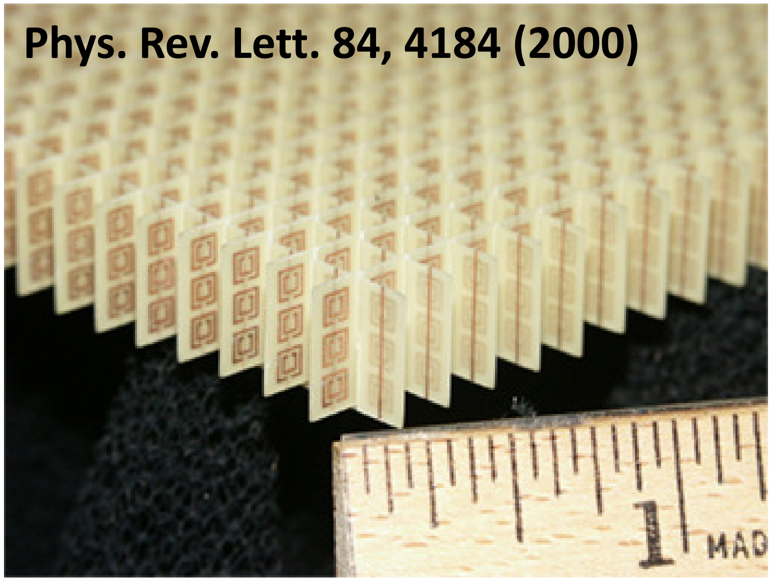
Mechanical Metamaterial

?

Wikipedia: Metamaterials are artificial materials engineered to have properties that may not be found in nature. They are assemblies of multiple individual elements fashioned from conventional microscopic materials such as metals or plastics, but the materials are usually arranged in periodic patterns. Metamaterials gain their properties not from their composition, but from their exactingly-designed structures. Their precise shape, geometry, size, orientation and arrangement can affect the waves of light or sound in an unconventional manner, creating material properties which are unachievable with conventional materials.

Metamaterials: Waves



Negative index metamaterial array configuration, which was constructed of copper split-ring resonators and wires mounted on interlocking sheets of fiberglass circuit board.

An acoustic lens made of soda cans can focus sound waves to a spot as small as 1/25th of a wavelength.

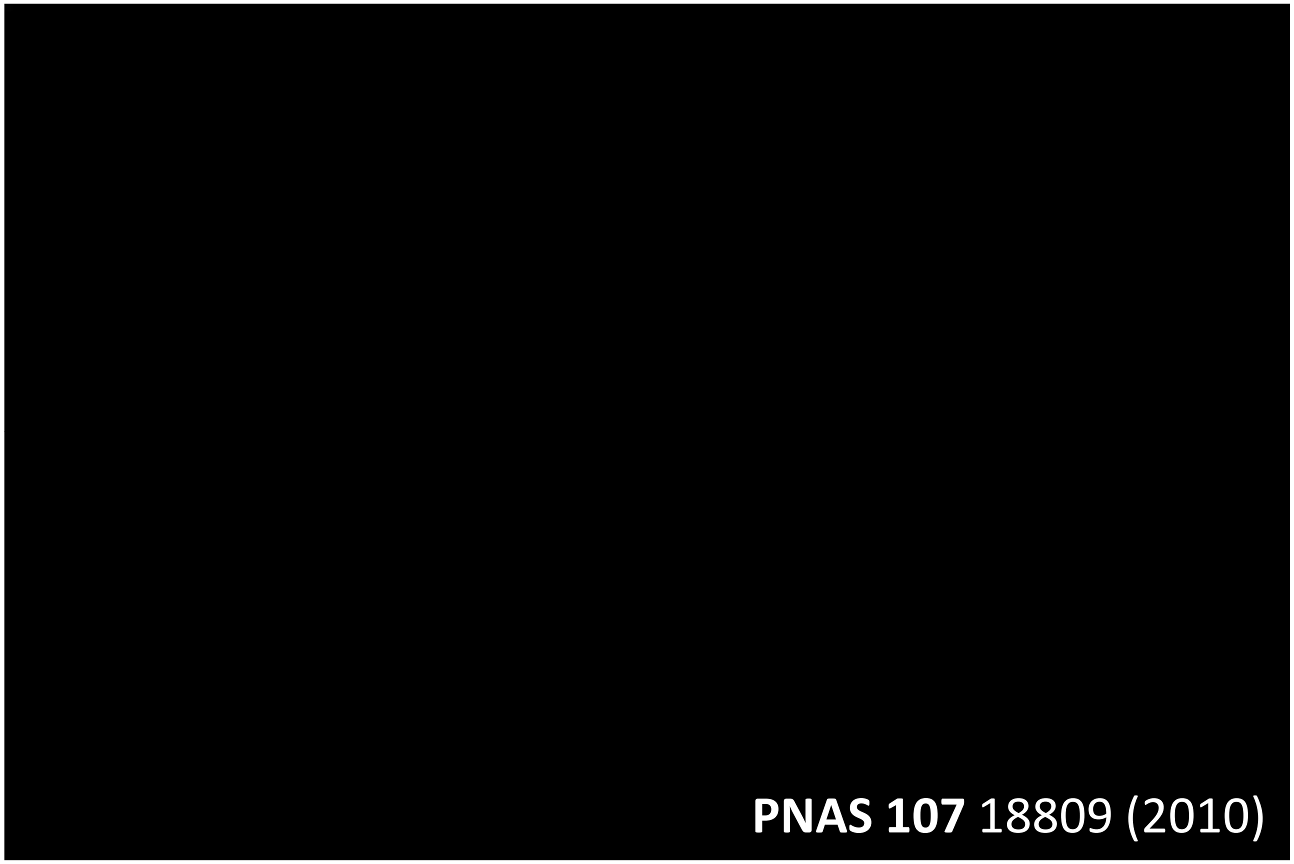
Vol. 334 no. 6058 pp. 962-965

Mechanical Metamaterial: UltraLight

Science 334, 962 (2011)

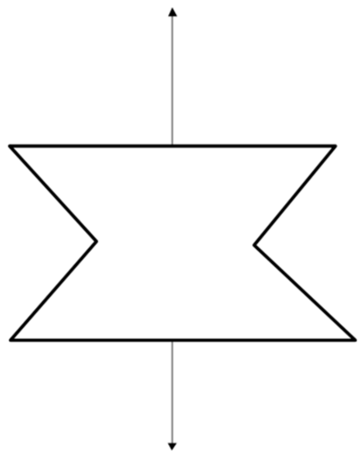
0.9 mg per cm^3
0.9 g per liter
0.9 kg per m^3

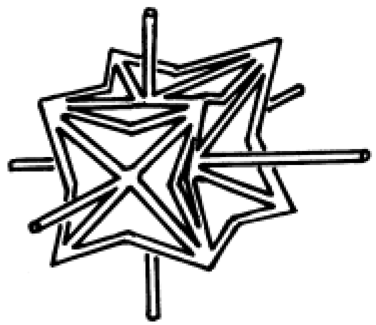
Mechanical Metamaterials: Responsive

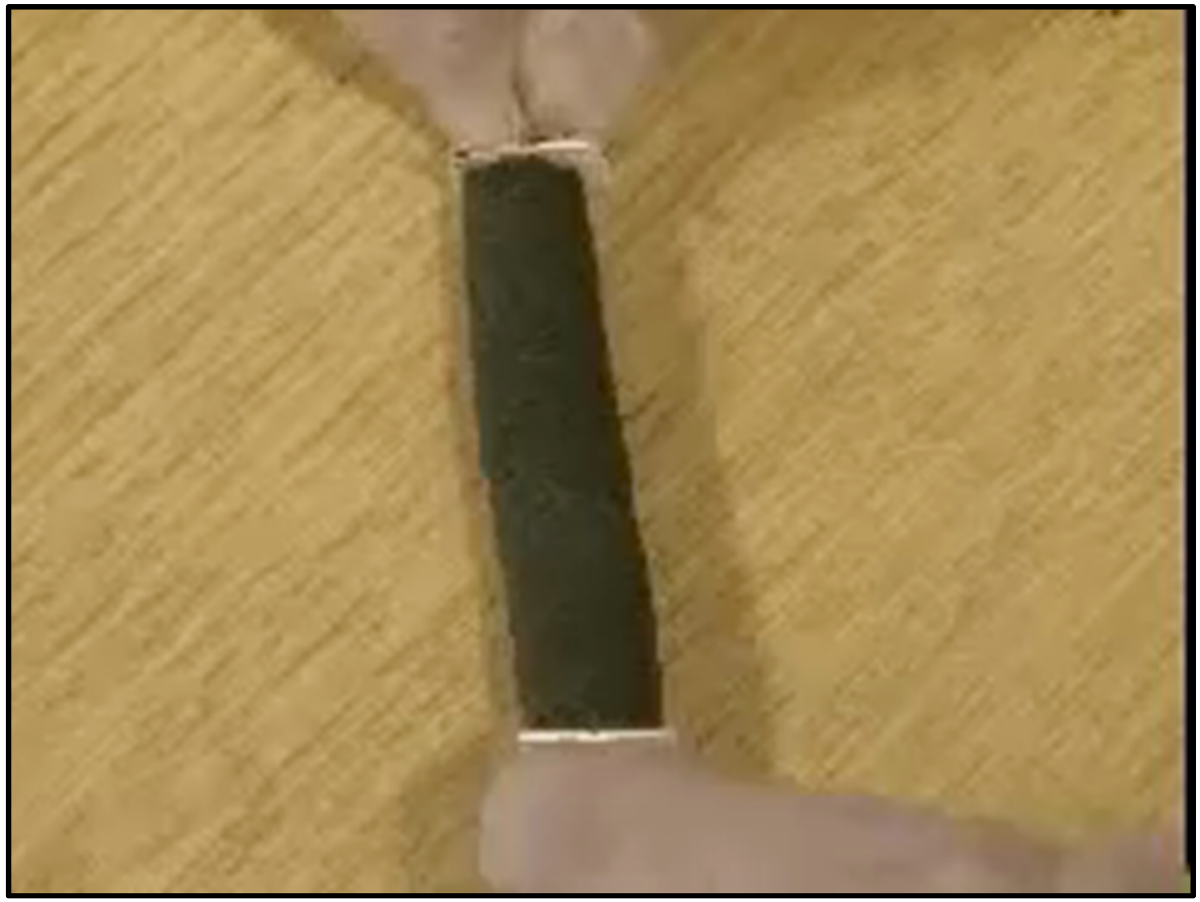


Frictional, simple: compression ~ strength

Mechanical Metamaterials: Auxetic

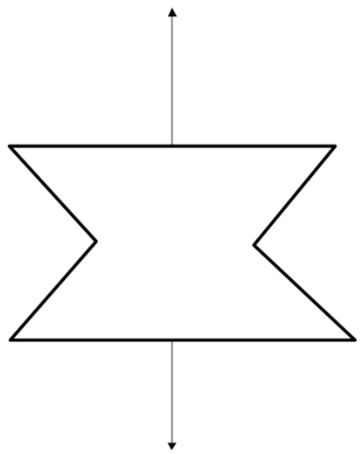


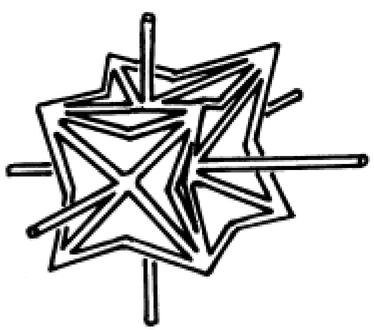


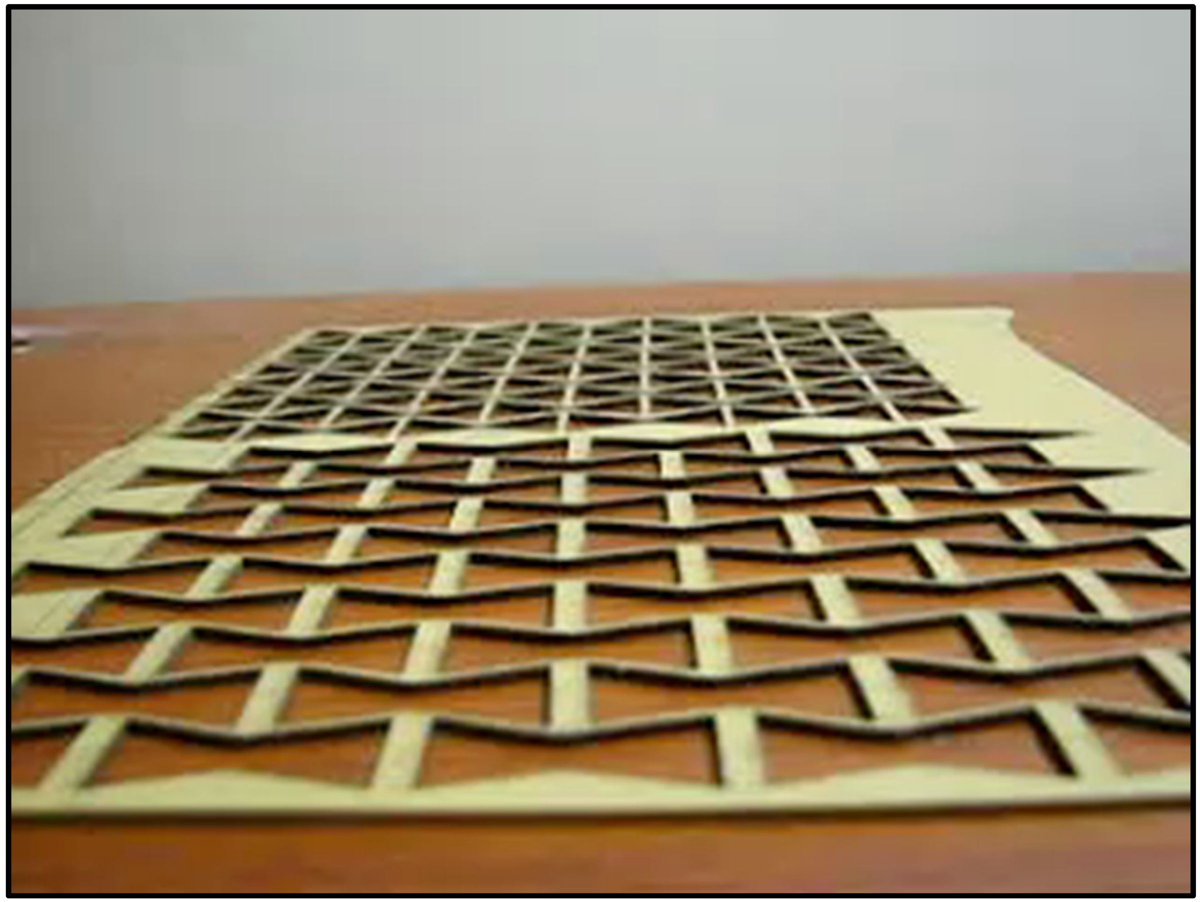


Science **235** 1038 (1987)

Mechanical Metamaterials: Auxetic



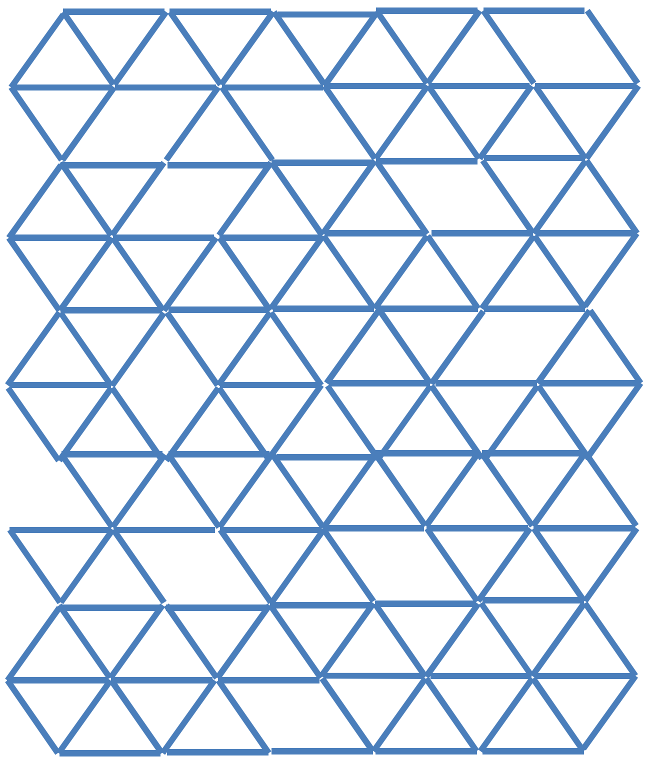




Science **235** 1038 (1987)

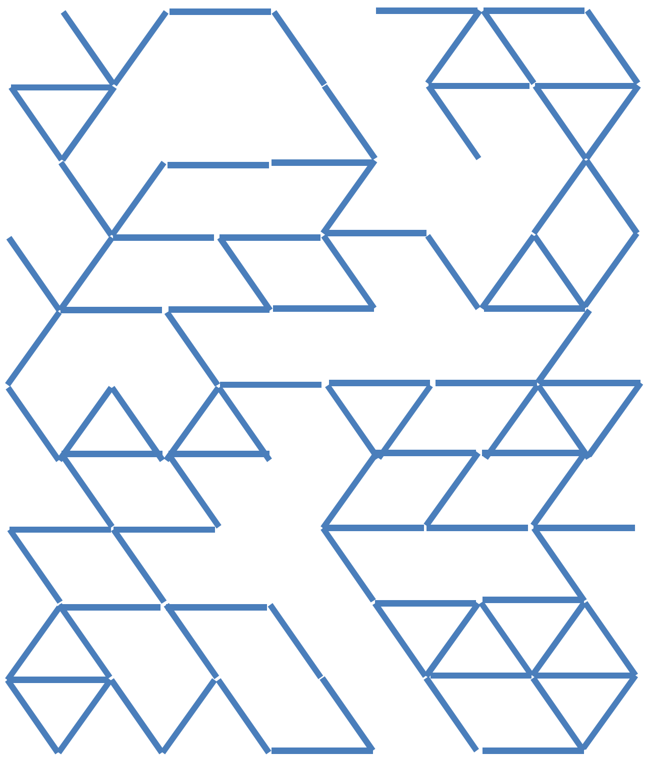
Mechanical Metamaterials

Here: Collective



Mechanical Metamaterials

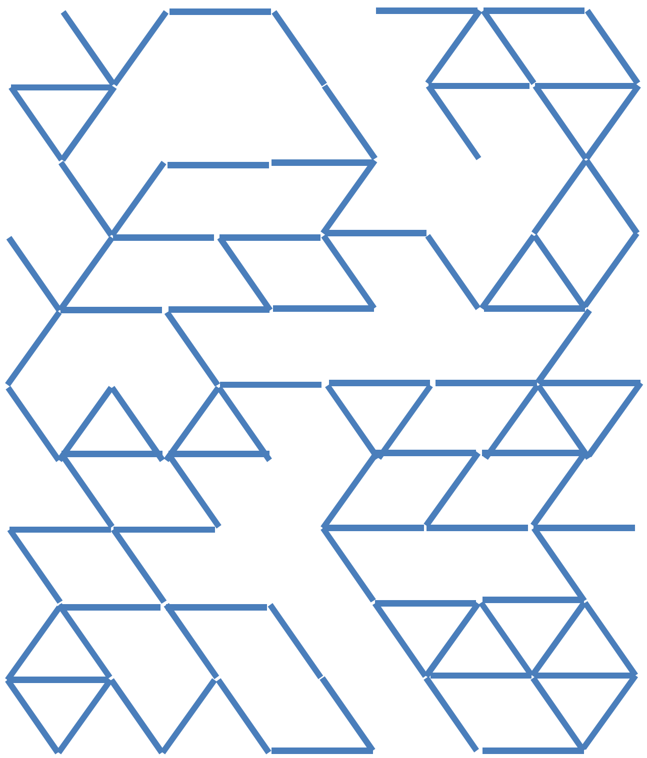
Here: Collective



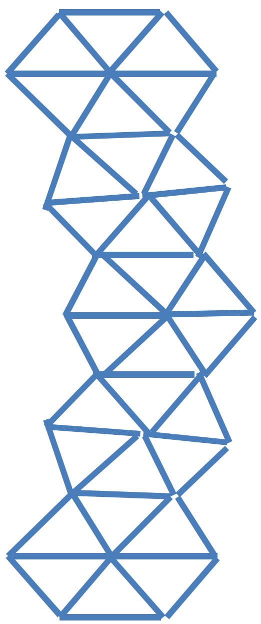
Rigidity Loss

Mechanical Metamaterials

Here: Collective

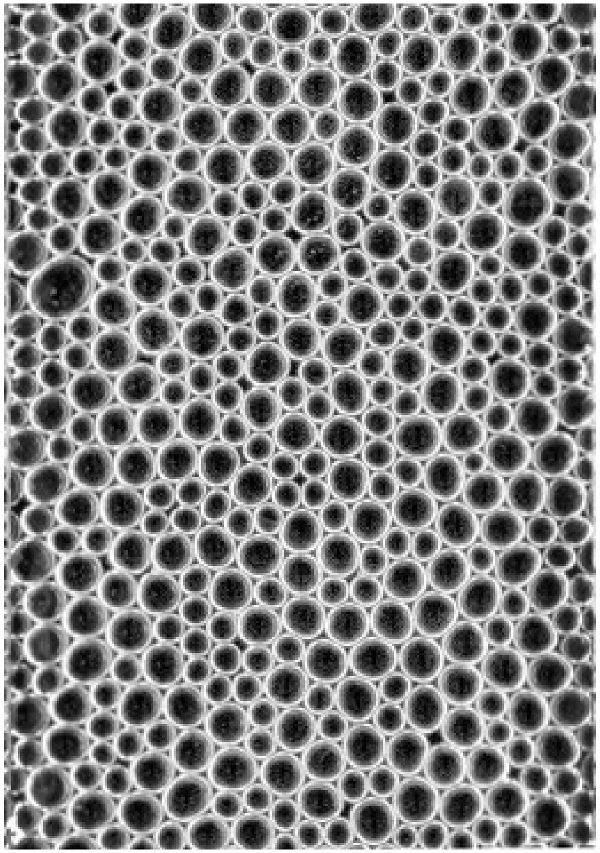


Rigidity Loss

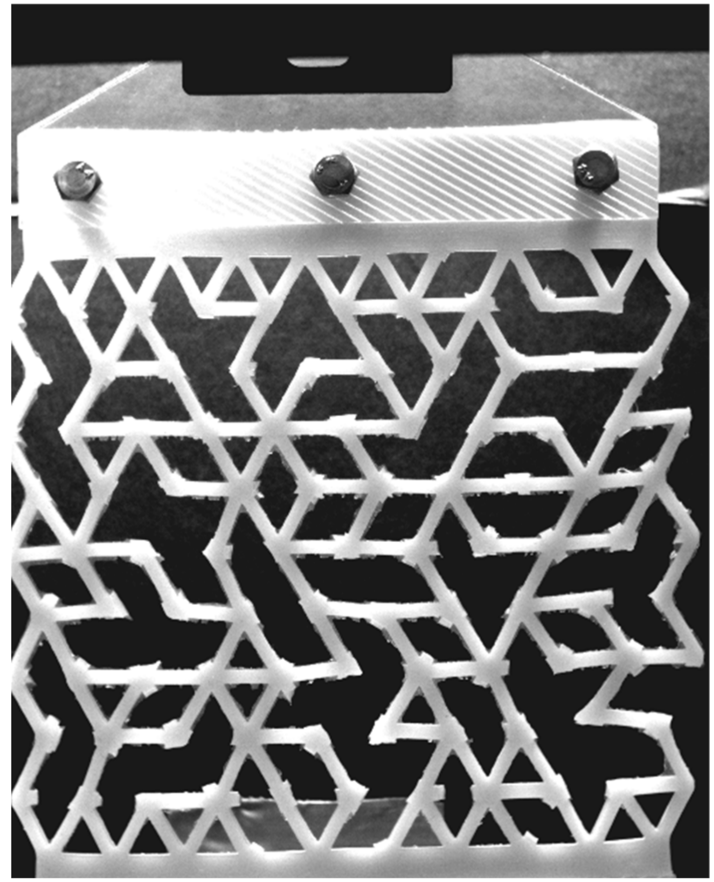


Instability

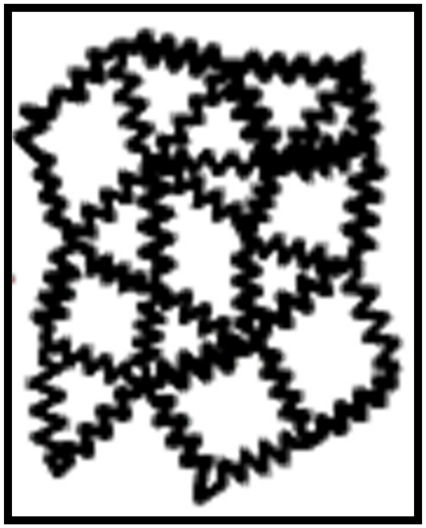
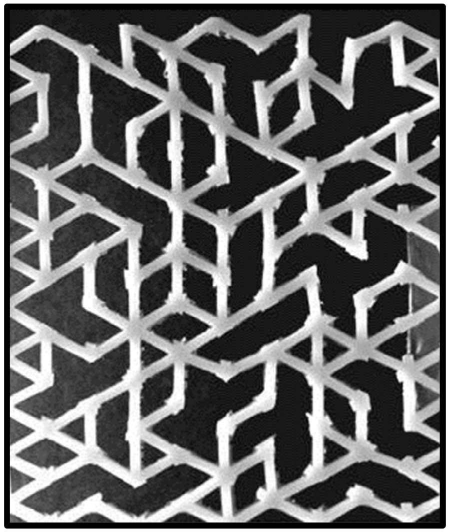
Usual Approach



Can you shape the feel of a thing?



