Shocks in fragile matter

Vincenzo Vitelli

Instituut-Lorentz for Theoretical Physics (Leiden)

Collaborators: L. Gomez, S. Ulrich, B. van Opheusden, A. Turner, N. Upadhyaya, M. van Hecke, R. van Loo, S.van den Wildenberg

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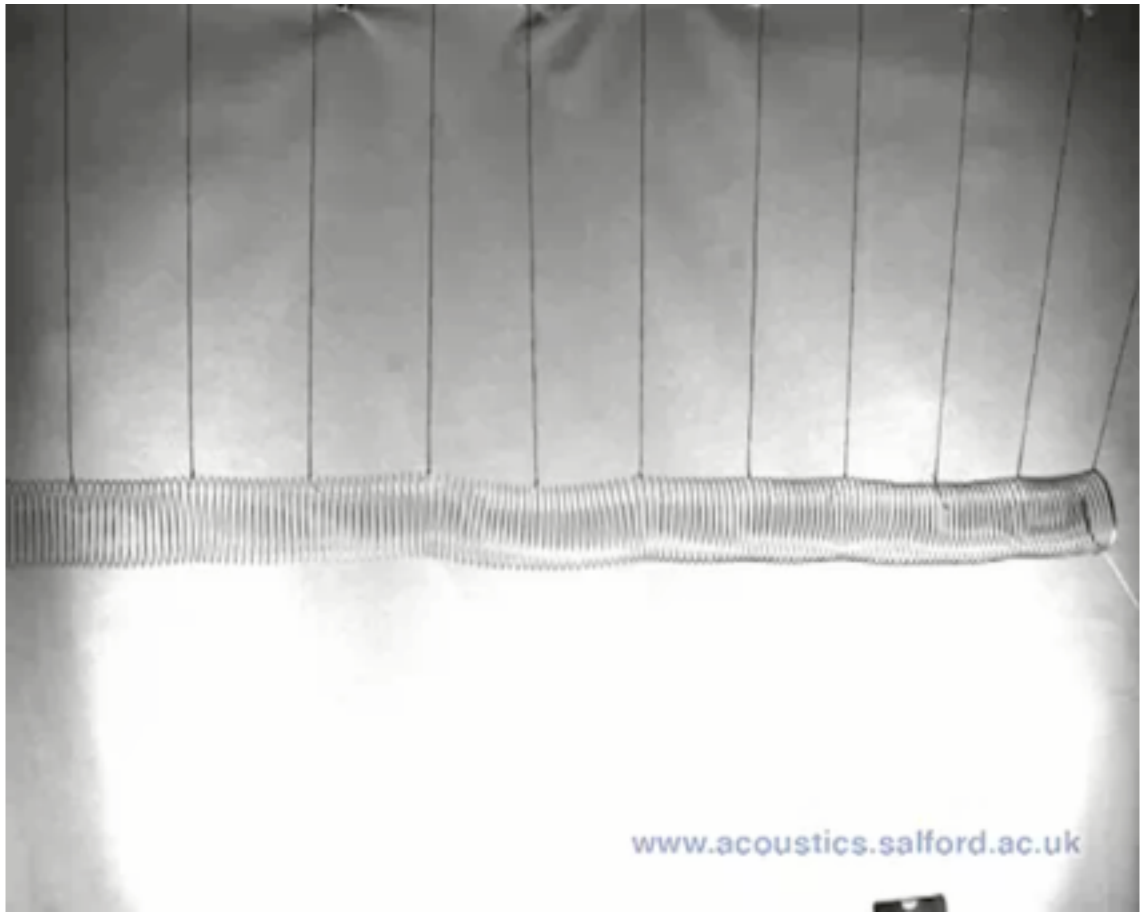

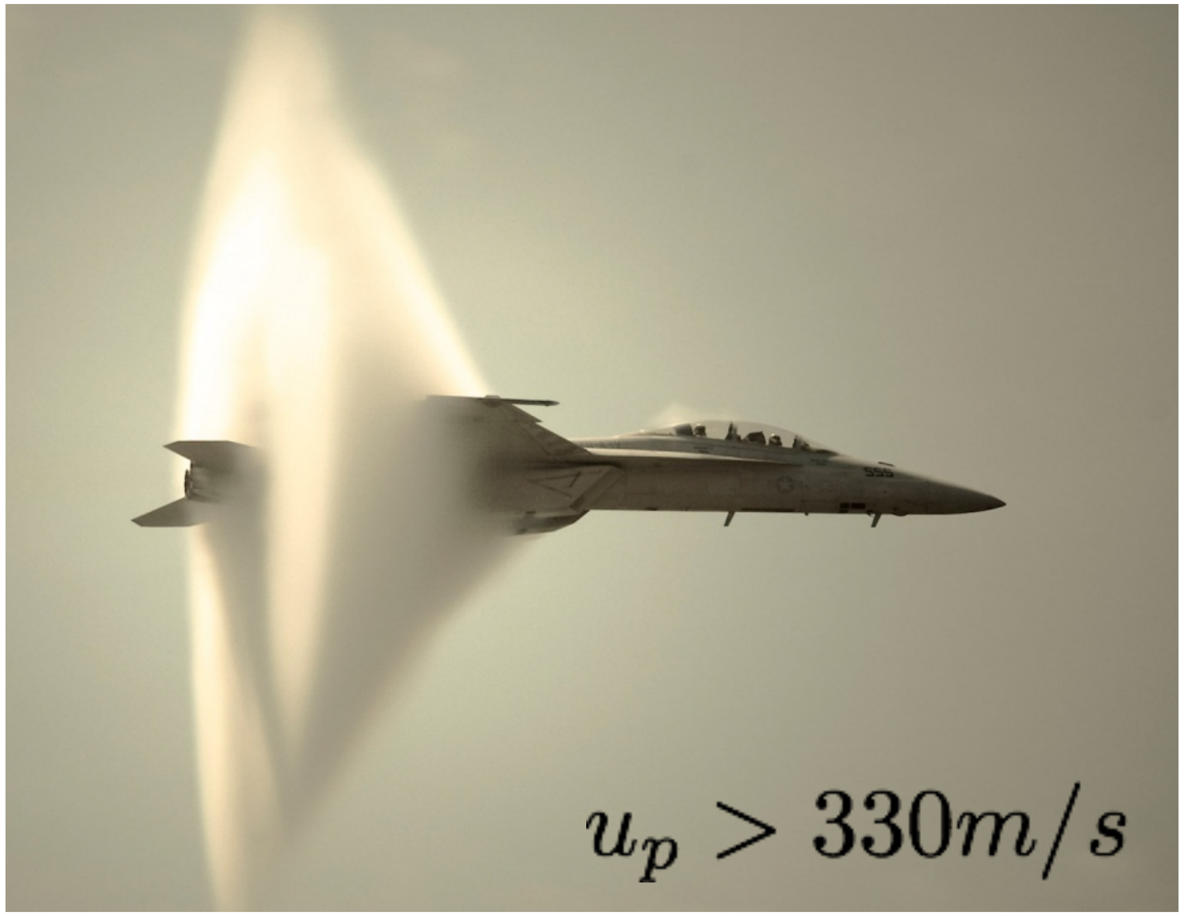

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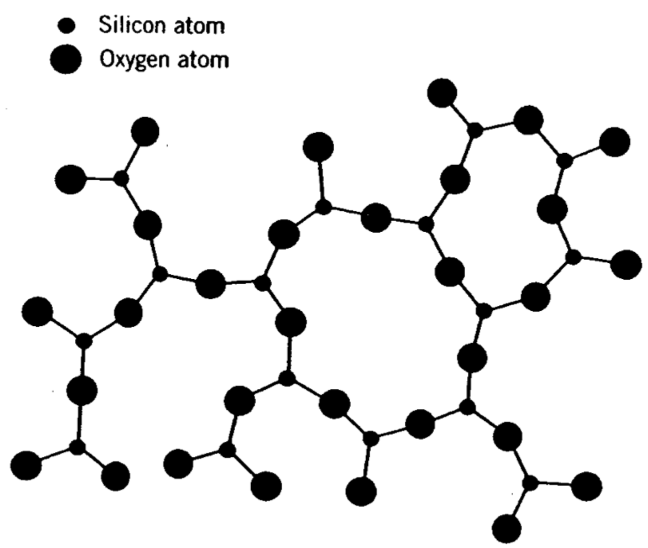
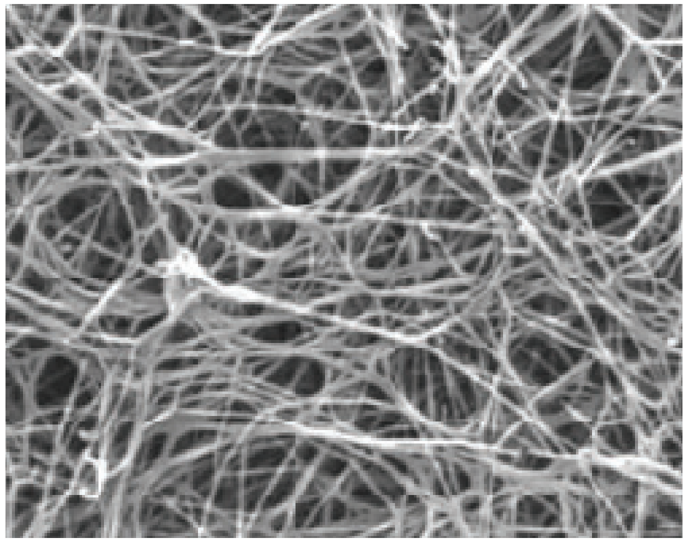
Fragile Matter

