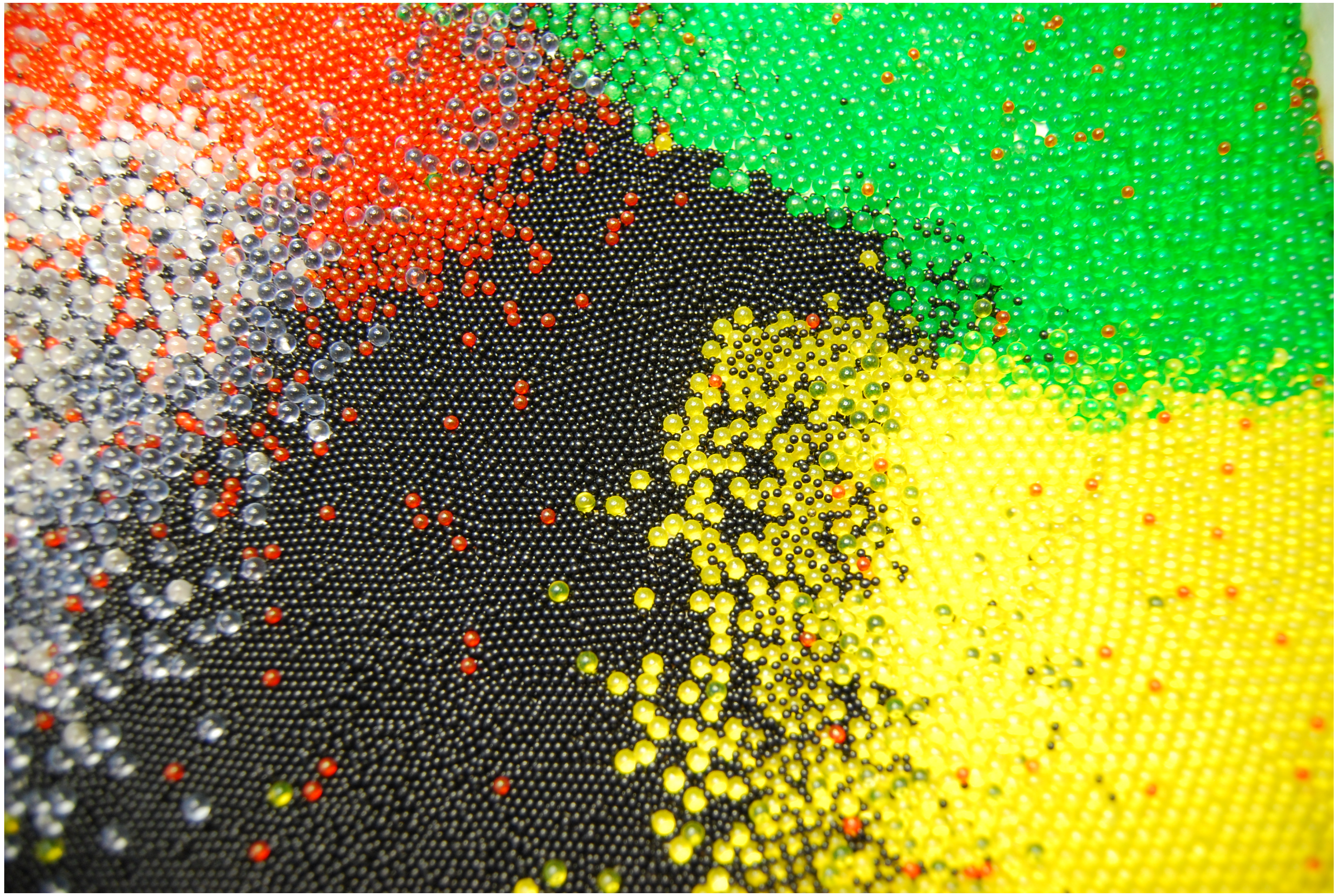
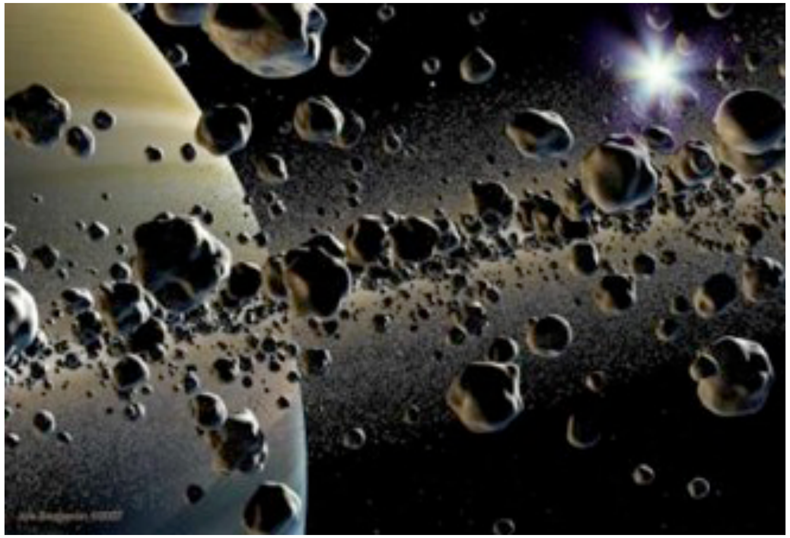
Granular materials: from physics to engineering applications