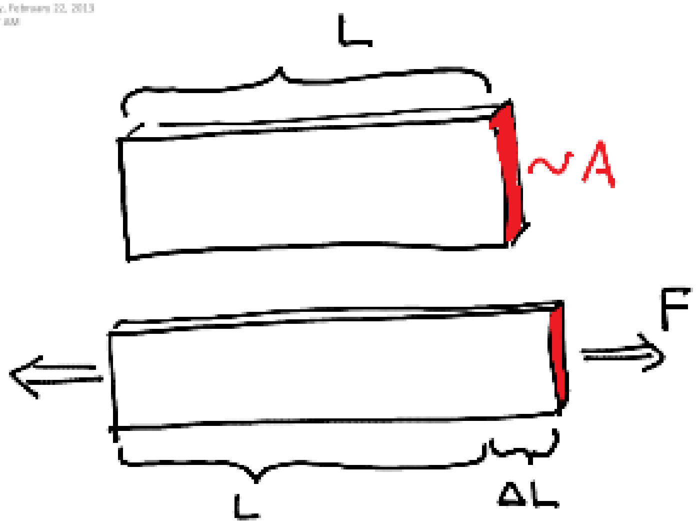
Mechanical Metamaterial

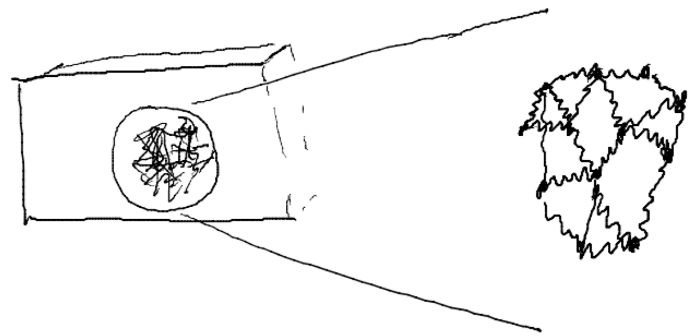
Linear and Nonlinear Response in Marginal Networks
Elastic Instabilities in Holey Sheets

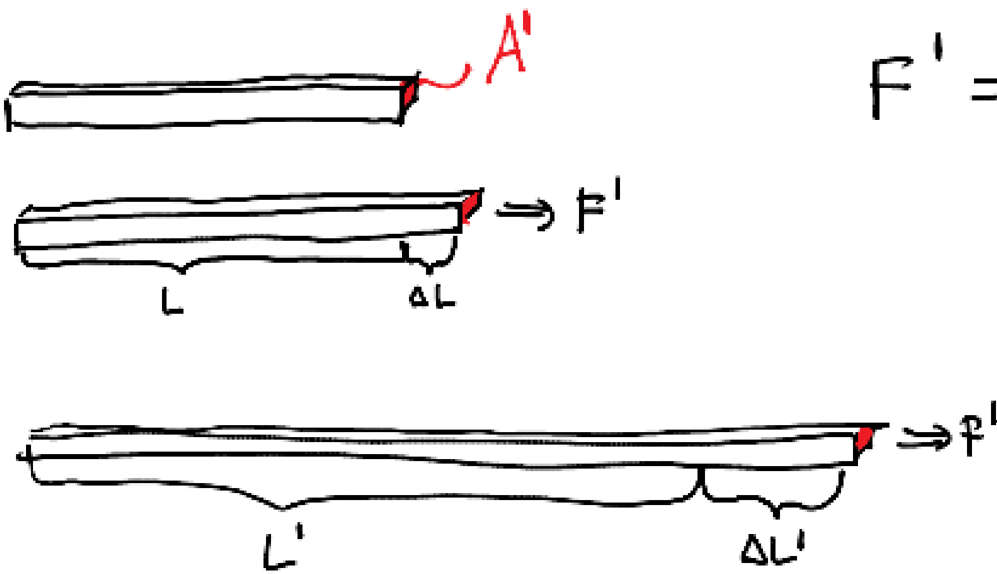
Elasticity
Jamming & Rigidity Percolation
Buckling, Snapback and Snapthrough

Elasticity 101



$$F = K\,\Delta L$$



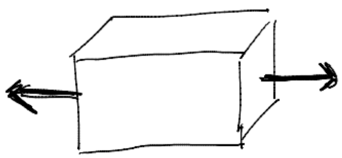


$$F' = A'/A \cdot F \Rightarrow$$

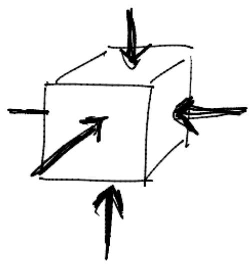
$$\frac{F'}{A'} = \frac{F}{A} = \sigma \text{ (STRESS)}$$

$$\frac{\Delta L'}{L'} = \frac{\Delta L}{L} = \gamma \text{ (STRAIN)}$$

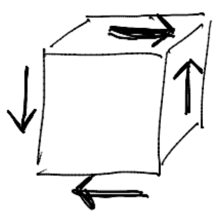
Elasticity 101



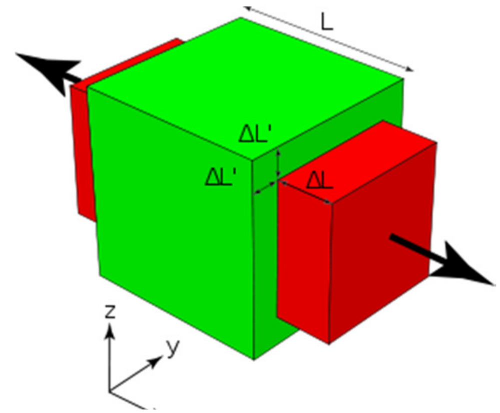
$$\sigma = Y \gamma$$



$$\sigma = K \gamma$$



$$\sigma = G \gamma$$



$$\nu = \frac{\Delta L'}{\Delta L}$$

ν

3 D $\quad G = \dfrac{Y}{2(1+\nu)} \quad K = \dfrac{Y}{3(1-2\nu)} \quad -1 \leq \nu \leq 0.5$

2 D $\quad G = \dfrac{Y}{2(1+\nu)} \quad K = \dfrac{Y}{2(1-\nu)} \quad -1 \leq \nu \leq 1$

Elasticity 101

Friday, February 22, 2013
11:58 AM

	$\boxed{3D}$	$\boxed{2D}$
$\sigma_{ij} = C_{ijkl}\,\gamma_{kl}$	$\underline{81}$	$\underline{16}$
$\sigma_{ij} = \sigma_{ji}$ & $\gamma_{kl} = \gamma_{lk}$	$\underline{36}$	$\underline{9}$
$C_{ijkl} = C_{klij}$	$\underline{21}$	$\underline{6}$
ISOTROPIC	$\underline{\underline{2}}$	$\underline{\underline{2}}$

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij}$$

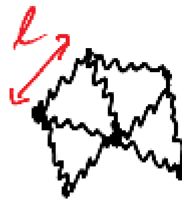
$$\varepsilon_{ij} = \frac{1}{2\mu}\sigma_{ij} - \frac{\nu}{E}\delta_{ij}\sigma_{kk} = \frac{1}{E}[(1+\nu)\sigma_{ij} - \nu\delta_{ij}\sigma_{kk}]$$

Elasticity 101

ISOTROPIC $\underline{\underline{2}}$ $\underline{\underline{2}}$

$\sigma = K \gamma$

$[K] = N m^{-2}$

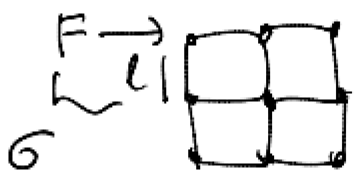


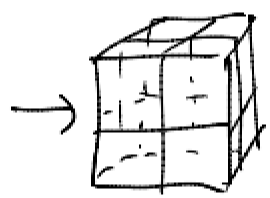
$\sim k \; [N m^{-1}]$

$K \sim k/\ell$ (3D)

$[K] = N m^{-1}$

$K \sim k$ (2D)





Elasticity 101

ISOTROPIC $\underline{\underline{2}}$ $\underline{\underline{2}}$

$\sigma = K \gamma$

$[K] = N m^{-2}$

$\sim k \; [N m^{-1}]$

$K \sim k/\ell$ (3D)

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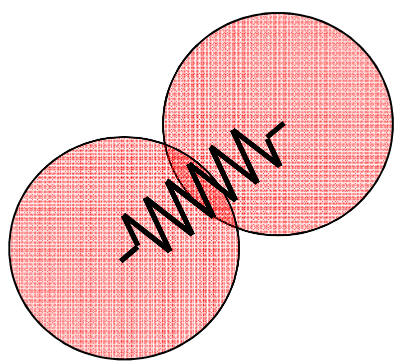
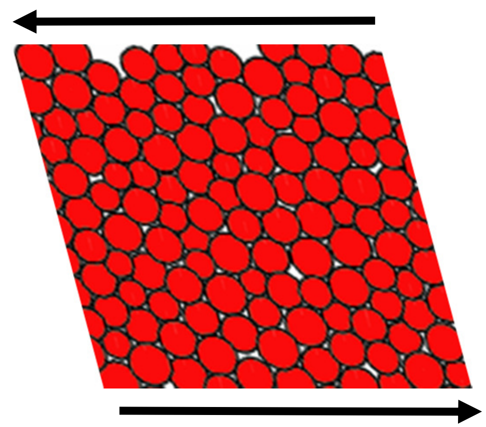


$\approx$



Elasticity 101

~~Stress State: Irrelevant~~

Linear Response

$\nu > -1$ /-0.5… $\nu > 0$

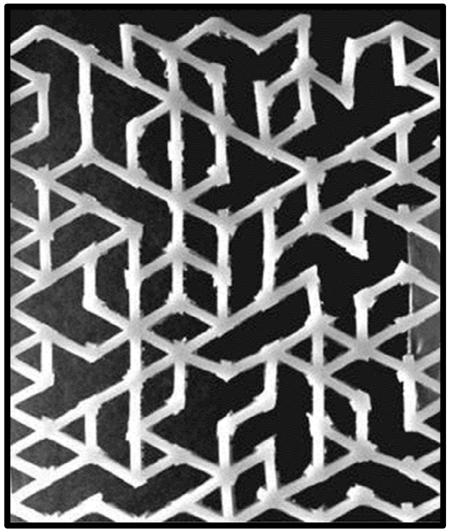
~~$G \approx K \approx k > 0$~~

Jamming

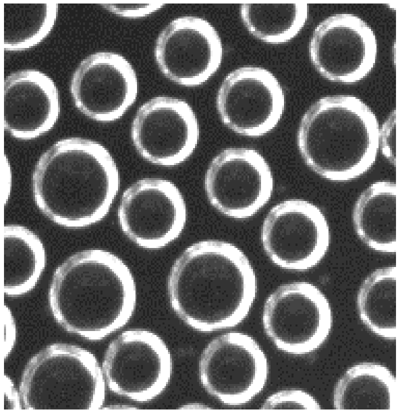
Marginal Networks

Holey Sheet

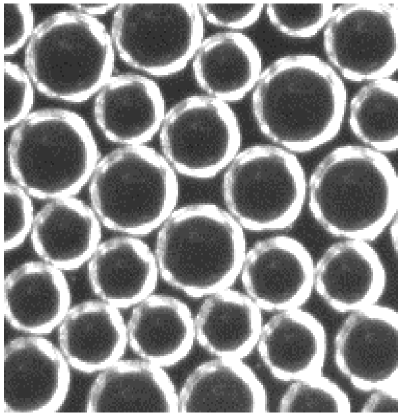
Neg. Compressibility?



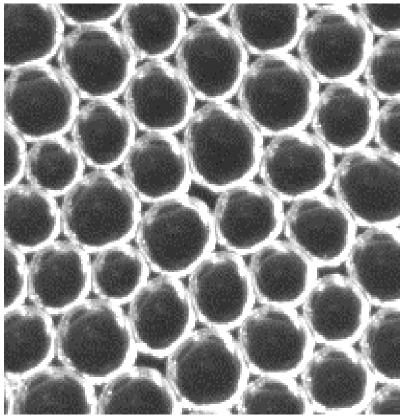
Jamming



Mechanical Vacua



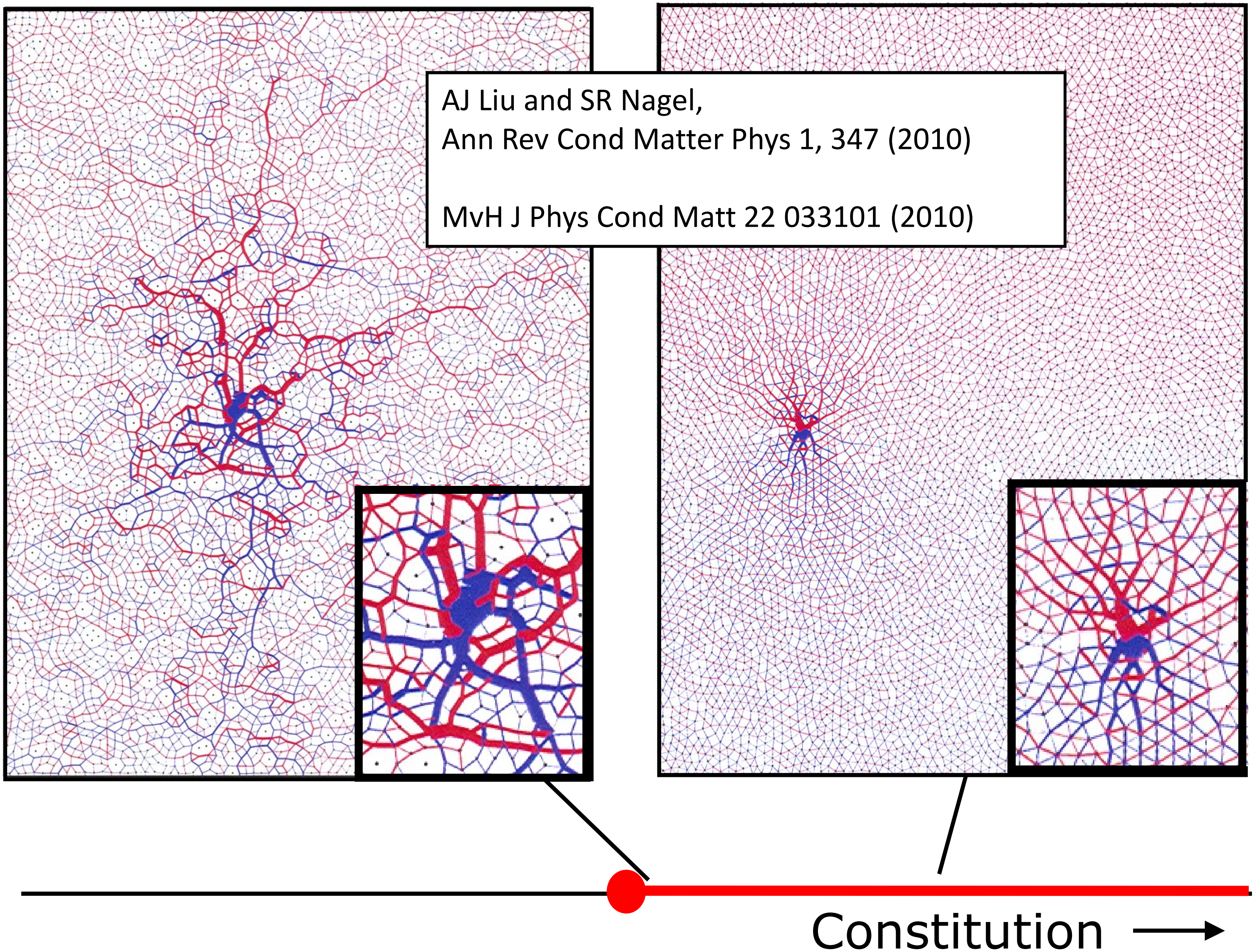
Marginal Point



Rigid

Constitution →

Jamming

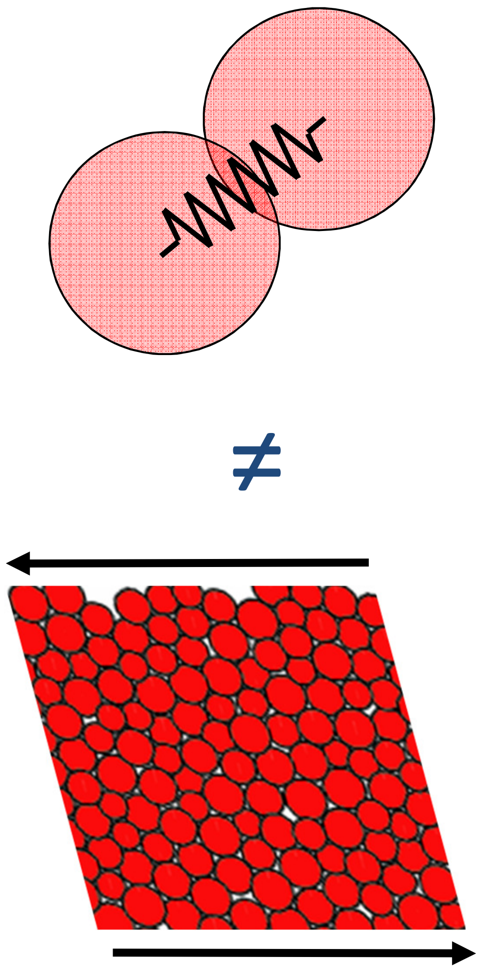


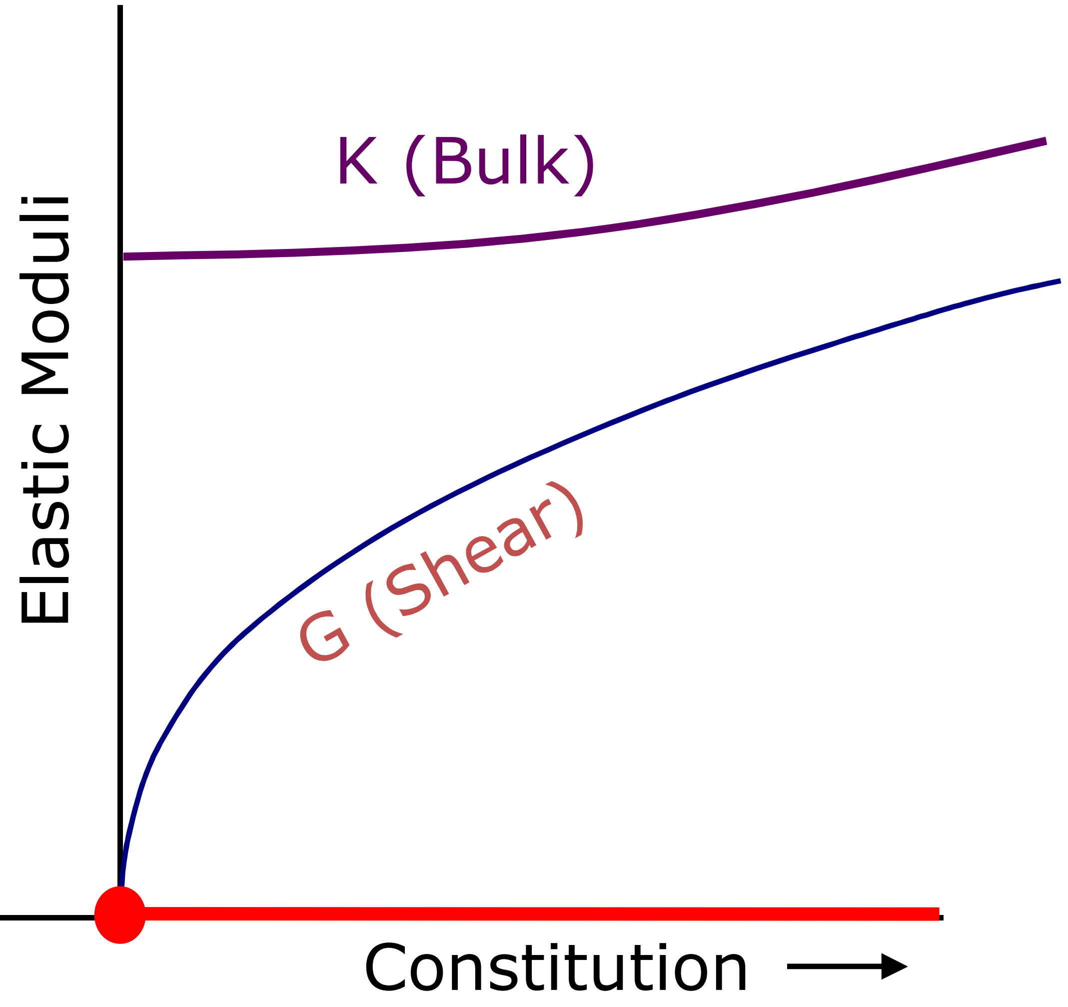
AJ Liu and SR Nagel,
Ann Rev Cond Matter Phys 1, 347 (2010)

MvH J Phys Cond Matt 22 033101 (2010)

Jamming

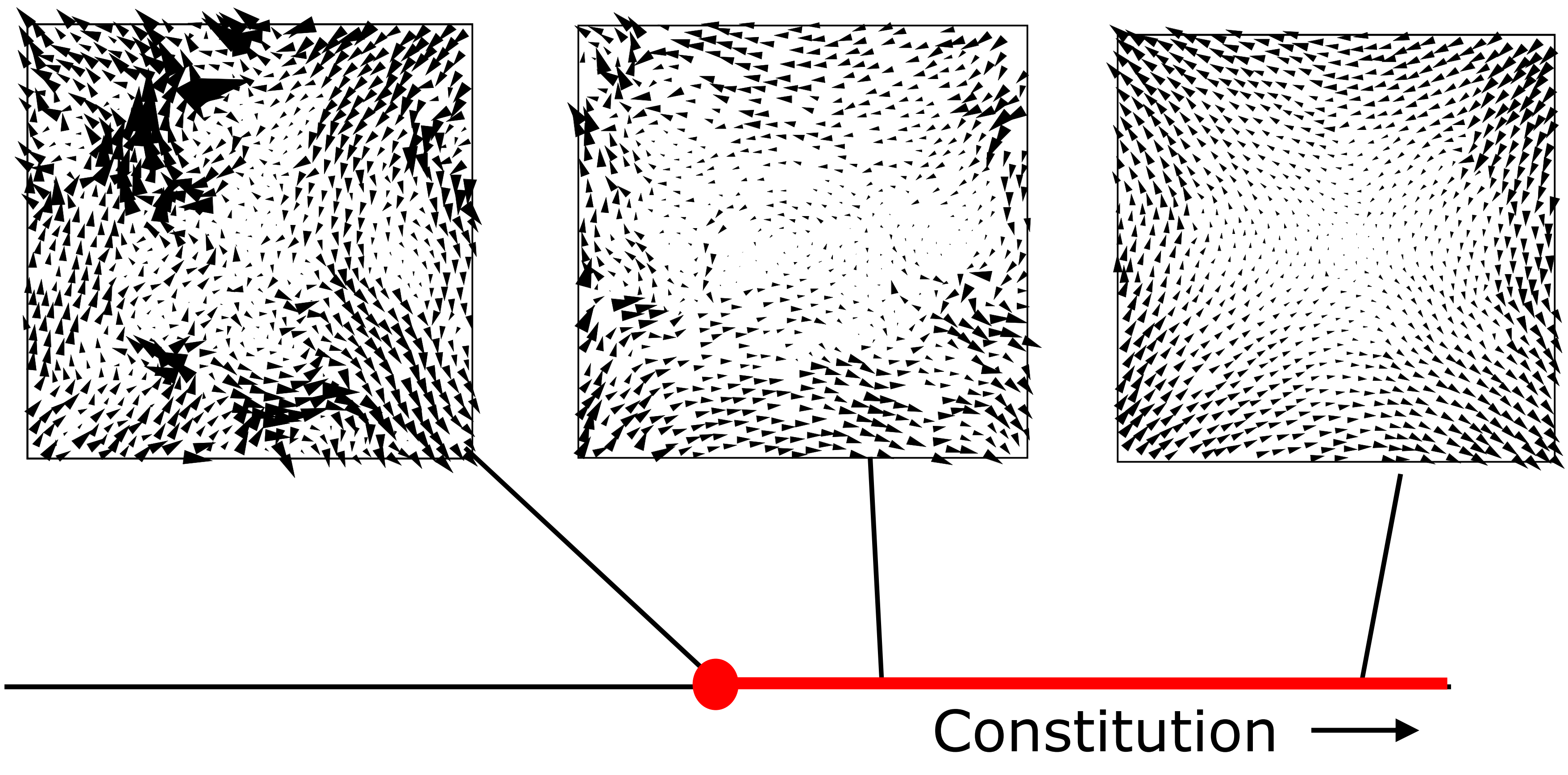
Bolton and Weaire 90, Durian 95, O'Hern et al 03



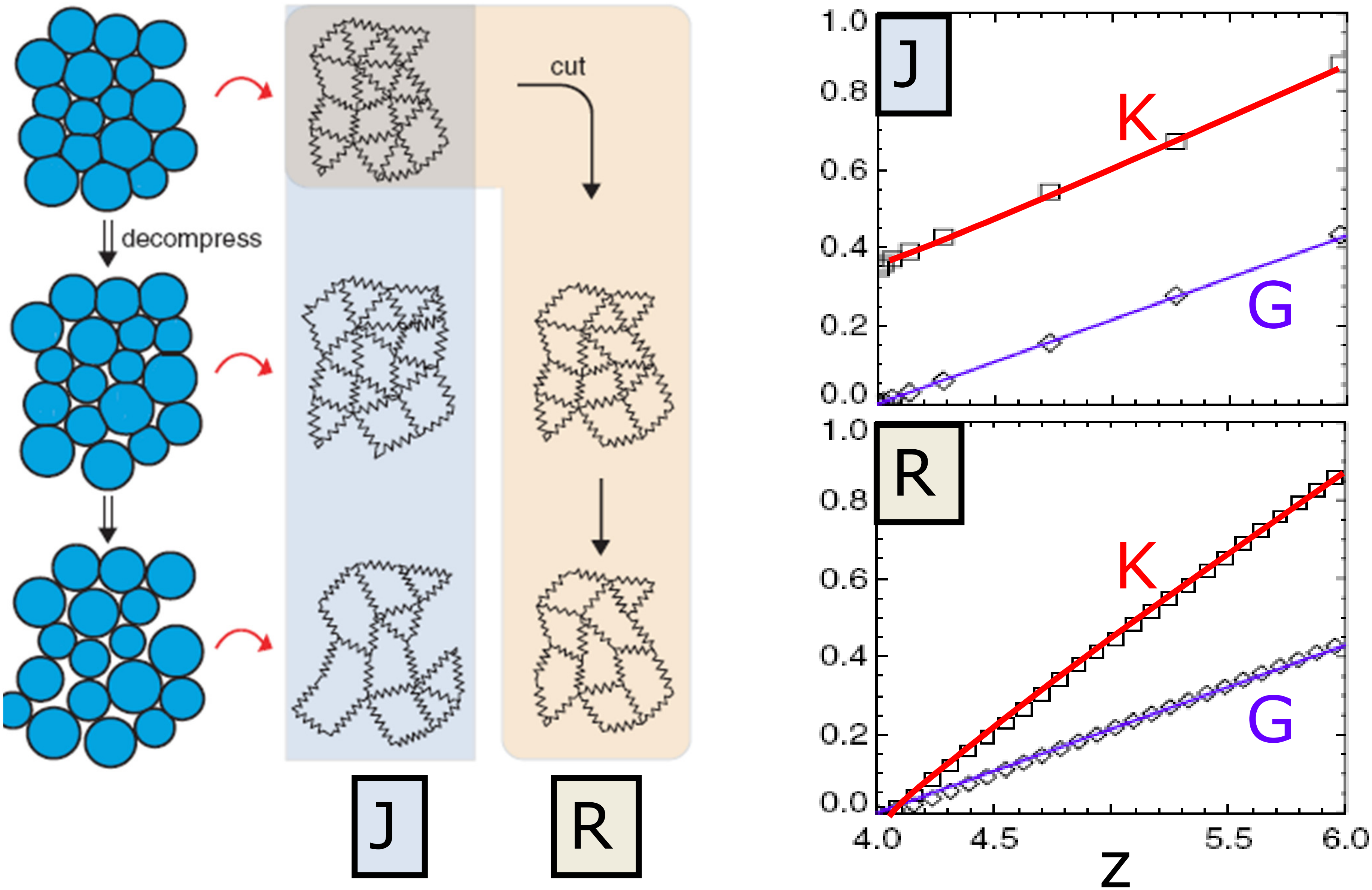


Jamming

Ellenbroek et al, 06/09

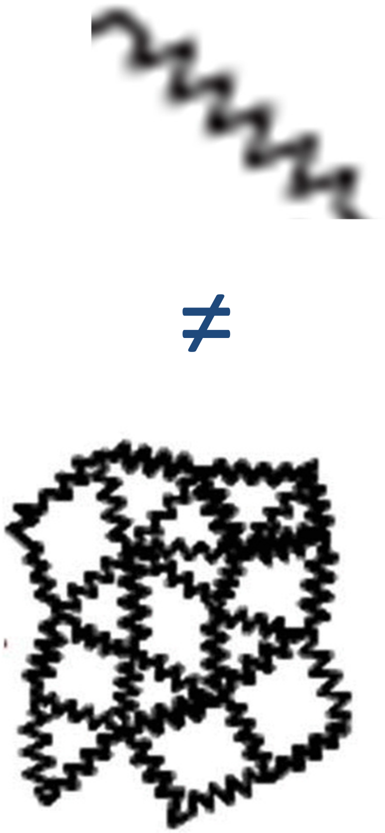


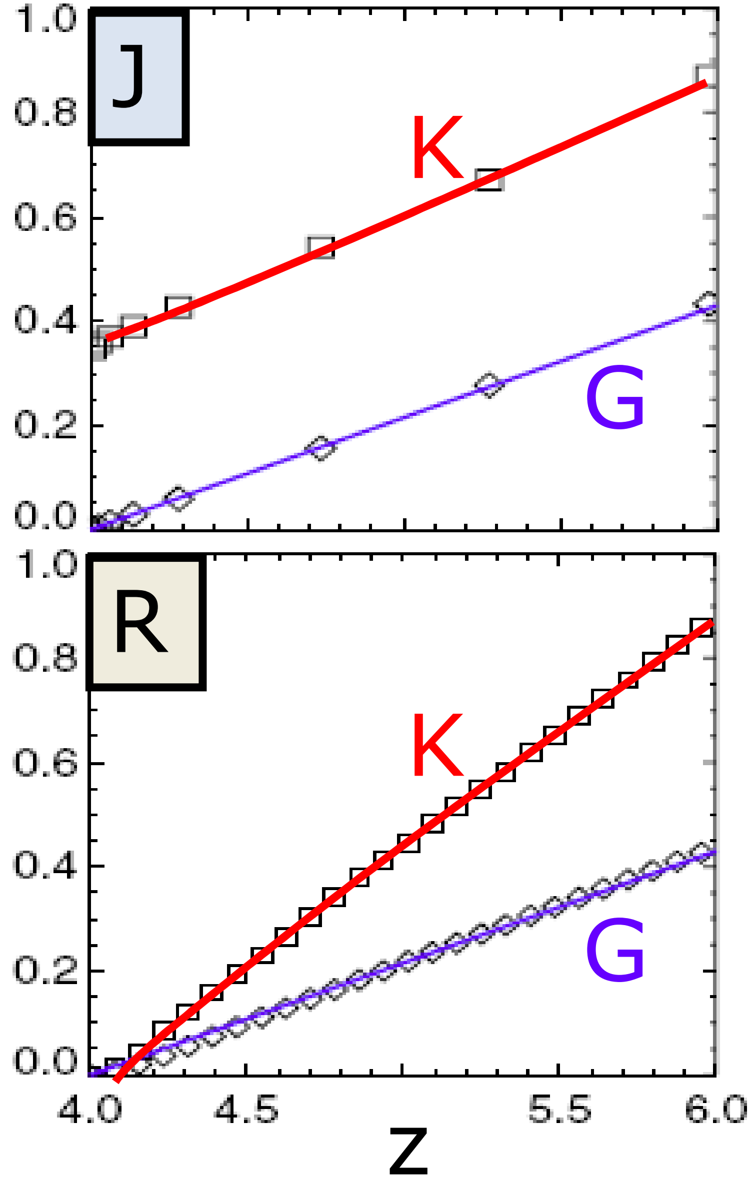
Jamming: Special Geometry



W. G. Ellenbroek, Z. Zeravcic, W. van Saarloos and MvH, EPL 87 34004 (2009)

Jamming: Special Geometry





W. G. Ellenbroek, Z. Zeravcic, W. van Saarloos and MvH, EPL 87 34004 (2009)

Jamming: Special Geometry

Jamming

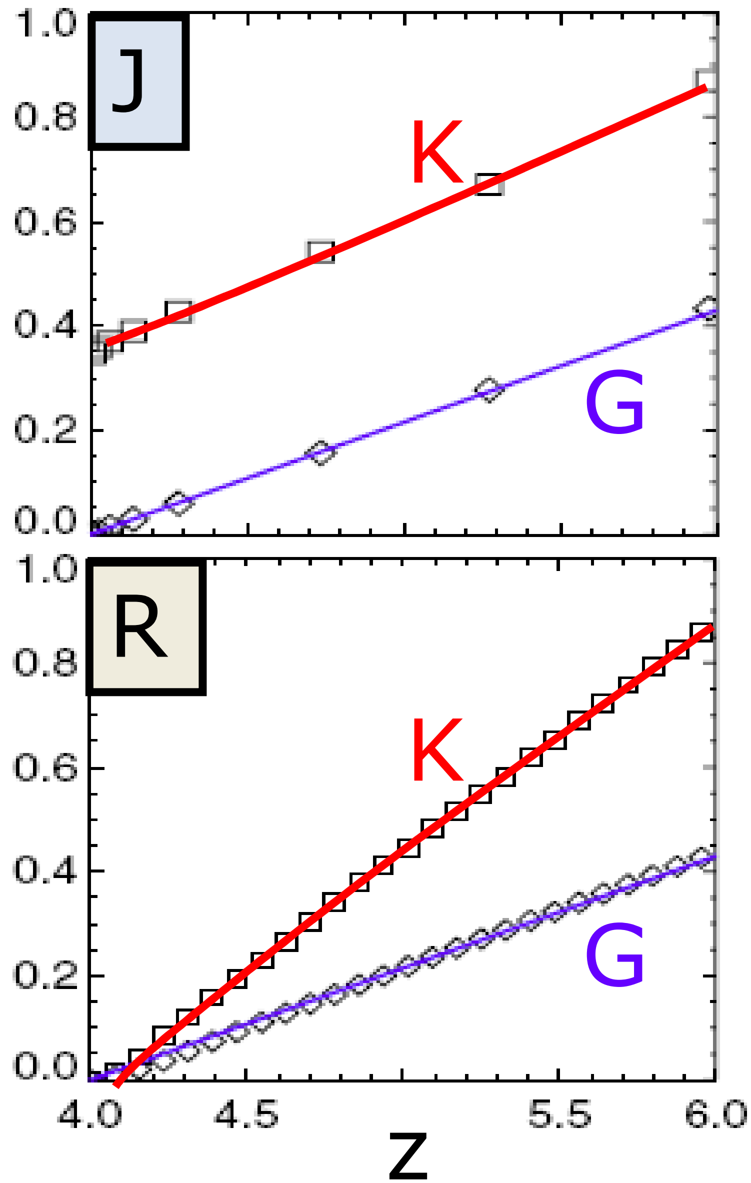
≠

Rig. Percolation

NonAffine:

