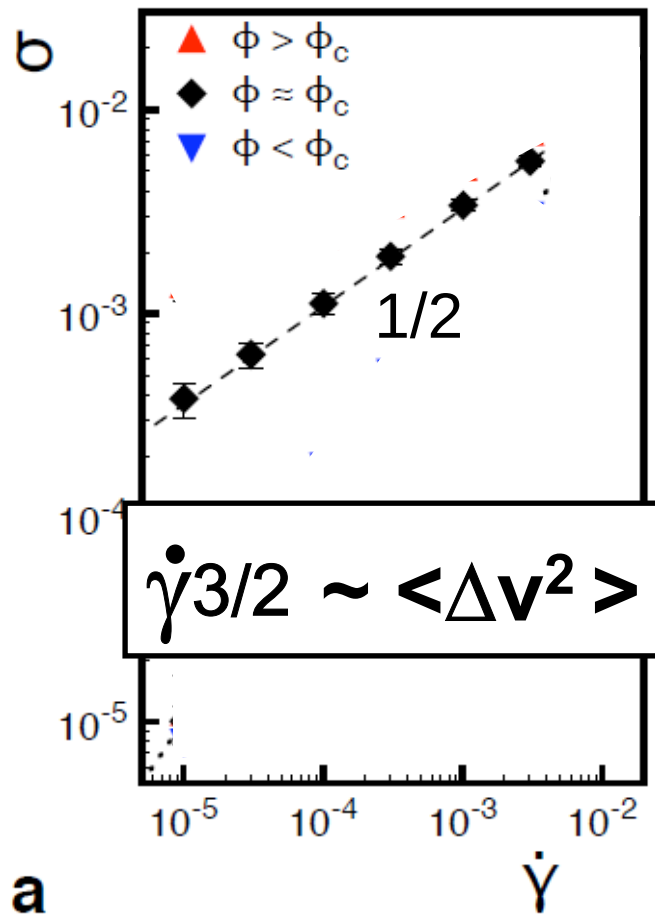
Energy Balance: Anomalous Fluctuations

$$\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$$



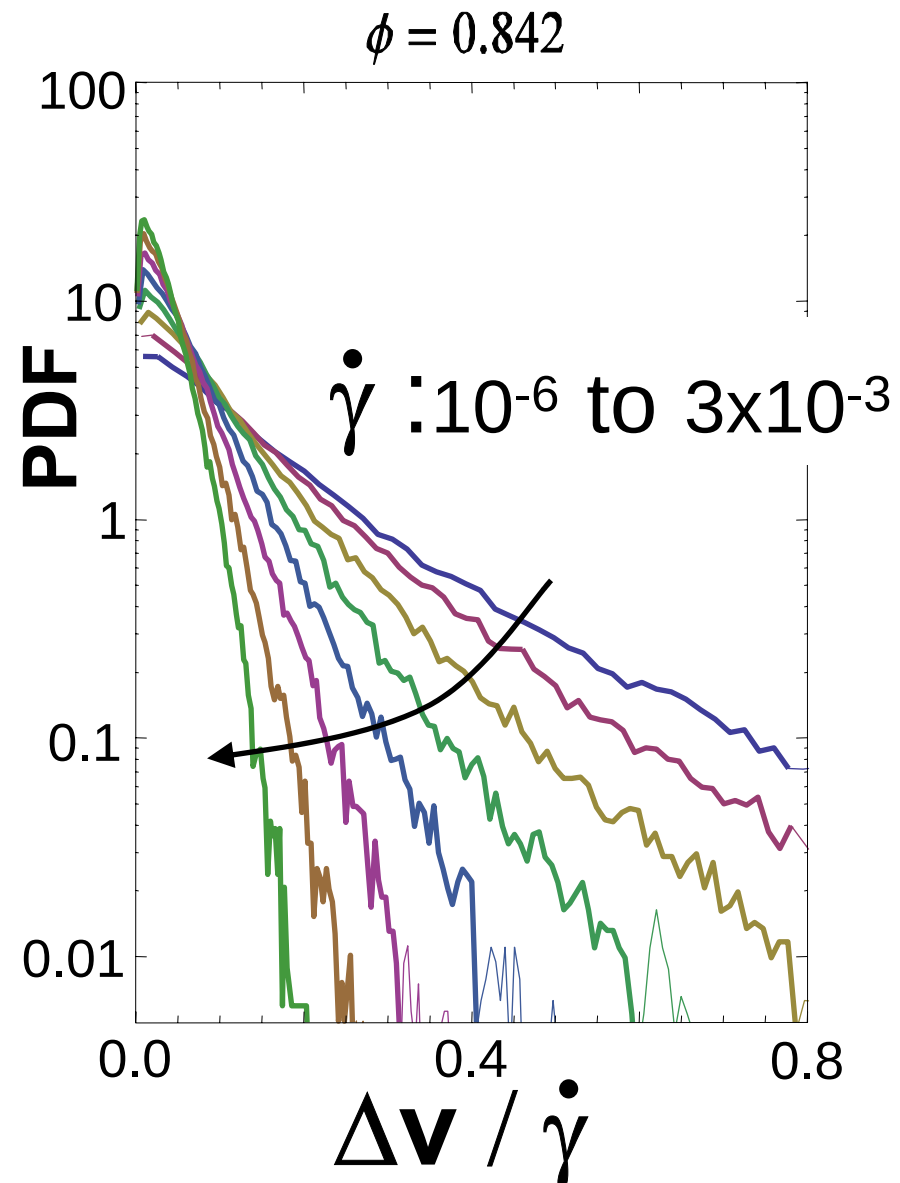
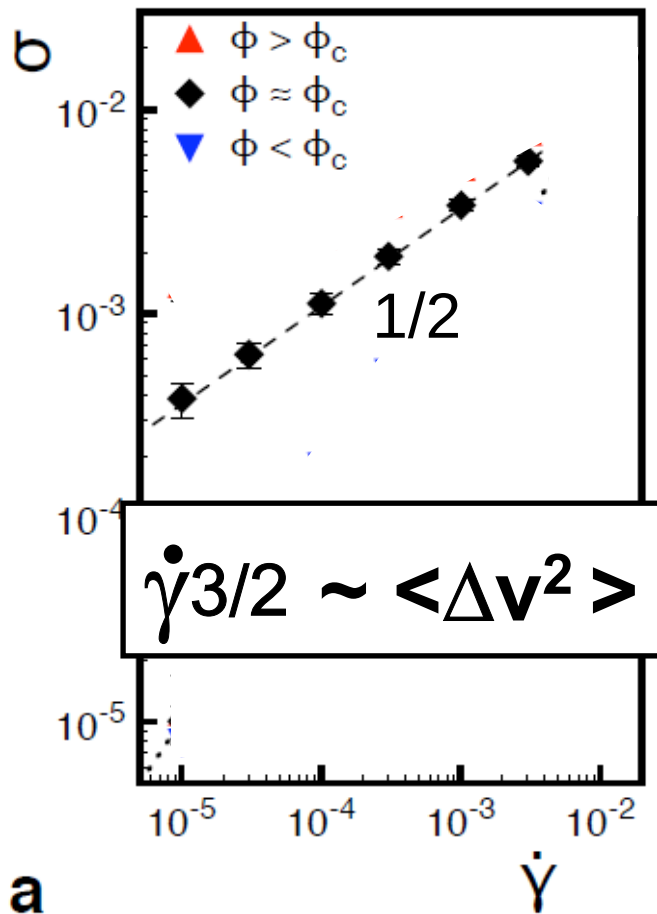
Energy Balance: Anomalous Fluctuations