Glasses

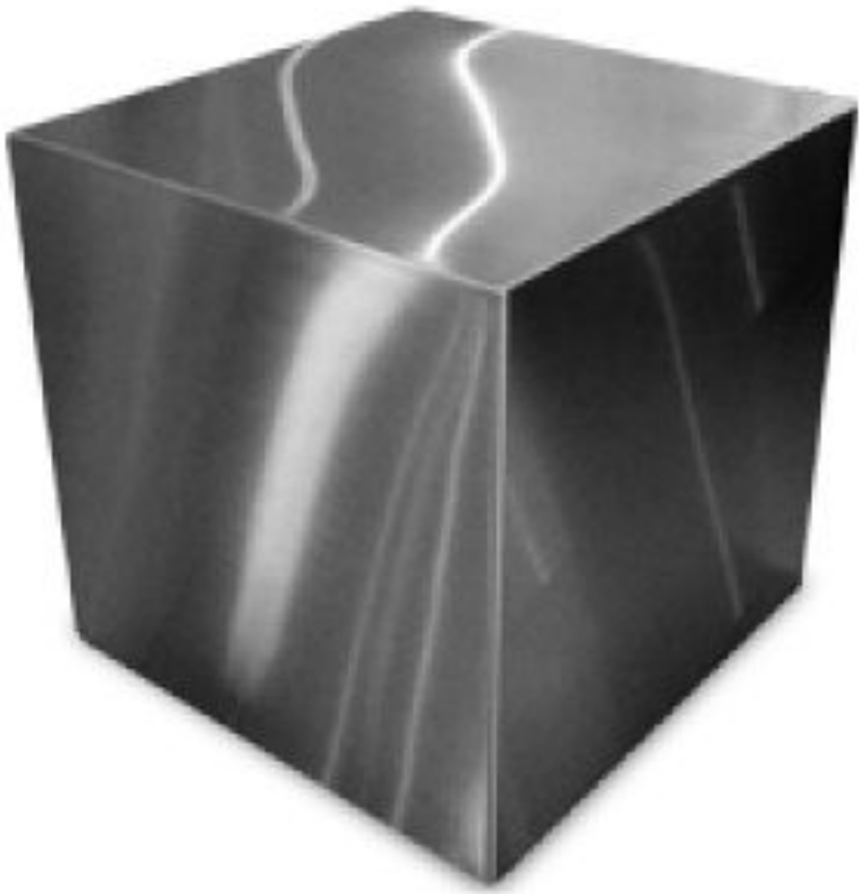


Polymer



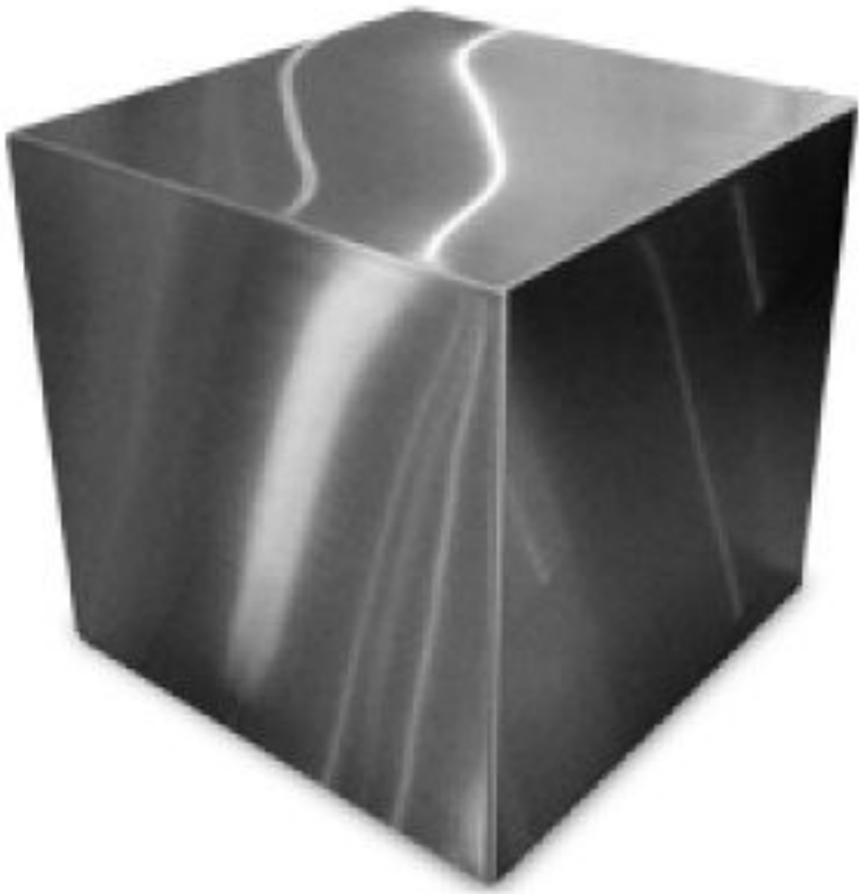
Granular media

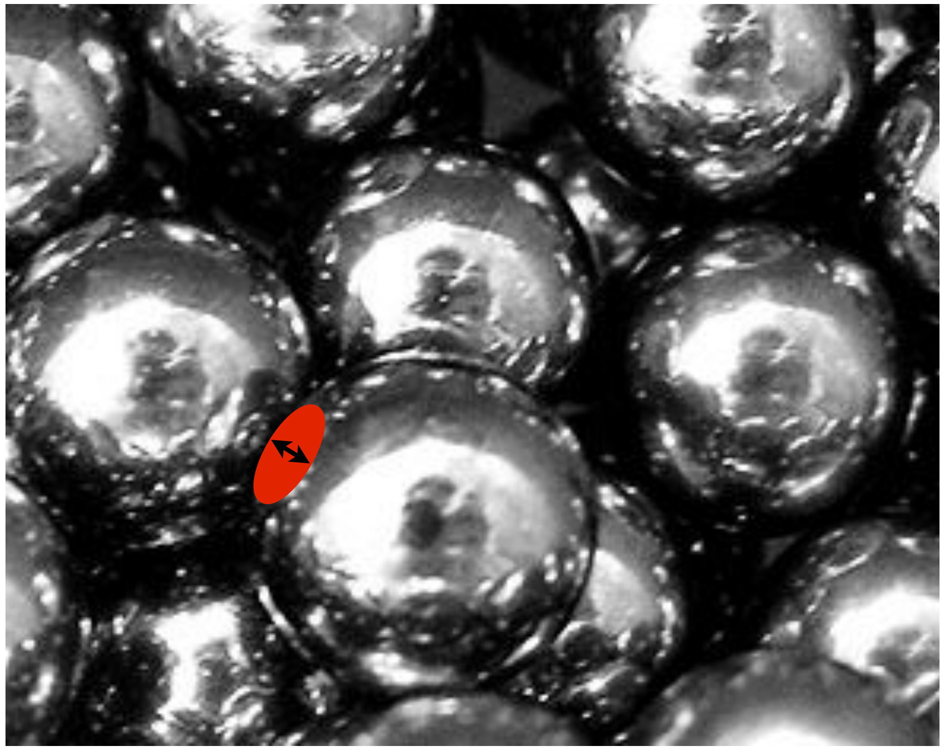
What is the origin of fragility ?



Sound speed 6000 m/s

The geometry of fragile objects

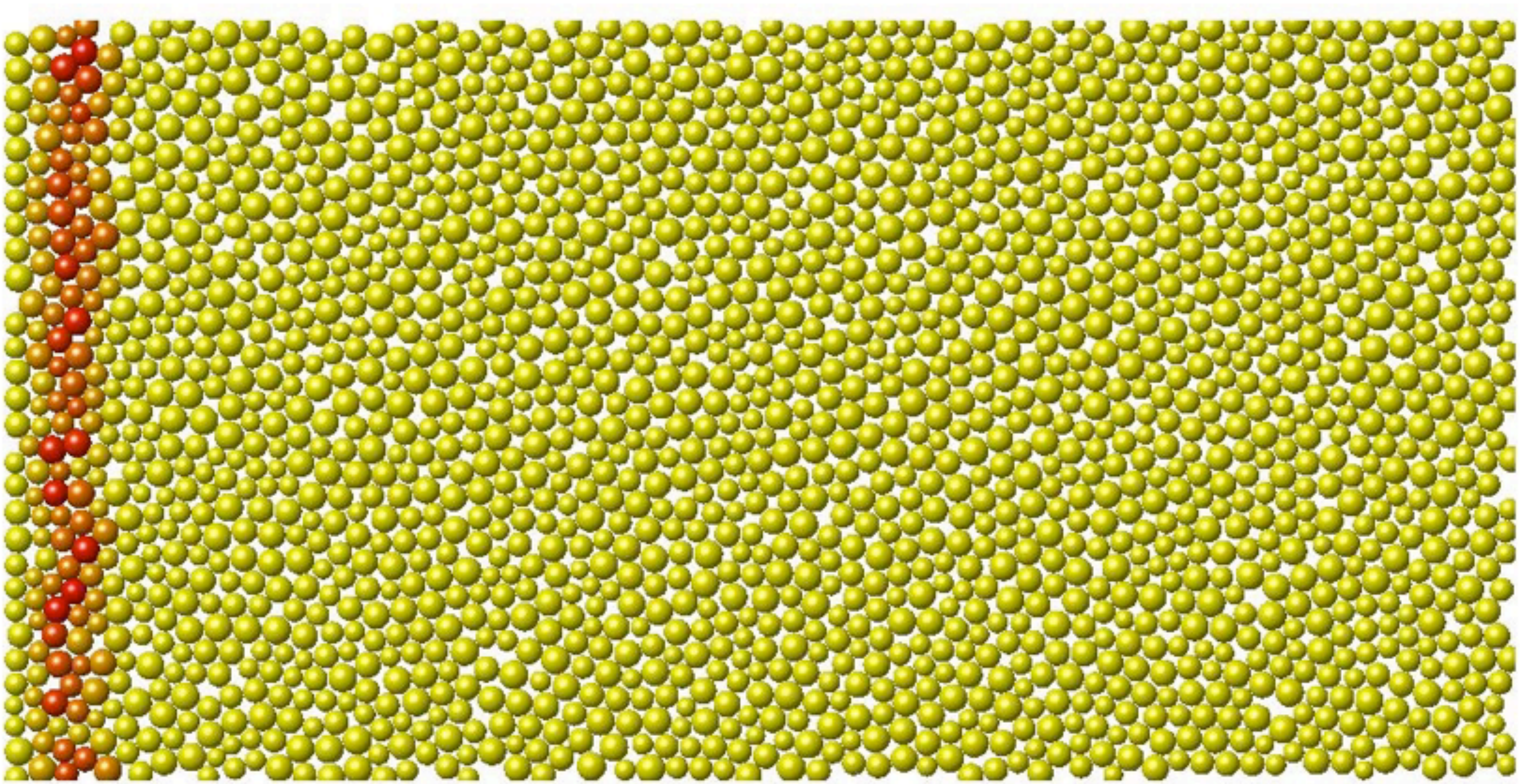




Sound speed 6000 m/s

Sound speed ≈ 0

Piston pushing at a constant speed



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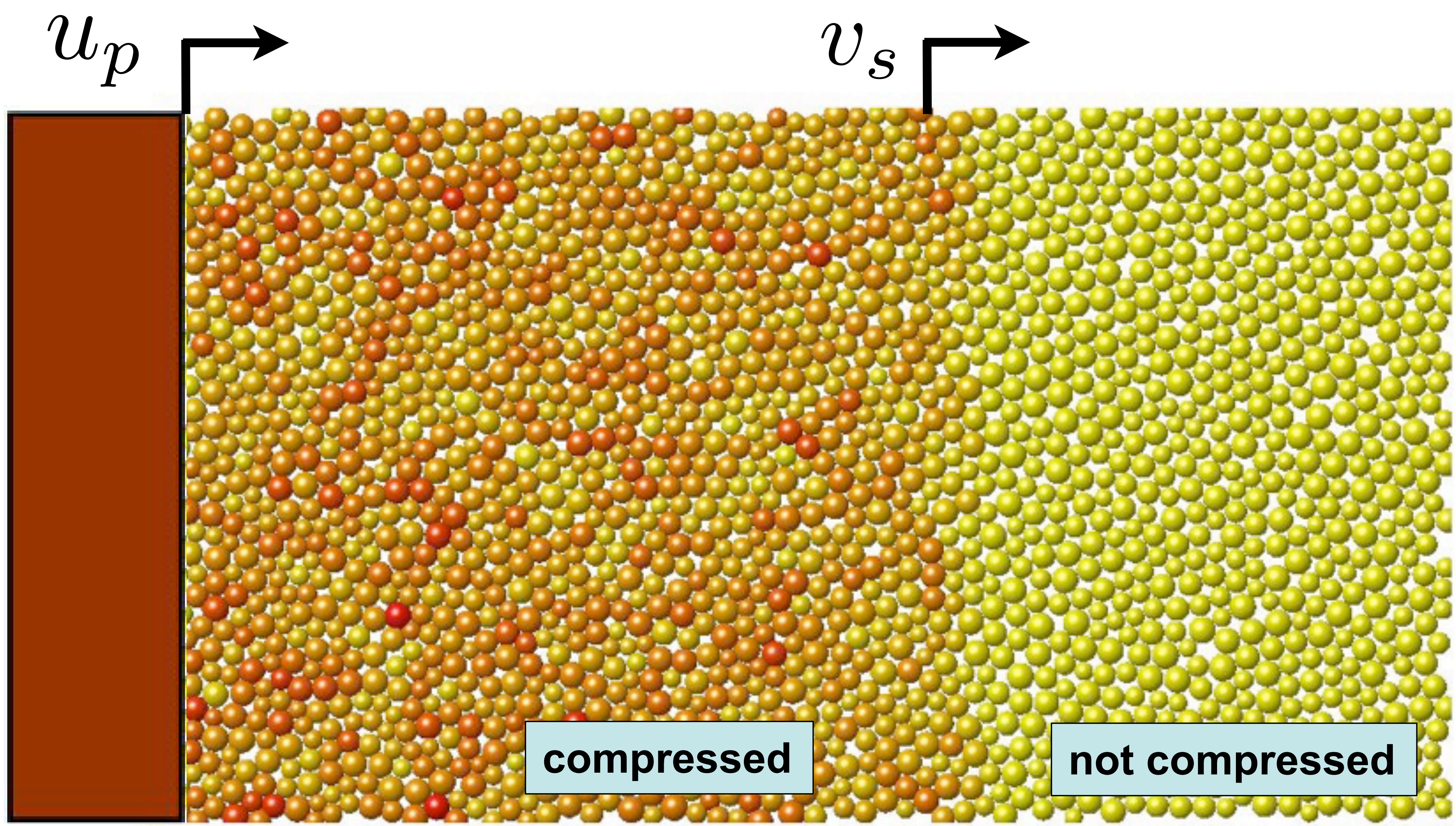
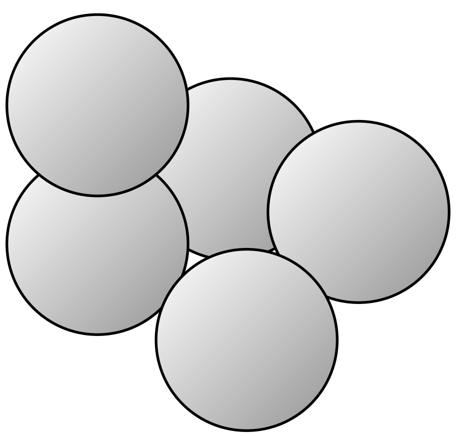
Piston pushing at a constant speed