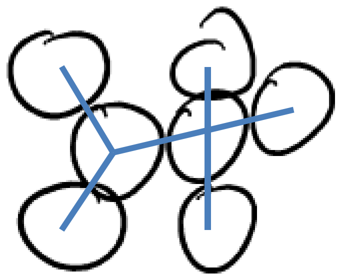
Part of the Story

Self Adjusting



W. G. Ellenbroek, Z. Zeravcic, W. van Saarloos and MvH, EPL 87 34004 (2009)

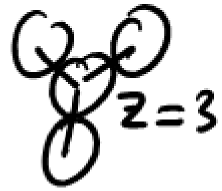
Contact Number



d : DIMENSION

N : # particles

z : CONTACT NR



* TOUCH. DOF: dN (coordinates)

CONSTR: $\frac{Nz}{2}$ (contacts)

$$dN \geq \frac{Nz}{2}$$

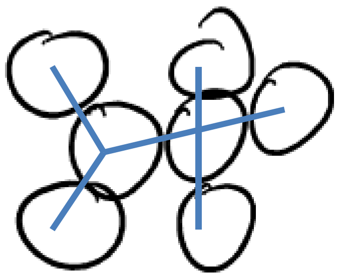
* FORCE BALANCE DOF: $\frac{Nz}{2}$ (forces)

Constr: dN (x, y, z -- force balance)

$$dN \leq \frac{Nz}{2}$$

$$z = 2d$$

Contact Number



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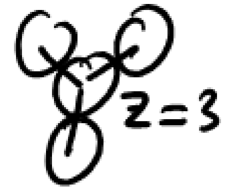
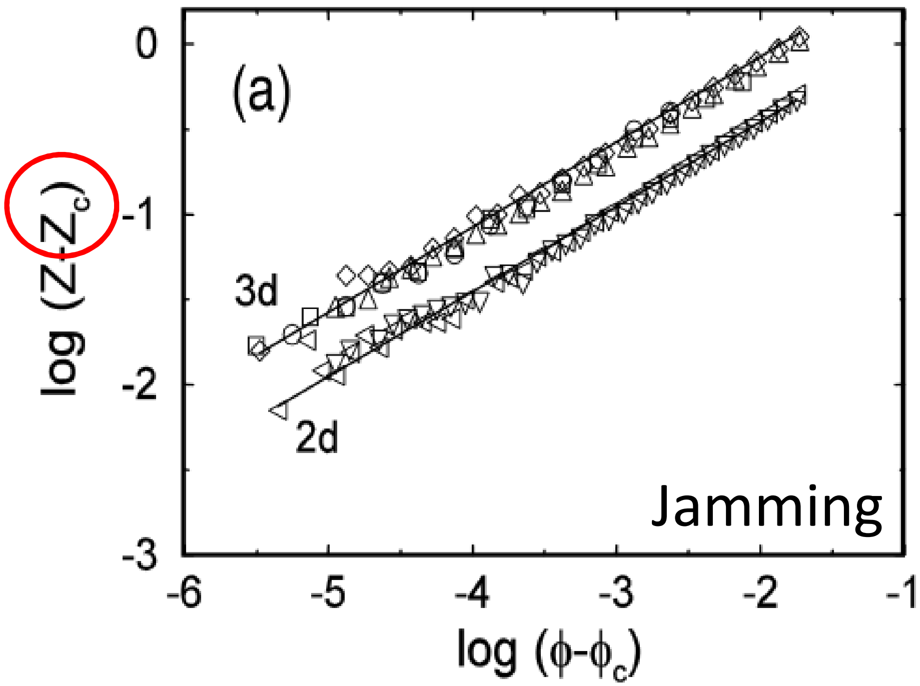
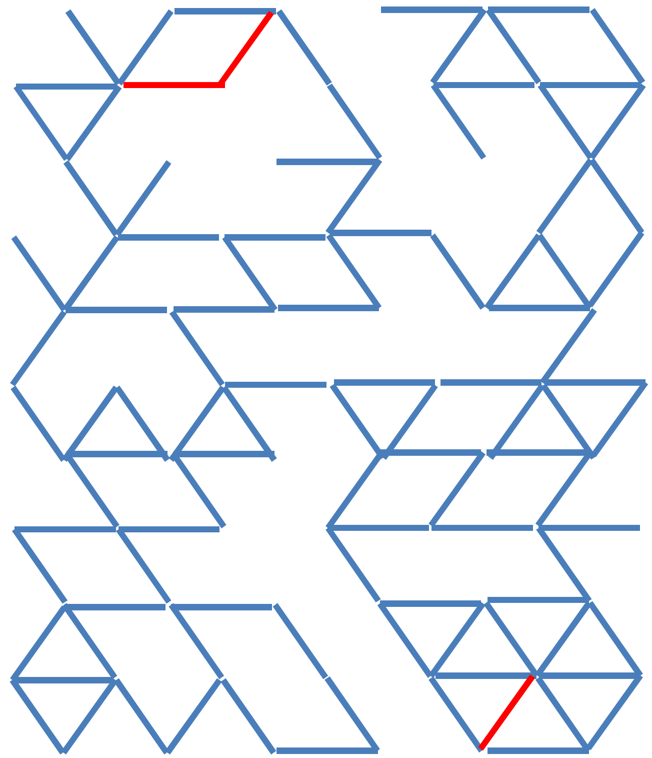


$\boxed{Z=2d}$

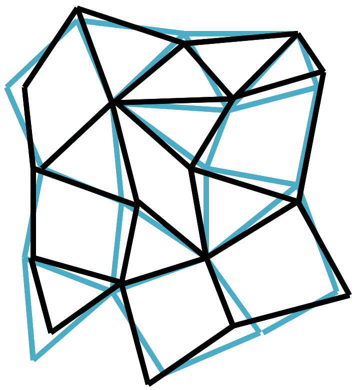


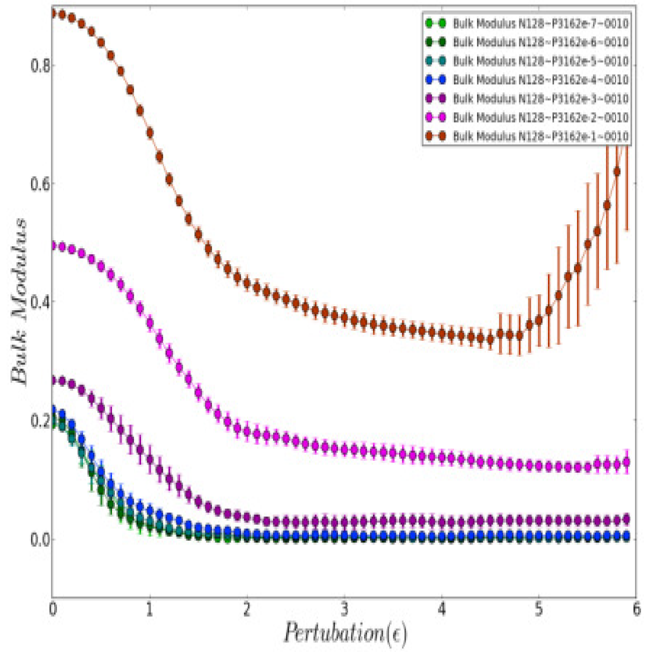
$\boxed{Z=2d}$

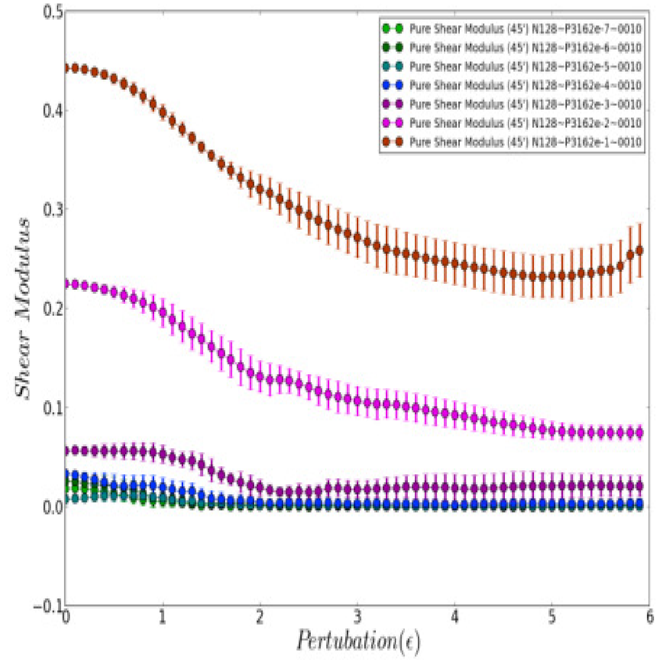


$\boxed{Z\approx 2d}$

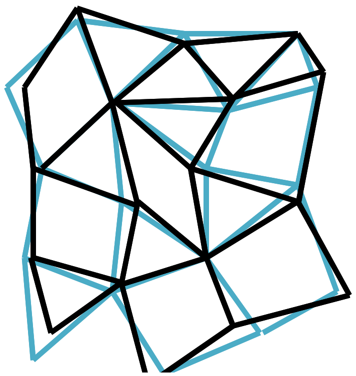
Perturbations

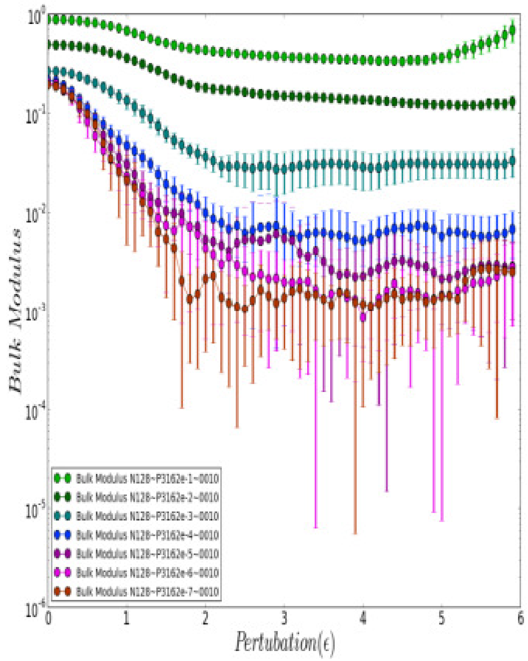


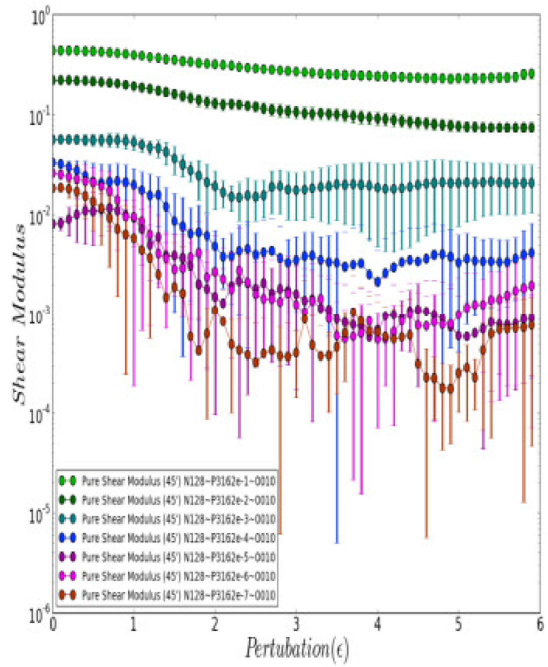


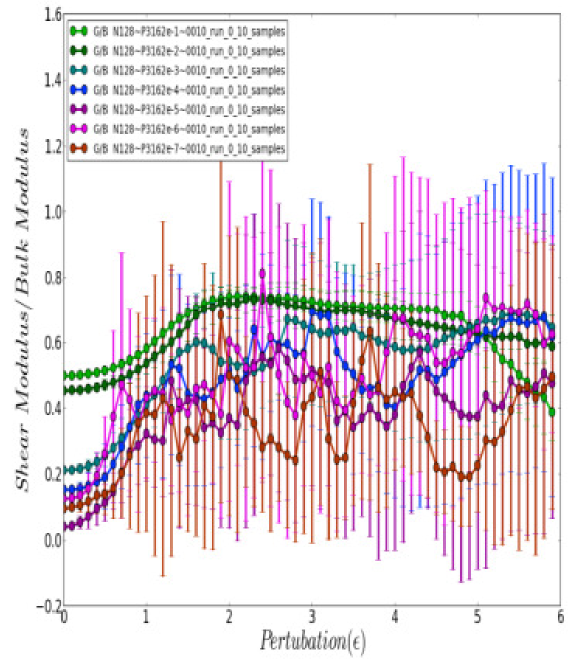


Perturbations







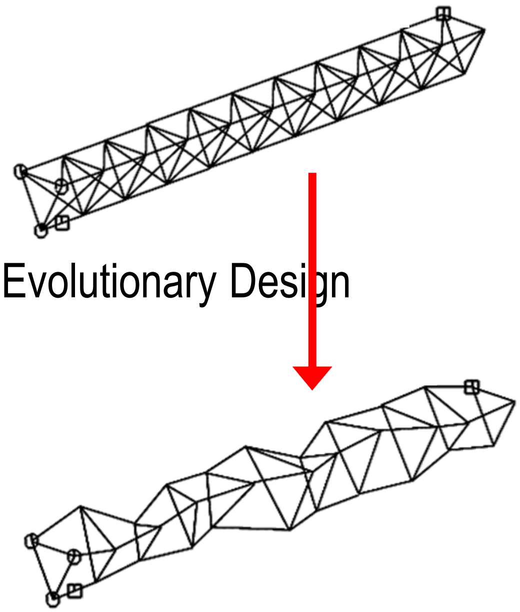


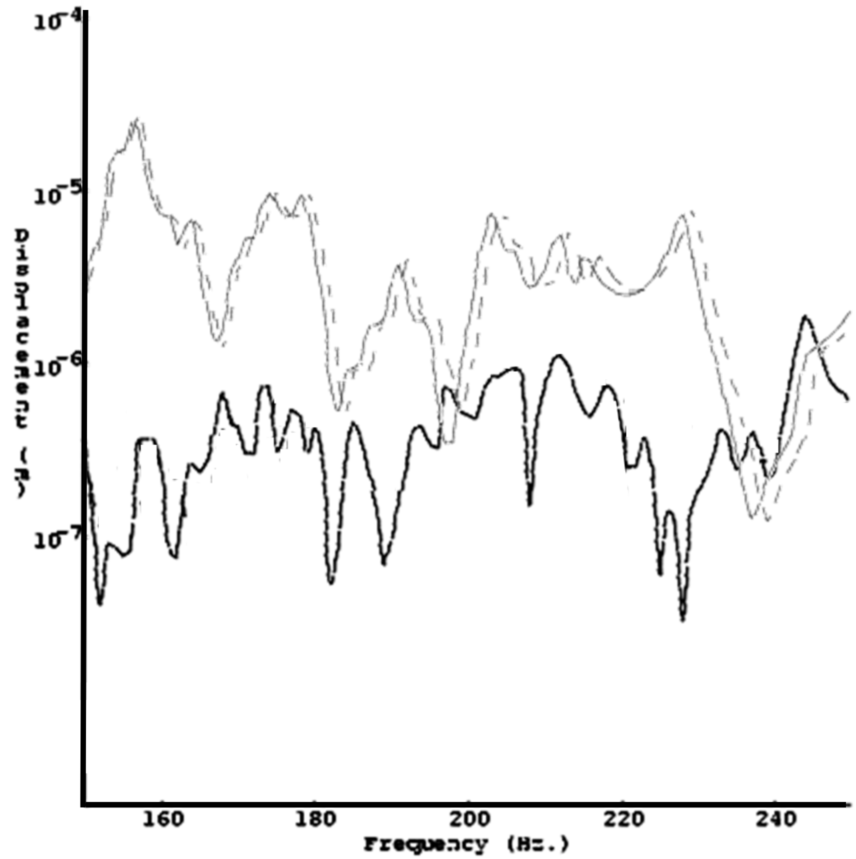
Bastiaan Florijn

Evolutionary Algorithms

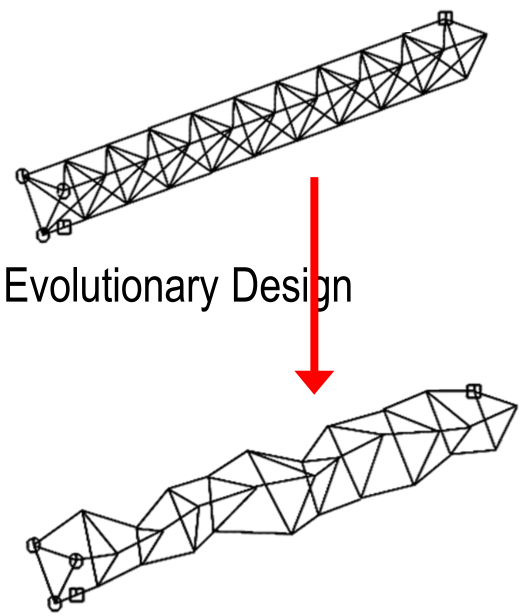
If jamming geometry is so special, can we mimic it?

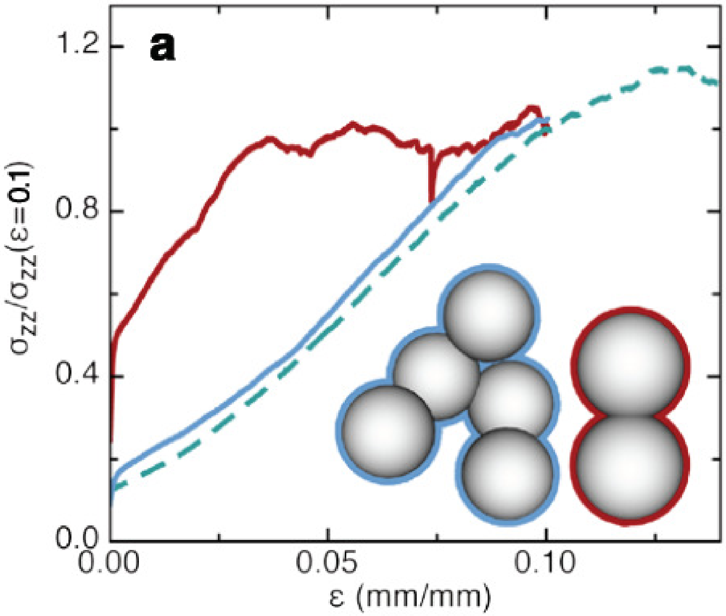
Evolutionary Algorithms



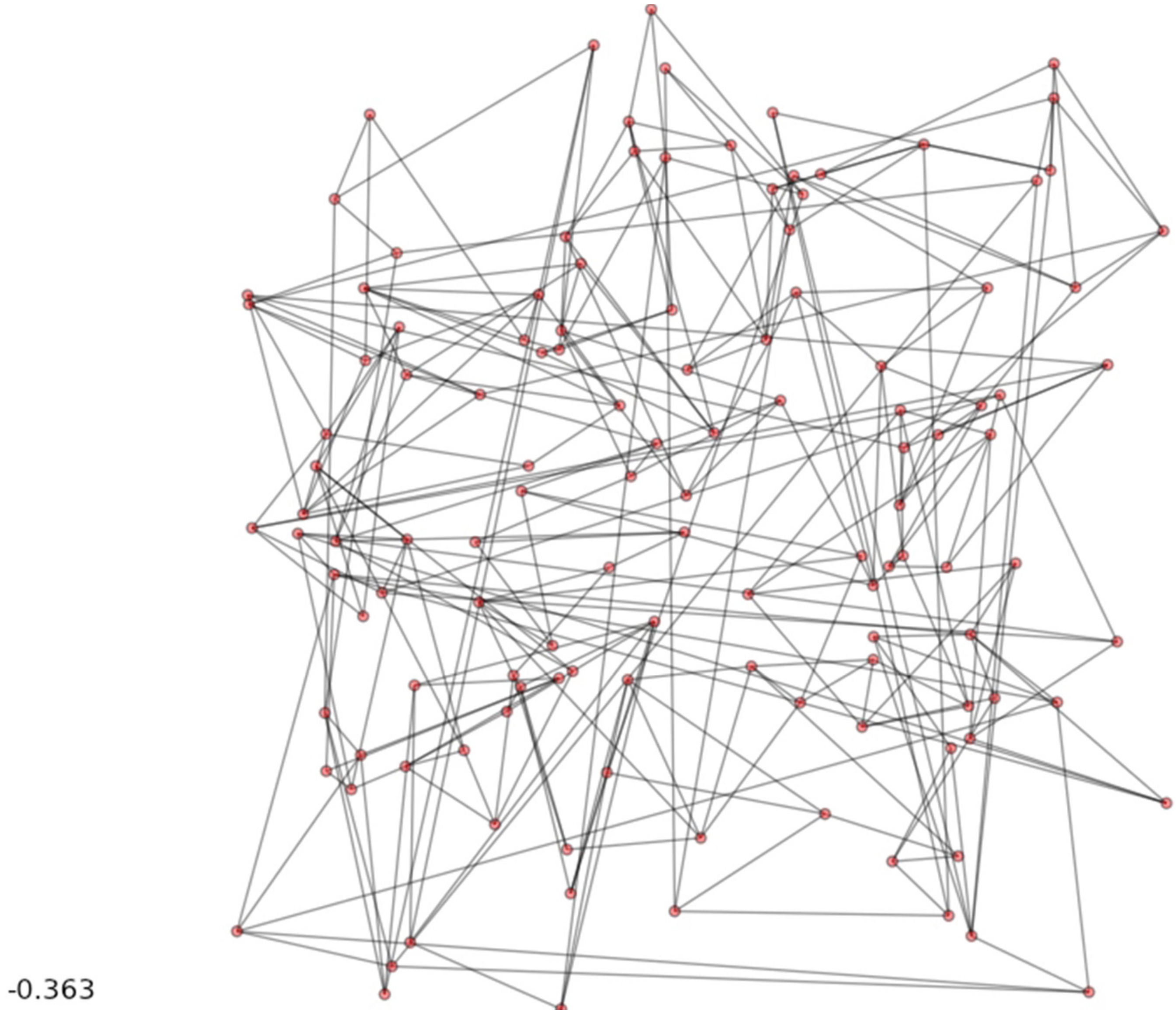


Evolutionary Algorithms





Evolutionary Algorithms



Marc Miskin

Metamaterials: Geometry

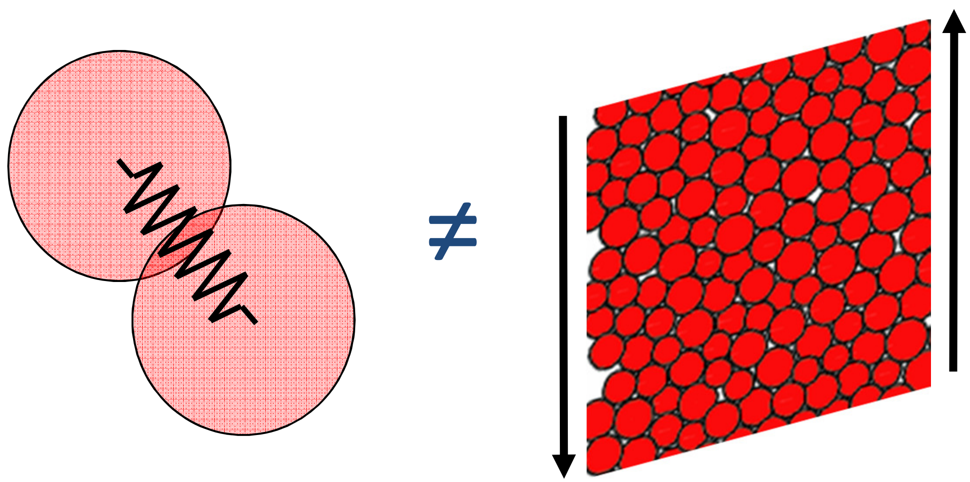
Structure Determines Linear Response

Connectivity

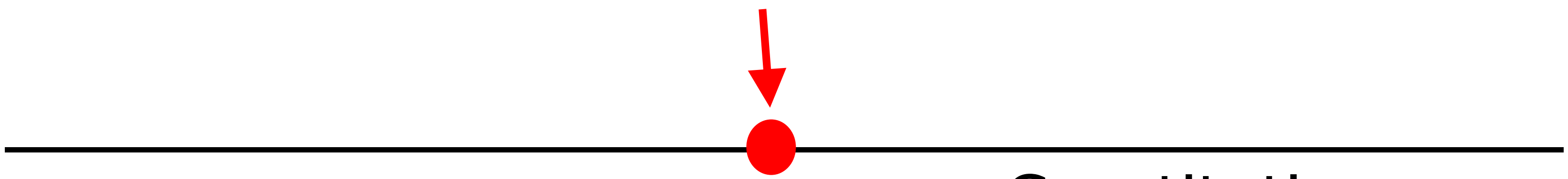
Geometry

Design Geometries

Marginal Point

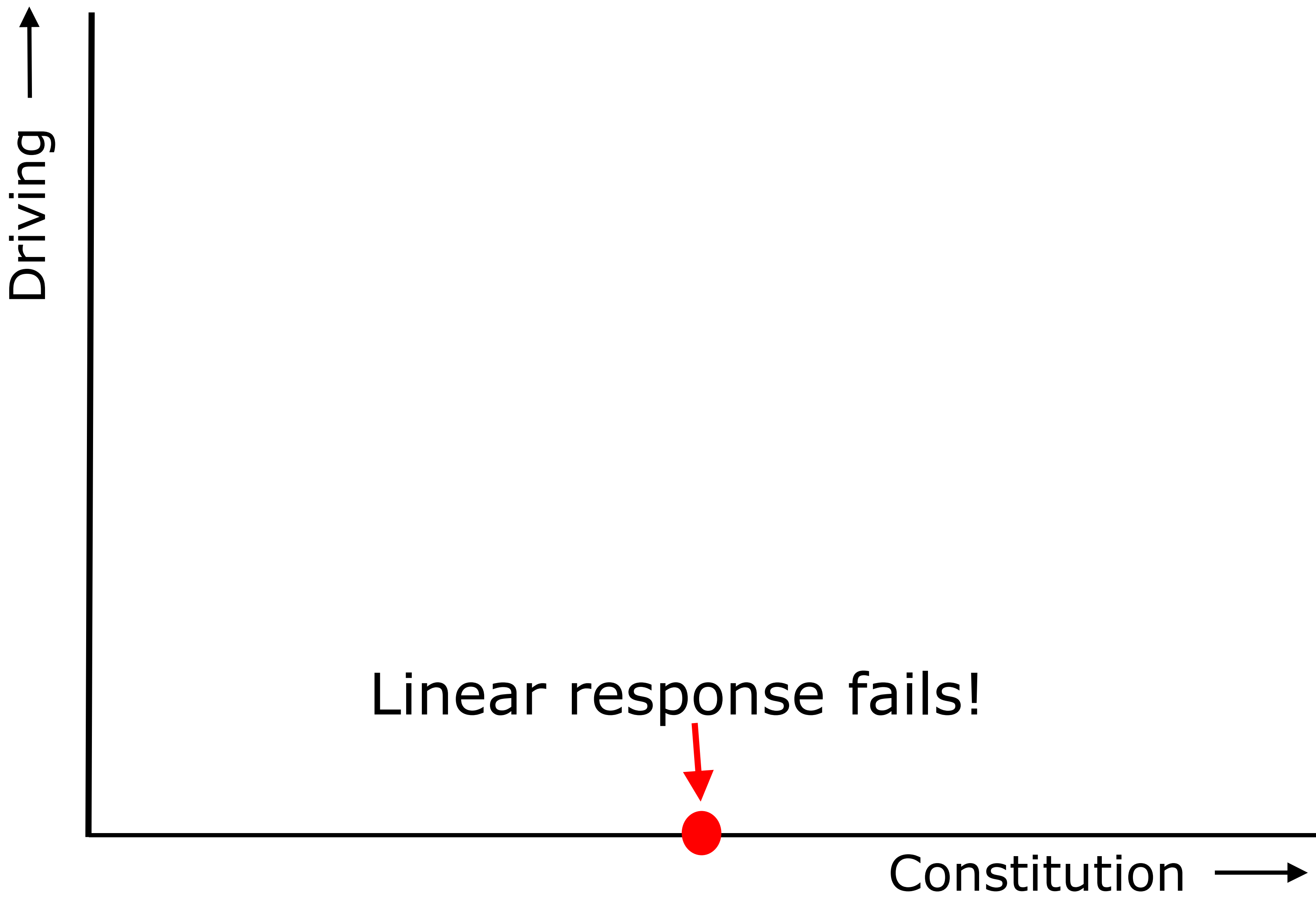


Disorder Rules!