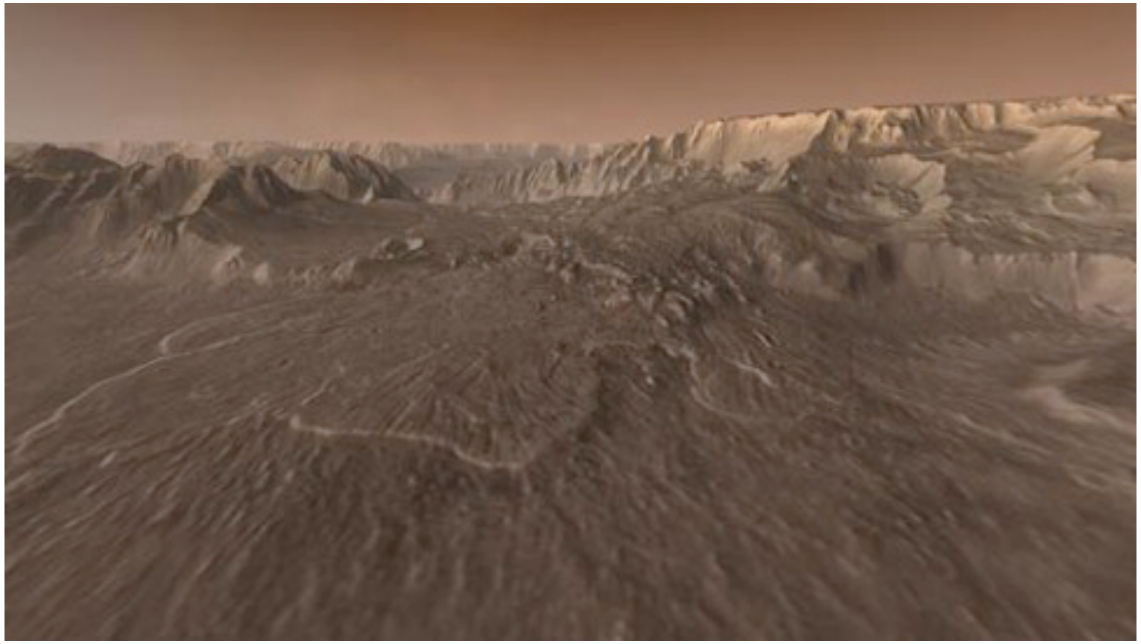
V. Magnanimo - MSM - University of Twente (NL)