$$¿ \quad v_S(u_p) \quad ?$$

Standard solid state approach

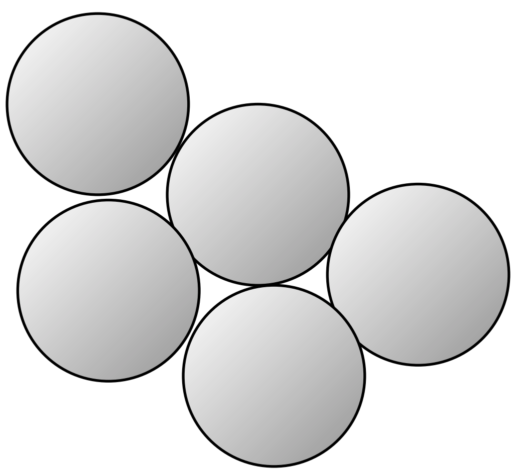
$$H = \ harmonic + {\color{red} anharmonic}$$



Phonons: excitations of rigid matter

Strong non-linearities

$$H = \cancel{harmonic} + \color{red}{anharmonic}$$



Shocks: "excitations" of fragile matter

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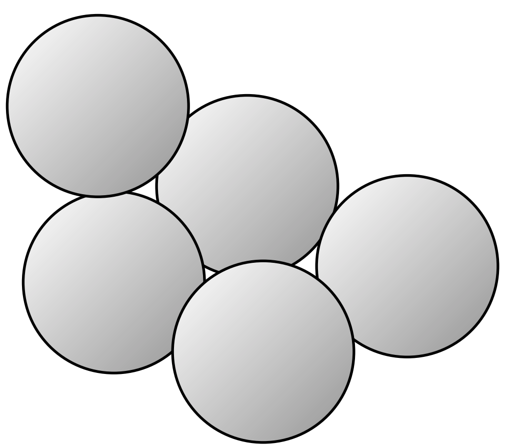
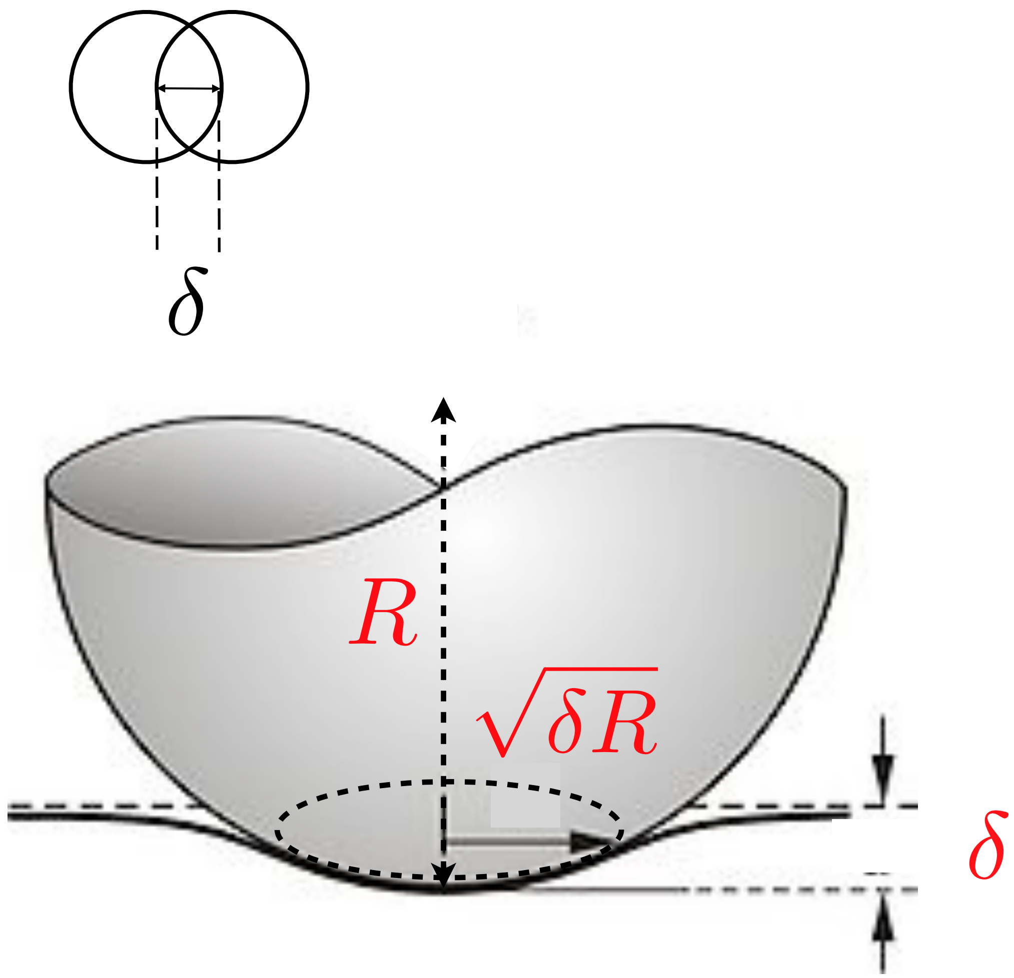
Strong non-linearities

$$H = harmonic + \color{red}{anharmonic}$$



Shocks: "excitations" of fragile matter

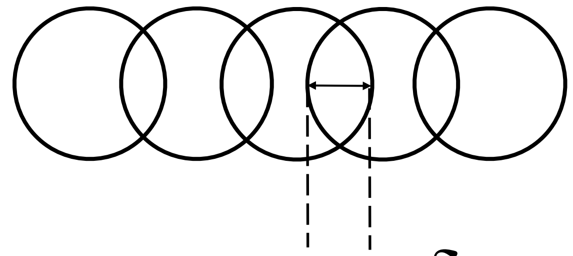
Contact Mechanics: Hertz law



typical strain	$\gamma \sim \frac{\delta}{\sqrt{\delta R}} \sim \delta^{\frac{1}{2}}$
typical stress	$\sigma \sim \gamma \sim \delta^{\frac{1}{2}}$
Force	$F \sim \sigma\delta \sim \delta^{\frac{3}{2}}$
Energy	$U \sim k \;\; \delta^{\frac{5}{2}}$

The material the grain is made of satisfies **linear** elasticity

Speed of sound: granular chain



$$x = \delta_0$$

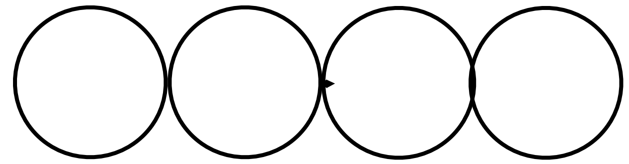
$$U \sim k\, x^{\frac{5}{2}}$$

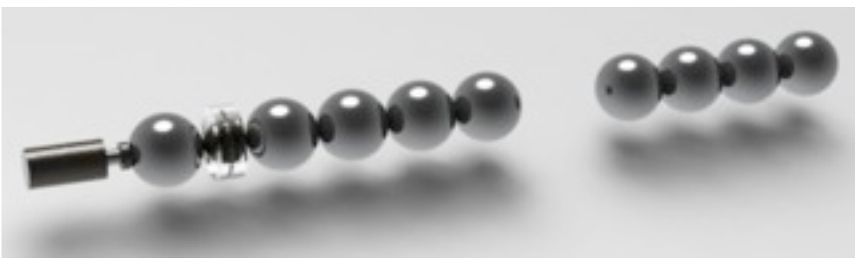
$$k_{eff} = \left.\frac{d^2U}{dx^2}\right|_{x = \delta_0}$$

$$c \sim \sqrt{k}\, \delta_0^{\,\frac{1}{4}}$$

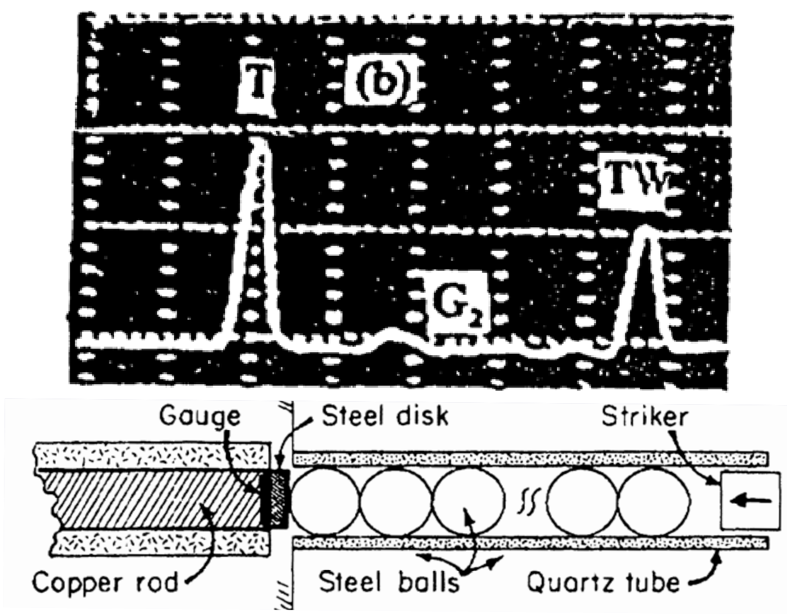
$$\sqrt{k_{eff}} \sim \delta_0^{\frac{\frac{5}{2}-2}{2}}$$

Solitons in the “sonic vacuum”





(Daraio Lab)