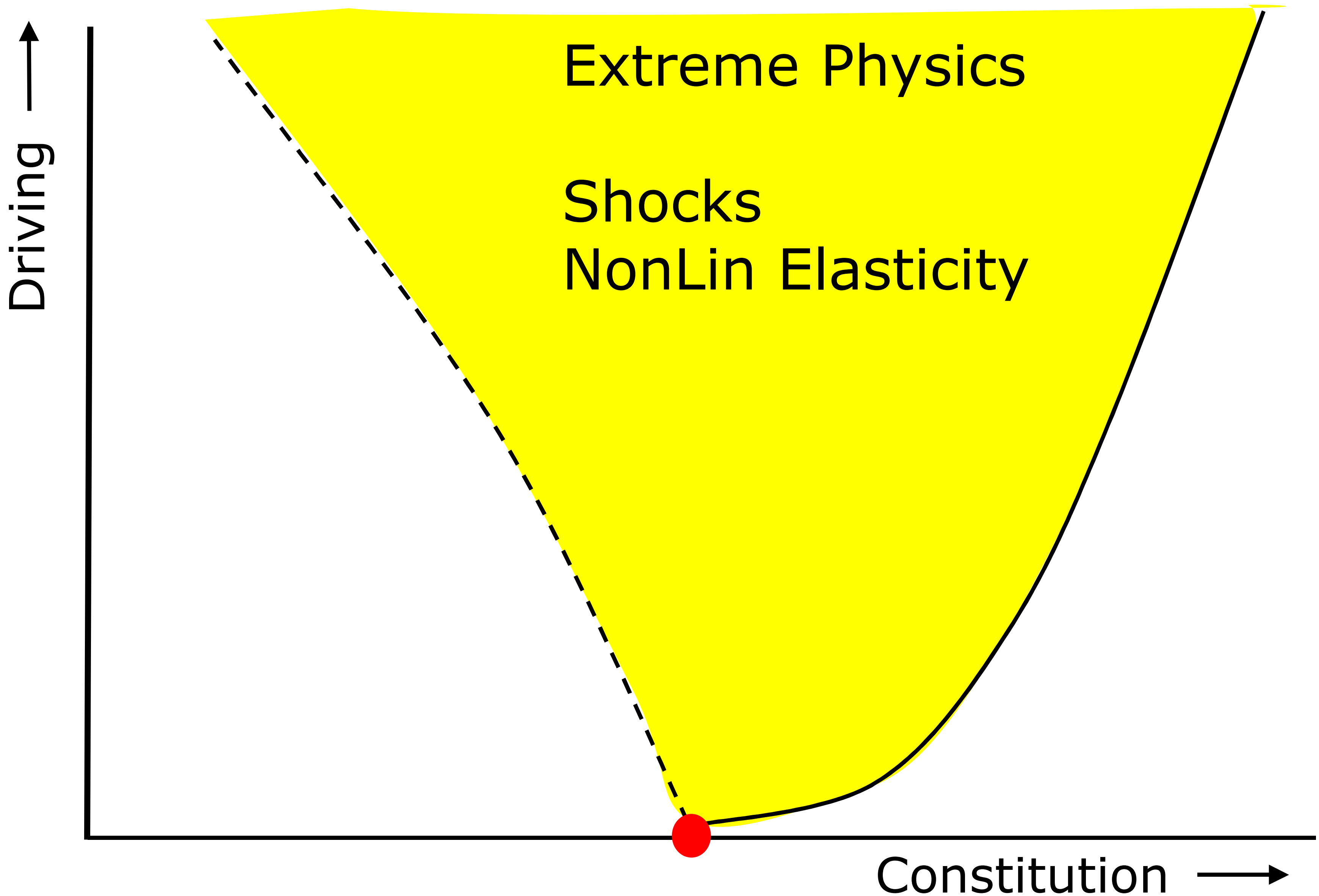


Constitution →

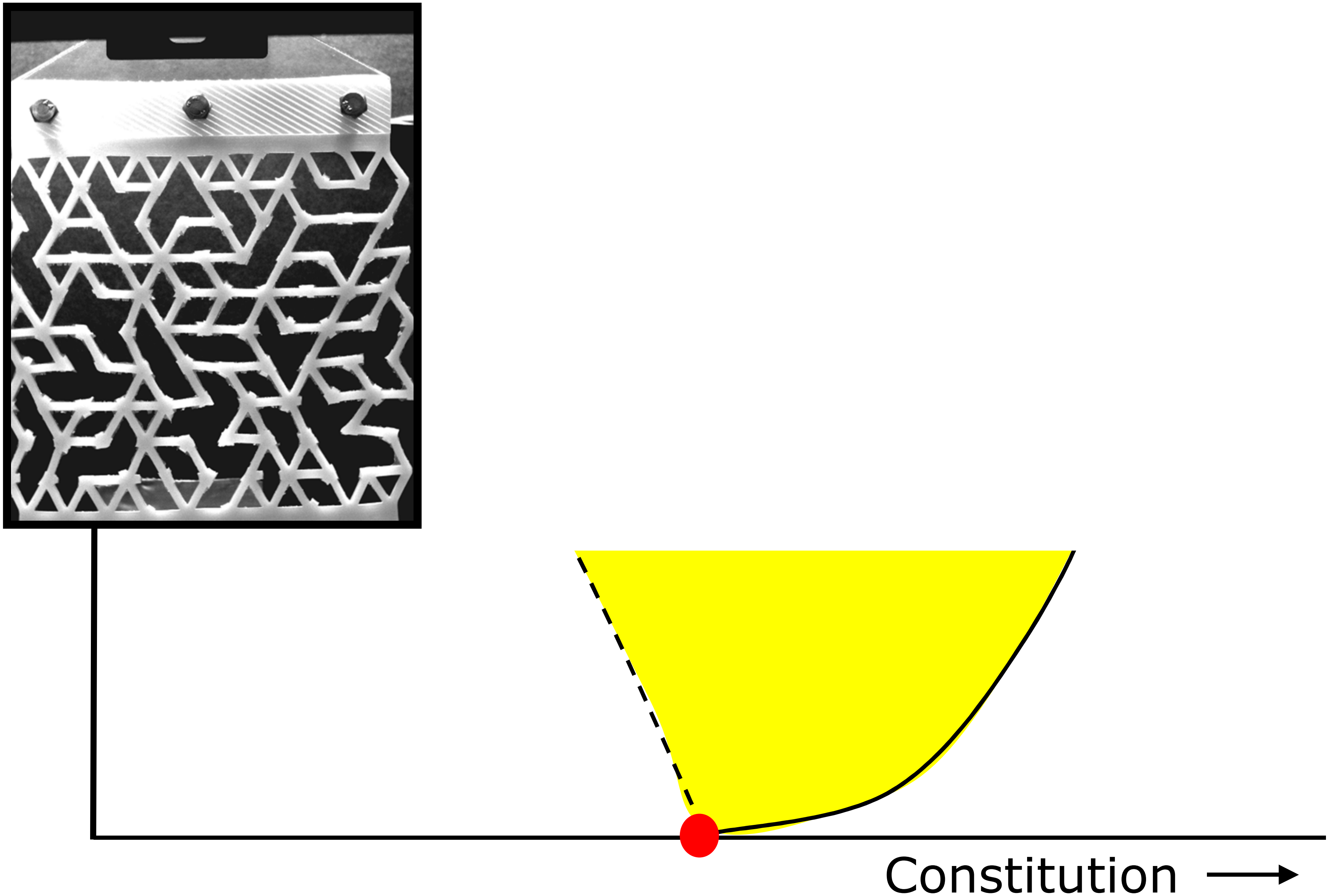
Close to falling apart...



Close to falling apart...

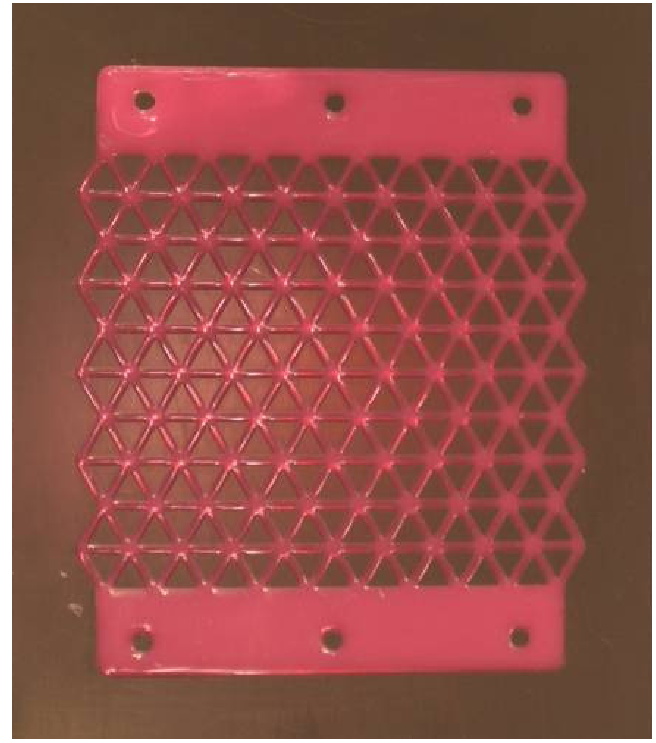


Close to falling apart...



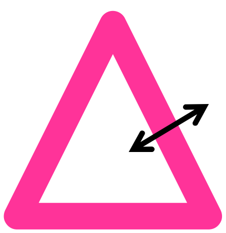
Nonlinear Networks

10 cm



10 cm

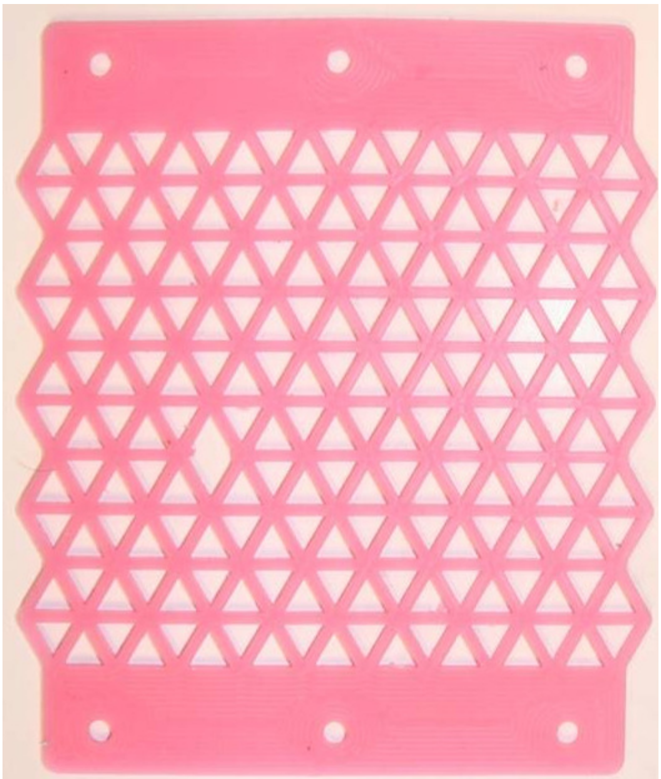
Young's Modulus 220 kPa

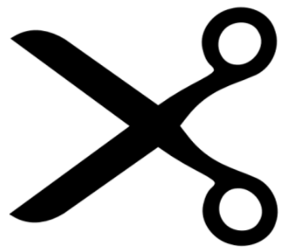


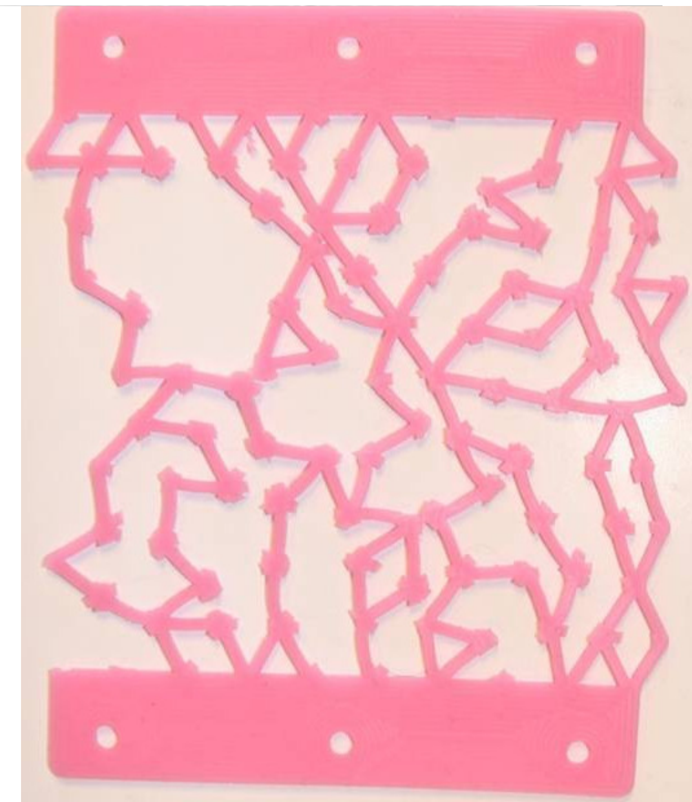
Thickness of struts 1 mm

Bastiaan Florijn

Nonlinear Networks



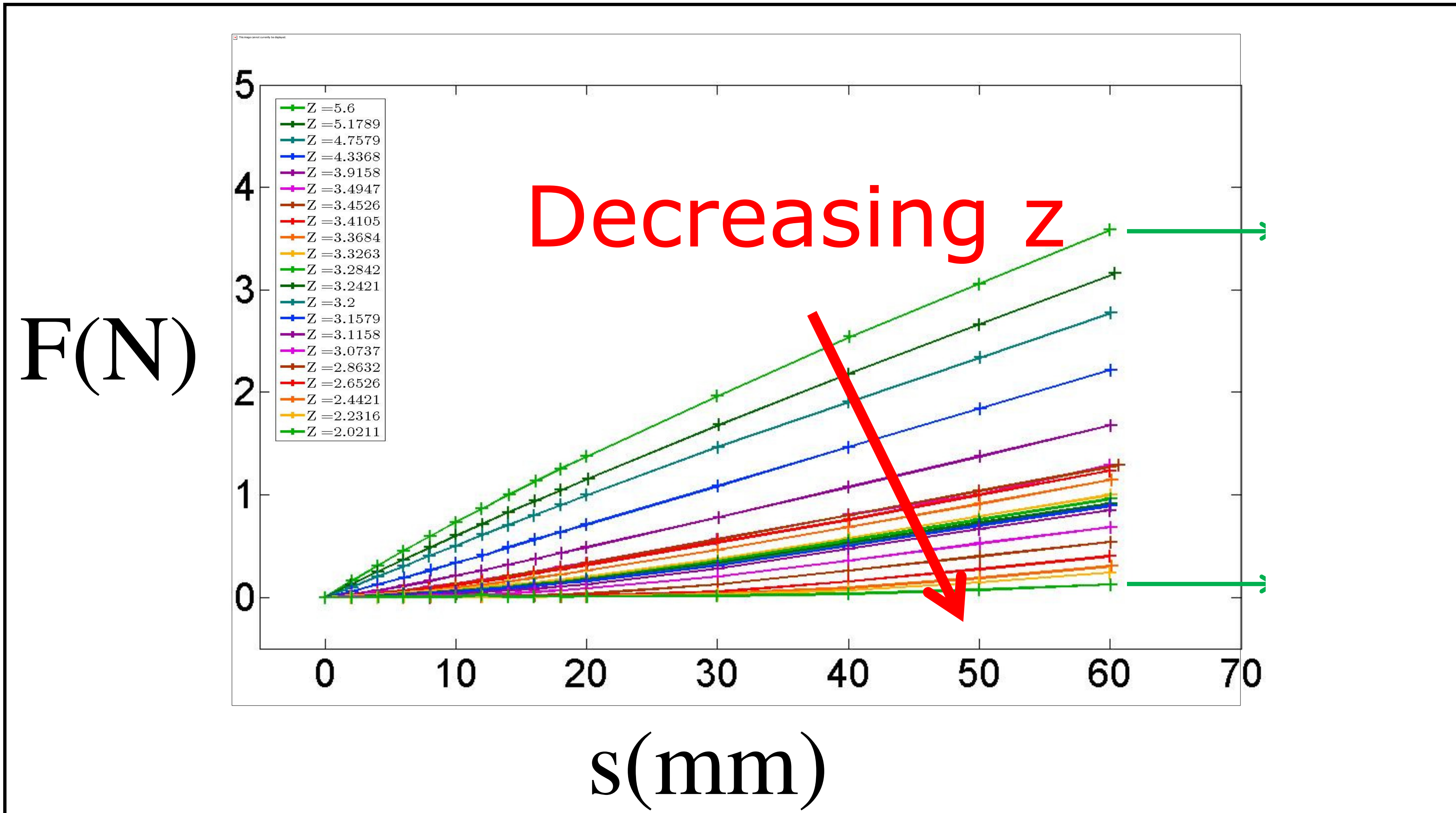




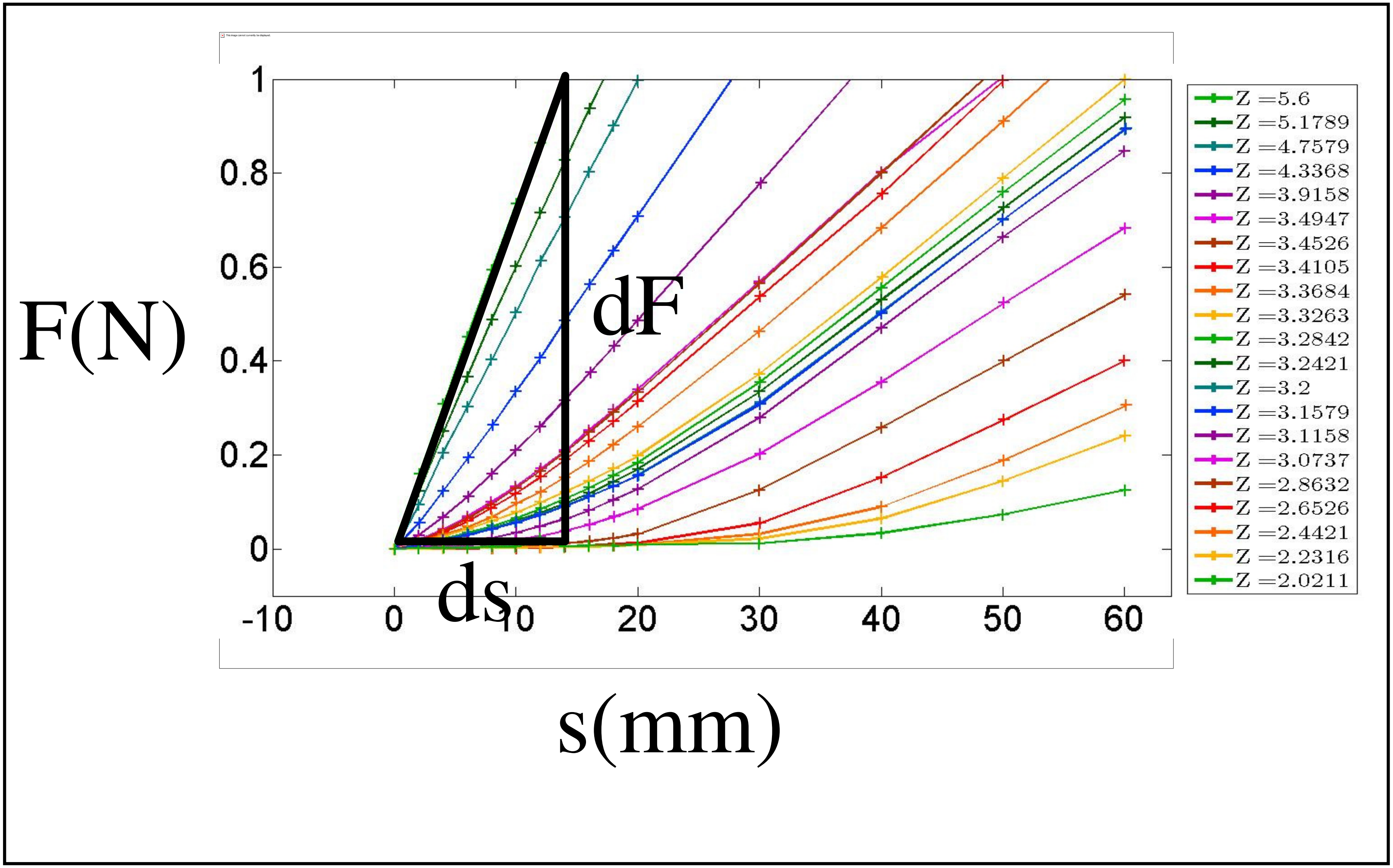
Z=5.6

Z=2.6

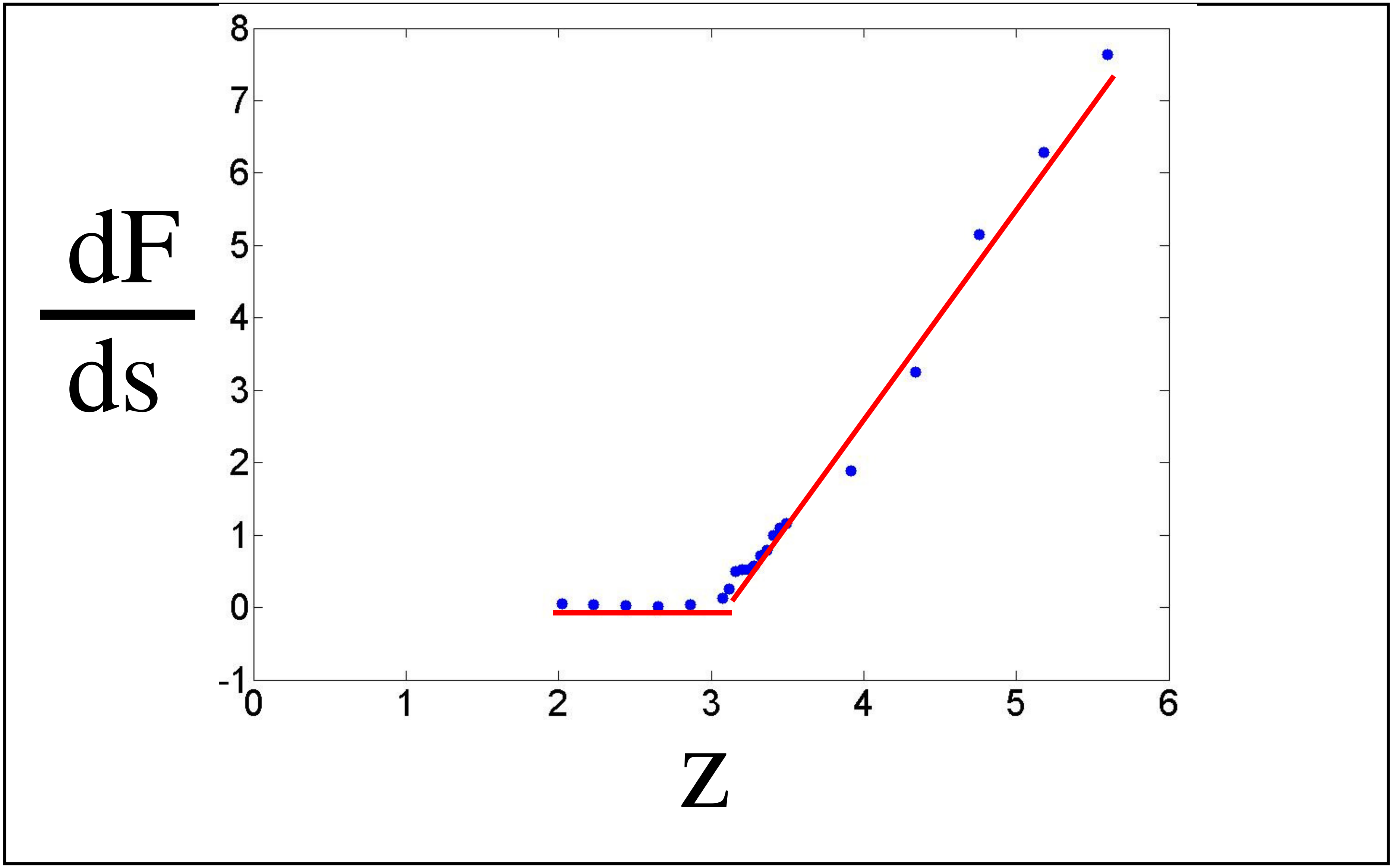
Nonlinear Networks



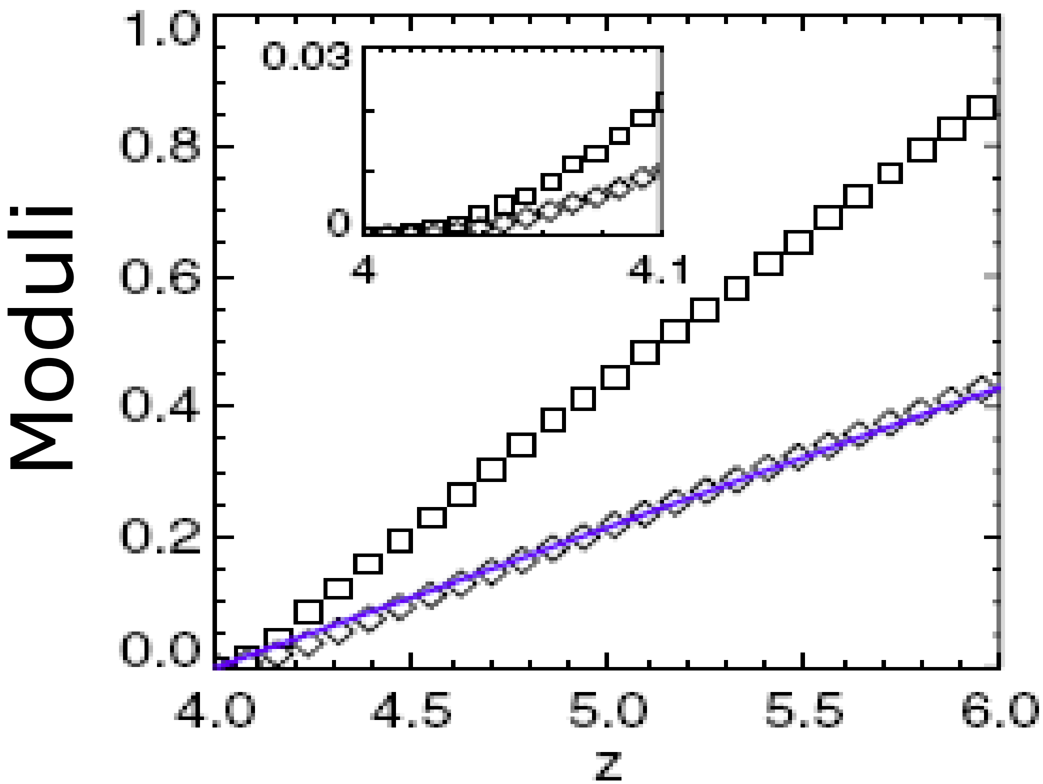
Nonlinear Networks



Nonlinear Networks

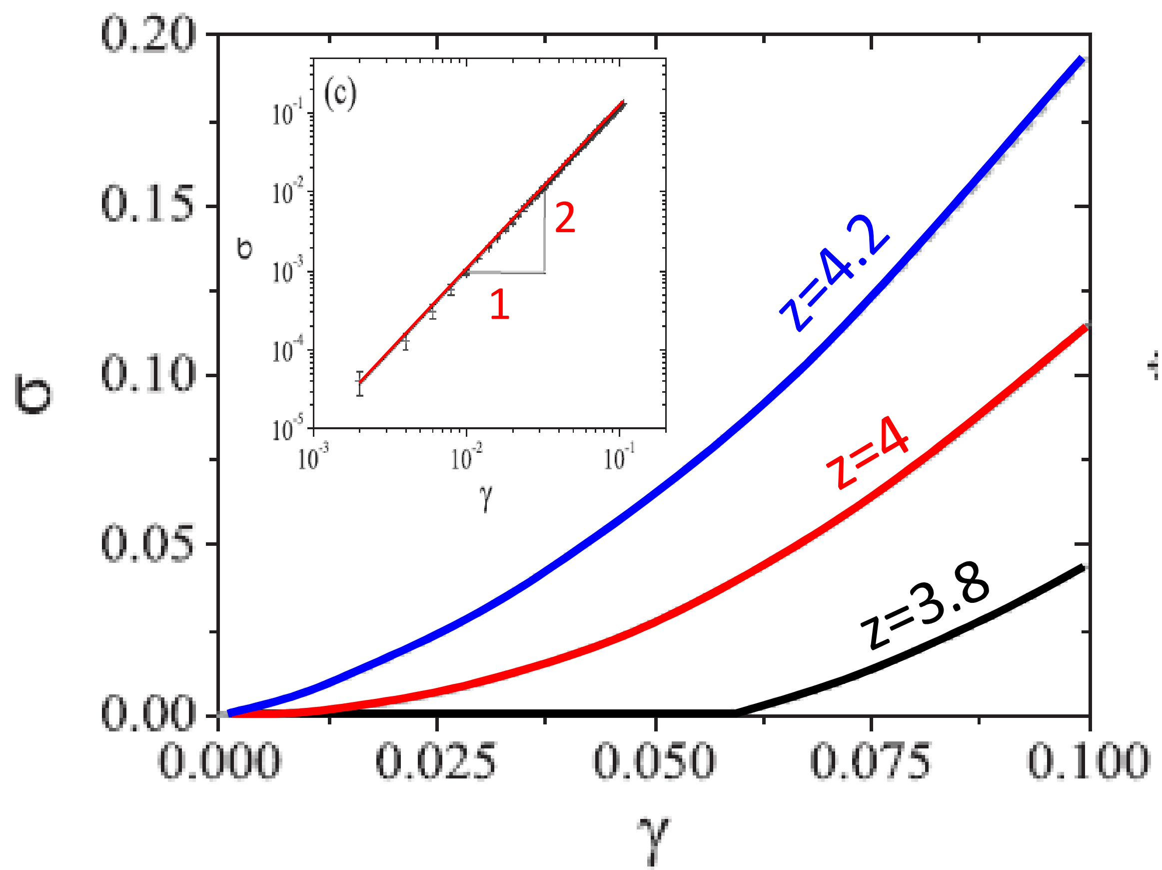


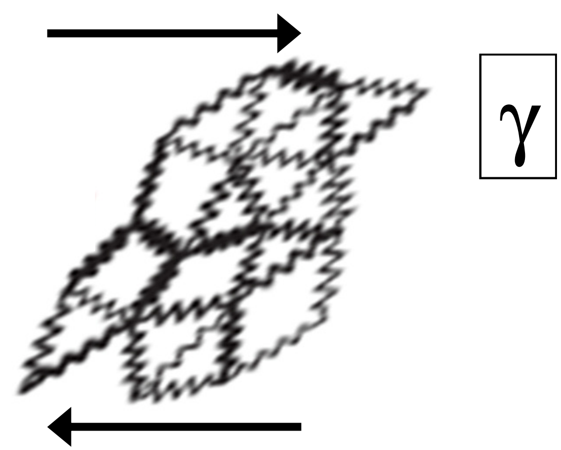
Intermezzo: Simulations of Spring Networks