$$\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$$



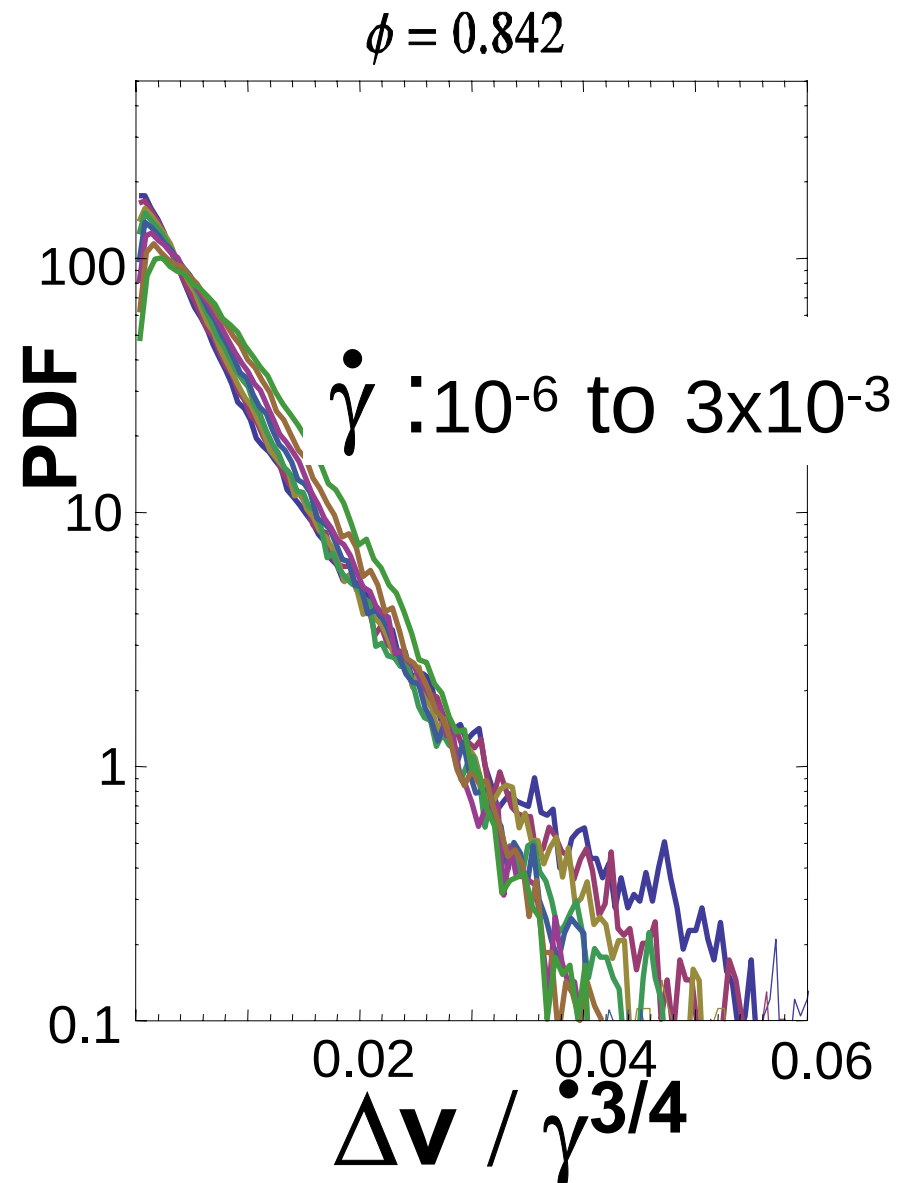
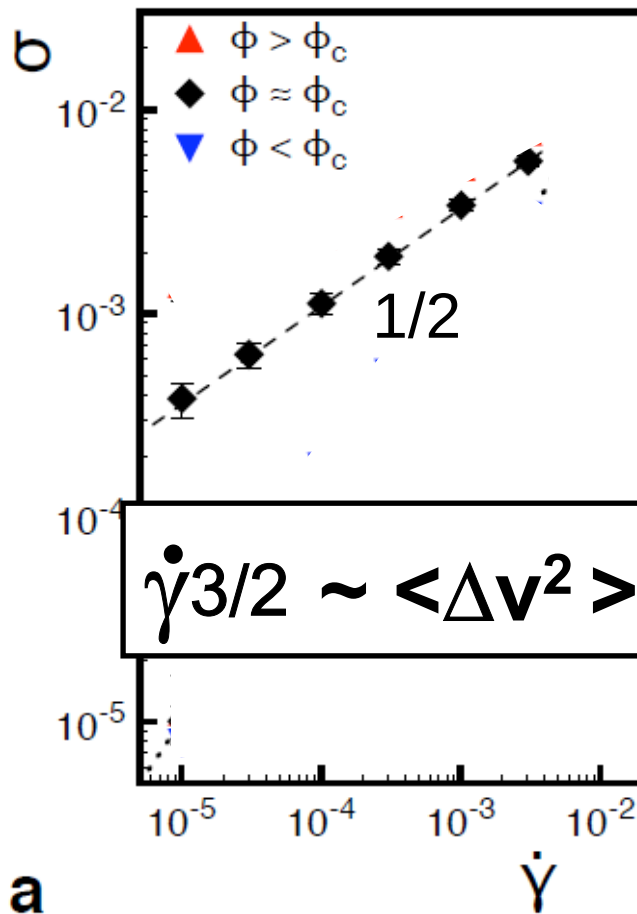
Energy Balance: Anomalous Fluctuations