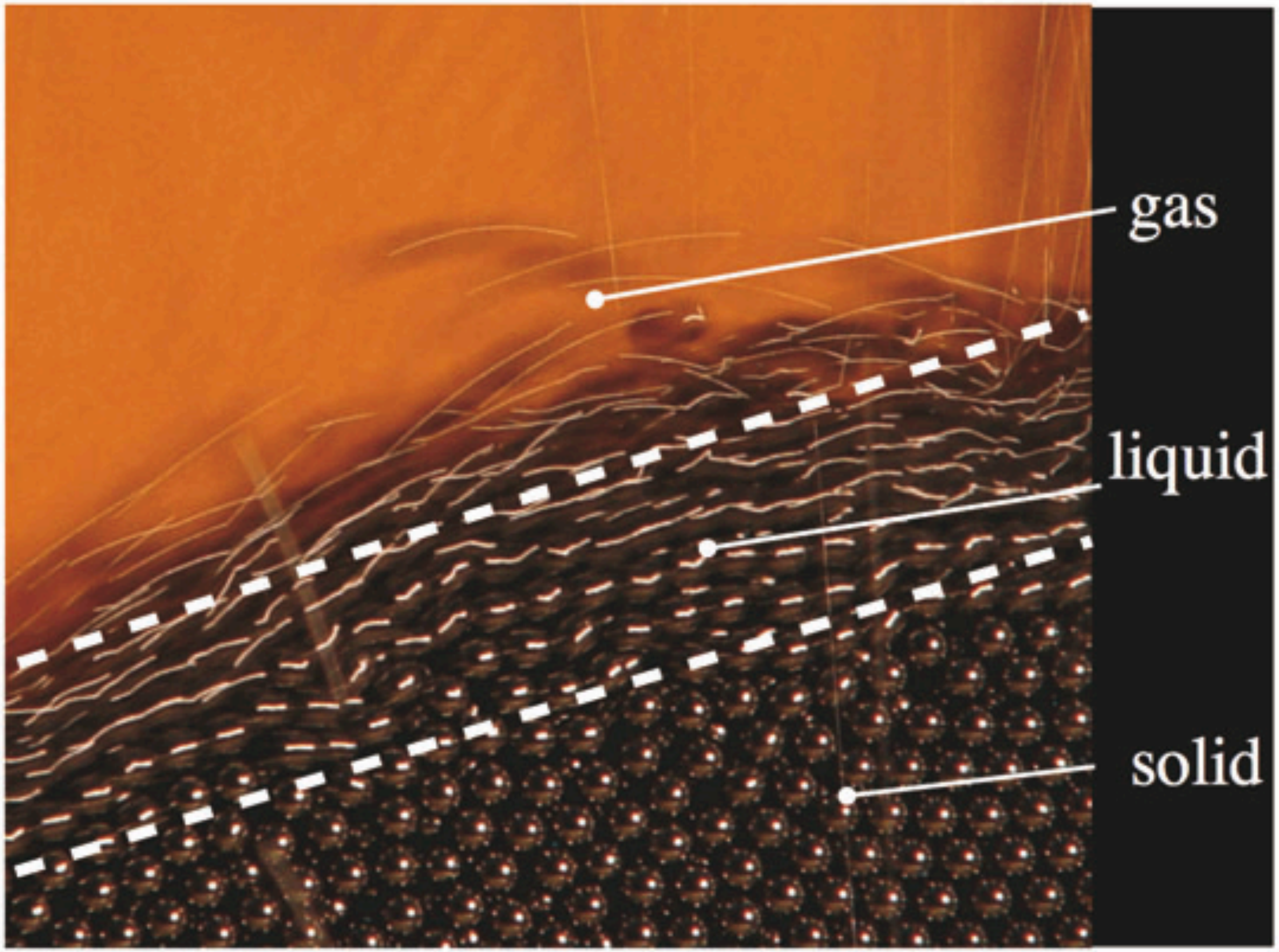
JMBC course
22nd March 2016

Granular material regimes



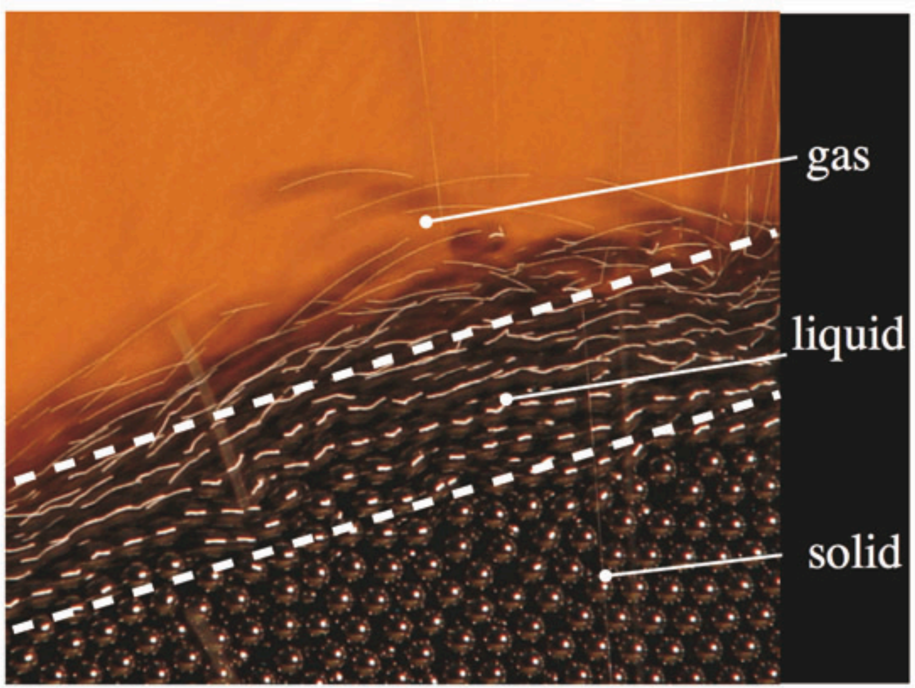


Solid+liquid+gas



[Forterre & Pouliquen, Annu. Rev. Fluid Mech. (2008)]

Solid+liquid+gas



Three regimes:

- **solid** – static
particles interact via frictional contacts

- **liquid** – dense, flow-like behavior
both collisions and friction

- **gas** – rapid dilute flow
particles interact via collisions

[Jaeger et al. (1996), Forterre & Pouliquen, Annu. Rev. Fluid Mech. (2008)]

Outline

- Introduction
- Internal force transmission
- Solid state
- Quasistatic regime and flow threshold
- Dense slow flows and inertial regime
- Collisional and rapid granular flows

References

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Granular material flows – An overview

Charles S. Campbell *

Aerospace and Mechanical Engineering, University of Southern California, Los Angeles, CA 90089-1453, USA

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References

Flows of Dense Granular Media

Yoël Forterre and Olivier Pouliquen

Institut Universitaire des Systèmes Thermiques Industriels, Centre National de la Recherche Scientifique, Université de Provence, 13453 Marseille cedex 13;
email: olivier.pouliquen@polytech.univ-mrs.fr

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Key Words

granular flows, rheology, friction, shallow water, instability, visco-plasticity

Abstract

We review flows of dense cohesionless granular materials, with a special focus on the question of constitutive equations. We first discuss the existence of a dense flow regime characterized by enduring contacts. We then emphasize that dimensional analysis strongly constrains the relation between stresses and shear rates, and show that results from experiments and simulations in different configurations support a description in terms of a frictional visco-plastic constitutive law. We then discuss the successes and limitations of this empirical rheology in light of recent alternative theoretical approaches. Finally, we briefly present depth-averaged methods developed for free surface granular flows.