V. Nesterenko, J. Appl. Mech. Tech. Phys. 5, 733 (1983).

$$U \sim k\ x^{\frac{5}{2}}$$

$$c \sim \sqrt{k}\,\delta_0{}^{\frac{1}{4}}$$

$$\delta_0 \to 0$$

$$\downarrow$$

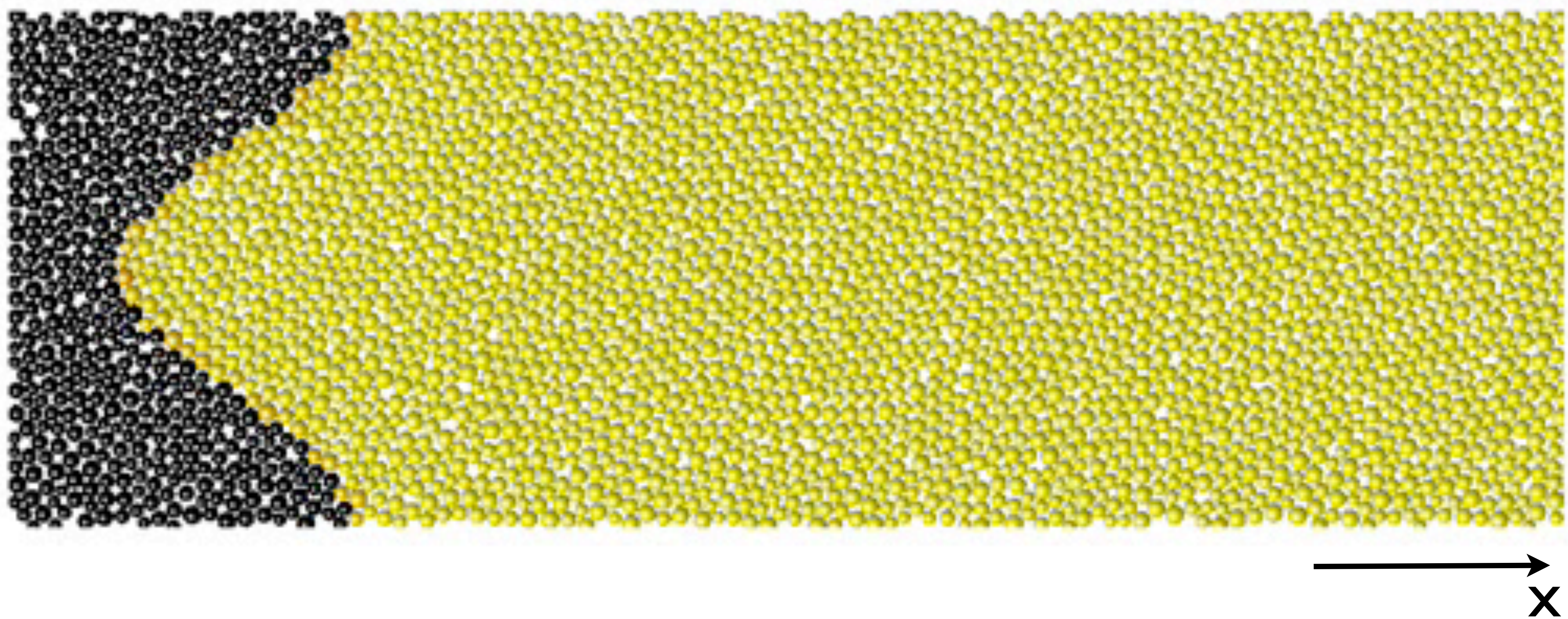
$$c \to 0$$

$$k_{eff} = \left.\frac{d^2U}{dx^2}\right|_{x\,=\,\delta_0}$$

$$\sqrt{k_{eff}} \sim \delta_0^{\frac{\frac{5}{2}-2}{2}}$$

Sound speed vanishes

Stable fronts: one dimensional model



$$L = \sum_n \frac{1}{2}\dot{u}_n^2 - \frac{A}{\alpha}(u_n - u_{n+1})^\alpha$$

$$\delta$$

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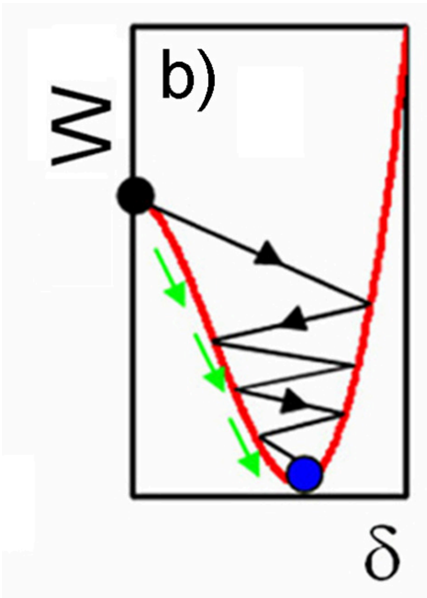
An equation of motion for shocks

$$\frac{R^2}{3}\delta_{ttxx} - \delta_{tt} + \frac{4R^2\varepsilon}{m}\left[\delta^{\alpha-1}\right]_{xx} = 0. \qquad \alpha = \frac{5}{2}$$

$$\delta(x,t) \;=\; \delta_0 + g(\tilde{x}), \qquad \tilde{x} \equiv x - v_S\, t.$$

$$\tfrac{1}{2}\delta_{\tilde{x}}^2 + W(\delta) = 0$$



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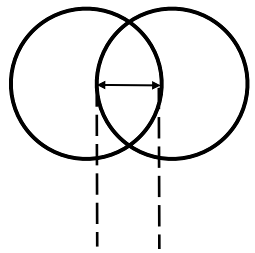
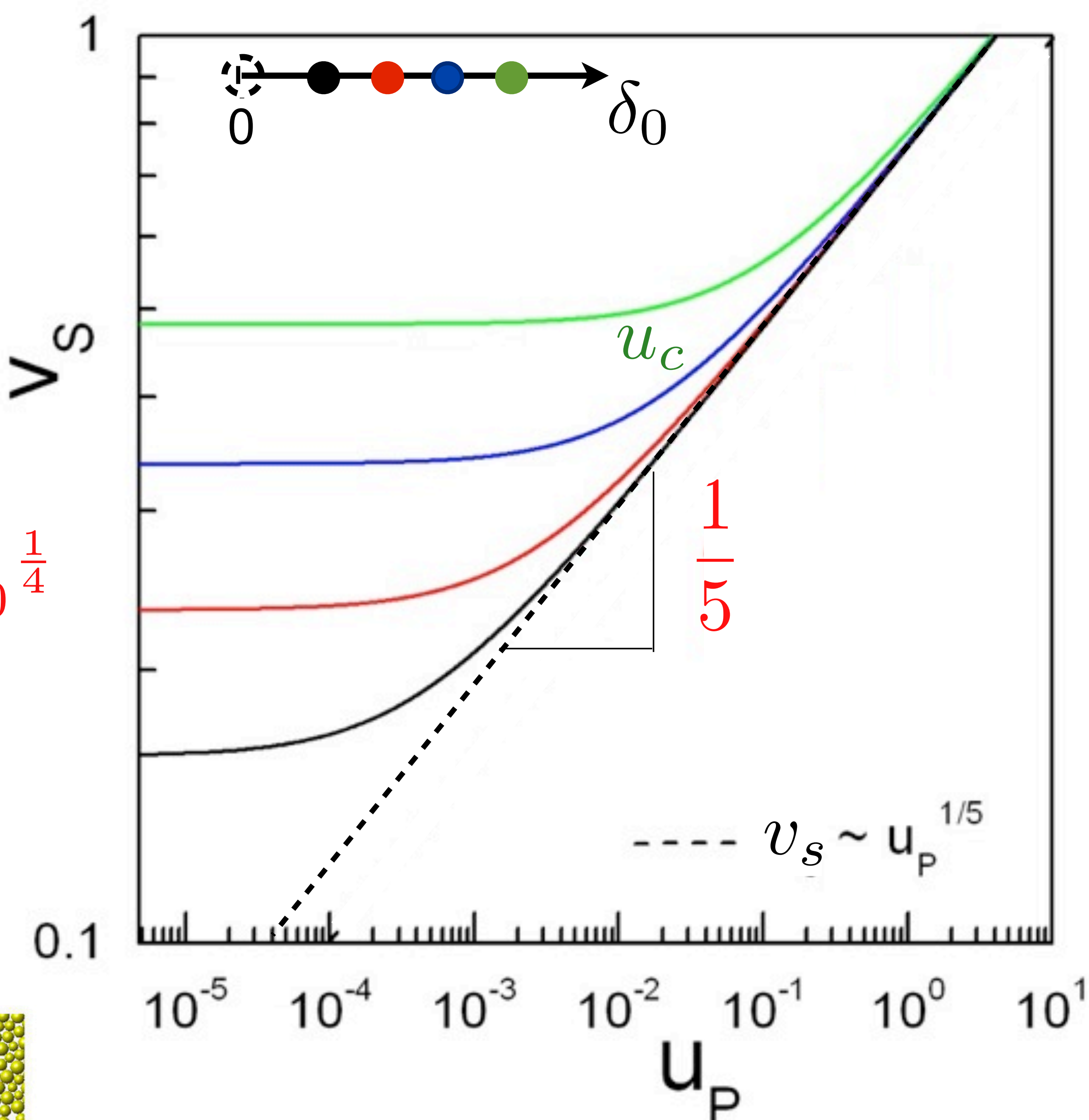
Non-linear waves and shocks