$$\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$$



Energy Balance: Anomalous Fluctuations

$$\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$$

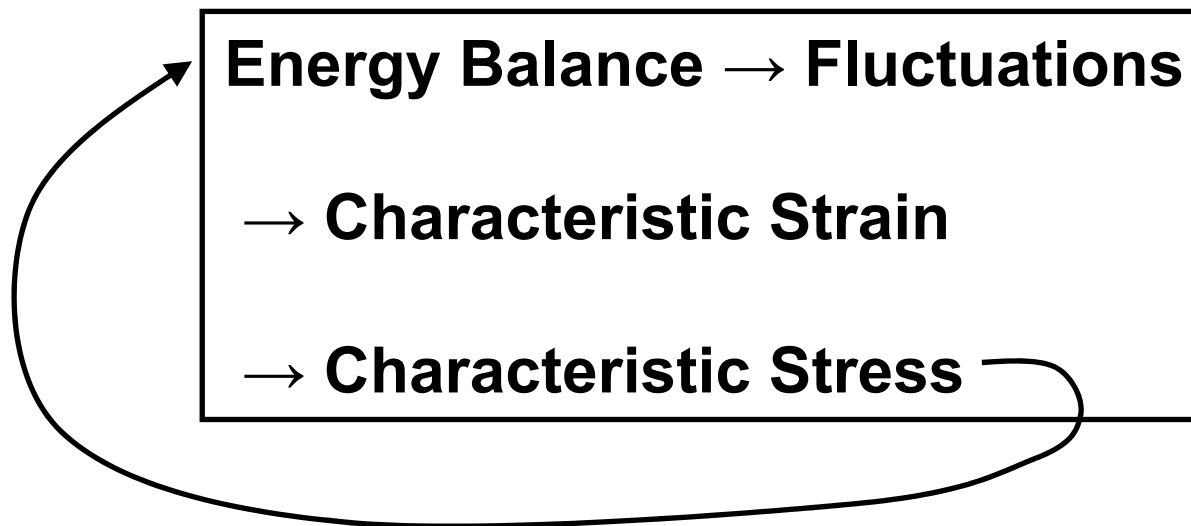


Rheology Near Jamming

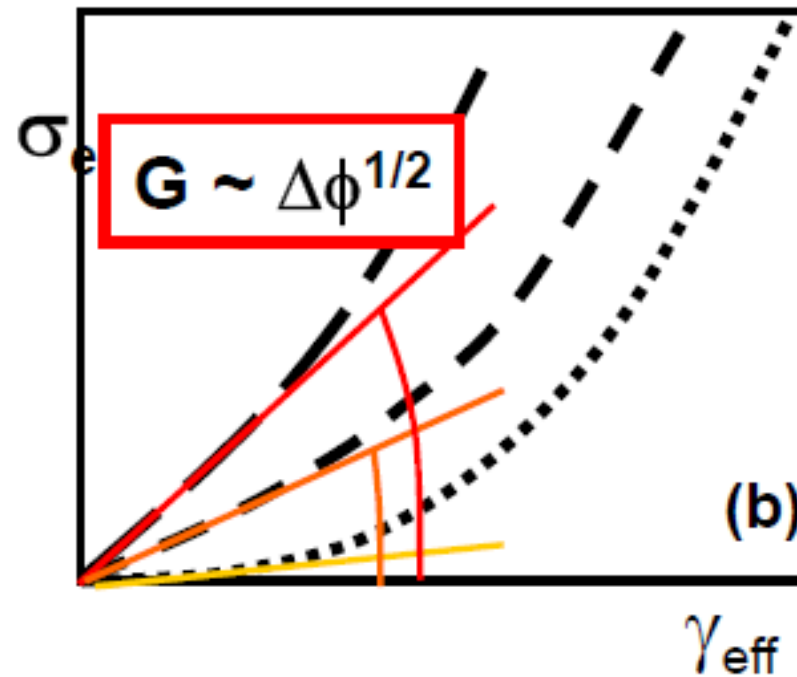
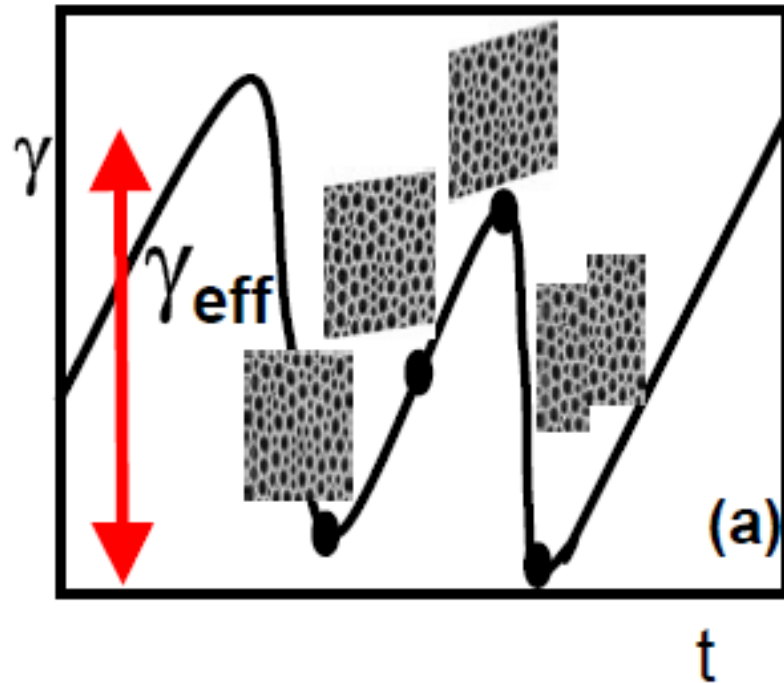
Microscopic Model