Internal forces transmission

Contact forces

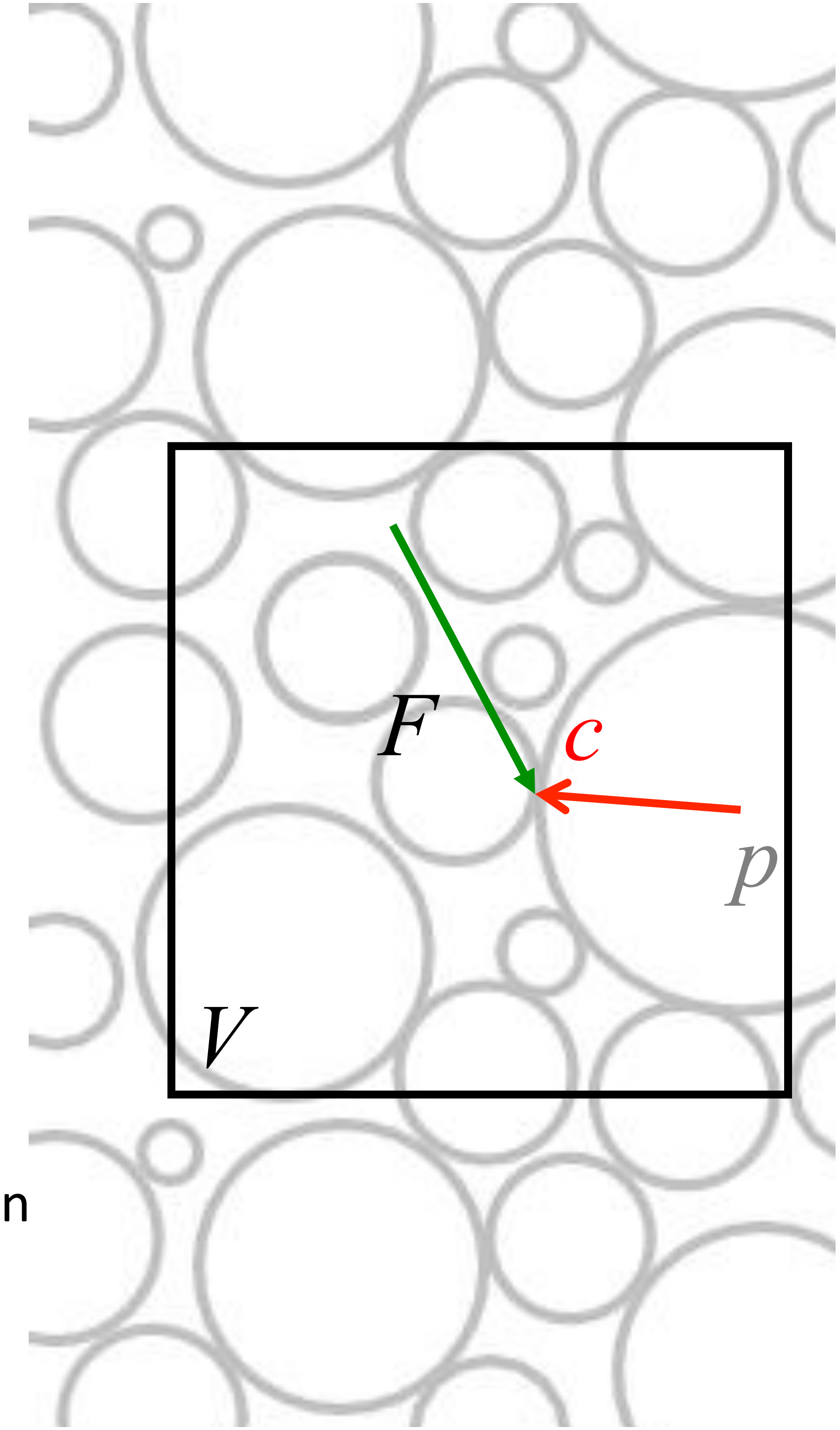
- Unique feature of granular material arises from internal force transmission
- Most fundamental microscopic property of granular materials: irreversible energy dissipation in the course of interaction collision between particles.

Micro-macro transition
Stress tensor

$$Q = \frac{1}{V}\sum_c \left(w_V^p\right) \boldsymbol{l}^{pc}\boldsymbol{F}^c$$

Any quantity:

- Scalar
- Vector
- Tensor: Stress



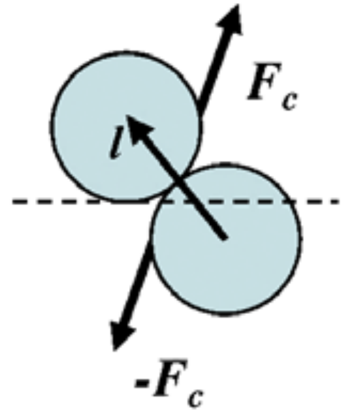
Overview of more complex formulations in [Weinhart et al. (2010)]