δ_0

$c \sim \sqrt{k}\,\delta_0^{\frac{1}{4}}$

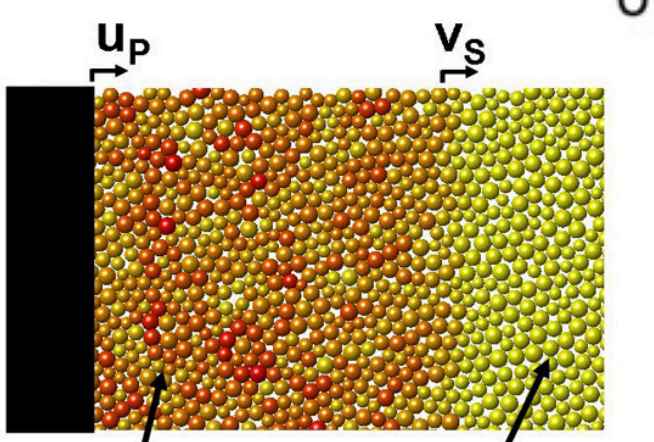


$v_s \sim \delta^{\frac{1}{4}}$

$Mu_p^2 \sim \delta^{\frac{5}{2}}$

$v_s \sim u_p^{\frac{1}{5}}$

$u_c \sim \delta_0^{\frac{5}{4}}$



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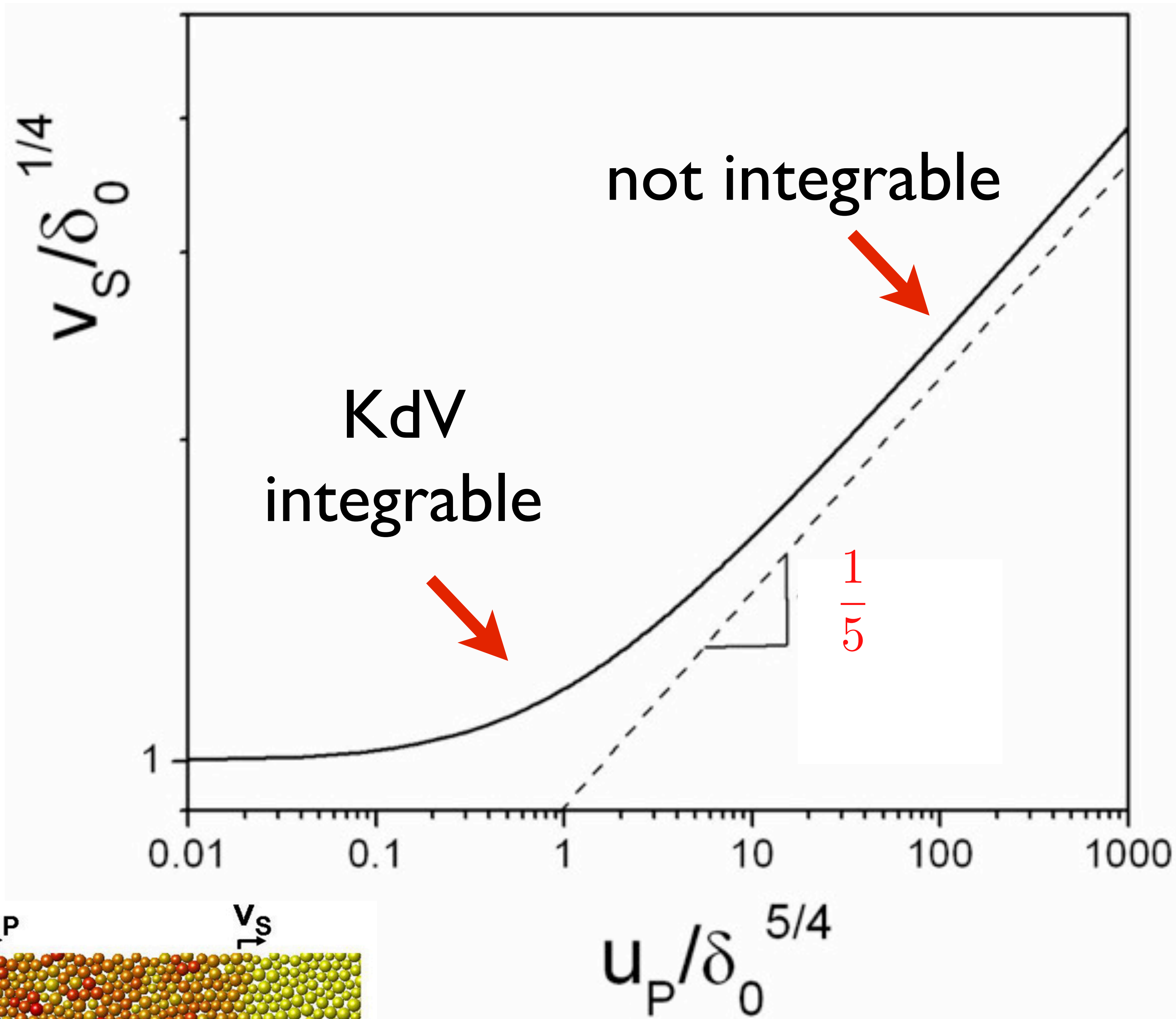
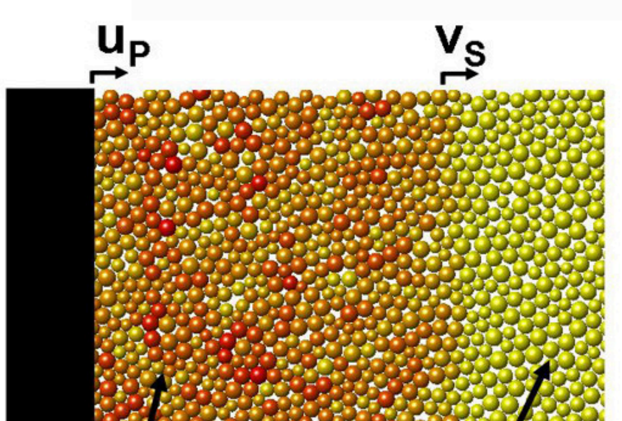
Collapse on single master curve





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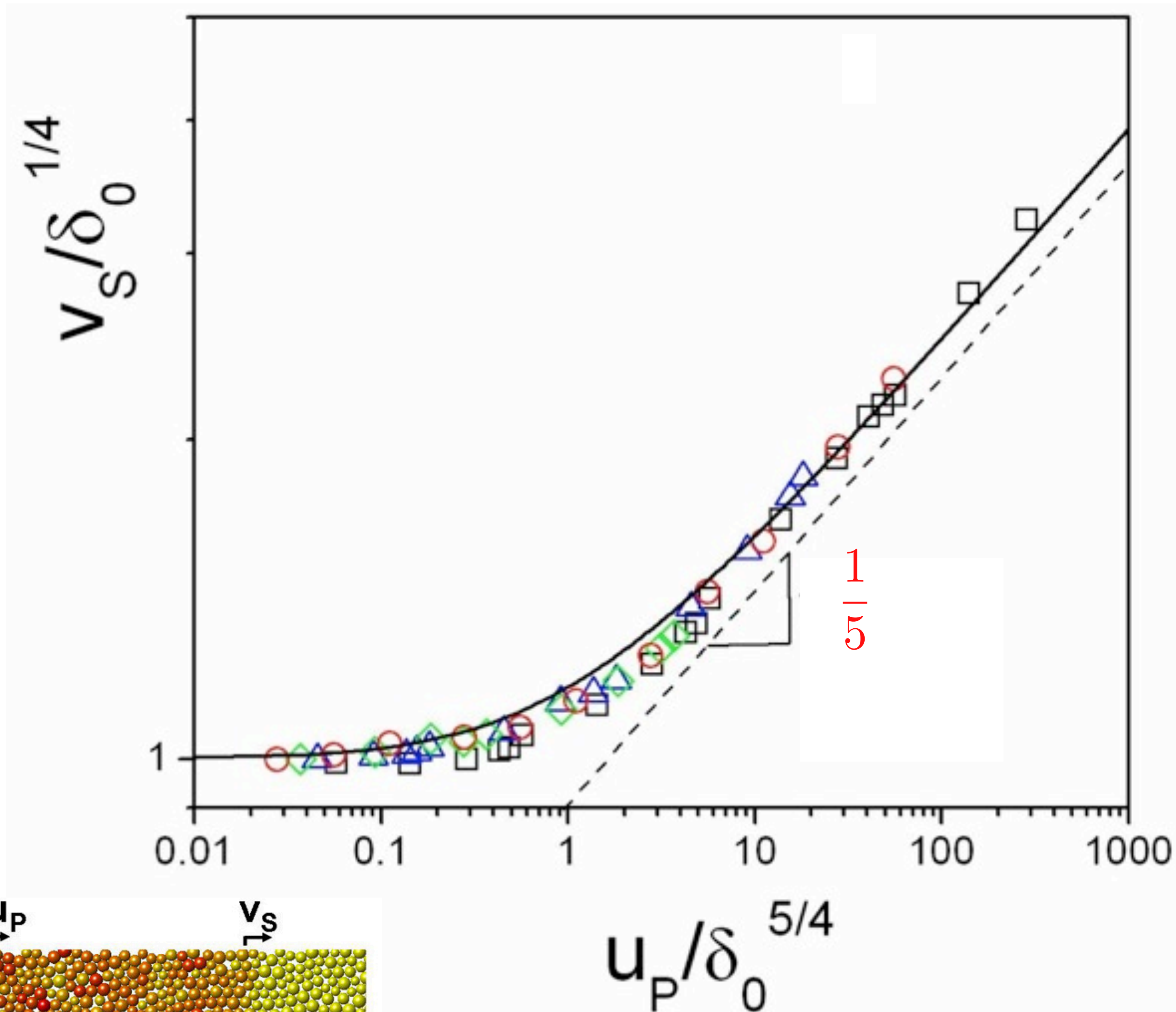
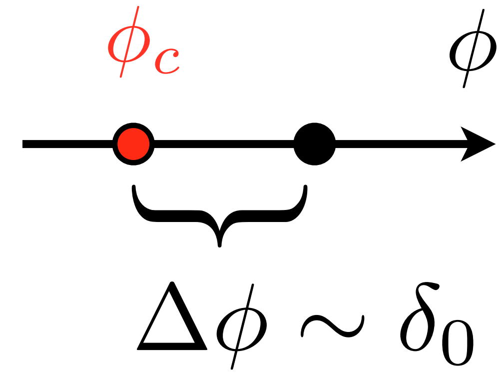
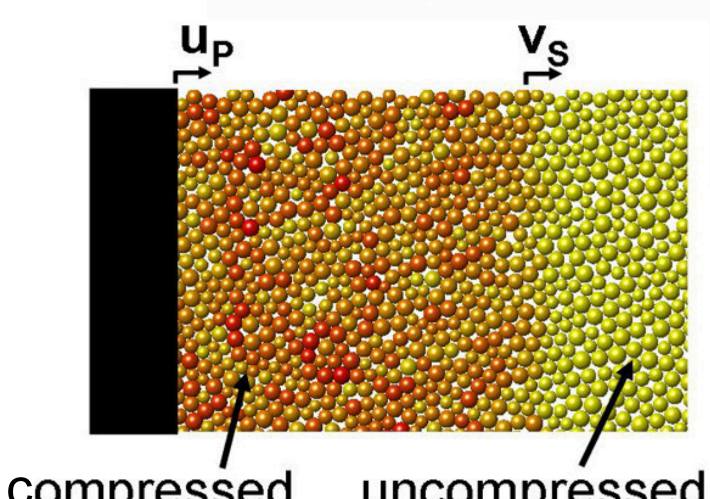
Comparison to simulations







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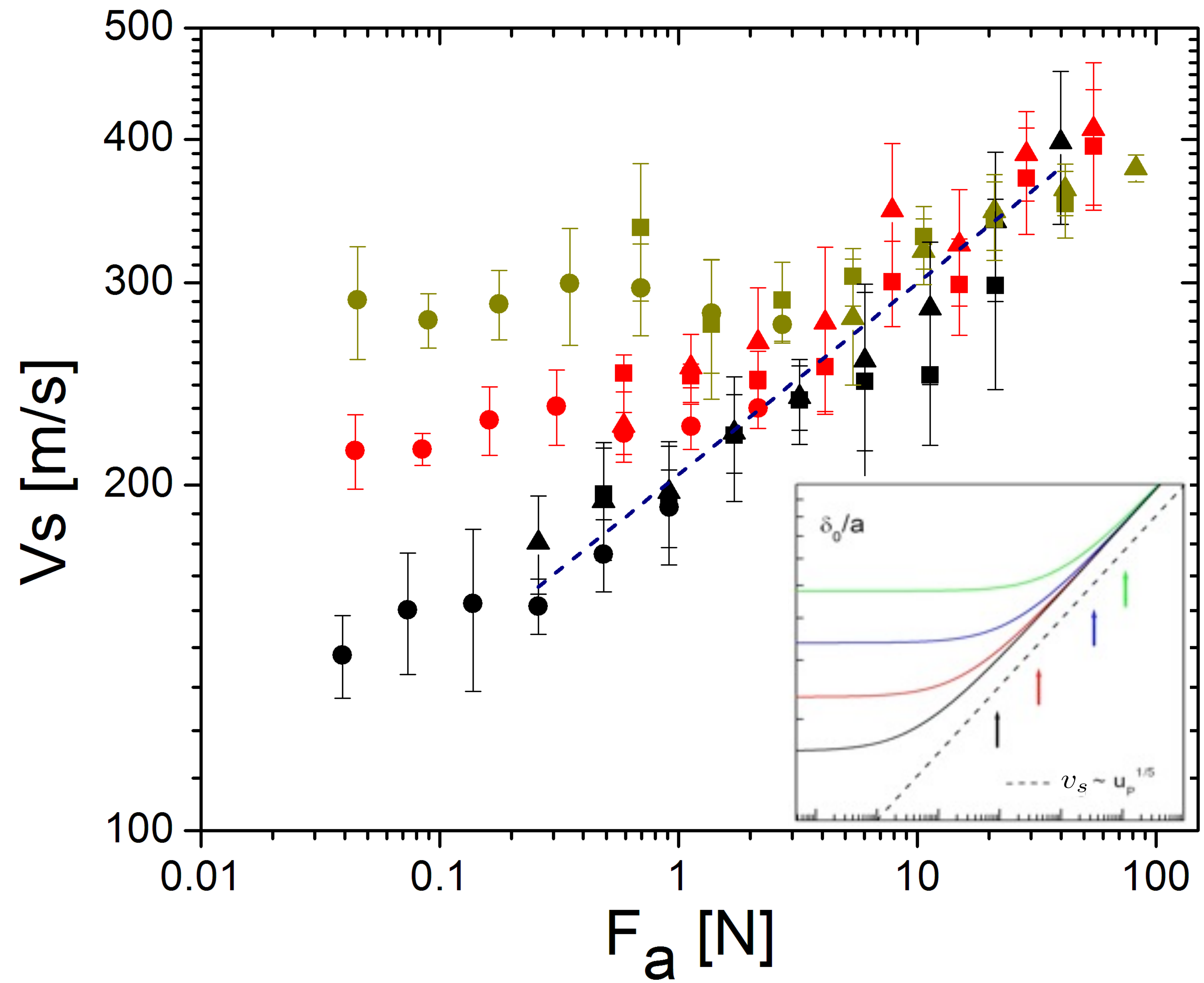
Preliminary experimental results