Energy Balance: Anomalous Fluctuations

Scaling Model for Rheology Near Jamming

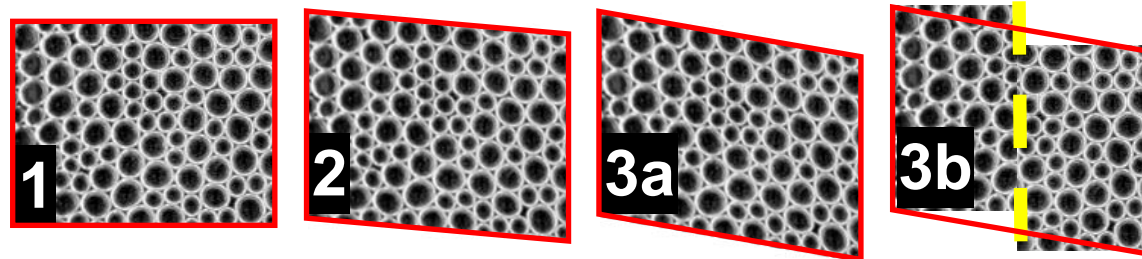


Scaling Model for Rheology Near Jamming



Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$$



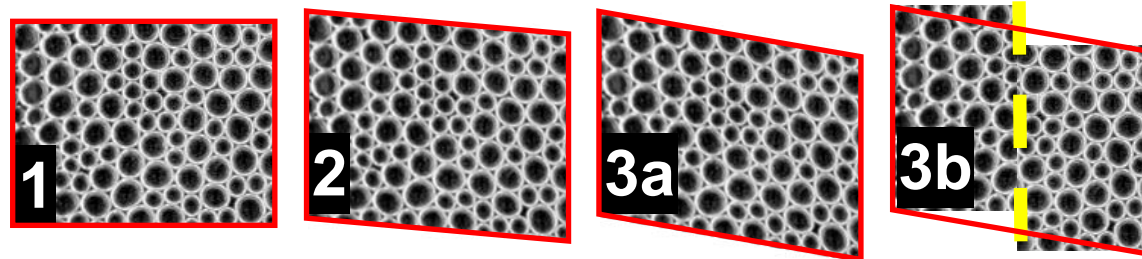
Strain ...

... and Relax

Timescale: $(d/\Delta v)$

Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$$



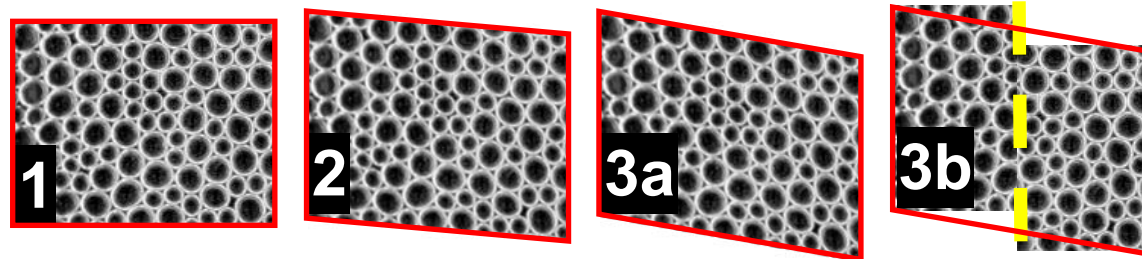
Strain ...

... and Relax

$$\text{Strainscale: } \dot{\gamma} (d/\Delta v)$$

Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$$



Strain ...

... and Relax

$$\gamma_c \sim \dot{\gamma} (d/\Delta v)$$

Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$$

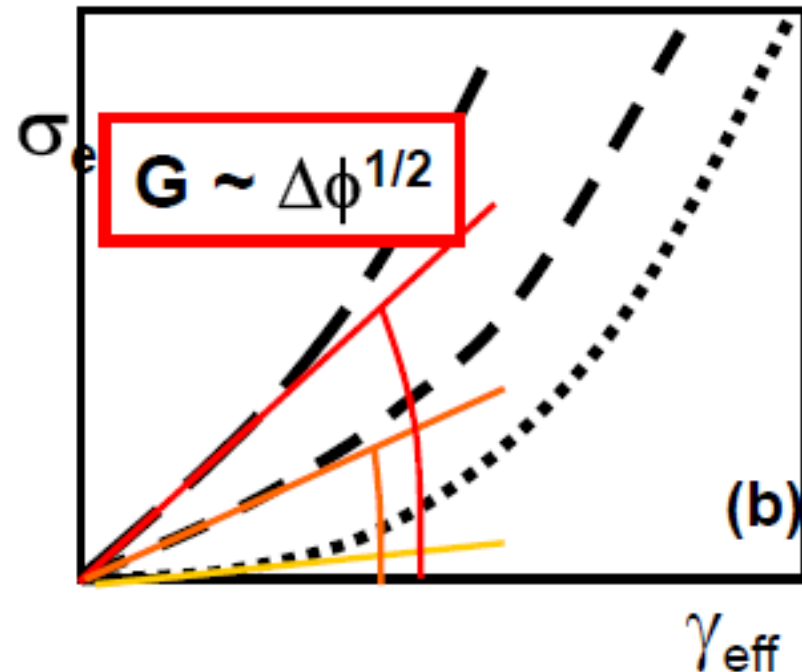
$$\gamma_c \sim \dot{\gamma} (d/\Delta \mathbf{v})$$

Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta v^2 \rangle$$

$$\gamma_c \sim \dot{\gamma} (d/\Delta v)$$

Stress?