W. G. Ellenbroek, Z. Zeravcic, W. van Saarloos and MvH, EPL 87 34004 (2009)

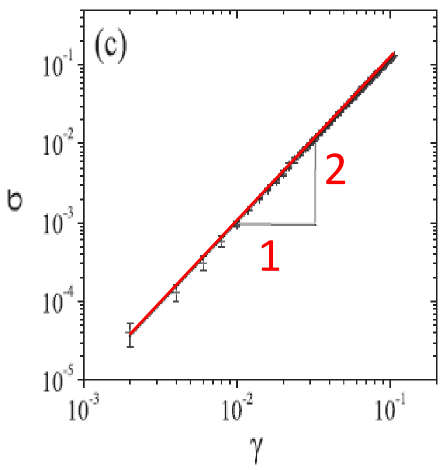
Intermezzo: Simulations of Spring Networks



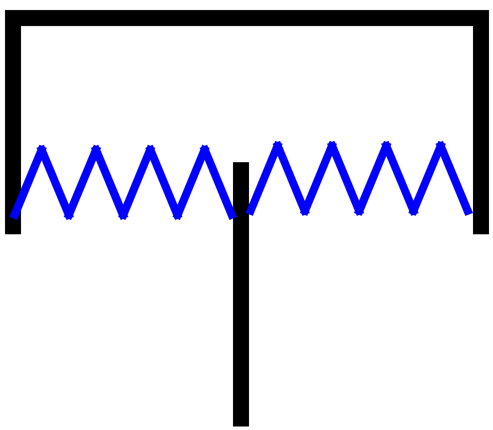


M Wyart, H Liang, A Kabla and L Mahadevan, PRL 101 215501 (2008)

You Should Be Shocked



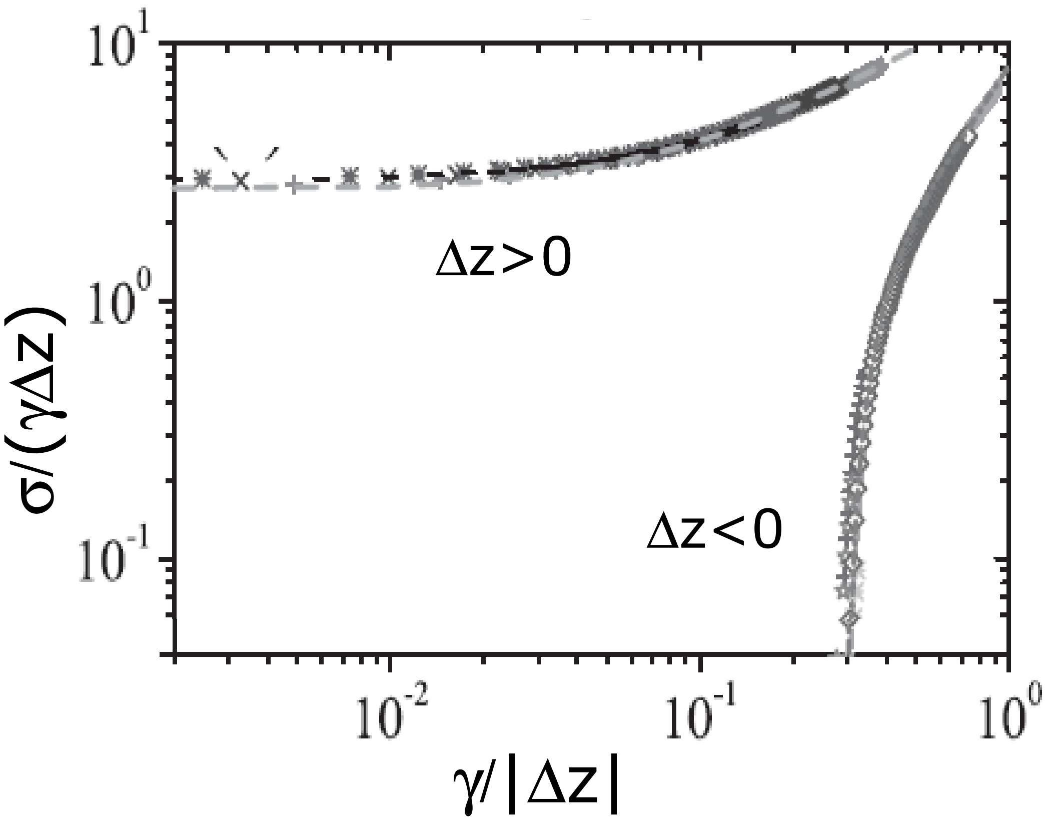
$$\boxed{\sigma \sim \gamma|\gamma|}$$

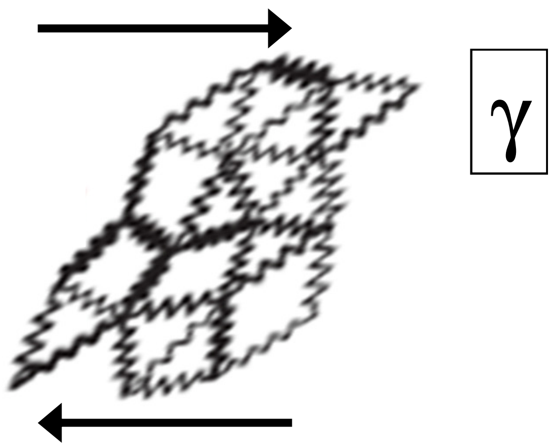


Tension : $\sigma \sim \gamma$

Slack : $\sigma \sim \gamma^3$

Intermezzo: Simulations of Spring Networks

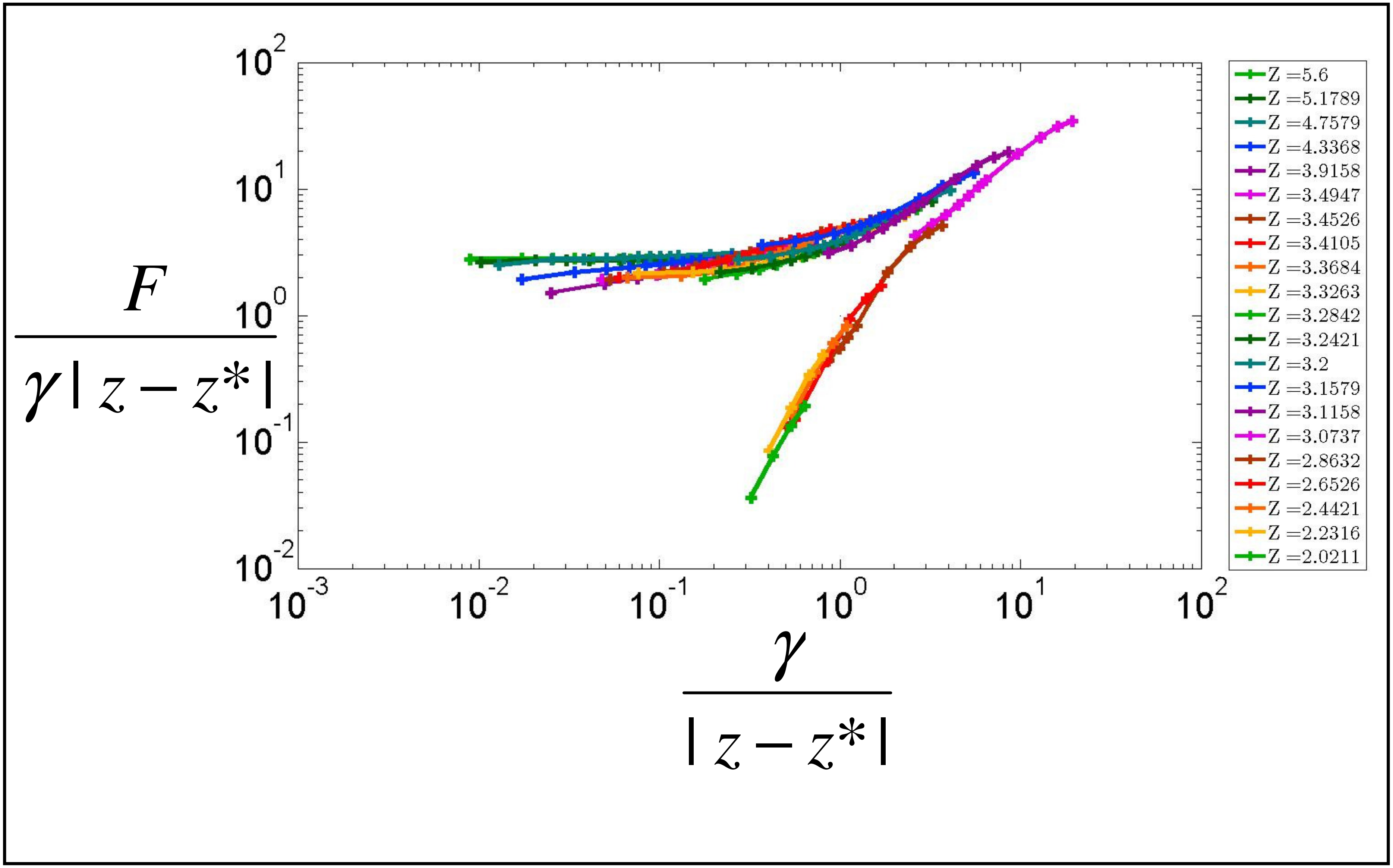




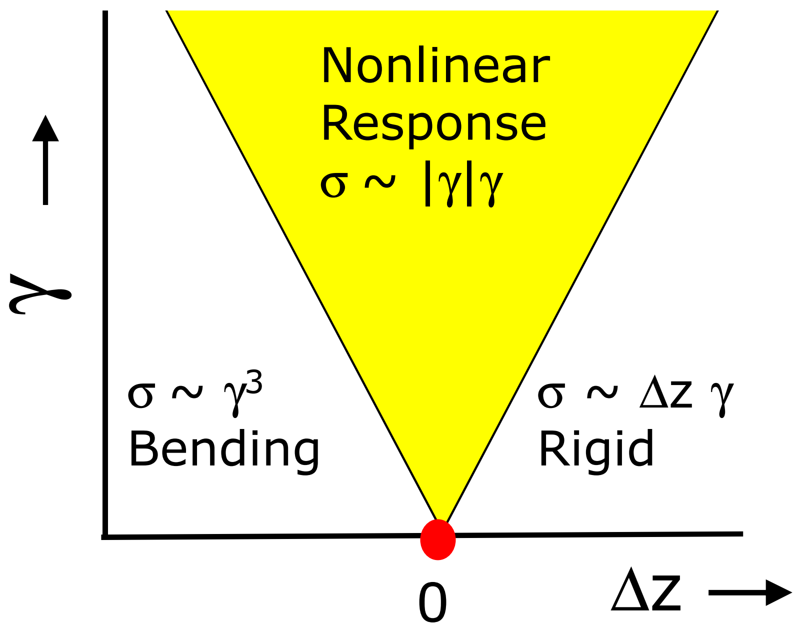
Linear:	$\sigma \sim \gamma\,\Delta z$
$\Delta z = 0$:	$\sigma \sim \gamma|\gamma|$

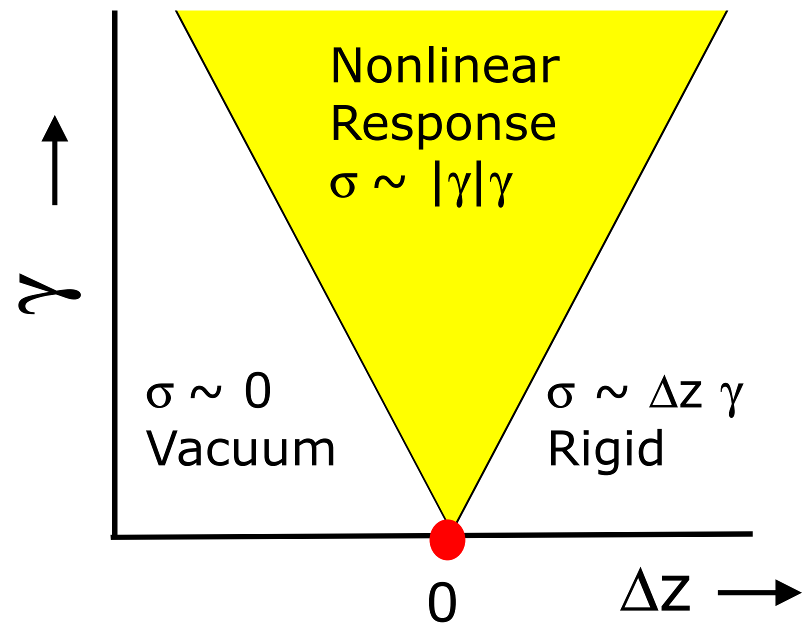
M Wyart, H Liang, A Kabla and L Mahadevan, PRL 101 215501 (2008)

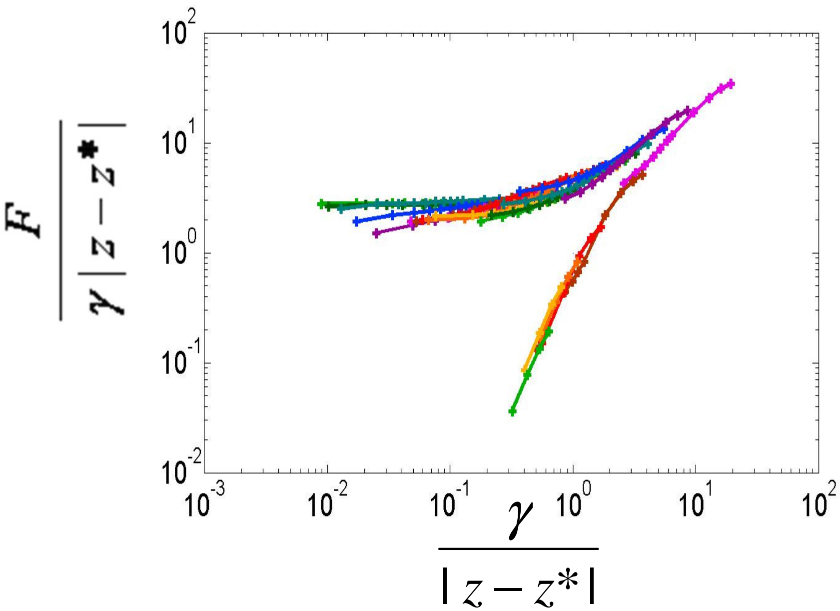
Nonlinear Networks

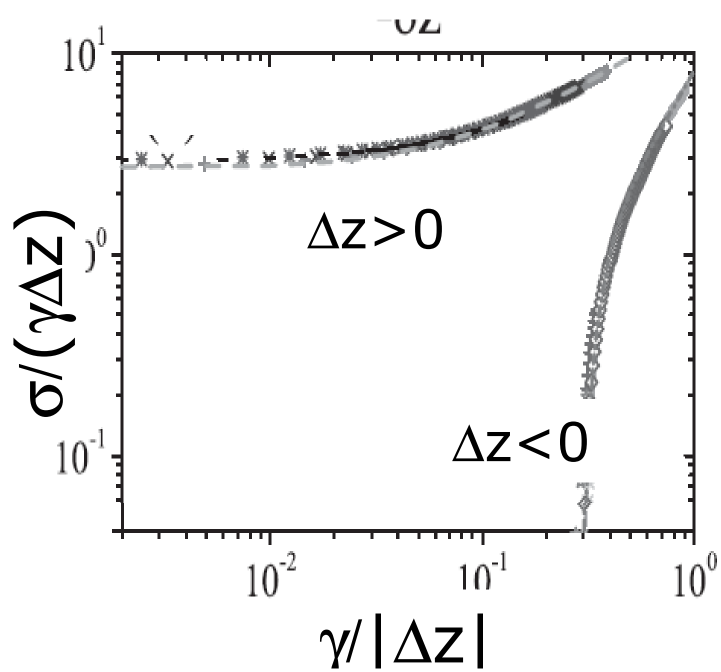


State Diagram

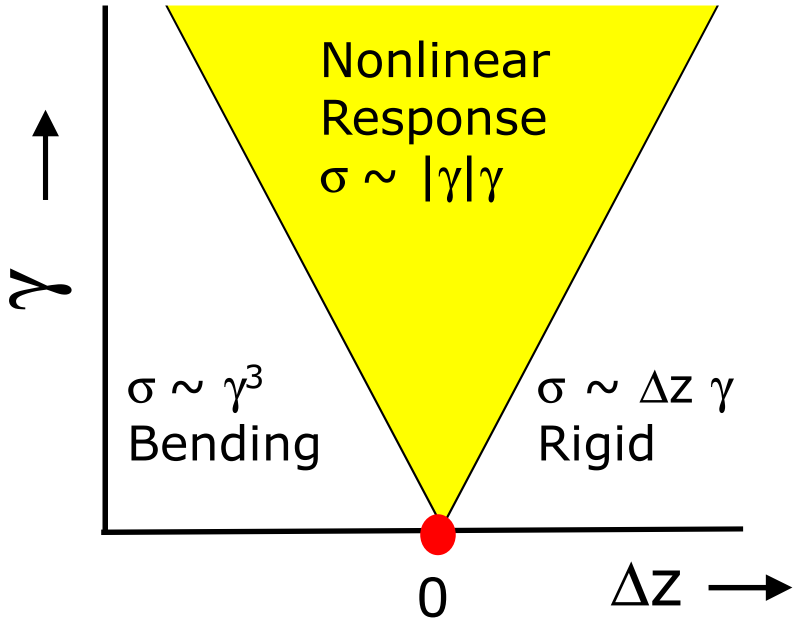


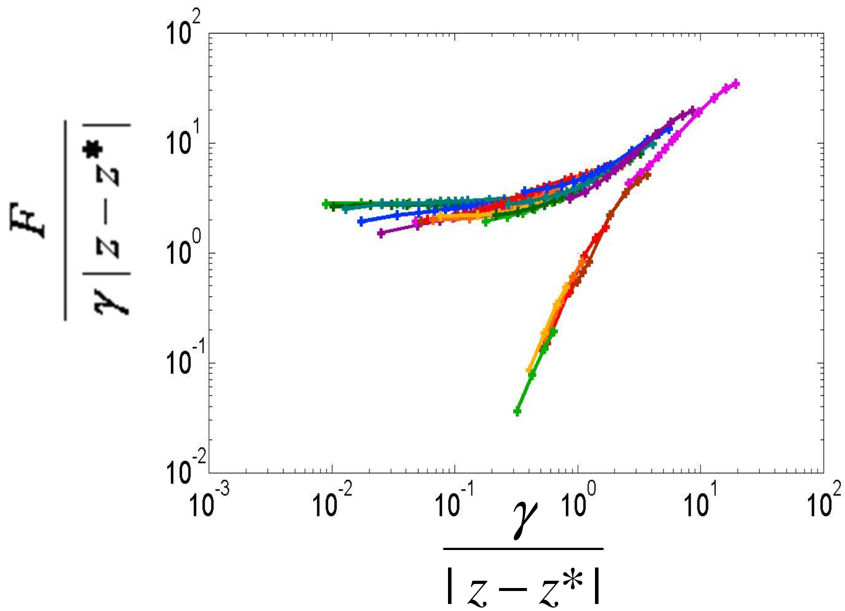






State Diagram





Nonlinear states out of Marginal Point

Linear: Constitution
Nonlinear: Driving

Collective Nonlinearity
– Material is Linear

Metamaterials: Nonlinear

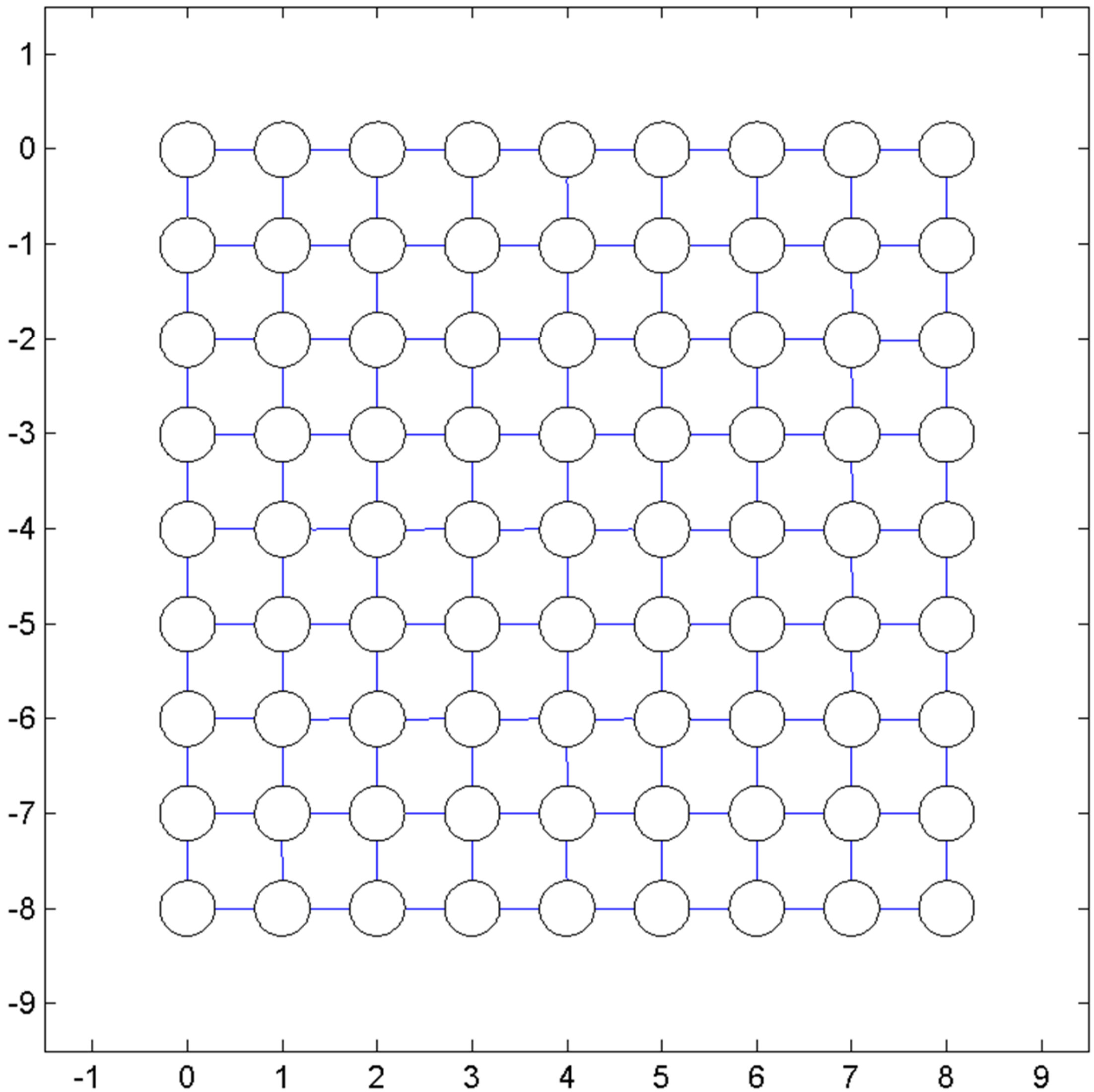
Nonlinear near Marginal Points
Cross Elasticity?

Holey Sheet

Mullin et al, PRL 99, 2007, 084301

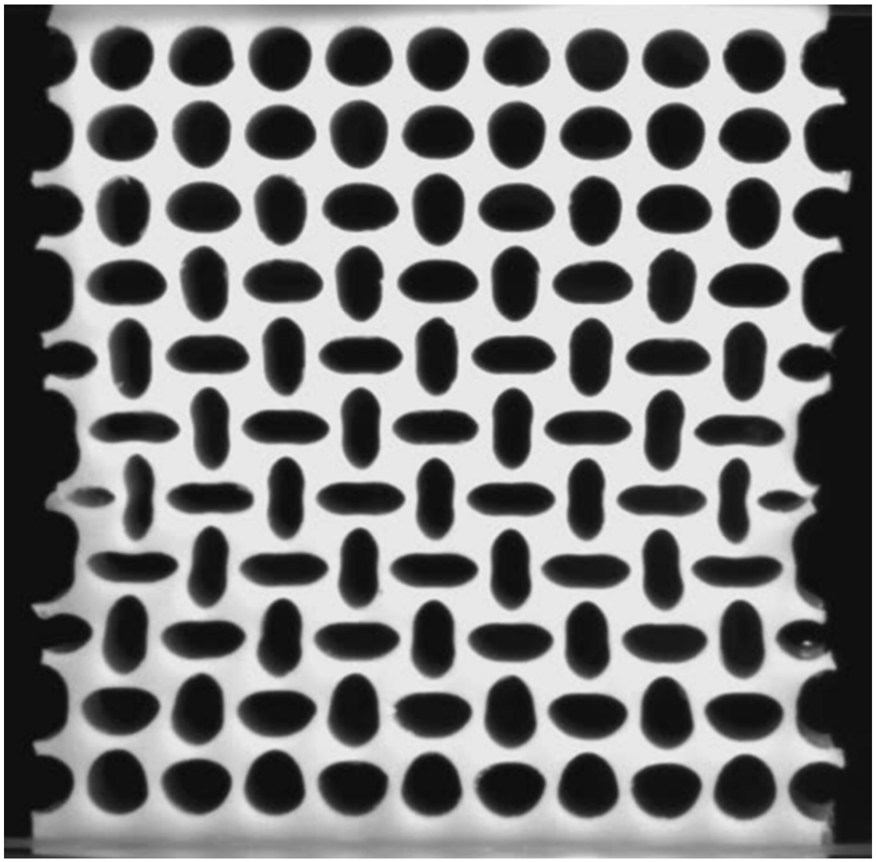
Bastiaan Florijn, Henk Imthorn, Robbin Bastiaansen, Corentin Coulais

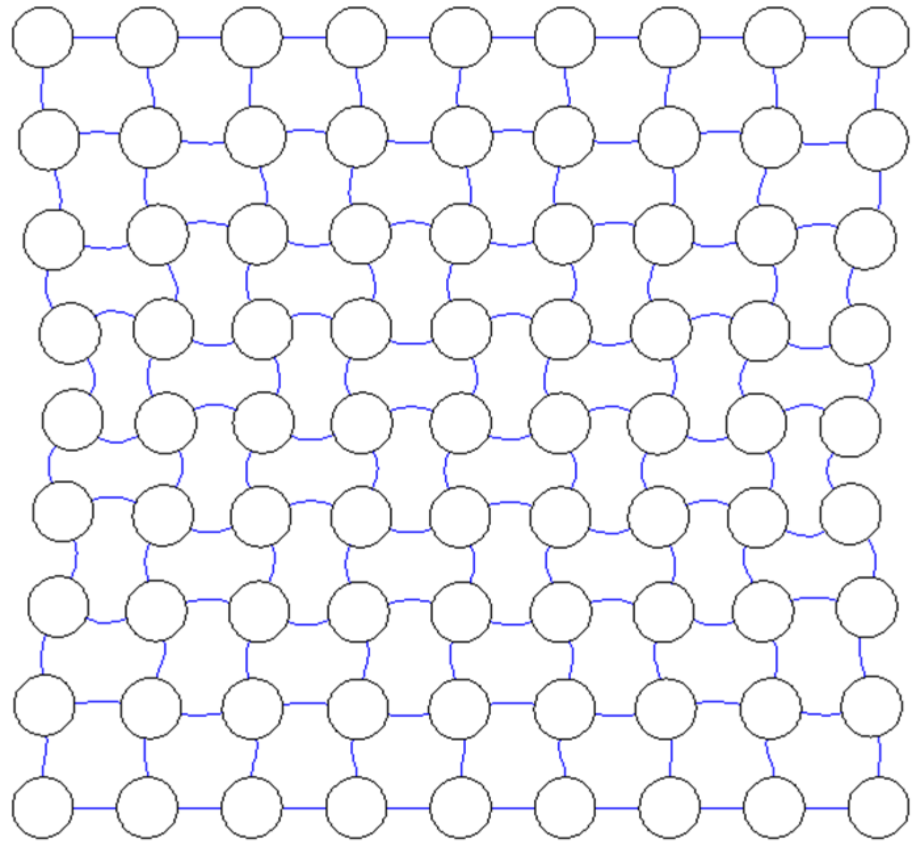
Holey Sheet



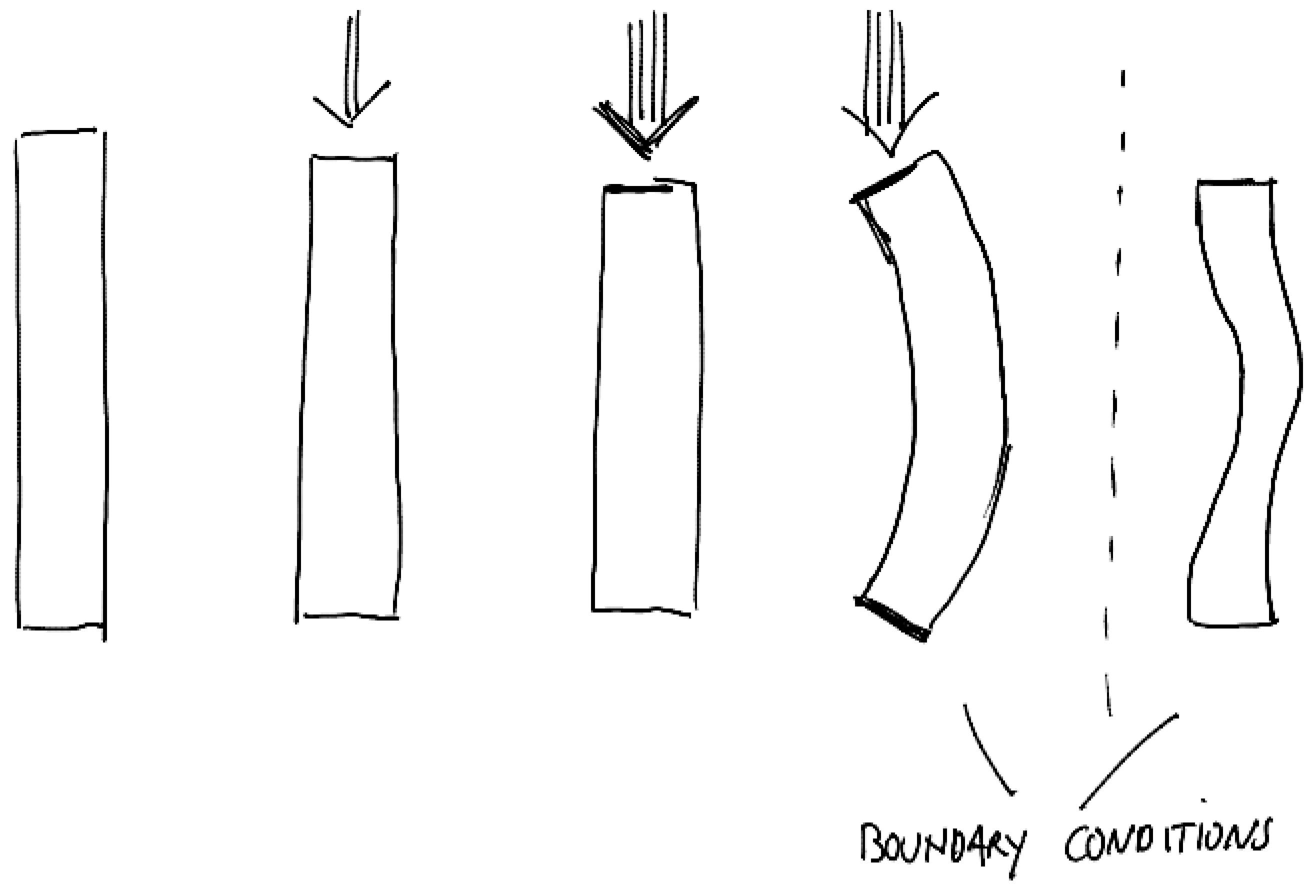
Kamrin, Priv Comm

Holey Sheet





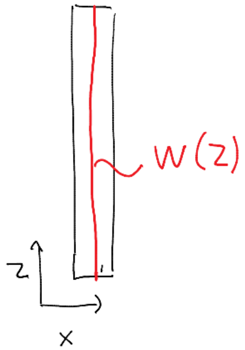
Buckling



COMPRESSIONAL LOADING VS BENDING

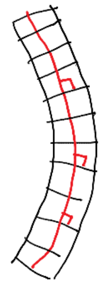
MULTIPLE EQUILIBRIA, INSTABILITIES, BIFURCATIONS

Buckling

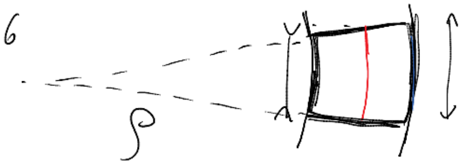


* small def.