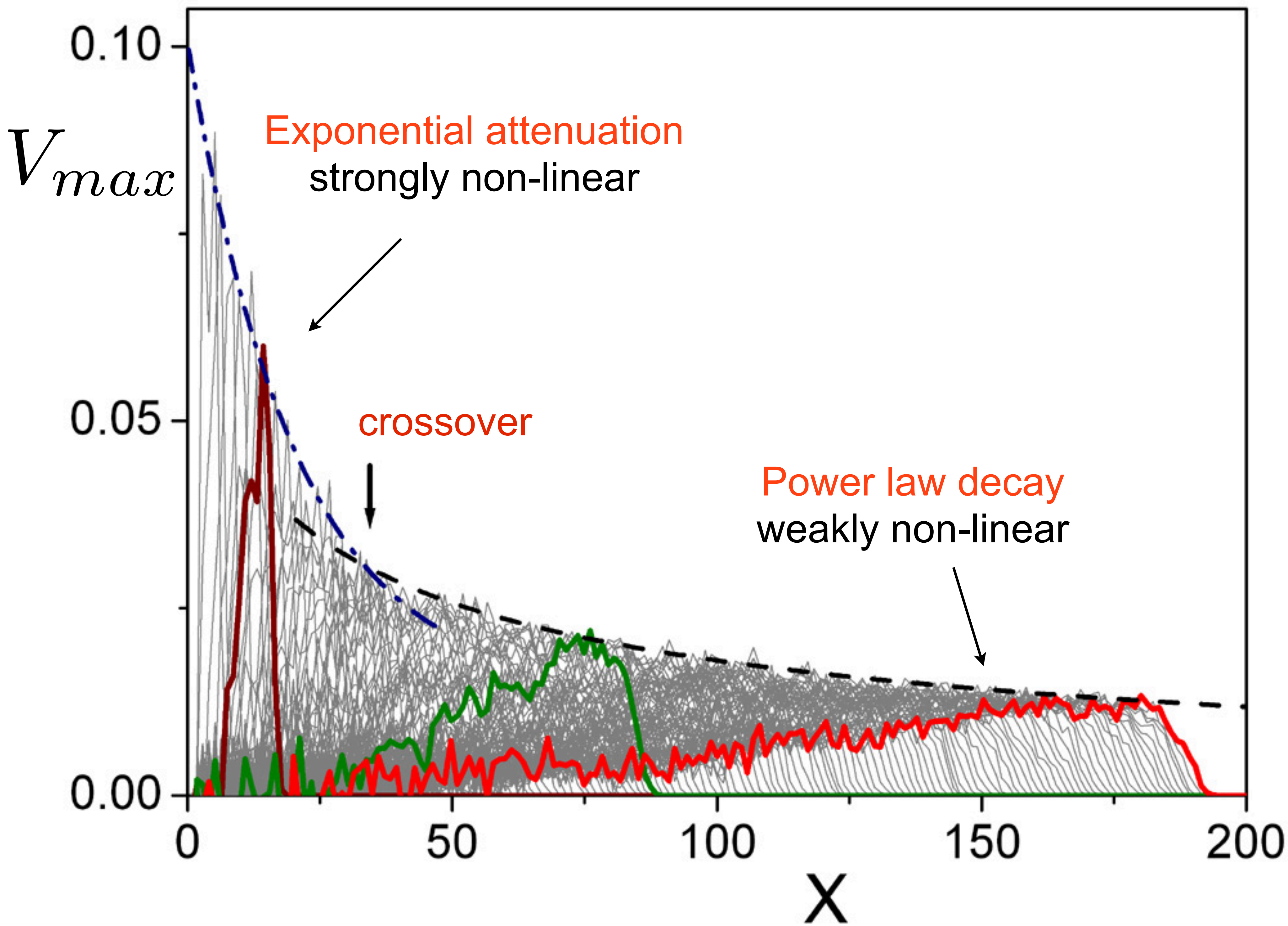
Van Hecke Lab:

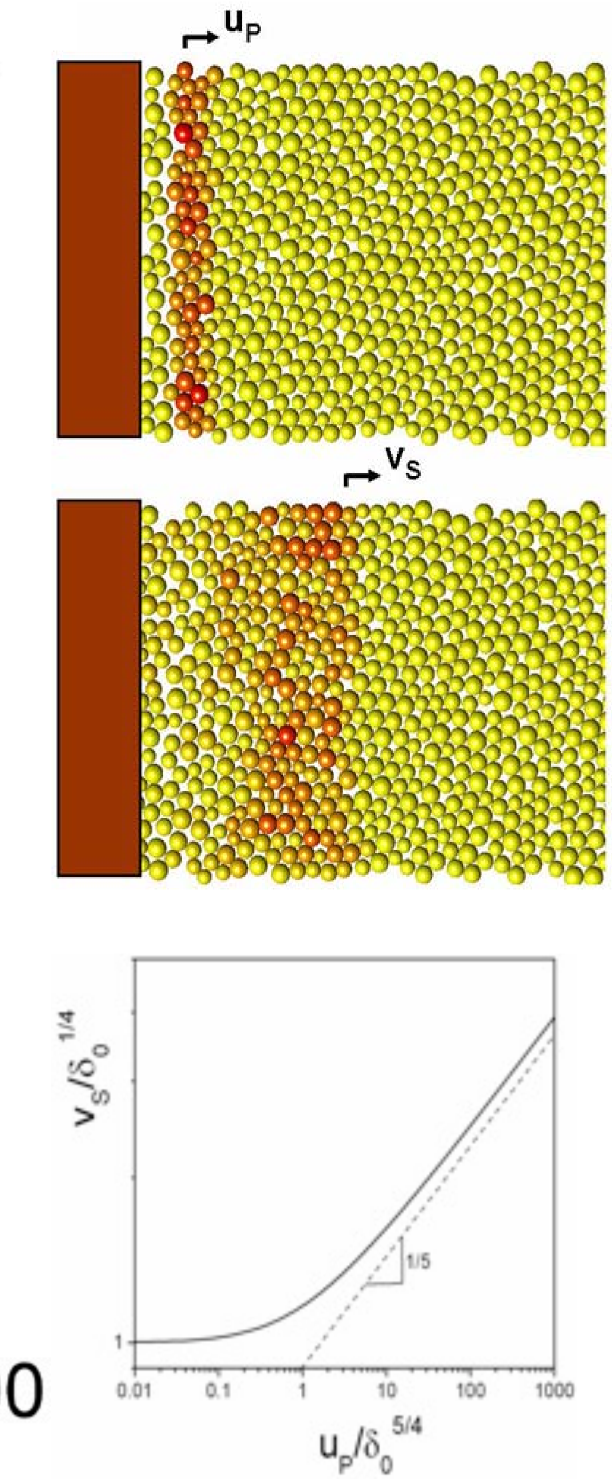
R. van Loo

S. van den Wildenberg

Dashed line has the predicted slope

Dynamic crossover

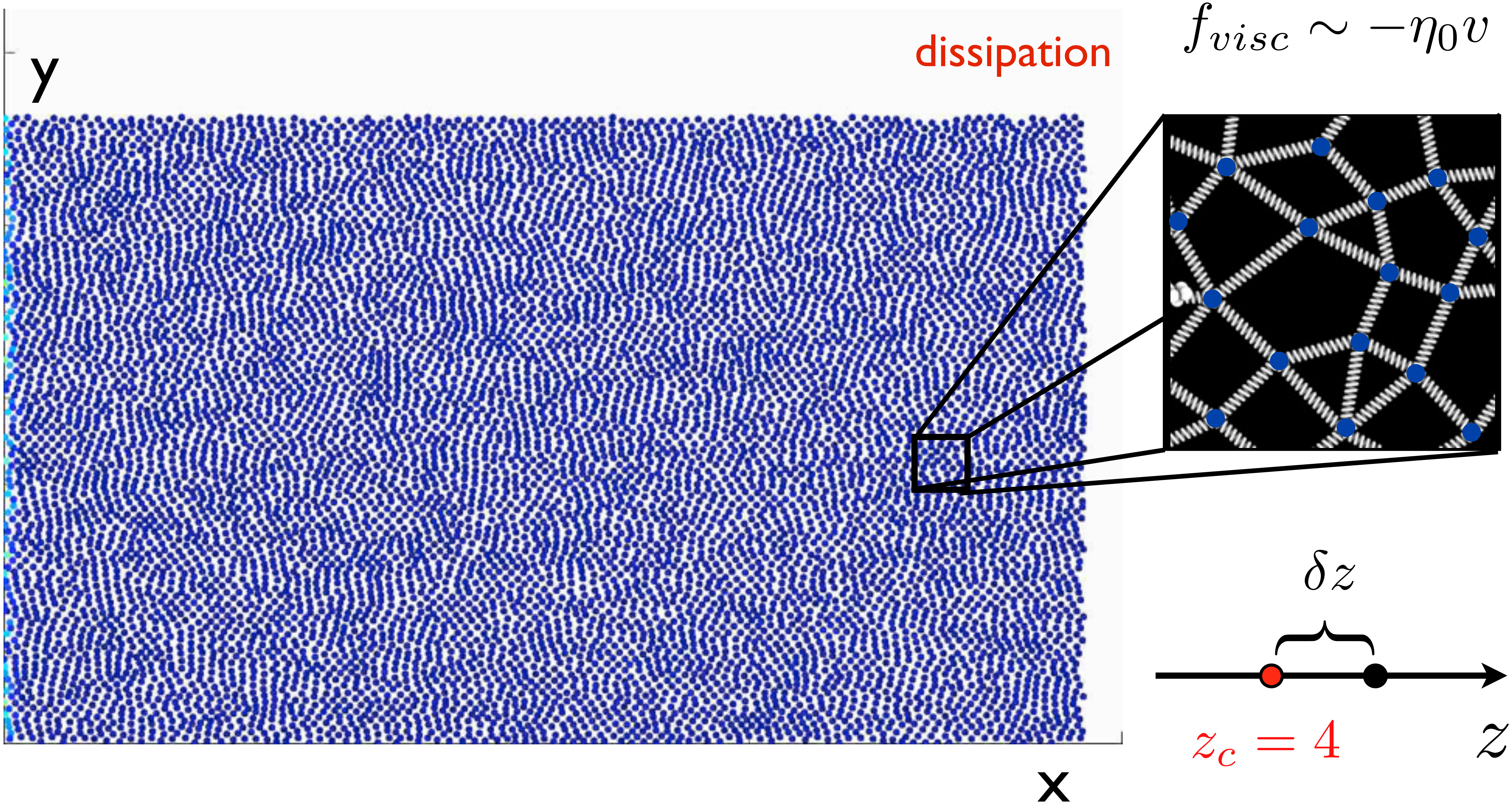




Gomez, Turner, Vitelli, unpublished.