Stress tensor

a. Contact stress tensor

Due to the force transmission across interparticle forces

$$\sigma_{ij}^{c} = \frac{1}{V}\sum_{C=1}^{Nc} F_i^{C} l_j$$

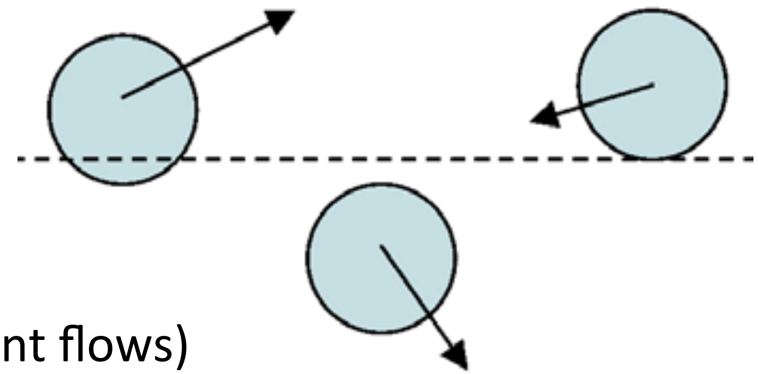


(a)

b. Streaming stress tensor

Due to the motion of a particle relative to the bulk material (Reynolds stress tensor in turbolent flows)

$$\sigma_{ij}^{s} = \frac{\rho_p \phi}{V}\sum_{p=1}^{Np} u'_i u'_j$$


(b)

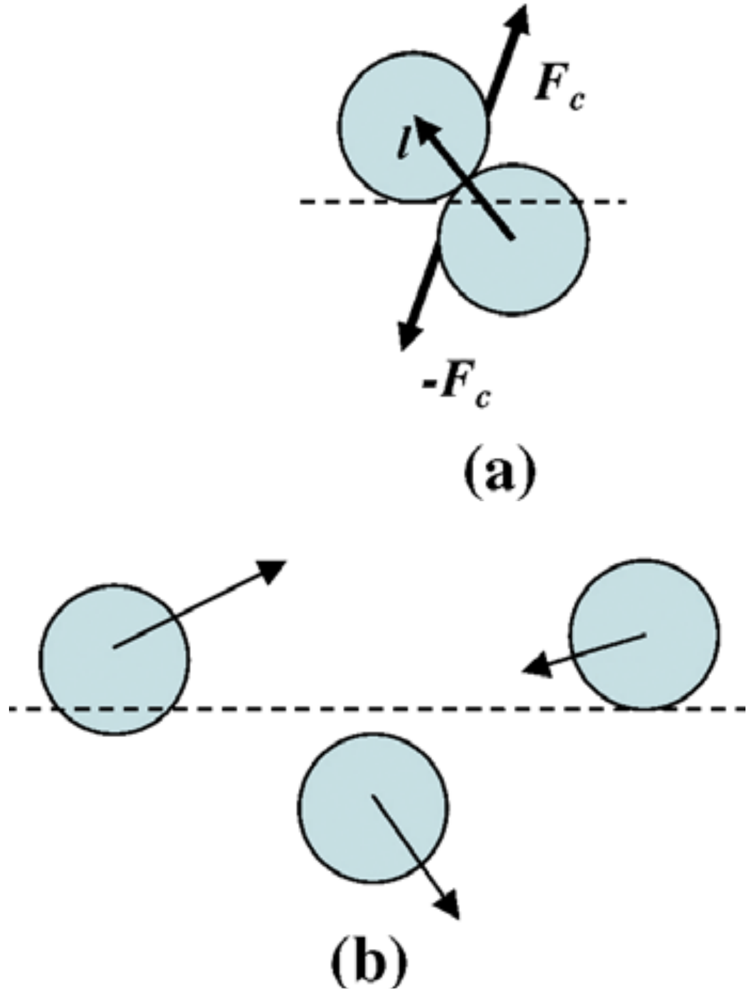
Stress tensor

a. Contact stress tensor

In hoppers, chutes, landslides: ϕ **> 50%**
is usually dominant in common granular flows