* $\sigma_{zz} >> \sigma_{xx}$



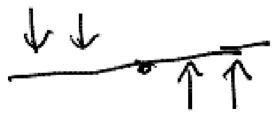
A) BENDING



$$\gamma_{zz} = -\frac{x}{\rho} \quad (1)$$

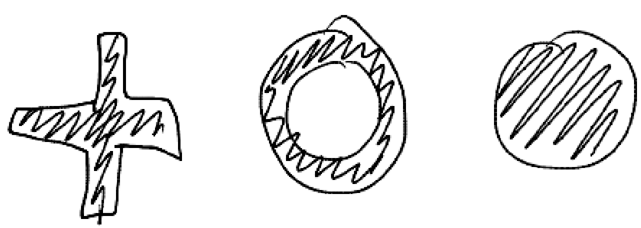
$$\sigma_{zz} = Y \gamma_{zz} \quad (2)$$

* BENDING MOMENT M

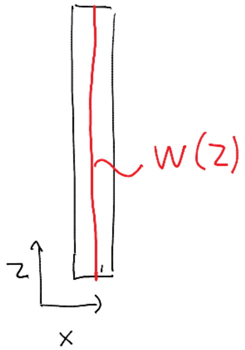


$$M = -\int dA \, x \, \sigma_{zz} \quad (3)$$

$$M = Y/\rho \underbrace{\int dA \, x^2}_{[m^4]\text{ : 2nd moment of Area}} := (Y/\rho) \underbrace{I}_{} \quad (4)$$

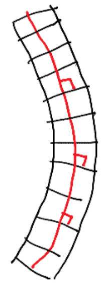


Buckling

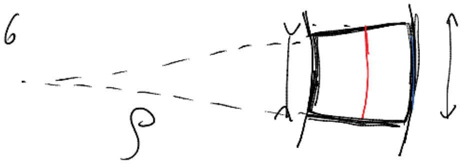


* small def.

* $\sigma_{zz} >> \sigma_{xx}$



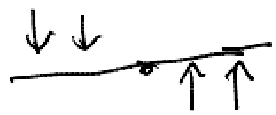
A) BENDING



$$\gamma_{zz} = -\frac{x}{\rho} \quad (1)$$

$$\sigma_{zz} = Y \gamma_{zz} \quad (2)$$

* BENDING MOMENT M



$$M = Y/\rho \int dA\, x^2 := (Y/\rho) J \quad (4)$$

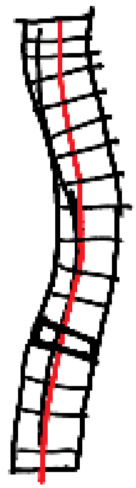
* $$\frac{1}{\rho} = \partial_z^2 w = w'' \quad (5)$$

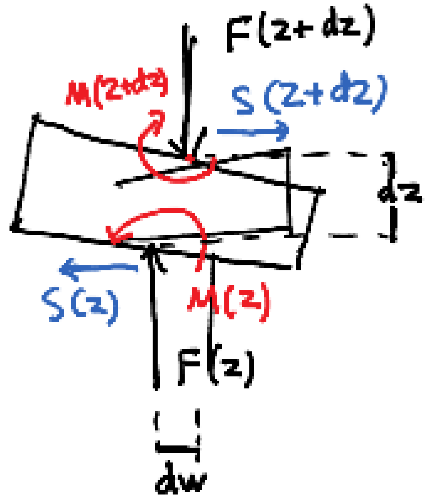
$$\boxed{M = Y J w'' \quad (6)}$$

Buckling

Sunday, February 24, 2013
10:06 AM

B





z: $F(z+dz) = F(z) \Rightarrow F' = 0$

x: $S(z+dz) = S(z) \Rightarrow S' = 0$ ①

Torque:

$M(z+dz) - M(z) = M'\,dz$

$S(z+dz)\cdot\frac{1}{2}dz + S(z)\cdot\frac{1}{2}dz \approx S(z)\,dz$

$F\,dw = F w'\,dz$

$\rightarrow M' + F w' + S = 0$ ②

① + ② : $M'' + F w'' = 0$

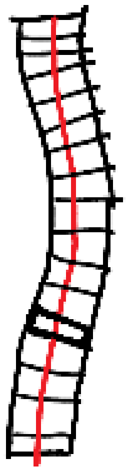
use $M = YJ\,w''$

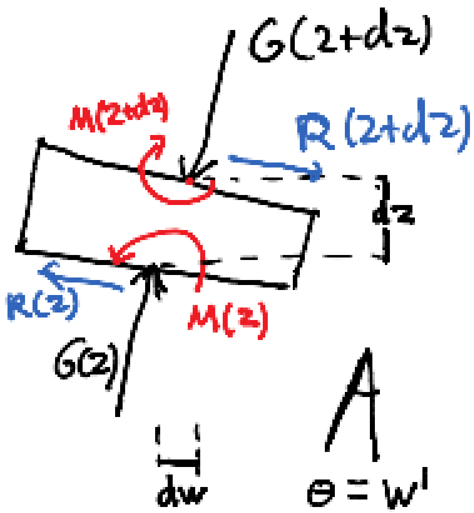
$$\Rightarrow \boxed{YJ\,w'''' + F w'' = 0} \quad ③$$

Buckling

Sunday, February 24, 2013
10:06 AM

B





$\theta = w'$

$\approx \sin\theta$

Z $\quad G(z+dz) = G(z) \Rightarrow G' = 0$

X $\quad -G(z+dz)\sin\theta(z+dz) + G(z)\sin\theta(z) + R(z+dz)\cos\theta(z+dz) - R(z)\cos(z)$

$\approx -G(z+dz)\,w'(z+dz) + G(z)\,w'(z) + R(z+dz) - R(z)$

$\Rightarrow -(Gw')' + R' = 0$

T $\quad R = -M'$

$\Big\} \quad Gw'' + M'' = 0$

$\uparrow$

$G \approx F \rightarrow$

$Fw'' + M'' = 0$

① + ② : $\quad M'' + Fw'' = 0$

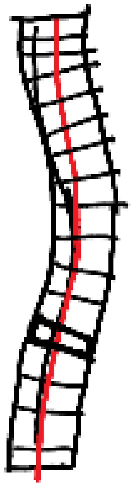
use $\quad M = YJ\,w''$

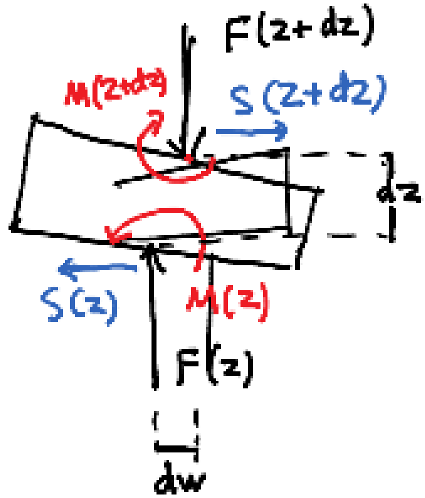
$\Rightarrow \boxed{YJ\,w'''' + F\,w'' = 0} \quad ③$

Buckling

Sunday, February 24, 2013
10:06 AM

B





z $\quad F(z+dz) = F(z) \Rightarrow F' = 0$

x $\quad S(z+dz) = S(z) \Rightarrow S' = 0 \qquad (1)$

Torque $\quad M(z+dz) - M(z) = M'\,dz$

$S(z+dz)\cdot\frac{1}{2}dz + S(z)\cdot\frac{1}{2}dz \approx S(z)dz$

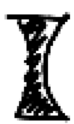
$F\,dw = F w'\,dz$

$\rightarrow \quad M' + F w' + S = 0 \qquad (2)$

(1)+(2): $M'' + F w'' = 0$

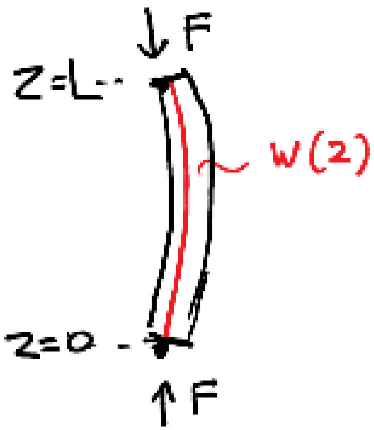
use $M = YJ w''$

$$\Rightarrow \boxed{YJ w'''' + F w'' = 0 \qquad (3)}$$

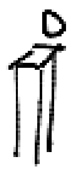
extensions:
- F not constant (gravity)
- Y or J not constant 
- precurved beam 

Buckling

ⓒ STABILITY

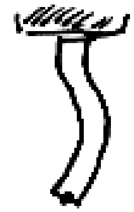


$$\left.\begin{aligned} YJ\, w'''' + F w'' &= 0 \\ W(z) &= \sin \tfrac{\pi}{L} z \\ w'''' &= \left(\tfrac{\pi}{L}\right)^4 \sin \tfrac{\pi}{L} z \\ w'' &= -\left(\tfrac{\pi}{L}\right)^2 \sin \tfrac{\pi}{L} z \end{aligned}\right\} \quad F_{cr} = YJ\left(\frac{\pi}{L}\right)^2$$

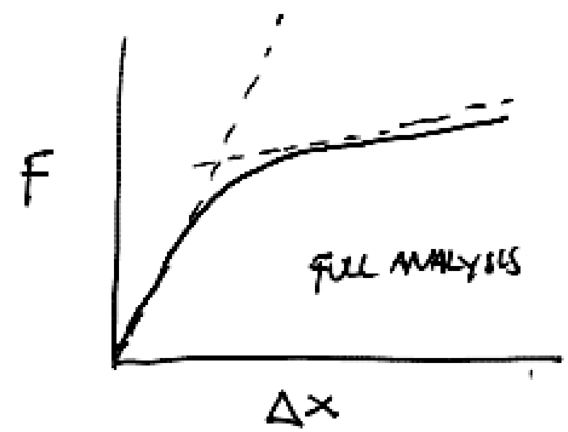
* square beam 

: $J = \frac{D^4}{12}$, $F_{cr} = \frac{\pi^2}{12} Y \left(\frac{D^2}{L}\right)^2$, $\sigma_{CR} = \frac{\pi^2}{12} Y \left(\frac{D}{L}\right)^2$

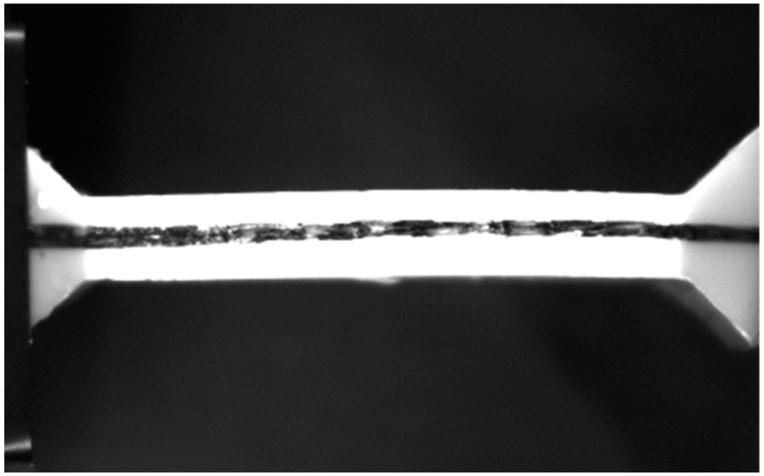
* STIFFER BOUNDARY : higher F_{cr}

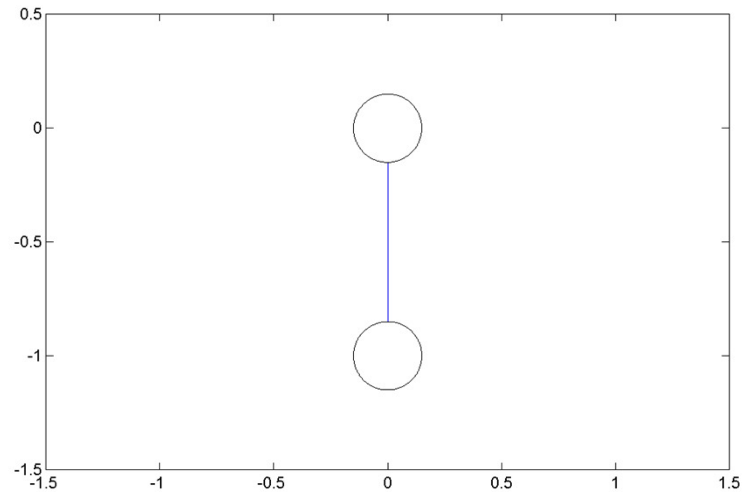


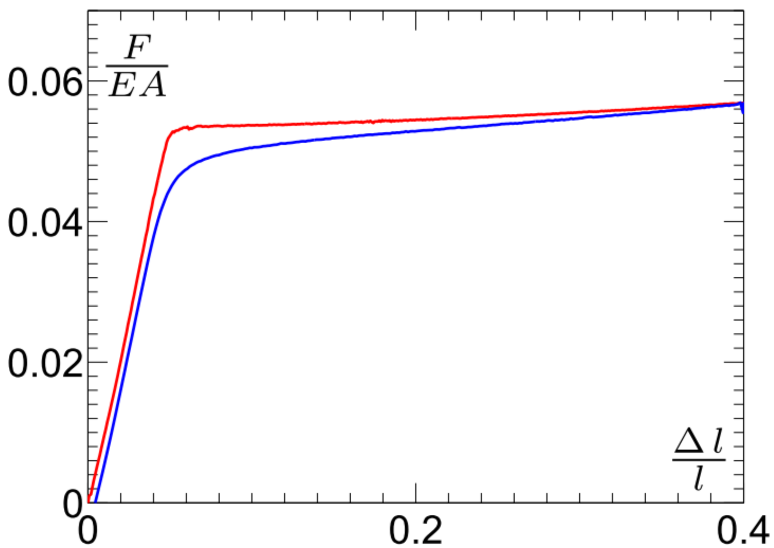
* IF P given, single beam: linear equation
* IN STRUCTURE: non lin eq (through bc)