Global non linearities: shear shocks in networks

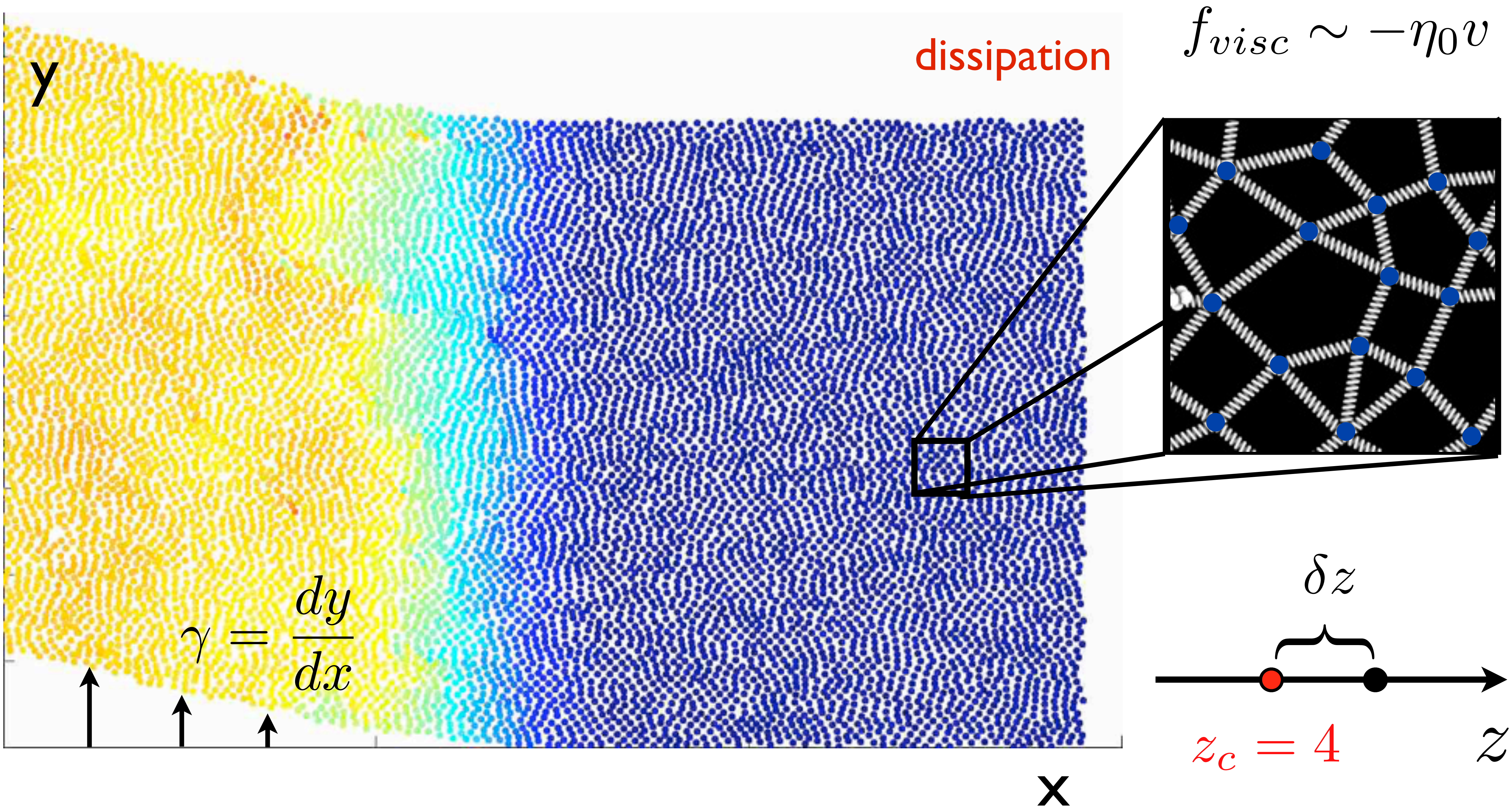
$$f_{visc} \sim -\eta_0 v$$



$$G \sim \delta z$$

S. Ulrich, N. Upadhyaya, B. van Opheusden, V. Vitelli, unpublished

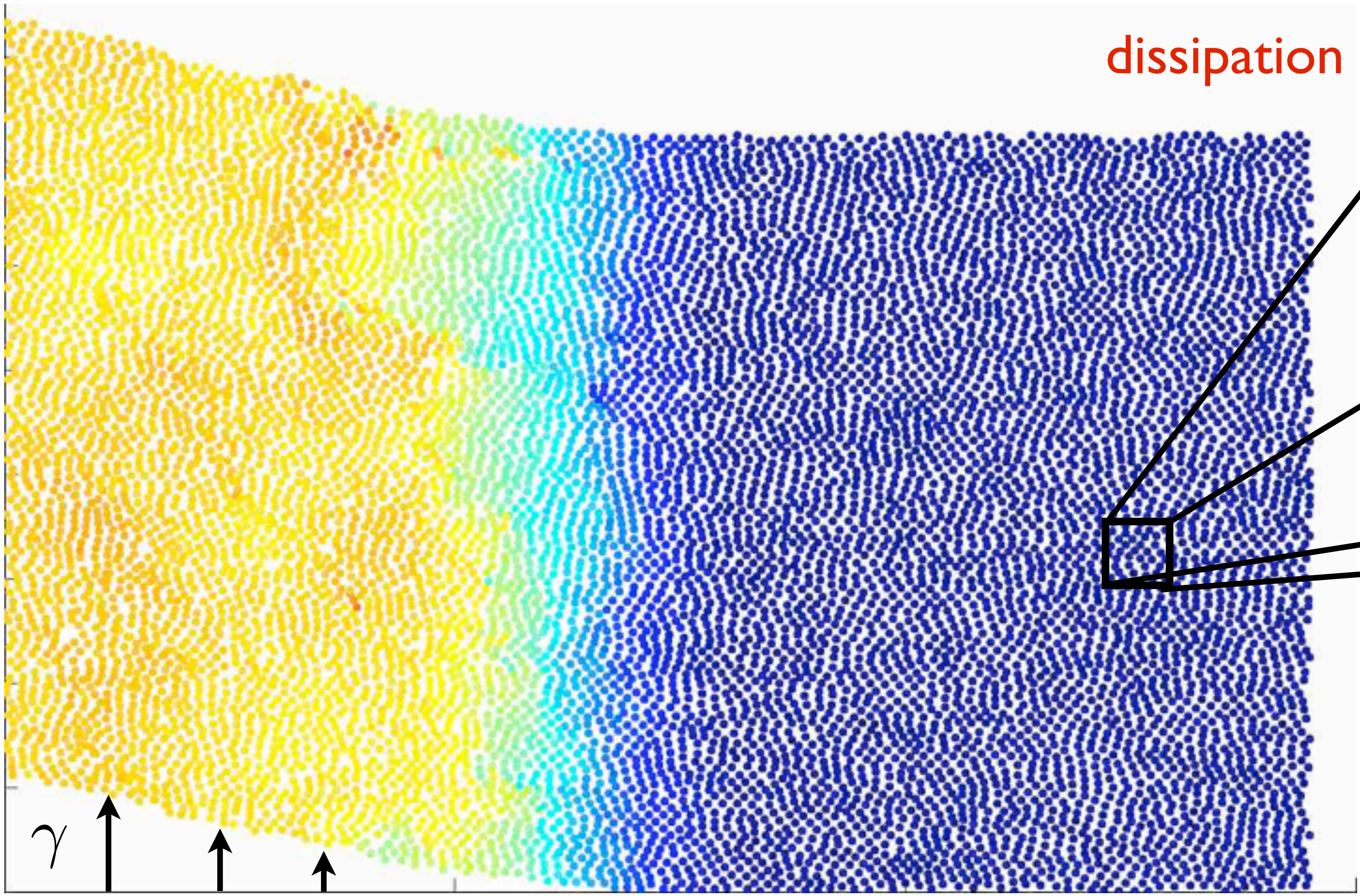
Global non linearities: shear shocks in networks



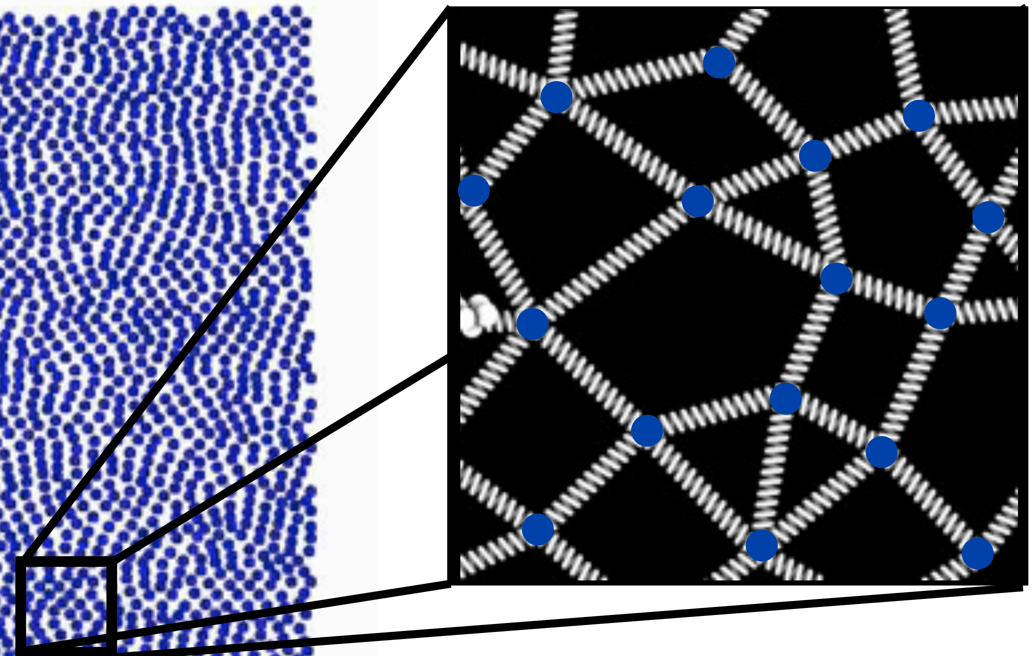
$G \sim \delta z$

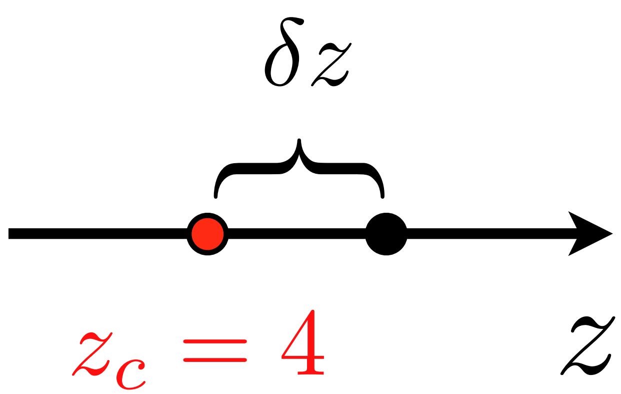
$$\frac{\partial \sigma}{\partial x} = \rho_0 \frac{\partial^2 y}{\partial t^2},$$