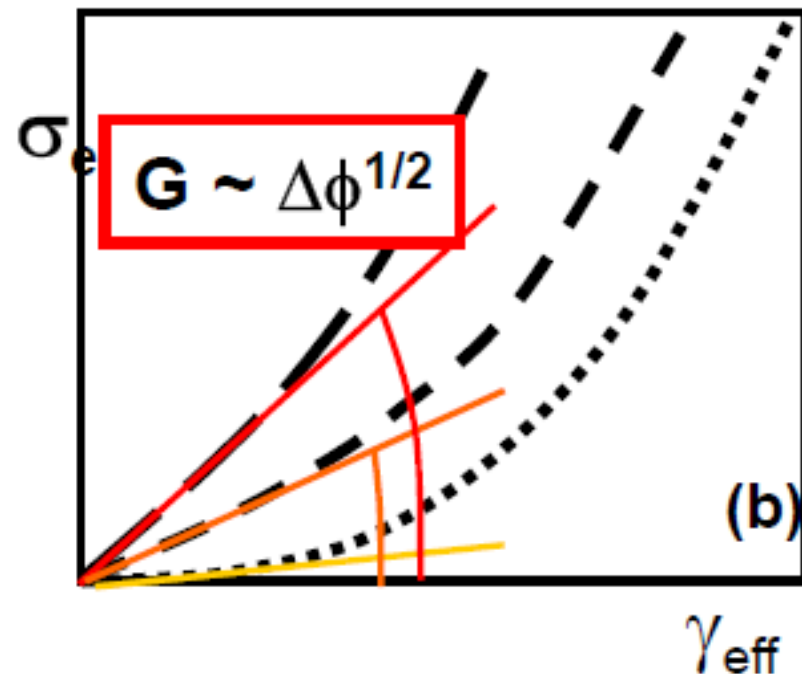


Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$$

$$\gamma_c \sim \dot{\gamma} / \Delta \mathbf{v}$$

$$\sigma \sim \Delta \phi^{1/2} \gamma_c + \gamma_c^2$$

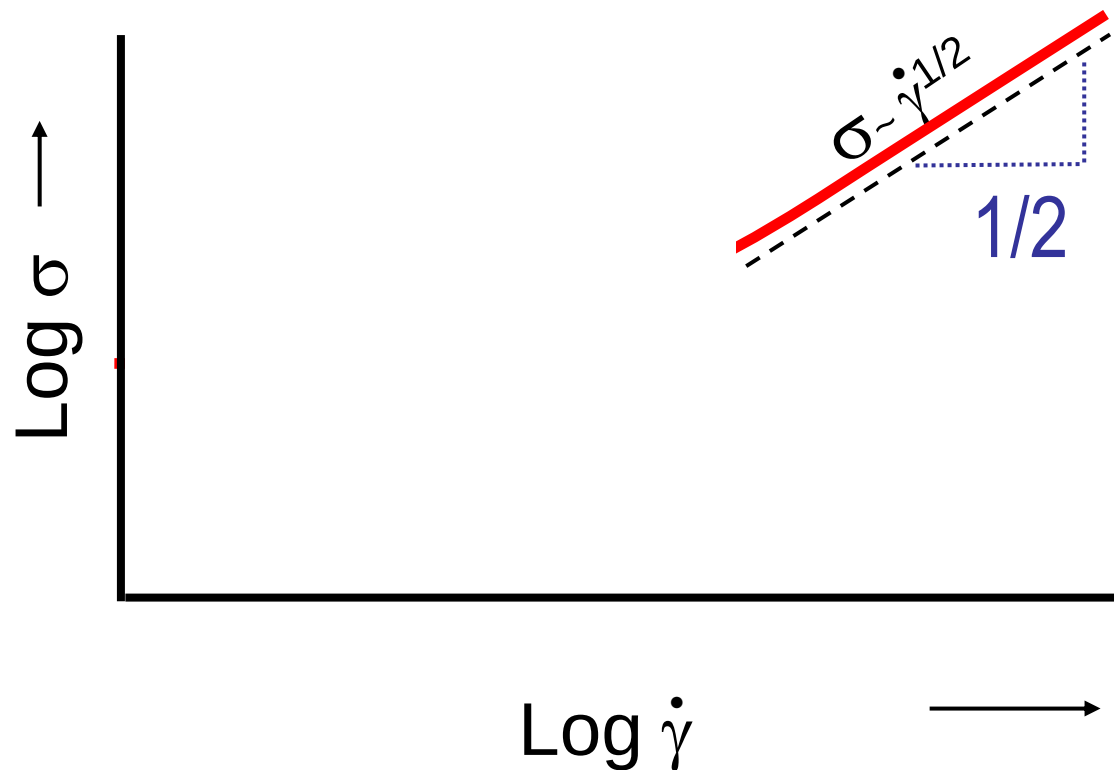


Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$$

$$\gamma_c \sim \dot{\gamma} / \Delta \mathbf{v}$$

$$\sigma \sim \Delta \phi^{1/2} \gamma_c + \gamma_c^2$$

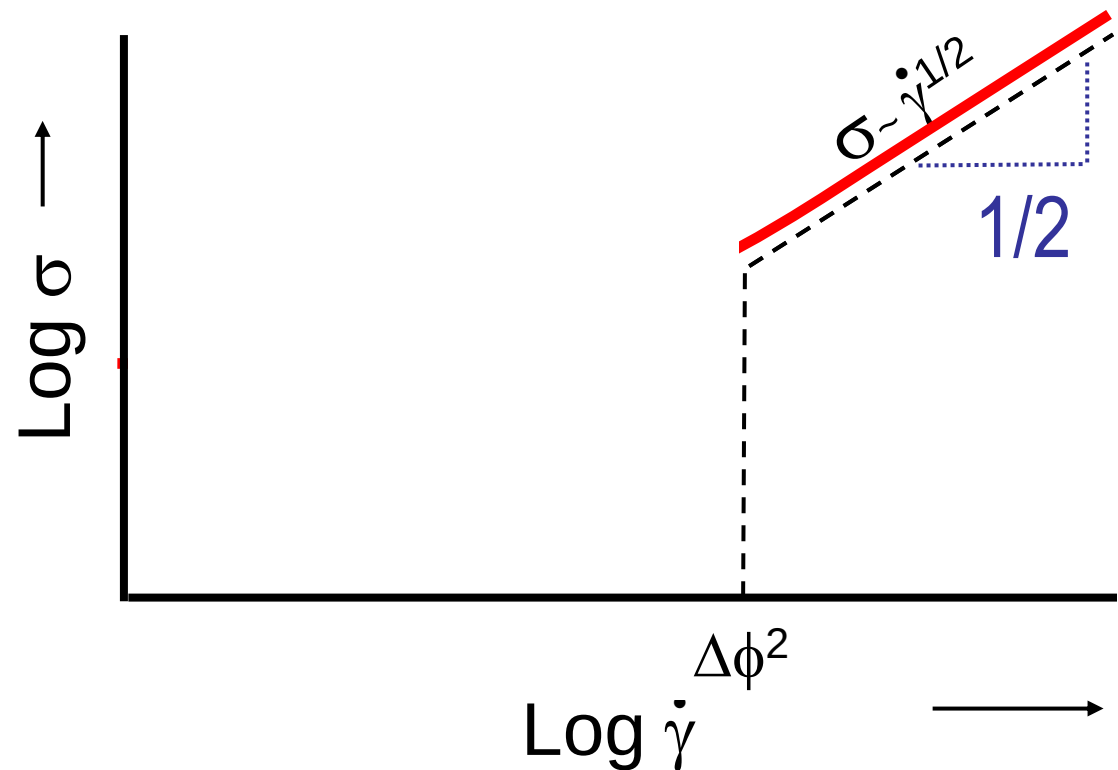


Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$$