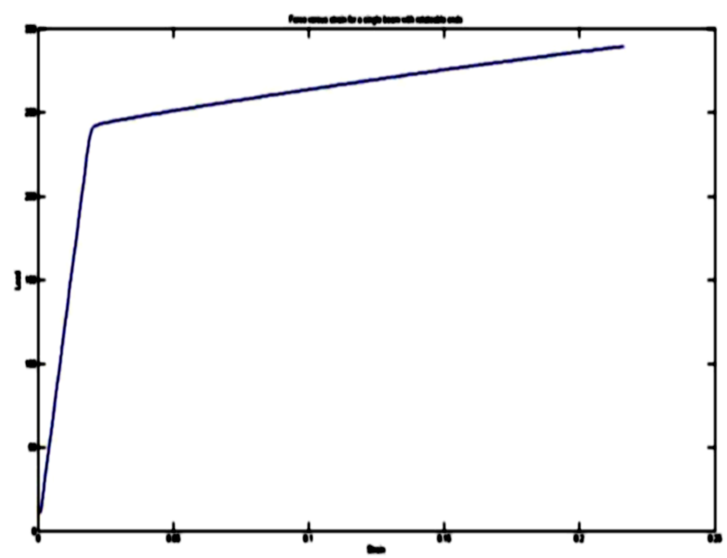


Buckling

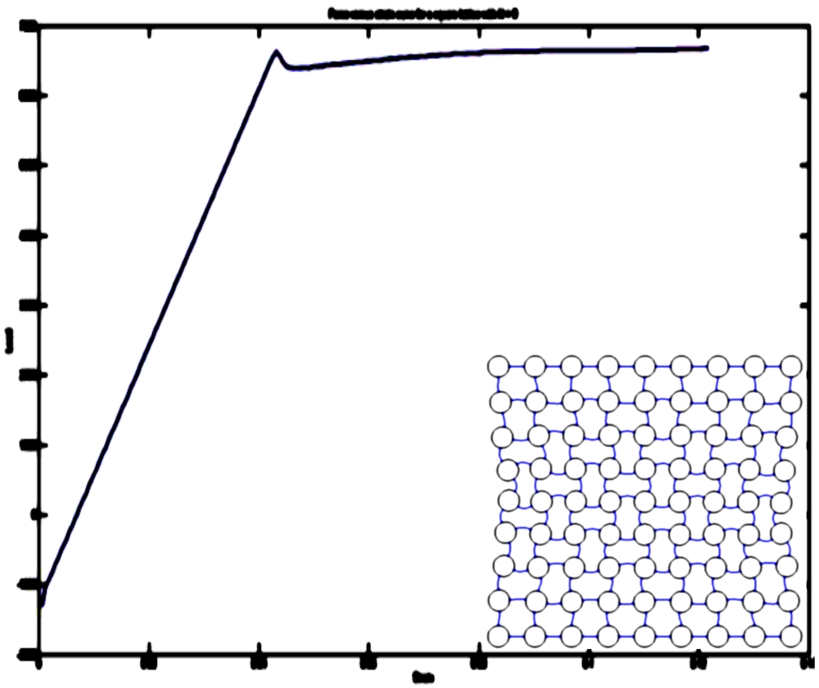


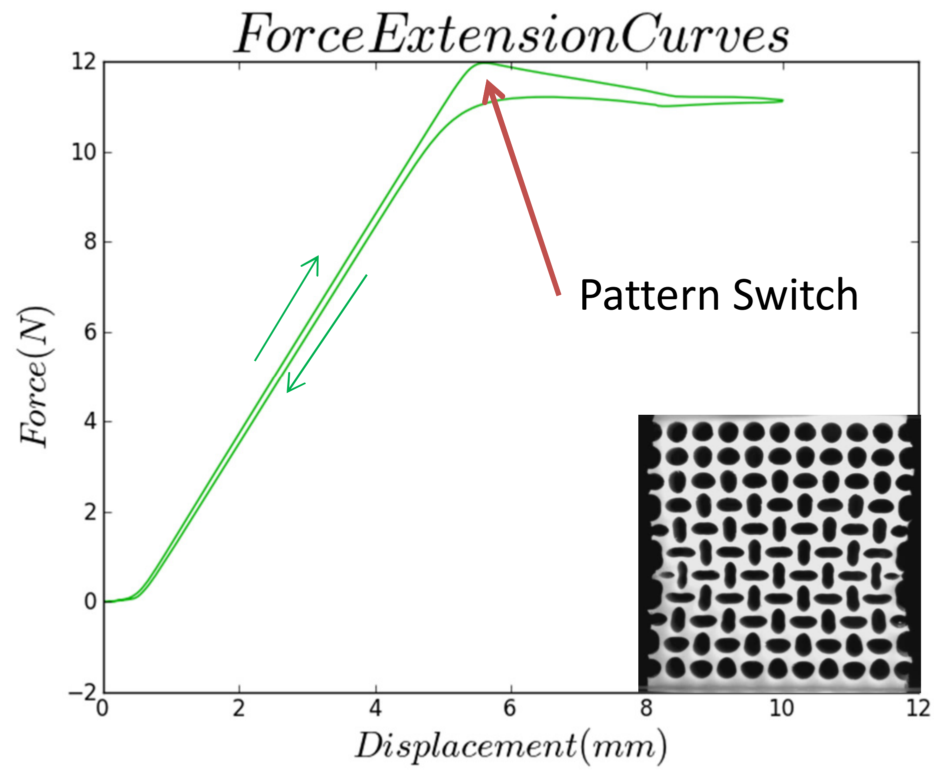

0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5


$\frac{F}{EA}$
0.06
0.04
0.02
0
0
0.2
0.4
$\frac{\Delta l}{l}$

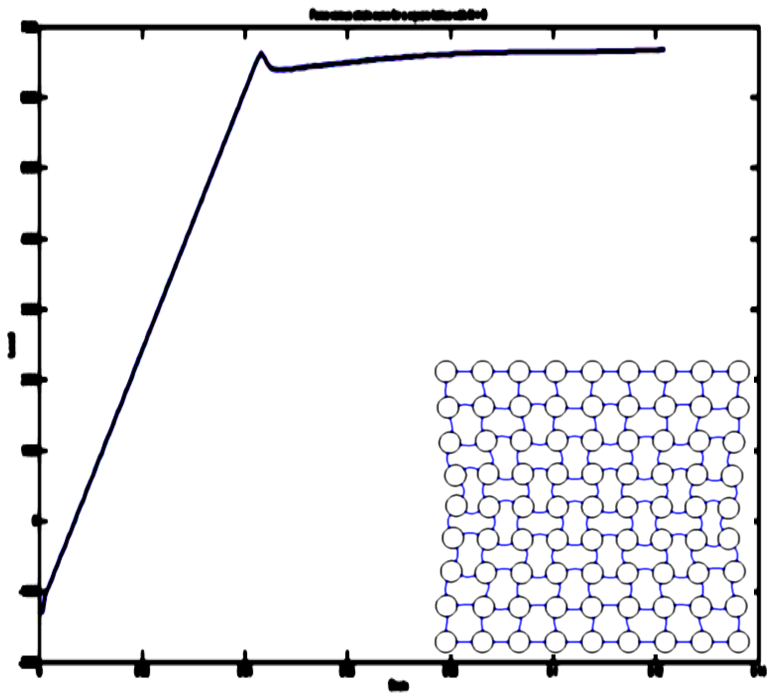


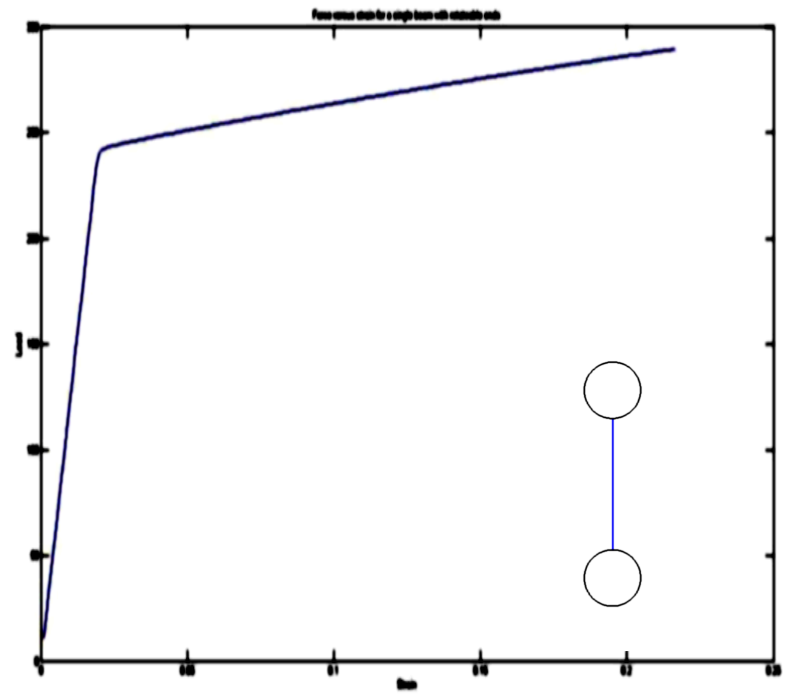
Buckling: Beammodel vs Holey Sheet



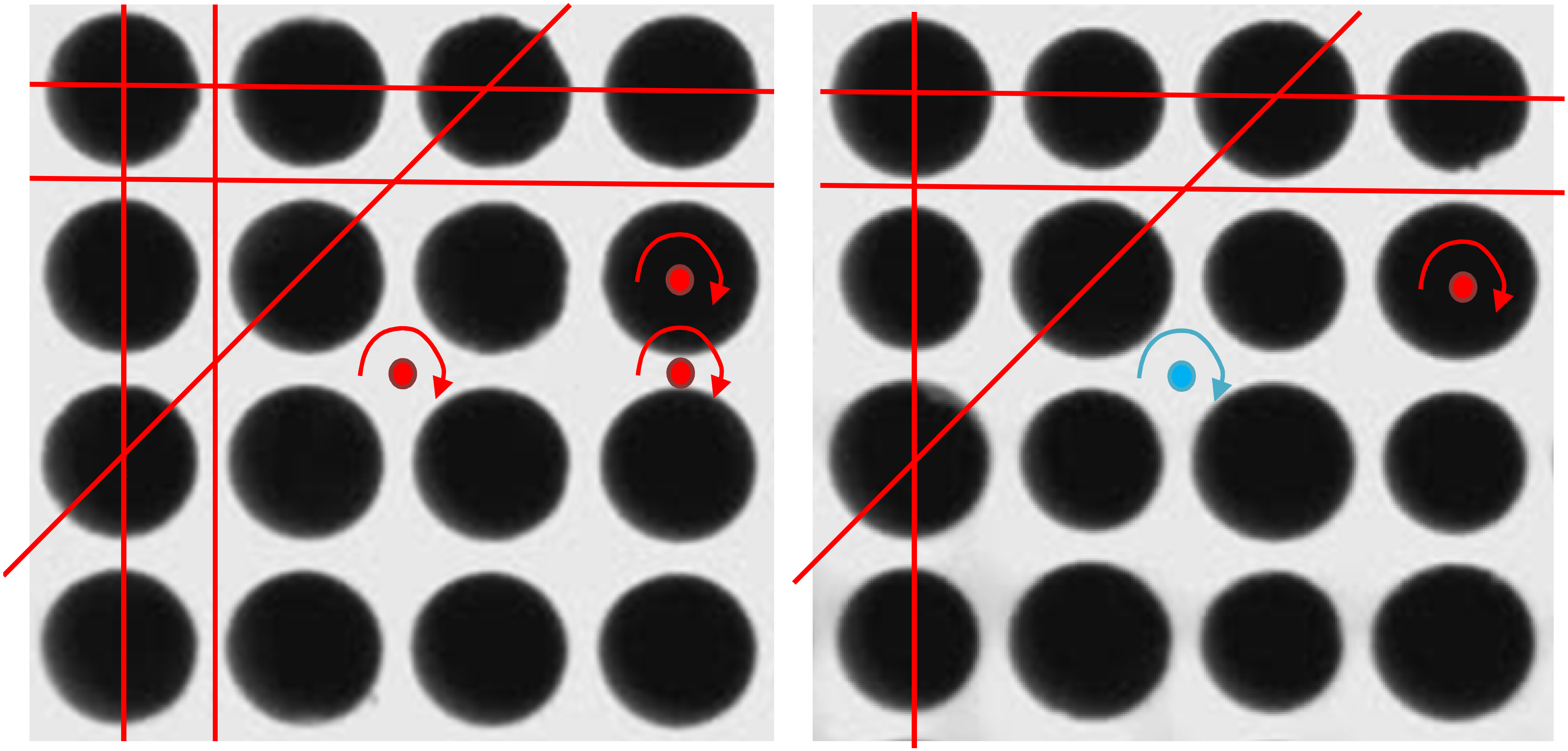


Buckling: Beammodel vs Beam

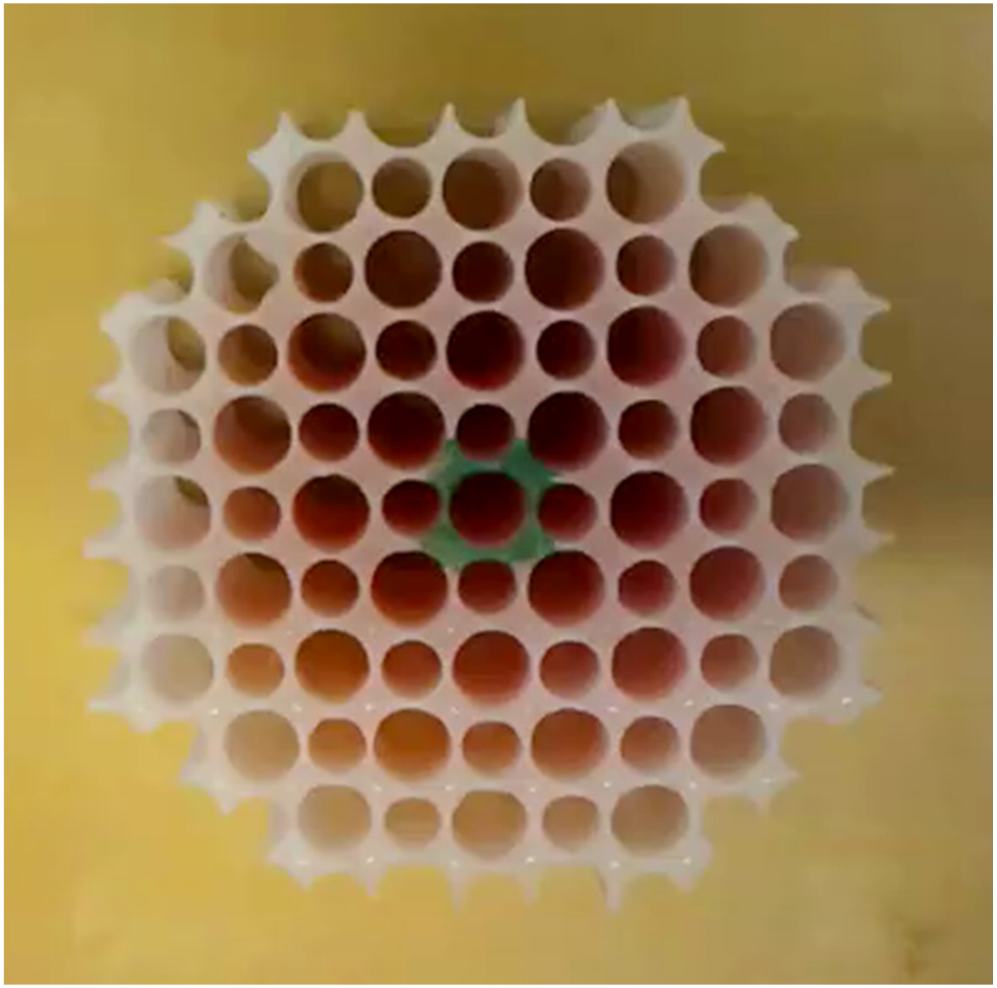




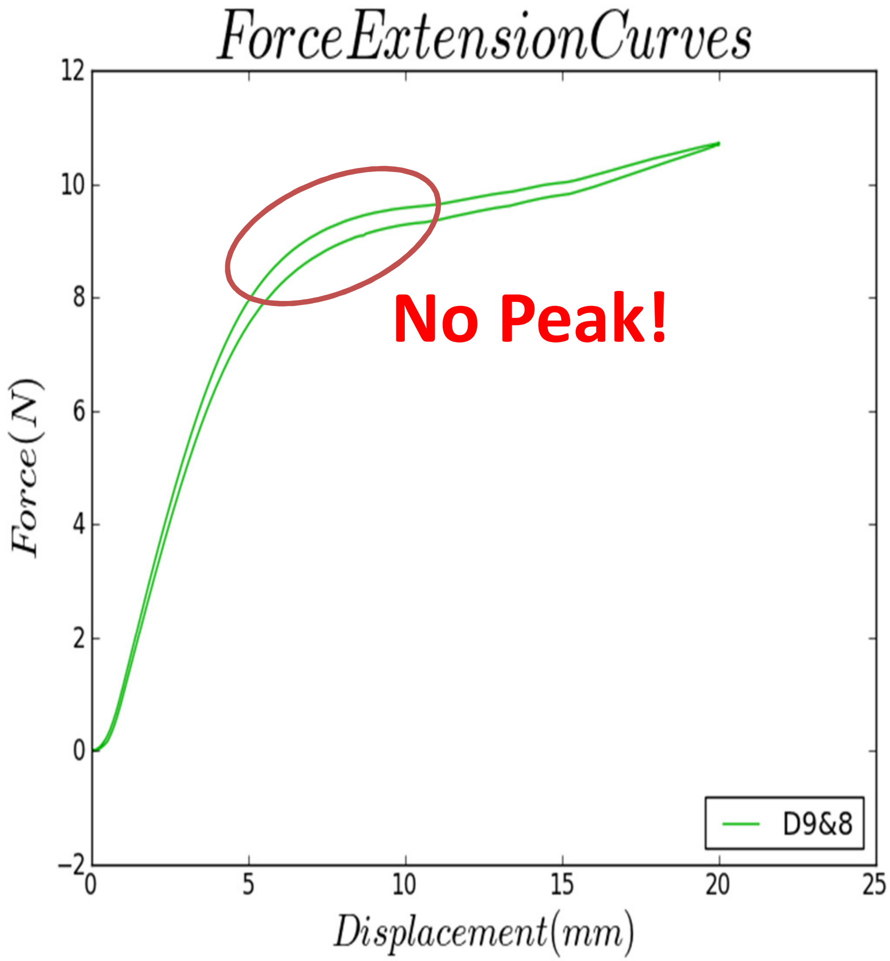
Biholar Sheets: Breaking Symmetry



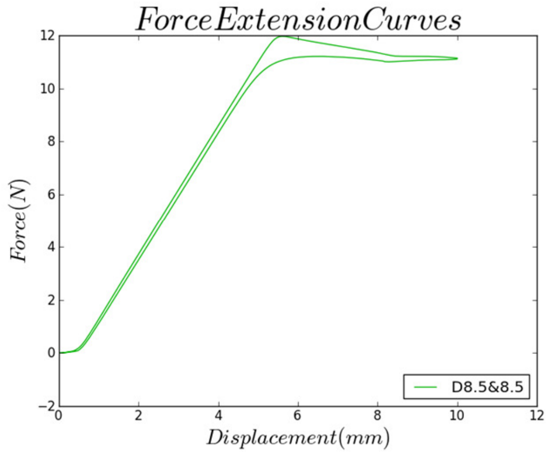
Biholar Sheets

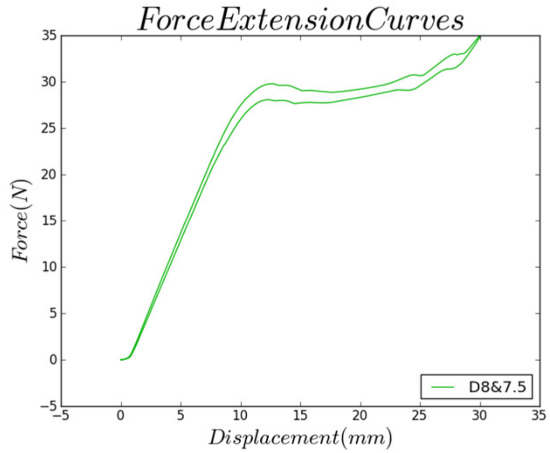


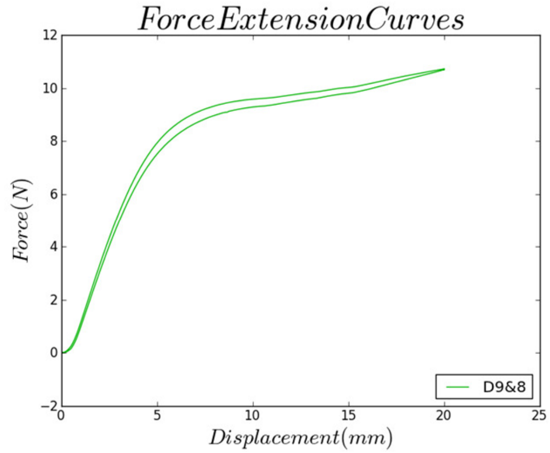
Biholar Sheets

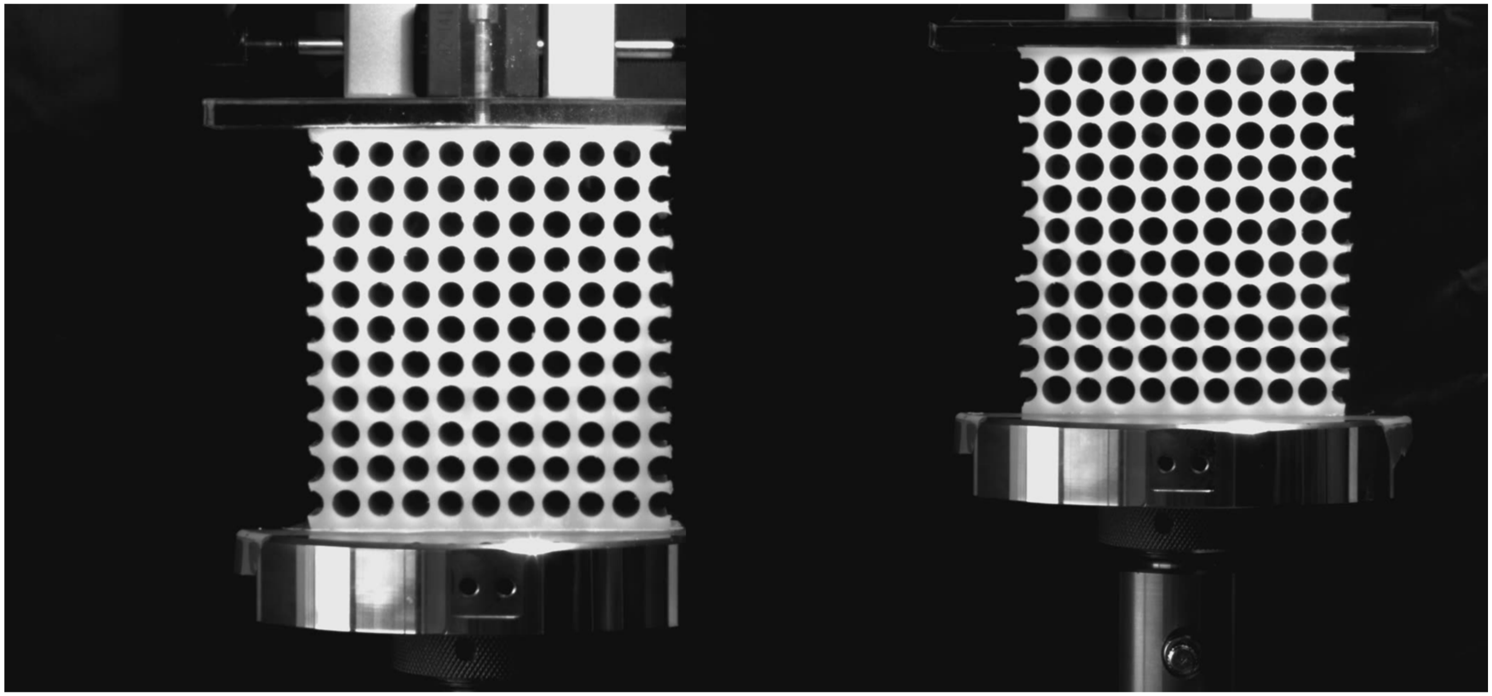


Weak Symmetry Breaking

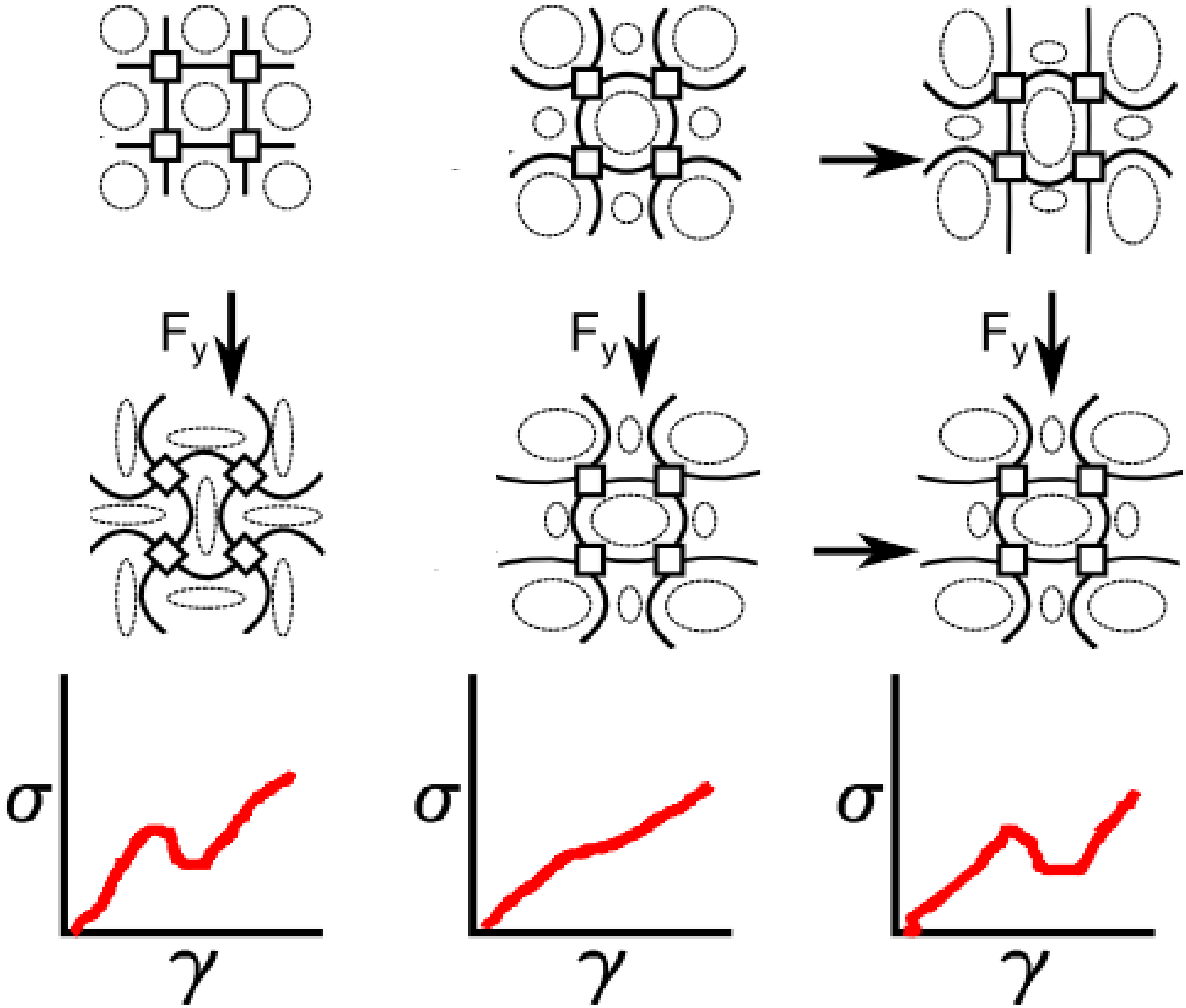




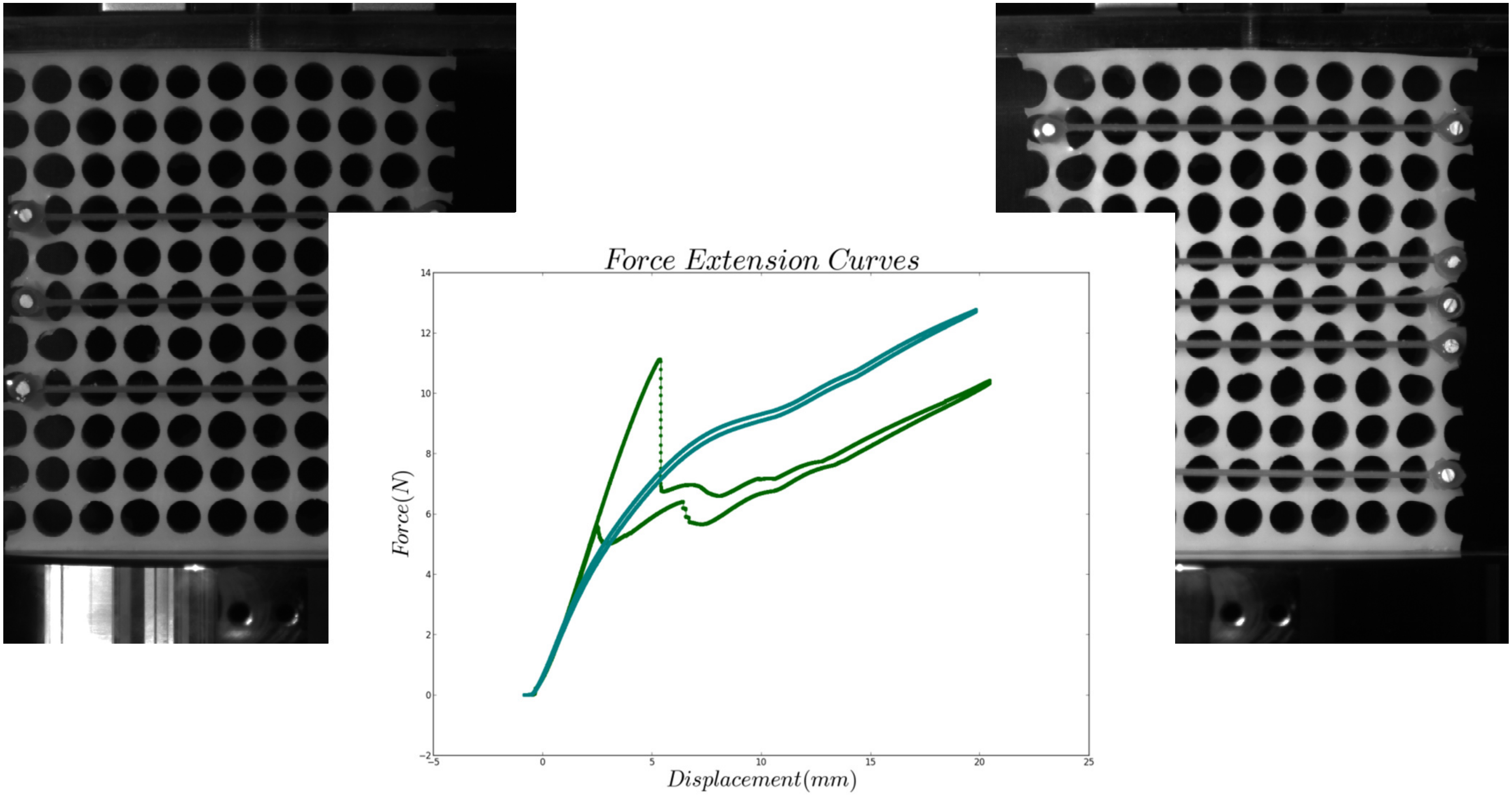




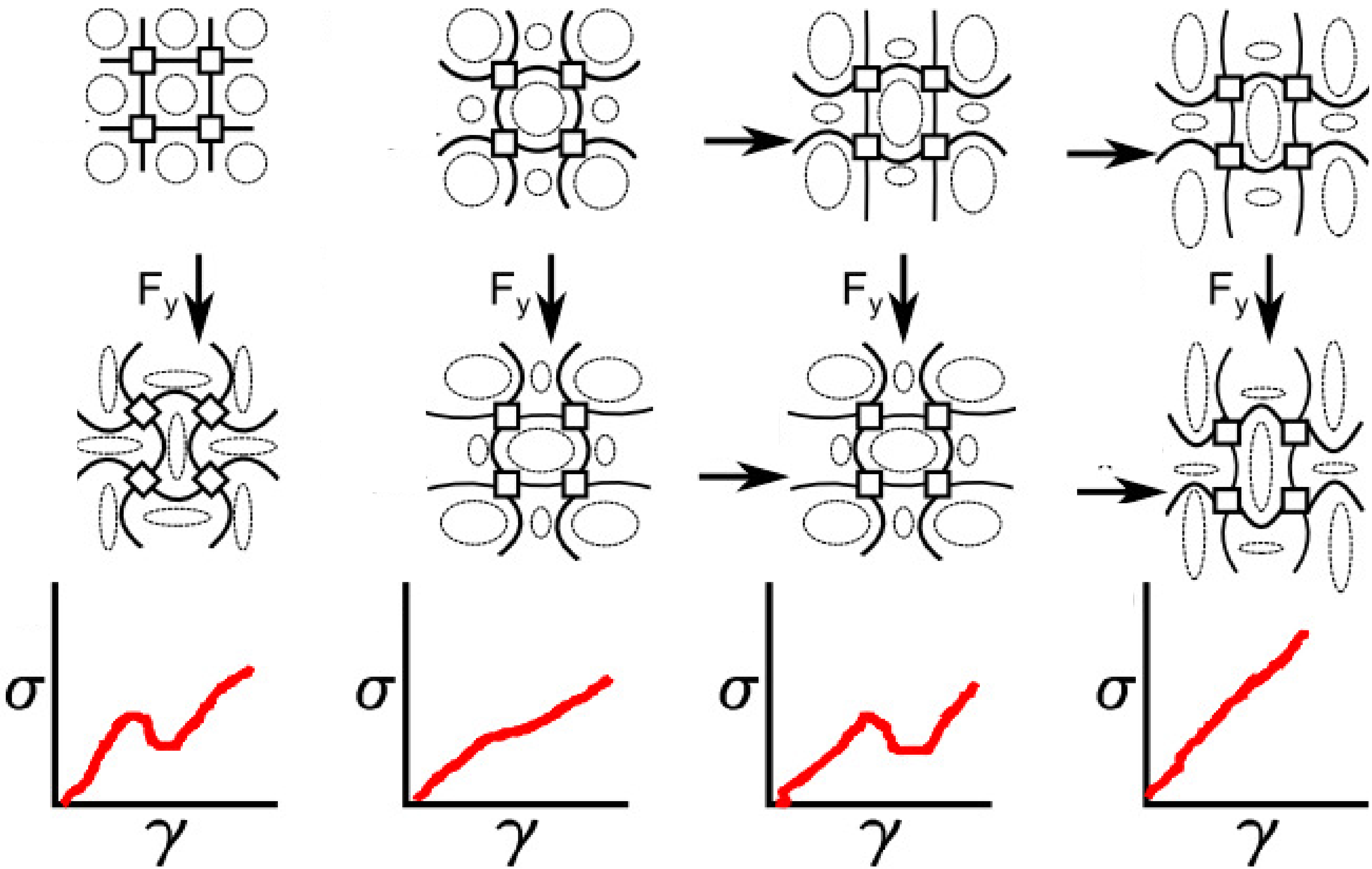
Weak Symmetry Breaking



Lateral Forcing



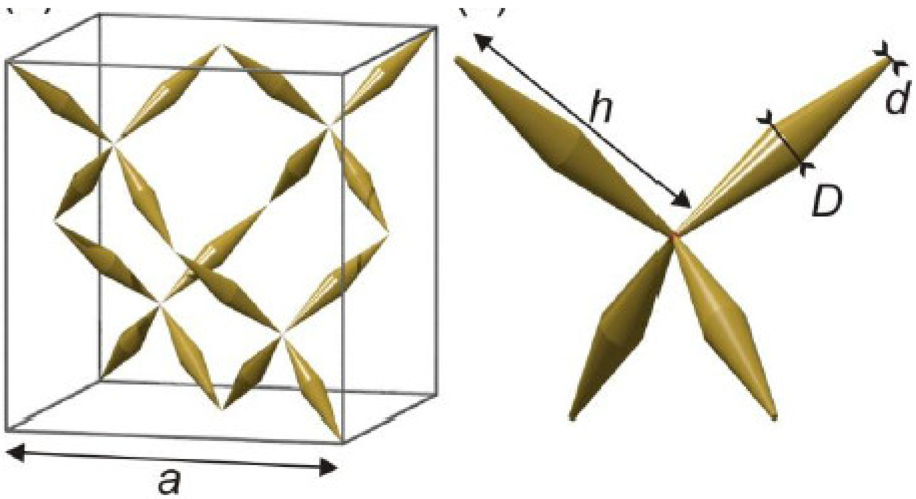
Weak Symmetry Breaking

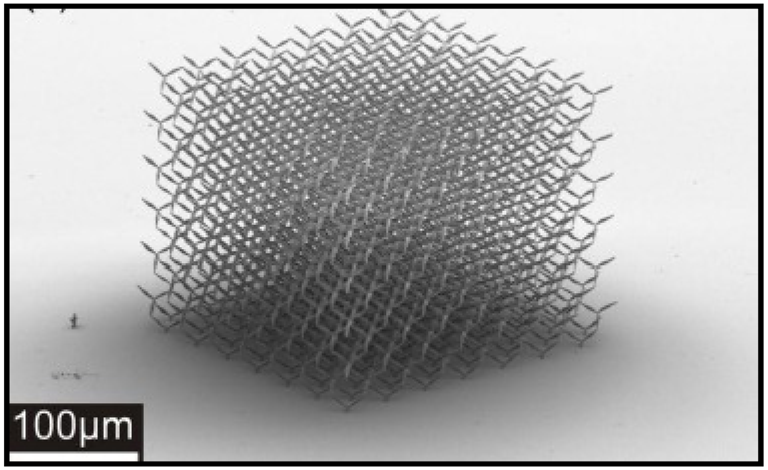


Metamaterials: Buckling

Buckling in Networks
Tunable Mechanical Response
Other Hole Patterns?

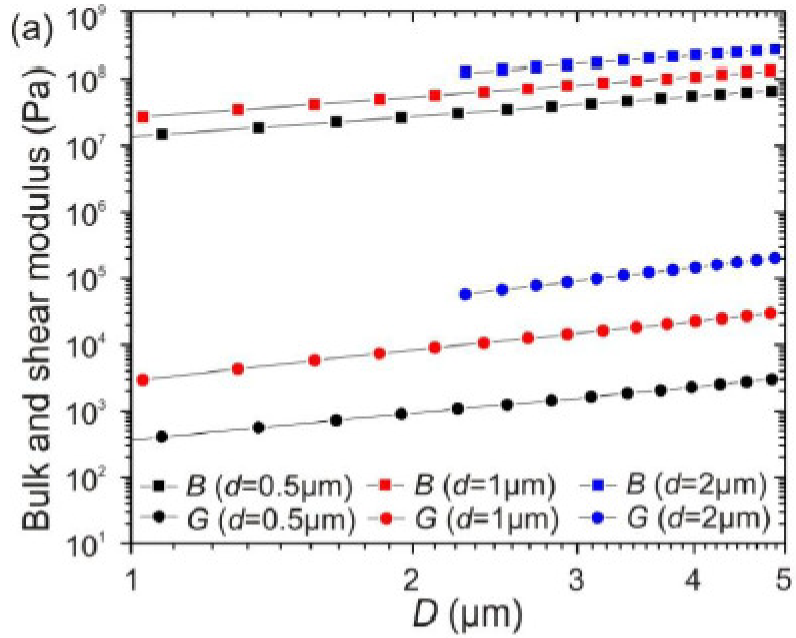
Outlook: G << K: Pentamode Materials



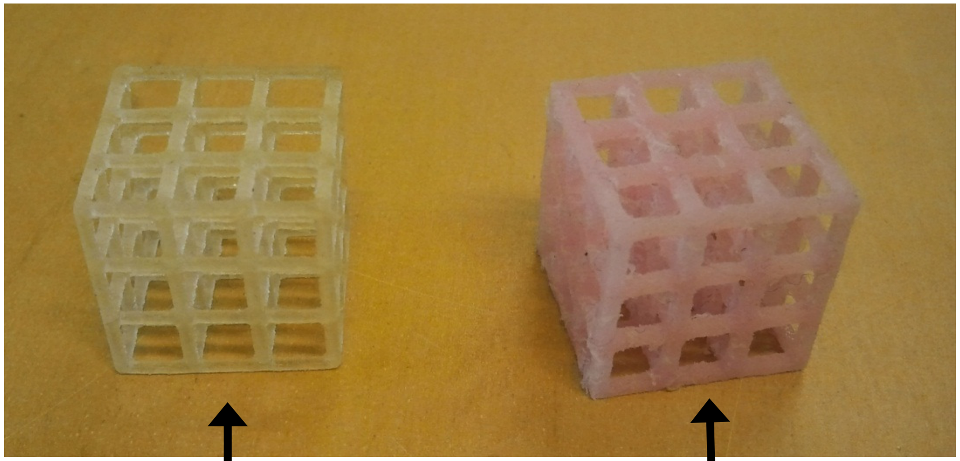


Acoustic Cloaking

Jammed Better?