Global non linearities: shear shocks in networks



$f_{visc} \sim -\eta_0 v$





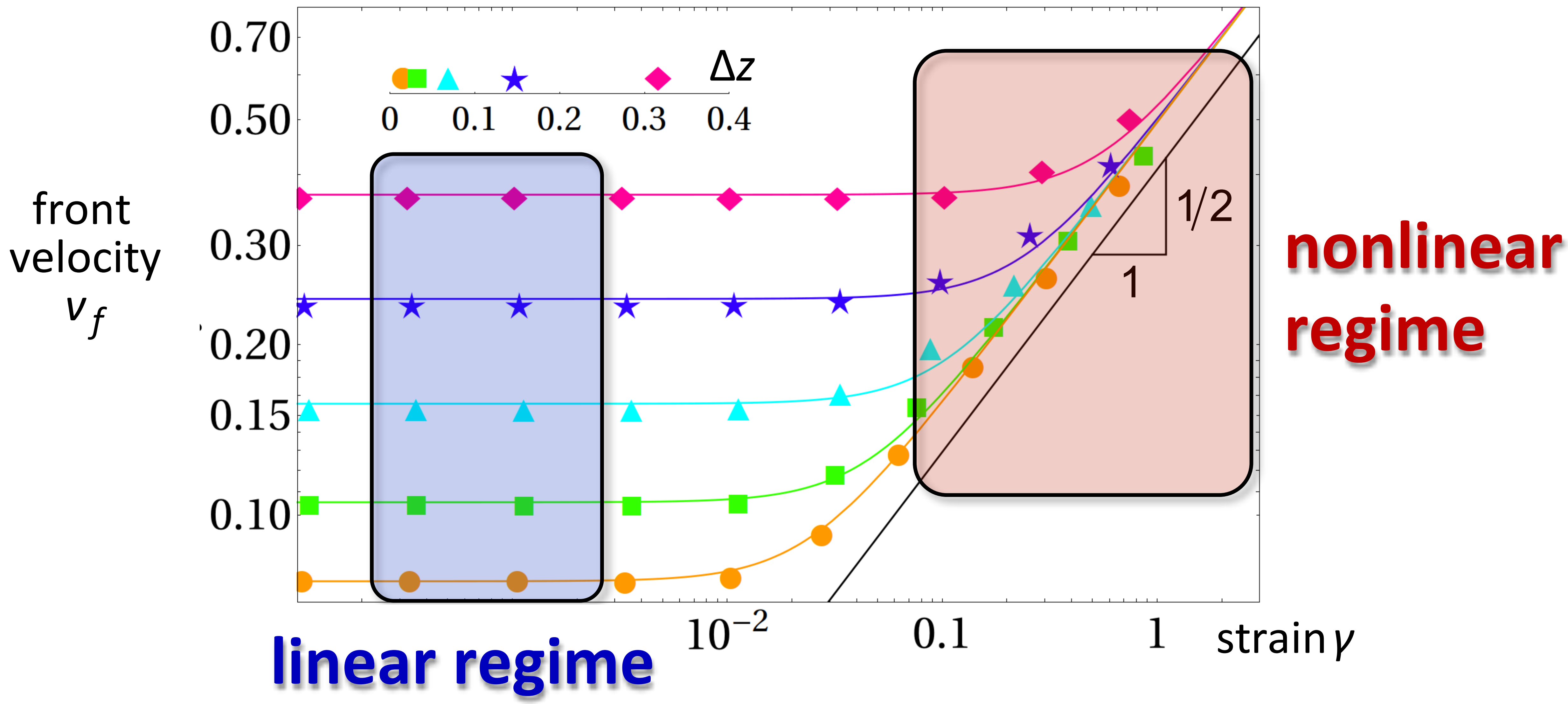
$G \sim \delta z$

$\eta \approx \frac{\eta_0}{\delta z}$

$$\sigma \sim G\gamma + k\ \gamma|\gamma| + \eta\frac{d\gamma}{dt}$$

Wyart et al. PRL 2009 Tighe et al. PRL 2009

Velocity of Shear Front



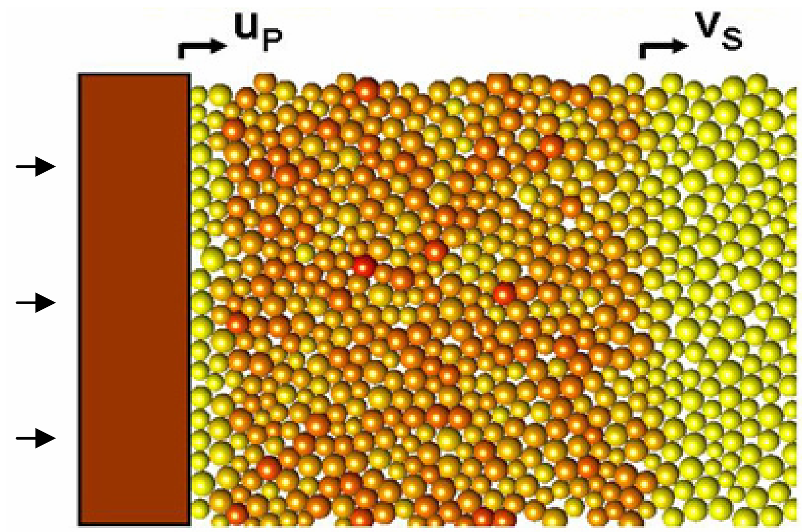
Linear regime vanishes

when $\Delta z \to 0$

Conclusion

At the critical point sound propagates by shocks only

Away from the critical point the same shocks control energy transport for large dynamical strains

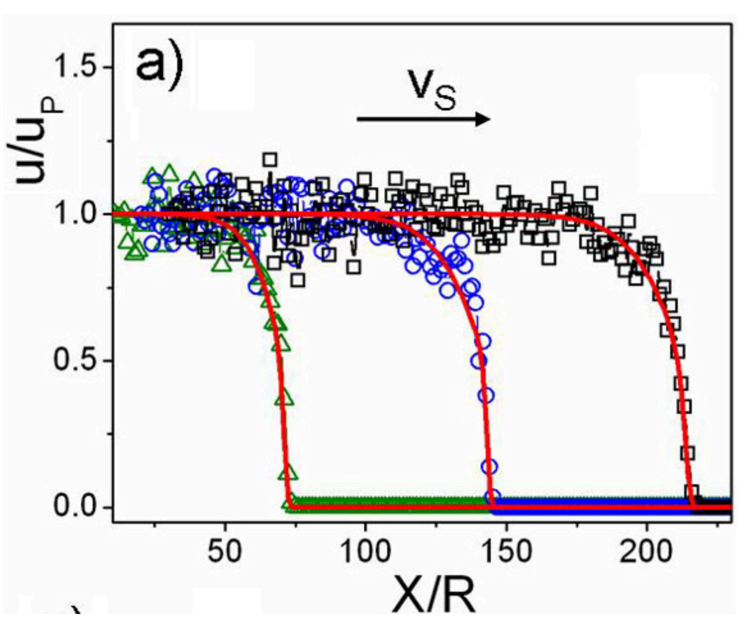


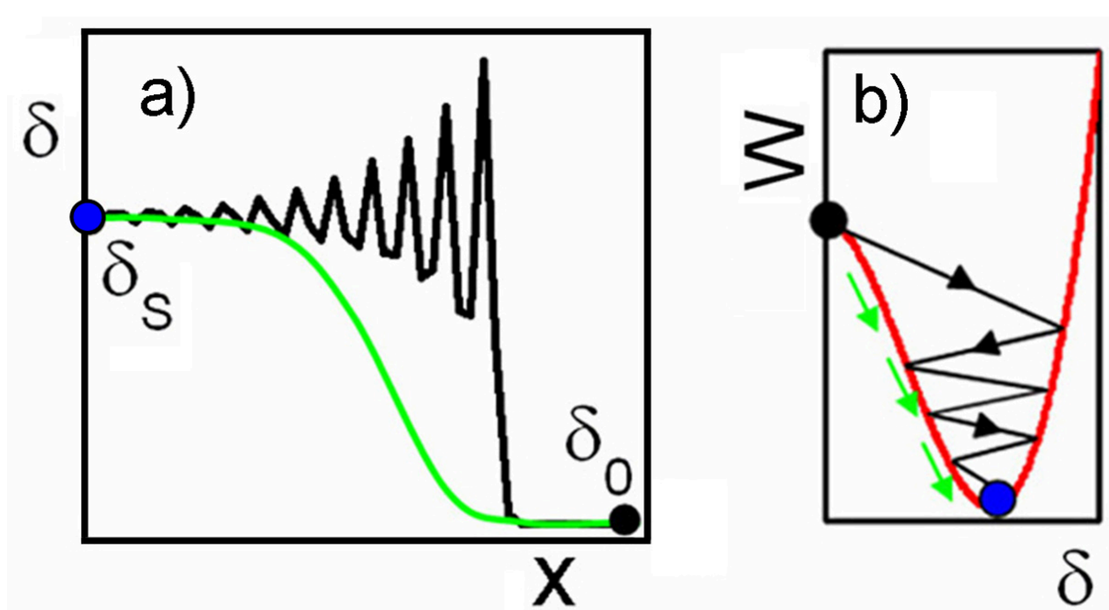
Disorder acts as an effective viscosity

$$\frac{R^2}{3}\delta_{ttxx} - \delta_{tt} + \frac{4R^2\varepsilon}{m}\left[\delta^{\alpha-1}\right]_{xx} = 0. \qquad \alpha = \frac{5}{2}$$

$$\delta(x,t) = \delta_0 + g(\tilde{x}), \qquad \tilde{x} \equiv x - v_S\, t.$$

$$\tfrac{1}{2}\delta_{\tilde{x}}^2 + W(\delta) = 0$$





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Non-linear wave equation for 1D front

$$L = \sum_n \frac{1}{2}\dot{u}_n^2 - \frac{1}{\alpha}(u_n - u_{n+1})^\alpha$$

Lagrangian

$$\boxed{u_{n+1} - u_n = \phi'(n + \frac{1}{2})}$$

Taylor expand LHS

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Non-linear wave equation for 1D front

$$L = \sum_n \frac{1}{2}\dot{u}_n^2 - \frac{1}{\alpha}(u_n - u_{n+1})^\alpha$$

Lagrangian

$$\boxed{u_{n+1} - u_n = \phi'(n + \frac{1}{2})}$$

$$\cancel{u_n - u_{n+1} = -\phi'(n) - \tfrac{1}{2}\phi''(n)} - \ldots$$

Non-linear wave equation for 1D front

$$L = \sum_n \frac{1}{2}\dot{u}_n^2 - \frac{1}{\alpha}(u_n - u_{n+1})^\alpha$$

Lagrangian

$$\boxed{u_{n+1} - u_n = \phi'(n + \frac{1}{2})}$$

Taylor expand LHS

$$u'(n + \tfrac{1}{2}) + \tfrac{1}{24}u'''(n + \tfrac{1}{2}) = \phi'(n + \frac{1}{2})$$

Invert it

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Non-linear wave equation for 1D front

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Taylor expand LHS

$$u'(n + \tfrac{1}{2}) + \tfrac{1}{24}u'''(n + \tfrac{1}{2}) = \phi'(n + \frac{1}{2})$$

Invert it

$$u(n + \frac{1}{2}) \approx \phi(n + \frac{1}{2}) - \frac{1}{24}\phi''(n + \frac{1}{2})$$

rewrite the kinetic term in L

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Non-linear wave equation for 1D front

$$L = \sum_n \frac{1}{2}\dot{u}_n^2 - \frac{1}{\alpha}(u_n - u_{n+1})^\alpha$$

Lagrangian

$$\boxed{u_{n+1} - u_n = \phi'(n + \frac{1}{2})}$$

Taylor expand LHS

$$u'(n + \tfrac{1}{2}) + \tfrac{1}{24}u'''(n + \tfrac{1}{2}) = \phi'(n + \frac{1}{2})$$

Invert it

$$u(n + \frac{1}{2}) \approx \phi(n + \frac{1}{2}) - \frac{1}{24}\phi''(n + \frac{1}{2})$$

rewrite the kinetic term in L

$$\boxed{\tfrac{1}{2}\dot{u}_n^2 \approx \tfrac{1}{2}\dot{\phi}(n)^2 - \tfrac{1}{24}\dot{\phi}(n)\dot{\phi}''(n)}$$

substitute in Lagrangian

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Non-linear wave equation for 1D front

$$L \approx \int \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{24}(\dot{\phi}')^2 - \frac{1}{\alpha}(-\phi')^\alpha \right) dx$$ Lagrangian

$$\boxed{u_{n+1} - u_n = \phi'(n + \frac{1}{2})}$$ Taylor expand LHS

$$u'(n + \tfrac{1}{2}) + \tfrac{1}{24}u'''(n + \tfrac{1}{2}) = \phi'(n + \frac{1}{2})$$ Invert it

$$u(n + \frac{1}{2}) \approx \phi(n + \frac{1}{2}) - \frac{1}{24}\phi''(n + \frac{1}{2})$$ rewrite the kinetic term in L

$$\boxed{\tfrac{1}{2}\dot{u}_n^2 \approx \tfrac{1}{2}\dot{\phi}(n)^2 - \tfrac{1}{24}\dot{\phi}(n)\dot{\phi}''(n)}$$ substitute in Lagrangian

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