b. Streaming stress tensor

can usually be neglected



Hertzian contact law

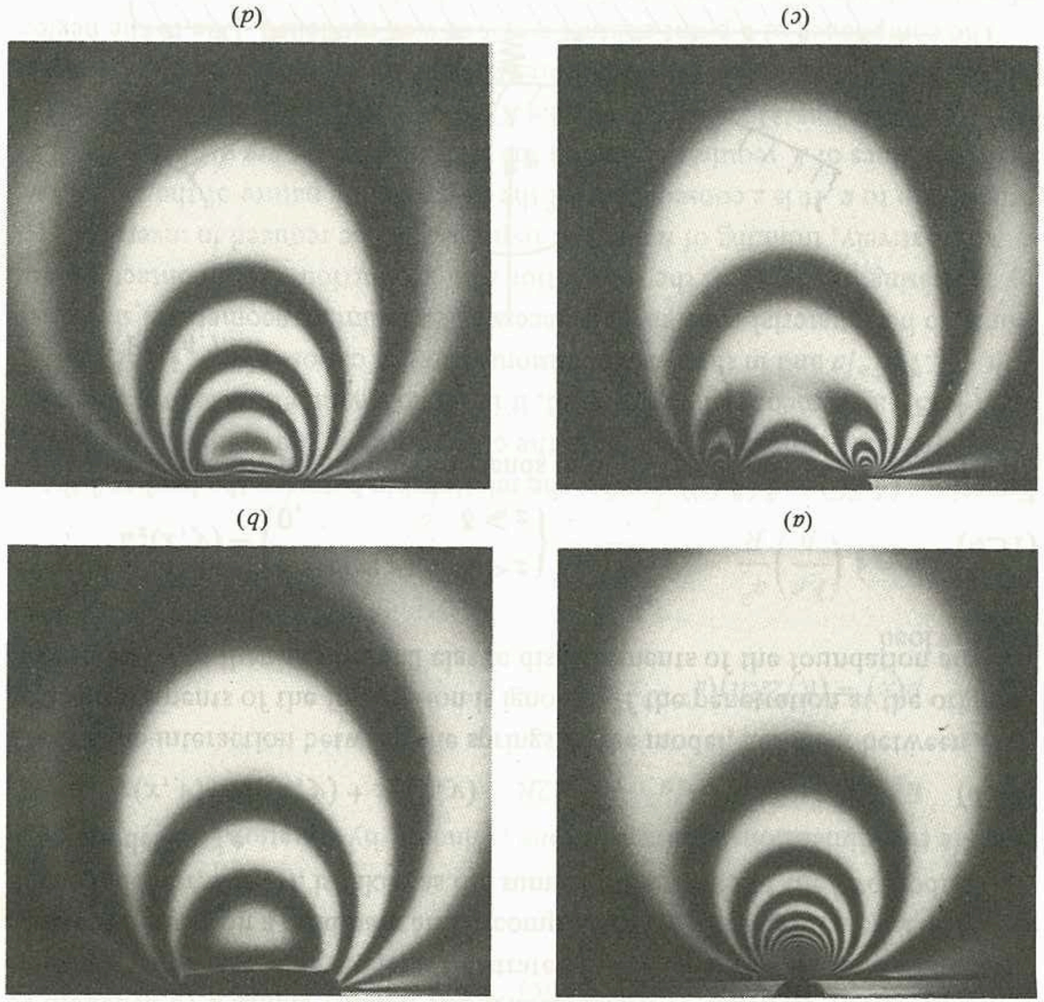
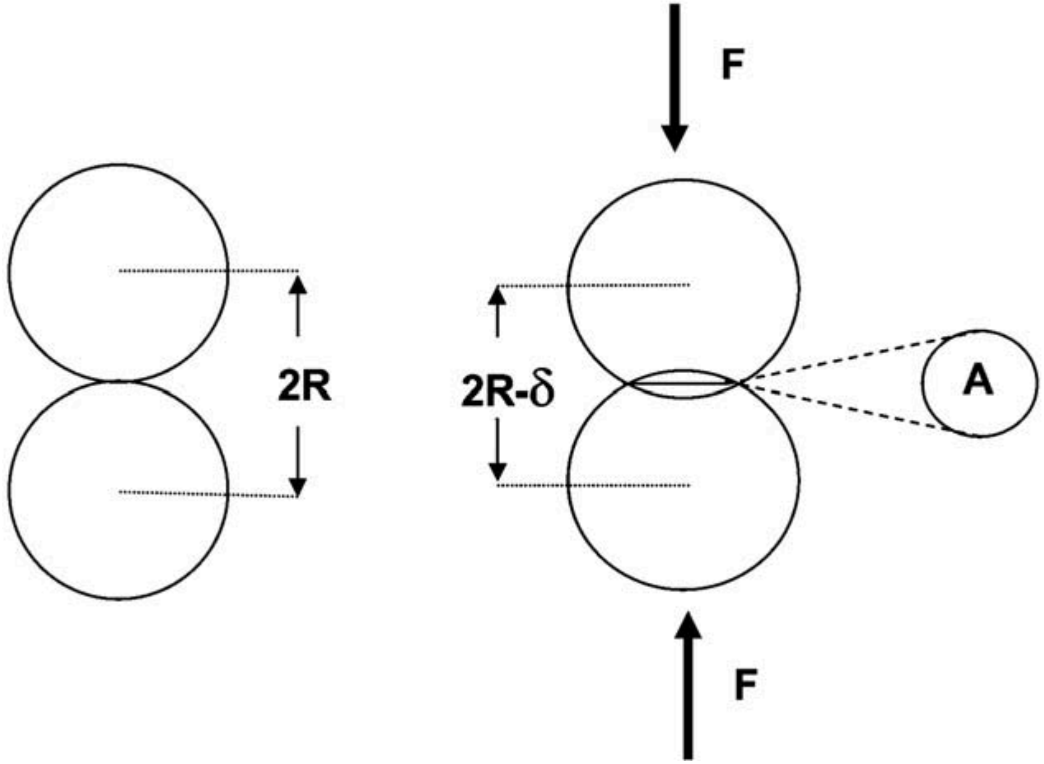


Fig. 4.6. Two-dimensional photo-elastic fringe patterns (contours of principal shear stress): (a) point load (§2.2); (b) uniform pressure (§2.5(a)); (c) rigid flat punch (§2.8); (d) contact of cylinders (§4.2(c)).