$$\gamma_c \sim \dot{\gamma} / \Delta \mathbf{v}$$

$$\sigma \sim \Delta \phi^{1/2} \gamma_c + \gamma_c^2$$

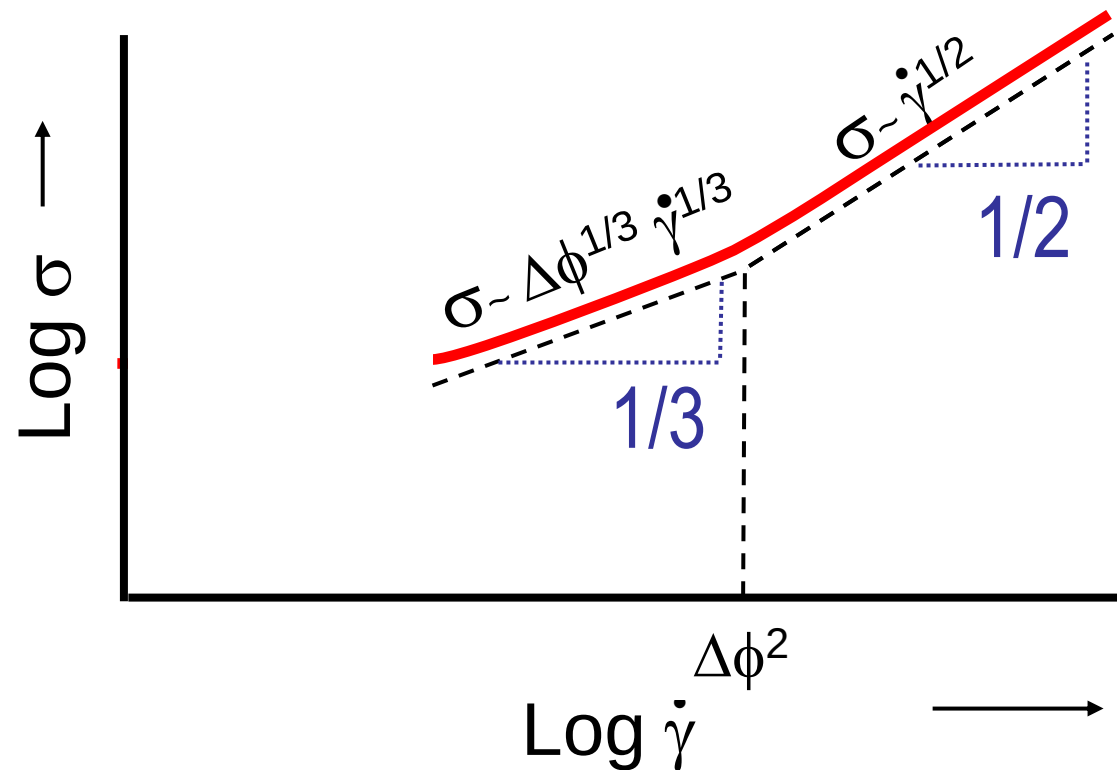


Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$$

$$\gamma_c \sim \dot{\gamma} / \Delta \mathbf{v}$$

$$\sigma \sim \Delta \phi^{1/2} \gamma_c + \gamma_c^2$$

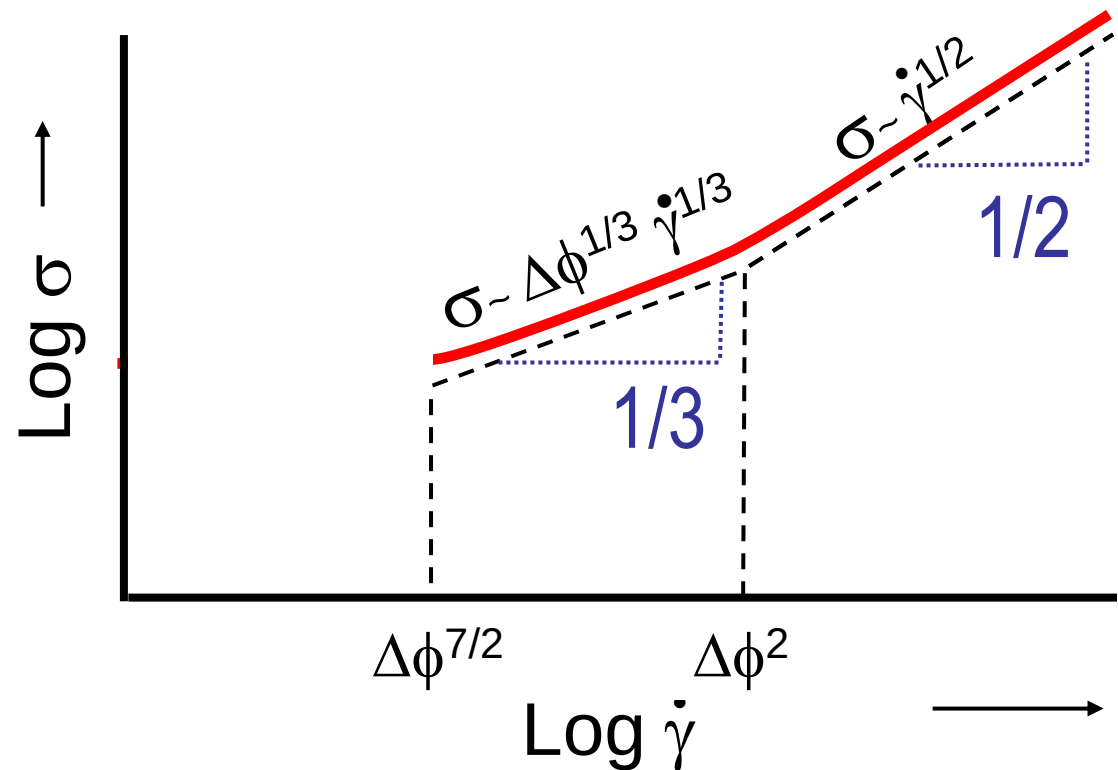


Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$$

$$\gamma_c \sim \Delta\phi + \dot{\gamma} / \Delta \mathbf{v}$$