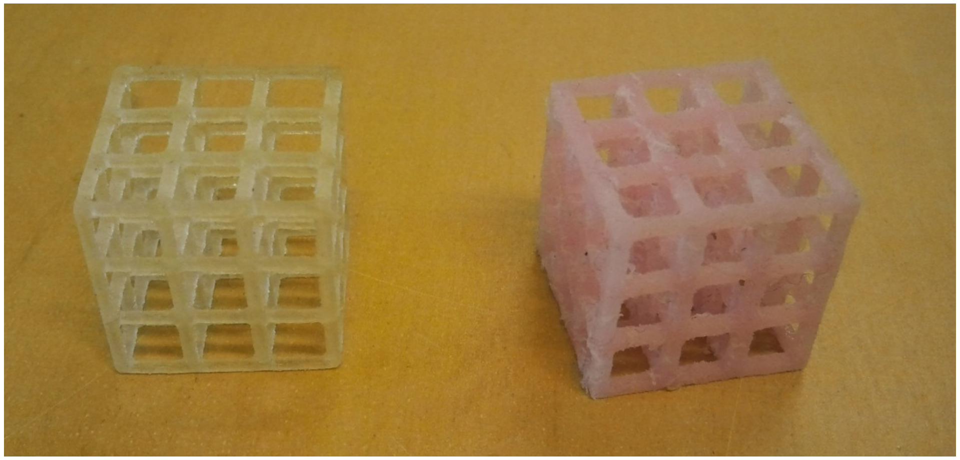
Outlook: 3D Elastica

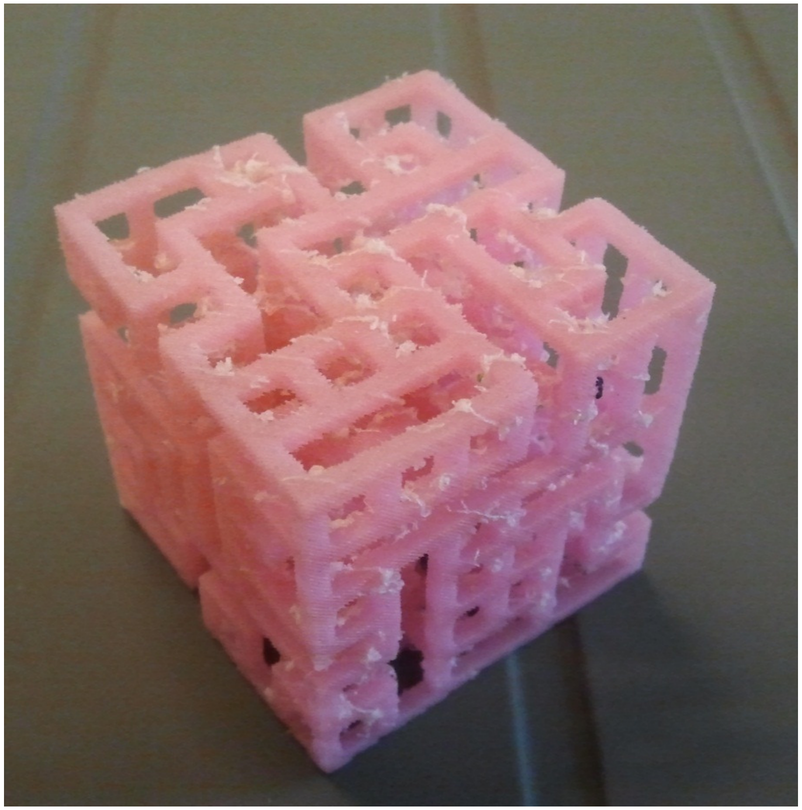


2k€ 3D Printing
(Soluble Mould)

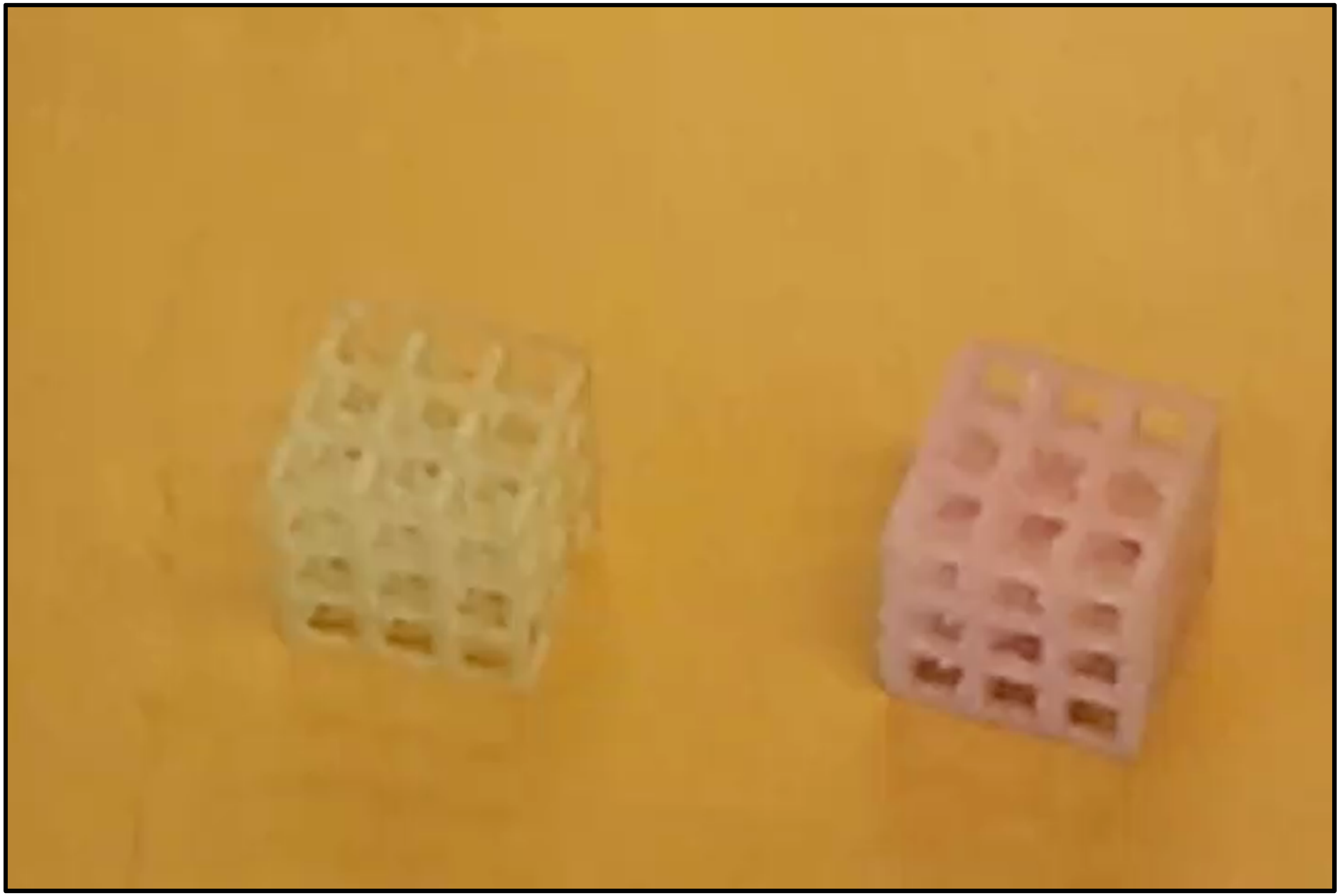
200k$ 3D Printing

Outlook: 3D Elastica

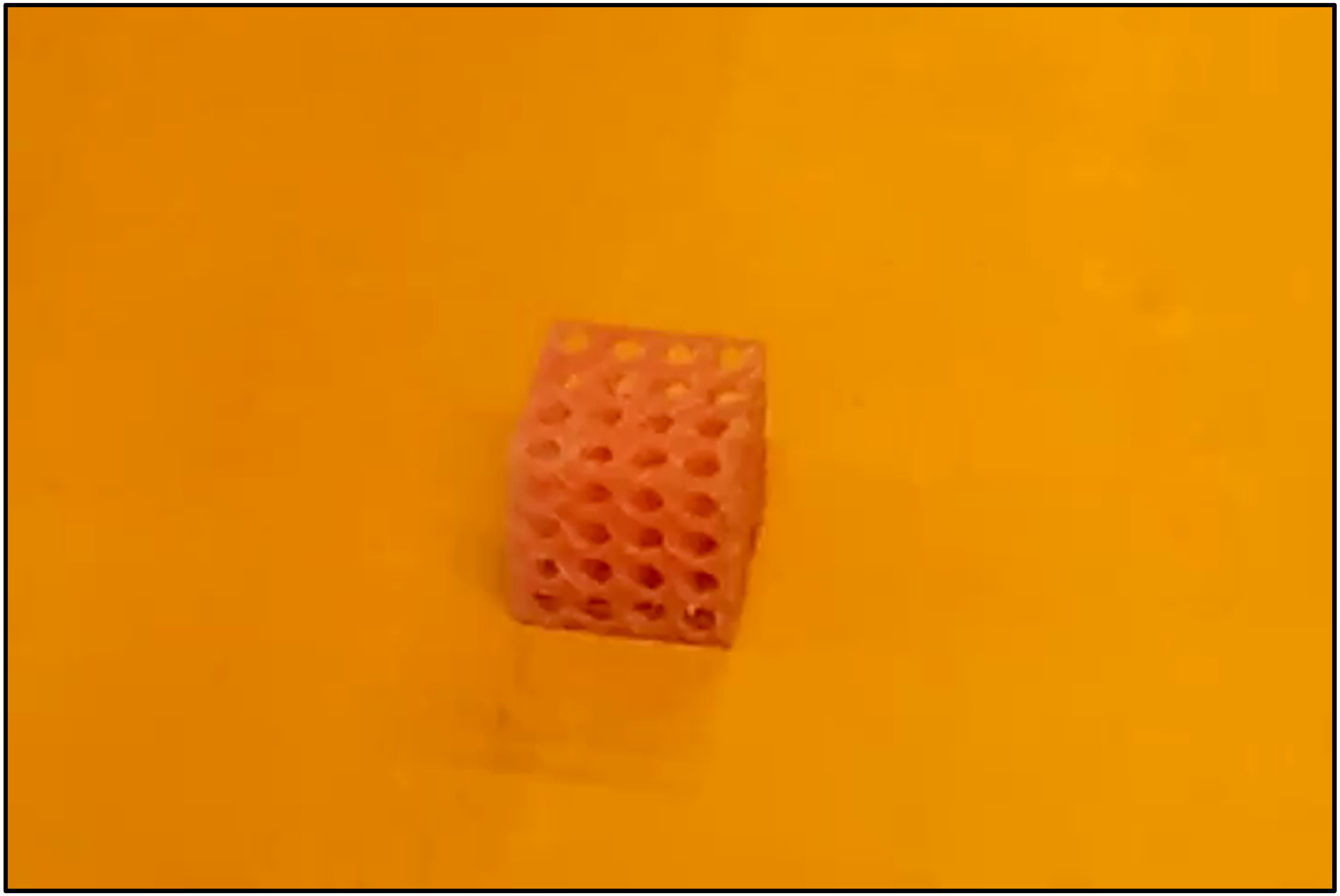




Outlook: 3D Elastica



Outlook: 3D Elastica

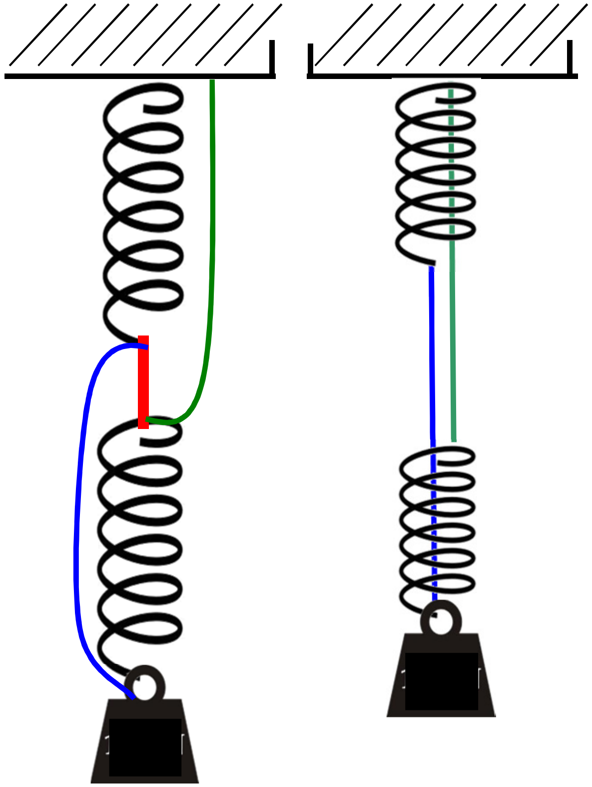


Outlook: Negative Compressibility?

ARTICLES
PUBLISHED ONLINE: 20 MAY 2012 | DOI: 10.1038/NMAT3331
nature materials

Mechanical metamaterials with negative compressibility transitions

Zachary G. Nicolaou[1] and Adilson E. Motter[1,2]*



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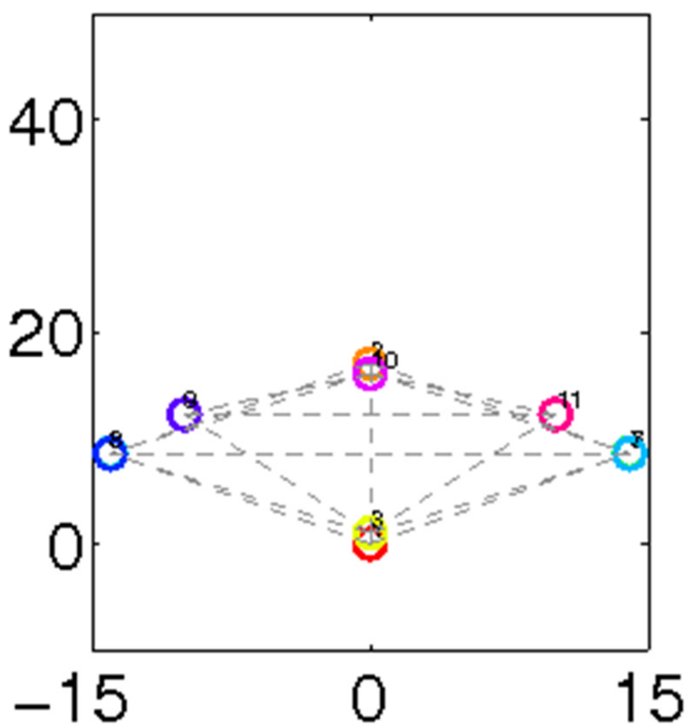
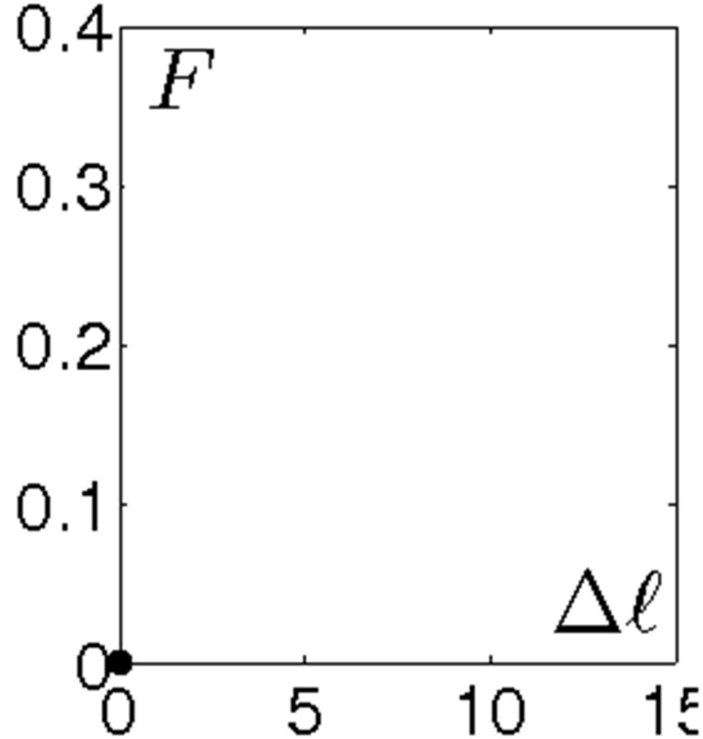
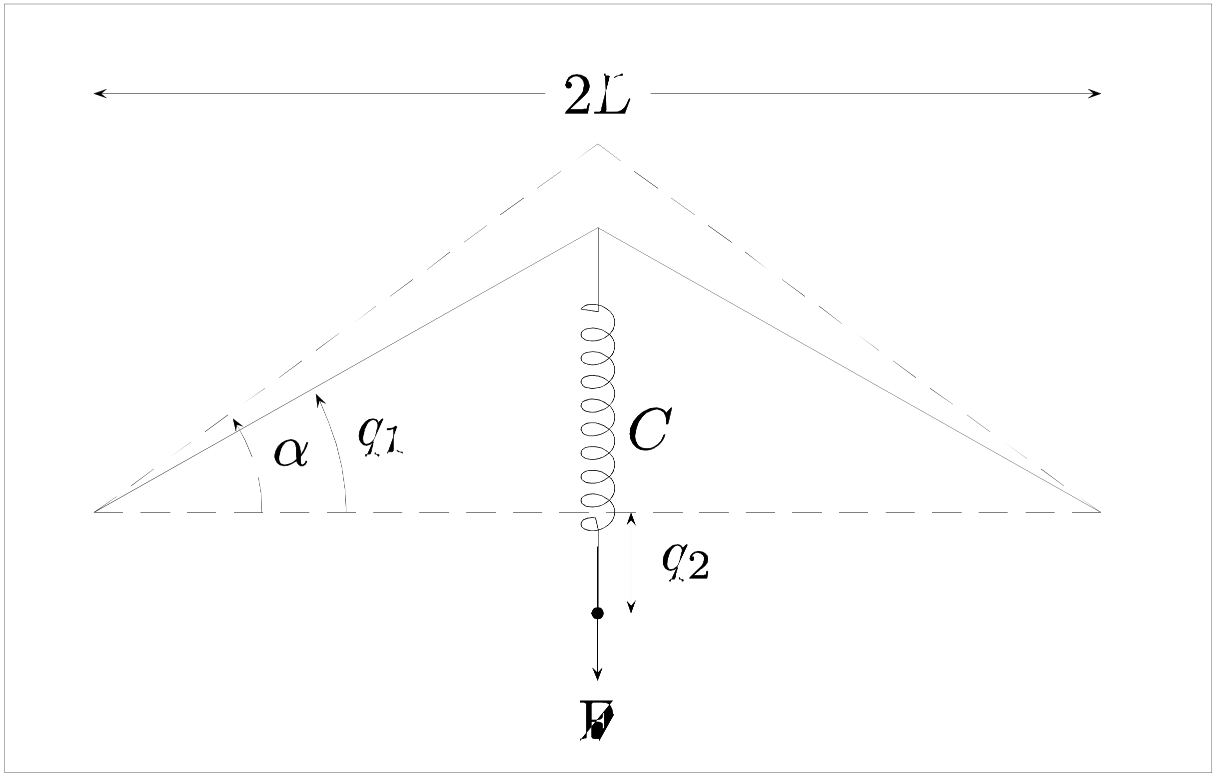
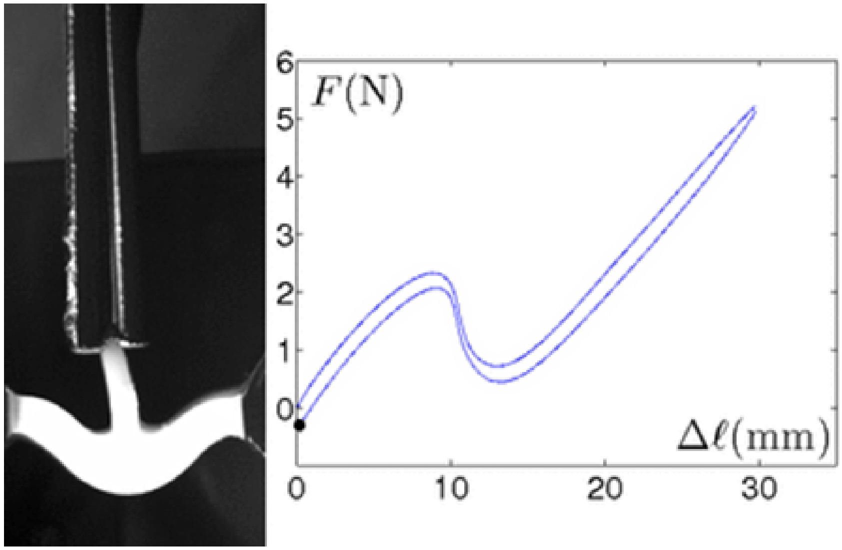


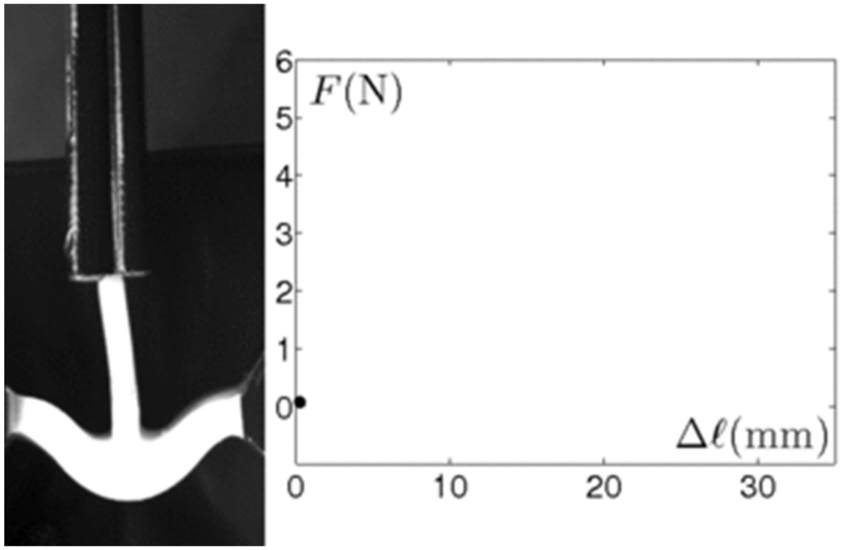


Outlook: Snapback and Snapthrough

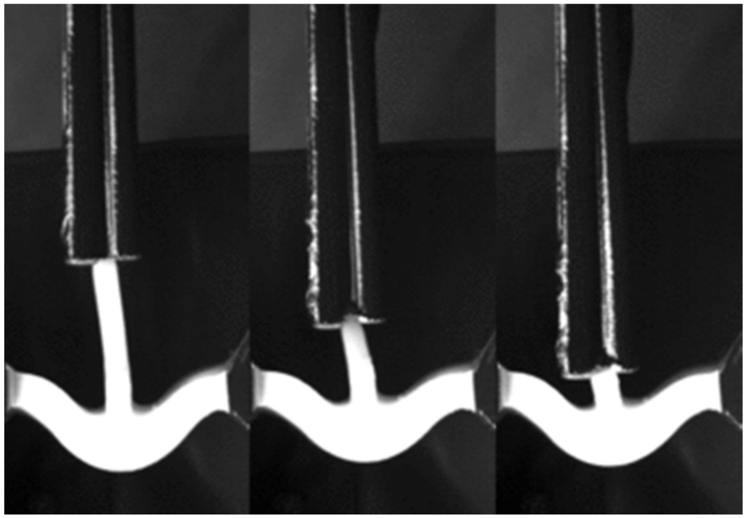


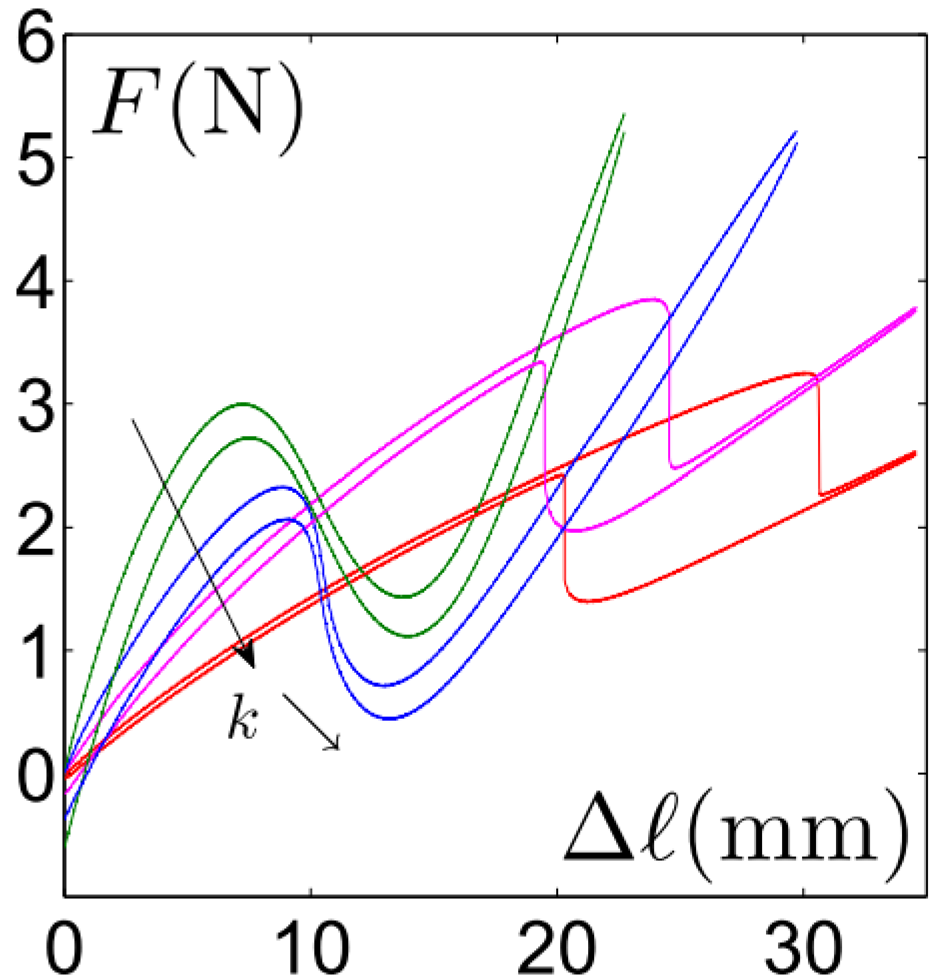
Outlook: Snapback and Snapthrough





Outlook: Snapback and Snapthrough





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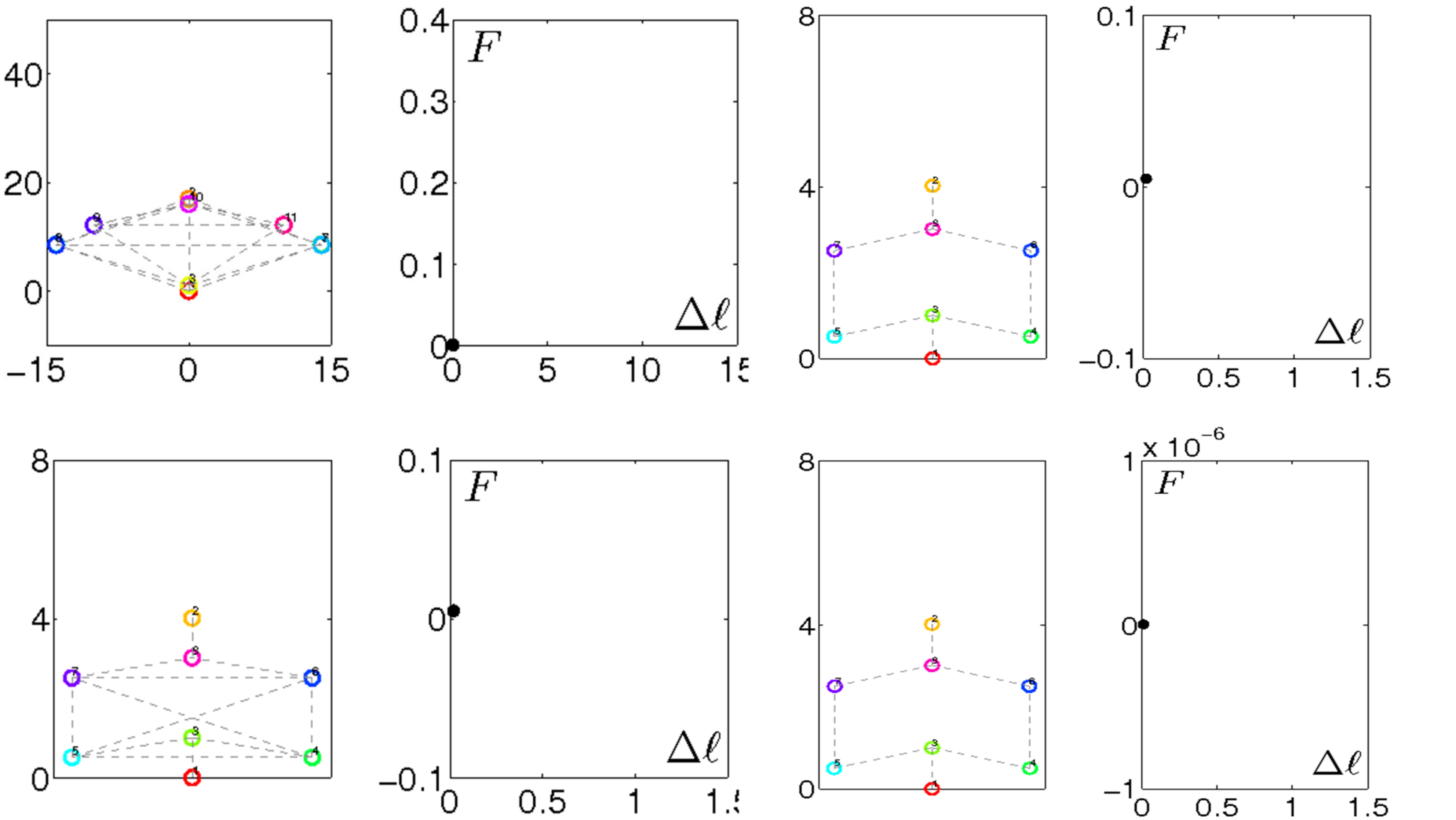

Metamaterials

Geometry
Marginal Points
Instabilities

Lots of open questions!