Hertzian contact law



Contact stiffness

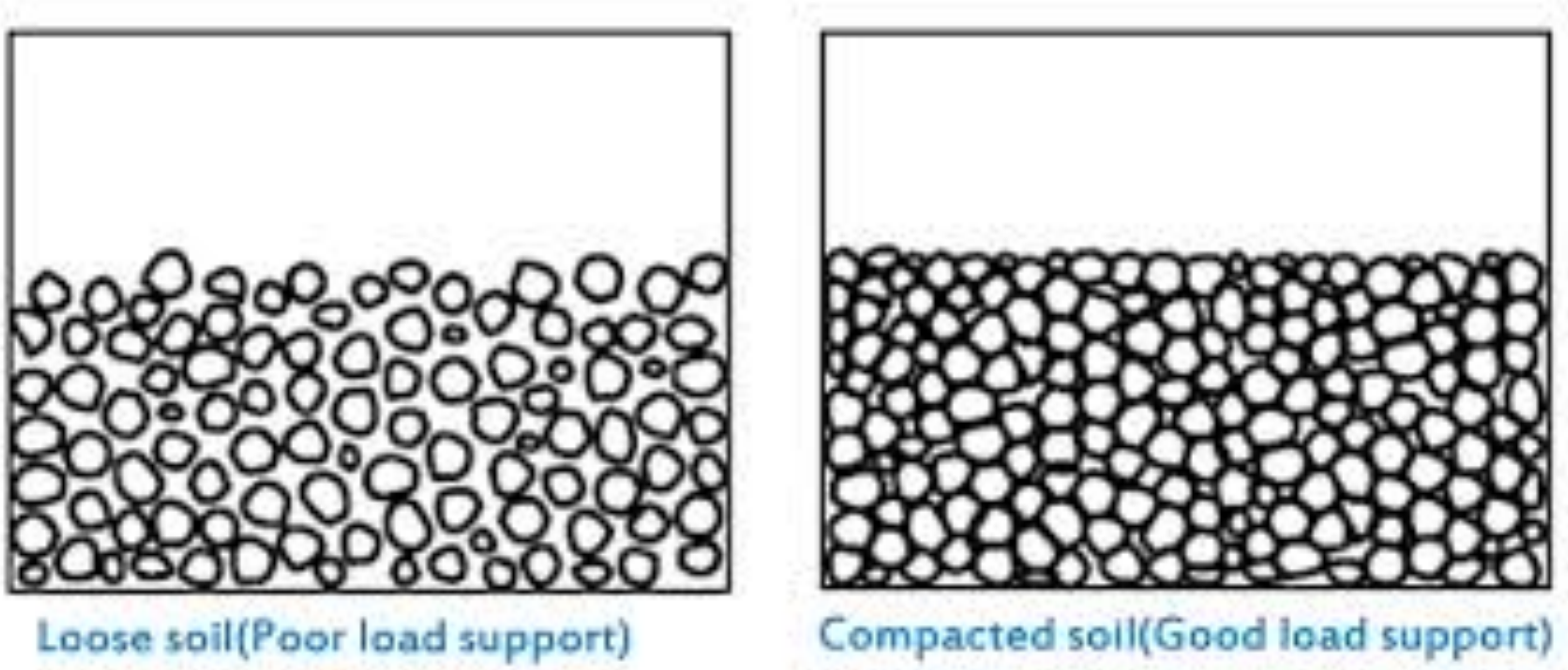
$$F = \frac{\mathbf{2}}{\mathbf{3}} R^{\mathbf{1/2}} \left(\frac{E_g}{\mathbf{1} - \nu_g^{\mathbf{2}}} \right) \delta^{\mathbf{3/2}} = k(\delta^{\mathbf{1/2}})\delta \qquad \rightarrow \qquad k \propto \delta \propto p$$