$$\sigma \sim \Delta\phi^{1/2} \gamma_c + \gamma_c^2$$

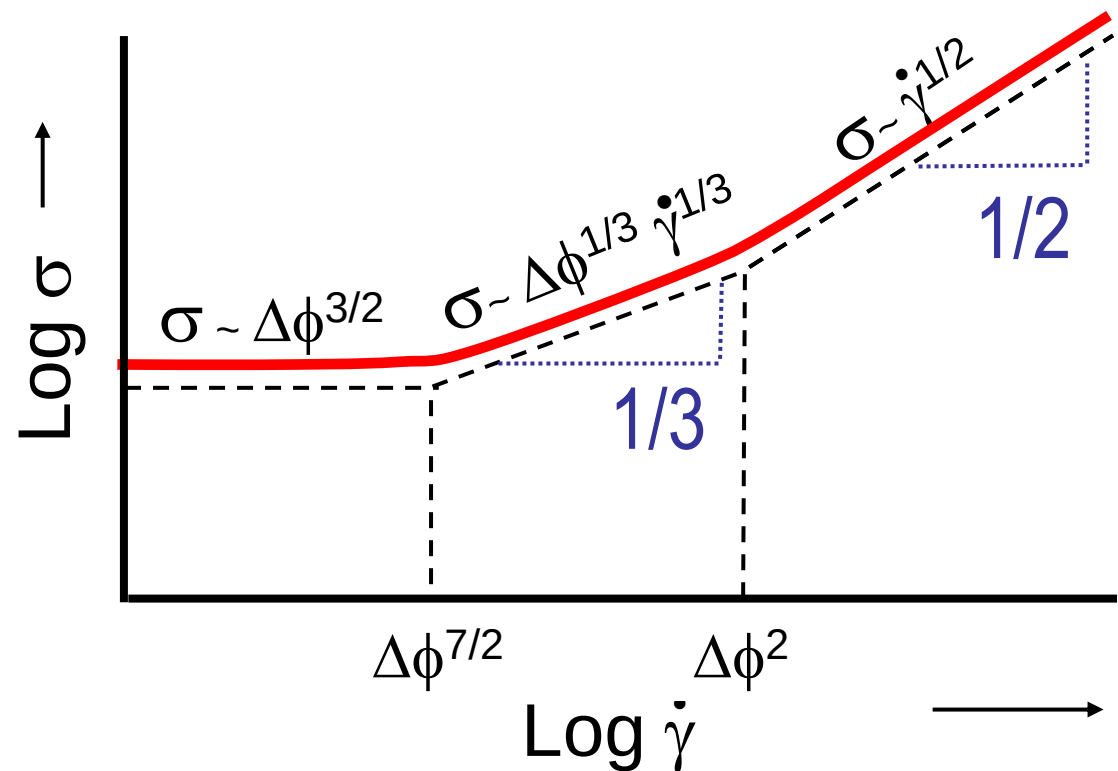


Scaling Model for Rheology Near Jamming

$$\sigma \dot{\gamma} \sim \langle \Delta \mathbf{v}^2 \rangle$$

$$\gamma_c \sim \Delta\phi + \dot{\gamma} / \Delta \mathbf{v}$$

$$\sigma \sim \Delta\phi^{1/2} \gamma_c + \gamma_c^2$$



Rheology Near Jamming

Microscopic Model

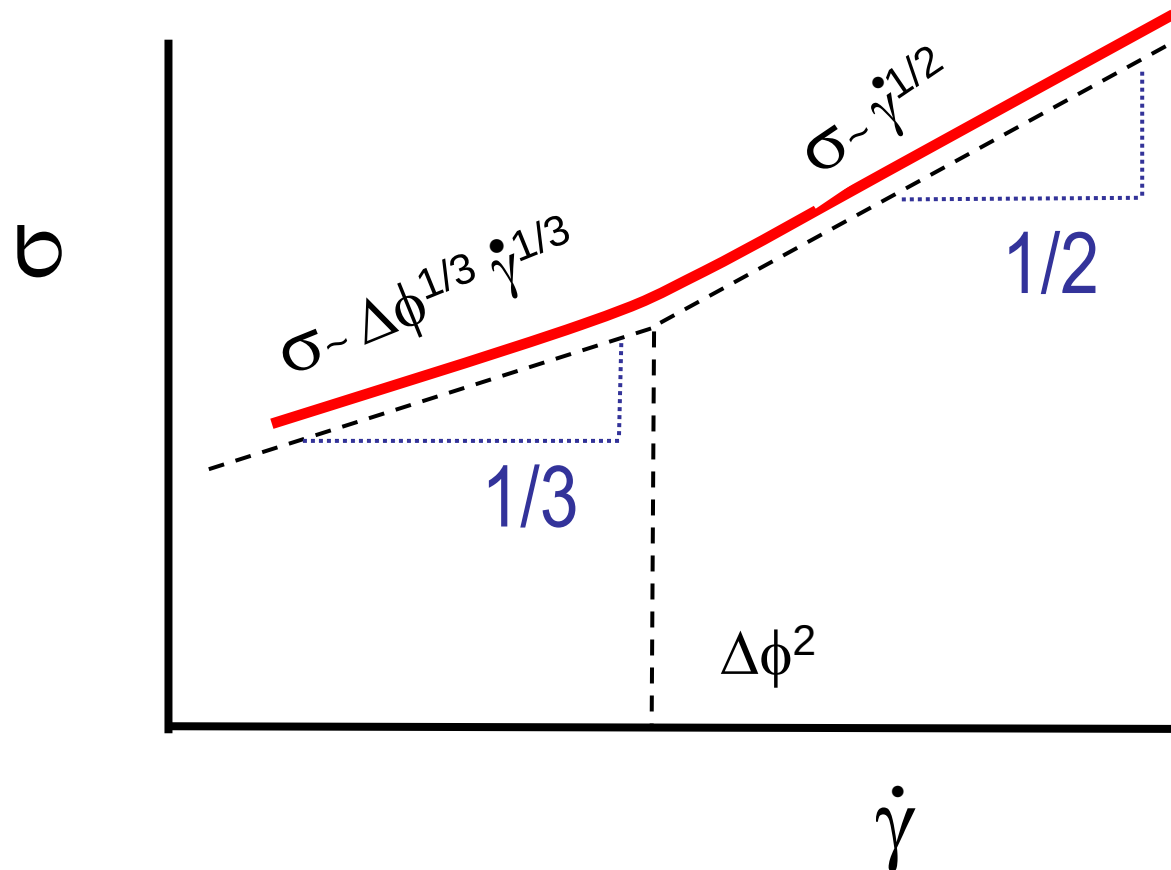
Energy Balance: Anomalous Fluctuations

Scaling Model for Rheology Near Jamming

DOES IT WORK?

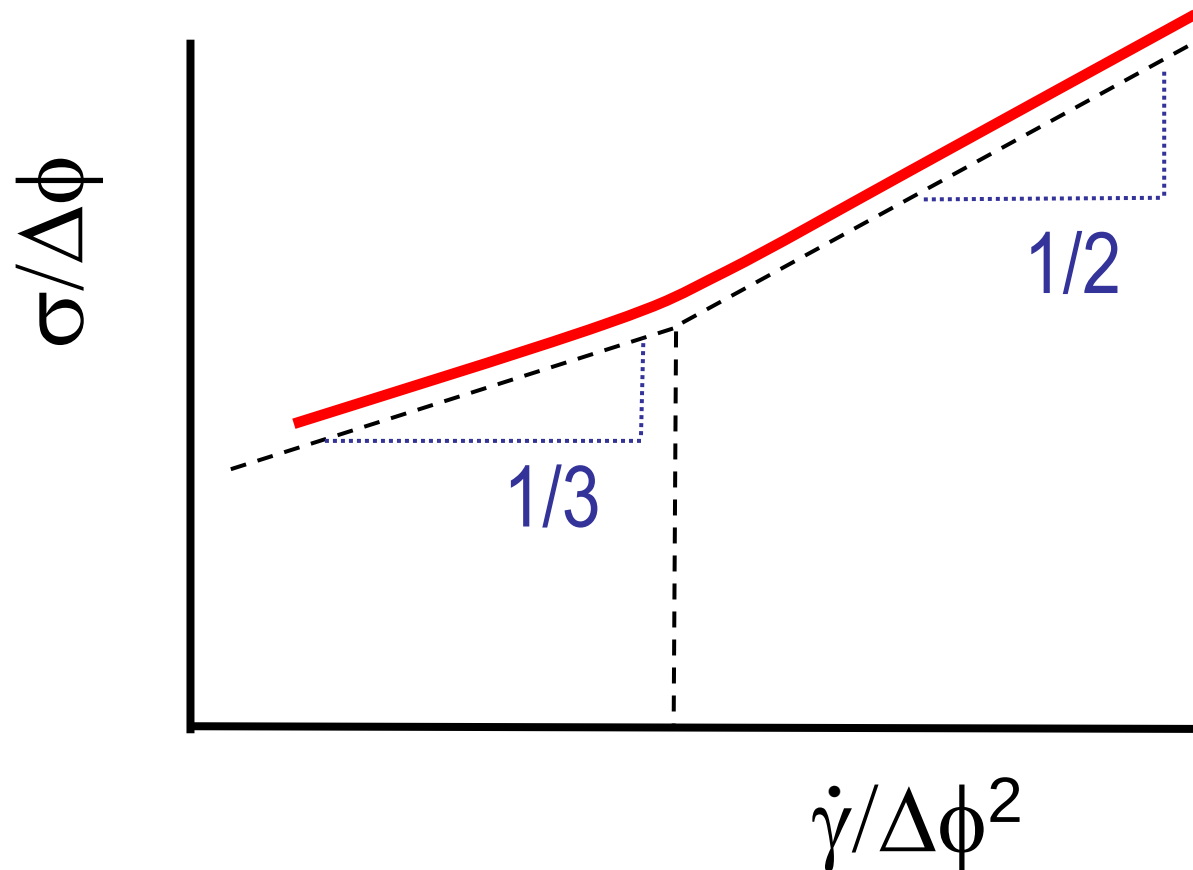
Scaling Model for Rheology Near Jamming

Data Collapse?

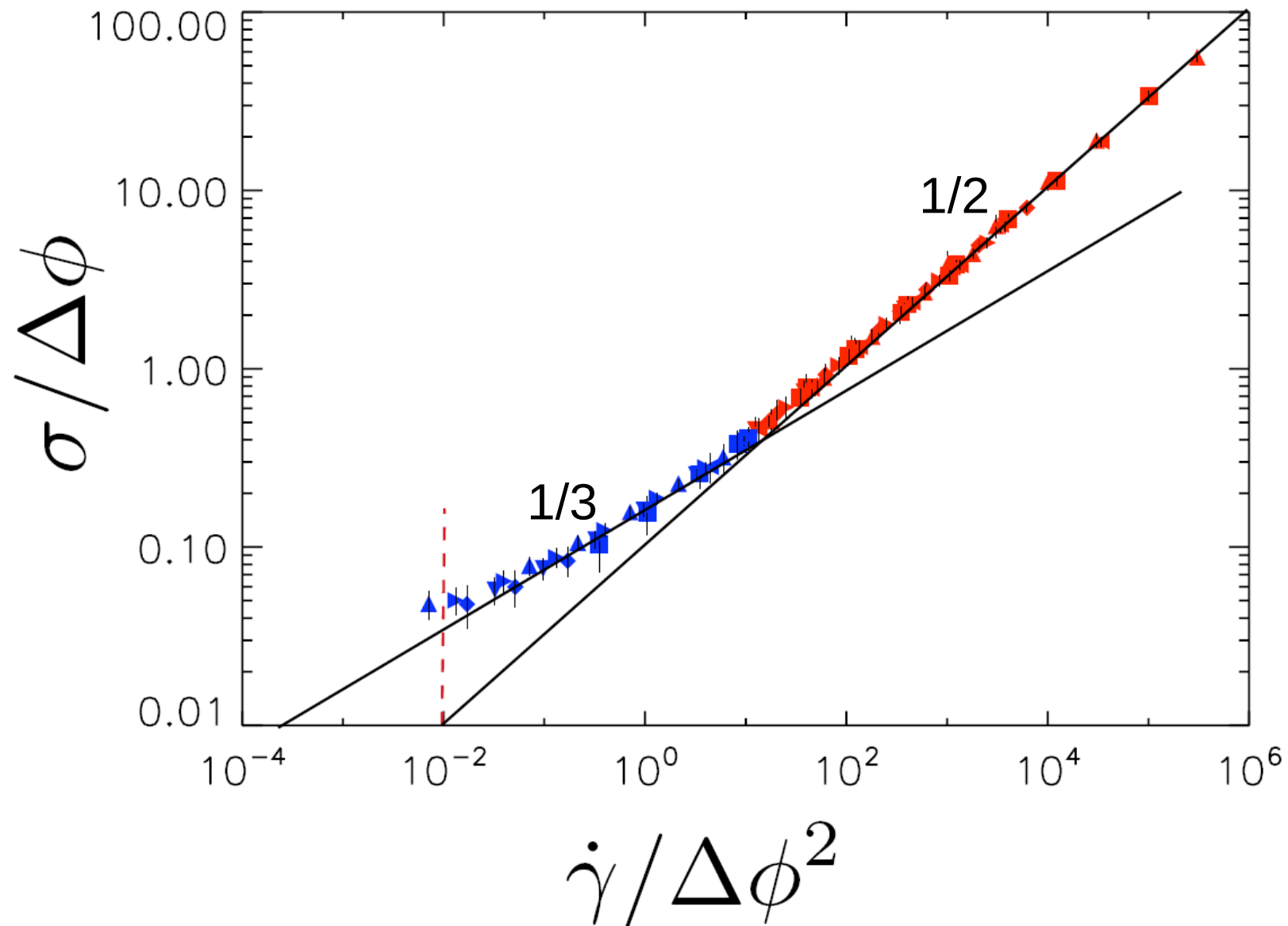


Scaling Model for Rheology Near Jamming

Data Collapse?



Scaling Model for Rheology Near Jamming



Outlook

Strong Fluctuations from Energy Balance

Predictive Model For Rheology

Predictions for general α (Foams: $\alpha \sim 2/3$, Rheo: **0.36**, Model: **4/11**)

Grains: $\alpha \sim 0$, global: 0!!!