the deformation δ increases
with applied pressure p

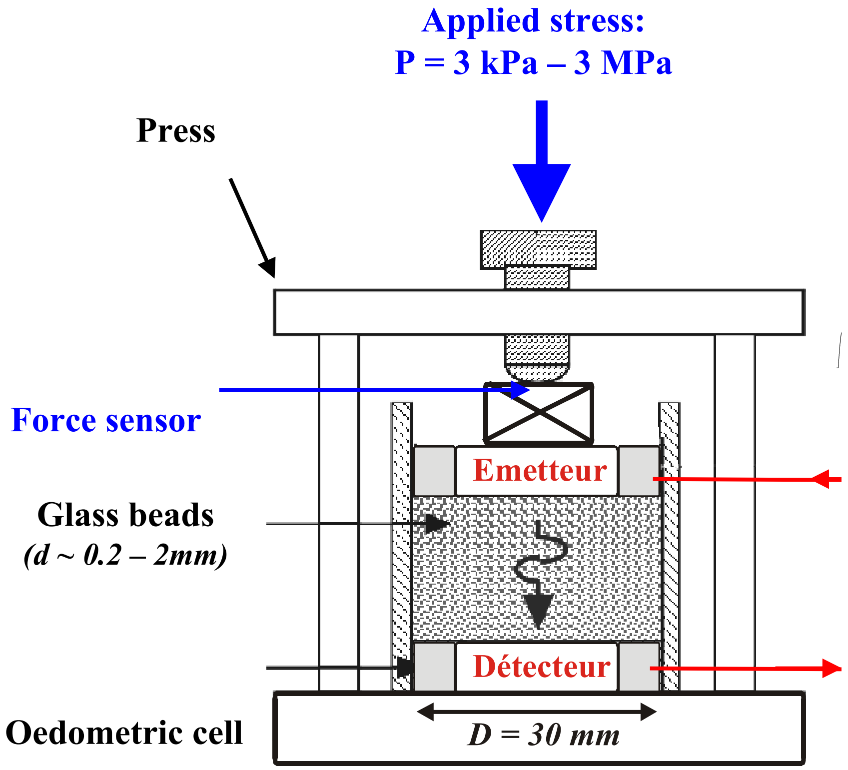
Solid state

Small strain (elastic) stiffness

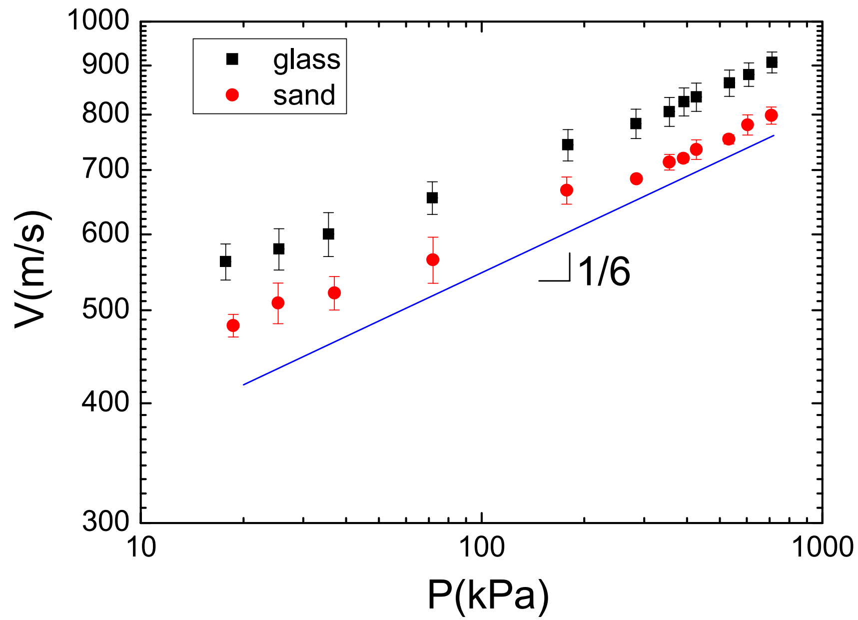
Classical solids: elastic stiffness is a material constant
Granular materials: elastic stiffness depends on **pressure** and **volume fraction**



Granular Elasticity: how to characterize it?

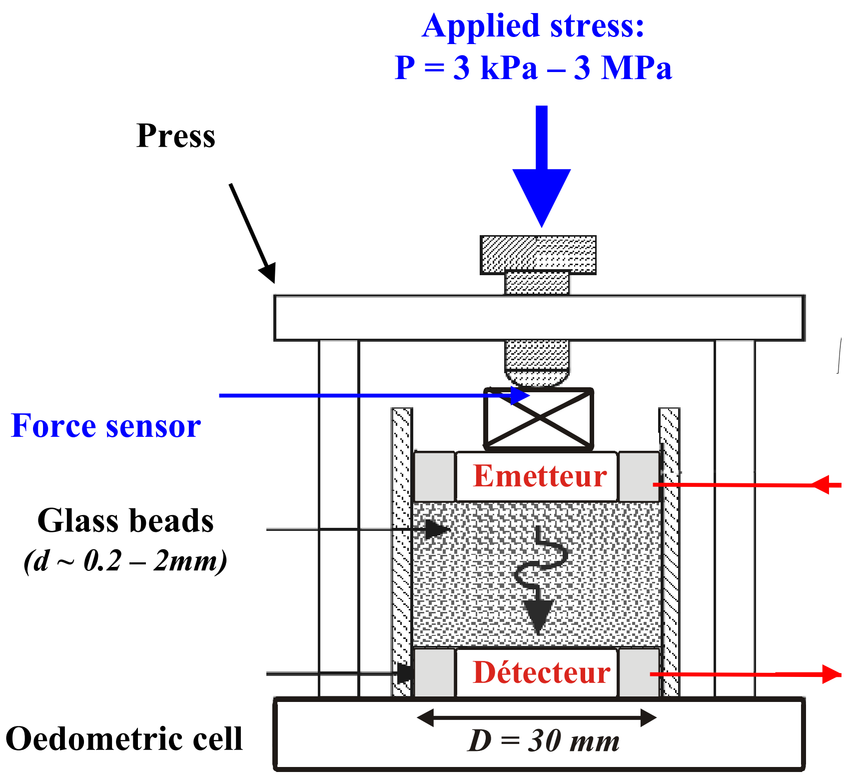


$$v_p = \sqrt{\frac{K + \frac{4}{3} G}{\rho}} \qquad v_s = \sqrt{\frac{G}{\rho}}$$



[Domenico (1977), Jia& Mills (2001), Wildenberg et al (2013),…]

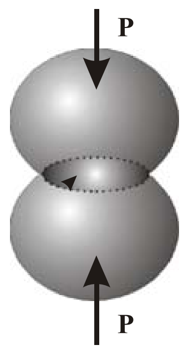
Granular Elasticity



$v_p = v_p\,(\phi,\ p)$ $v_s = v_s\,(\phi,\ p)$

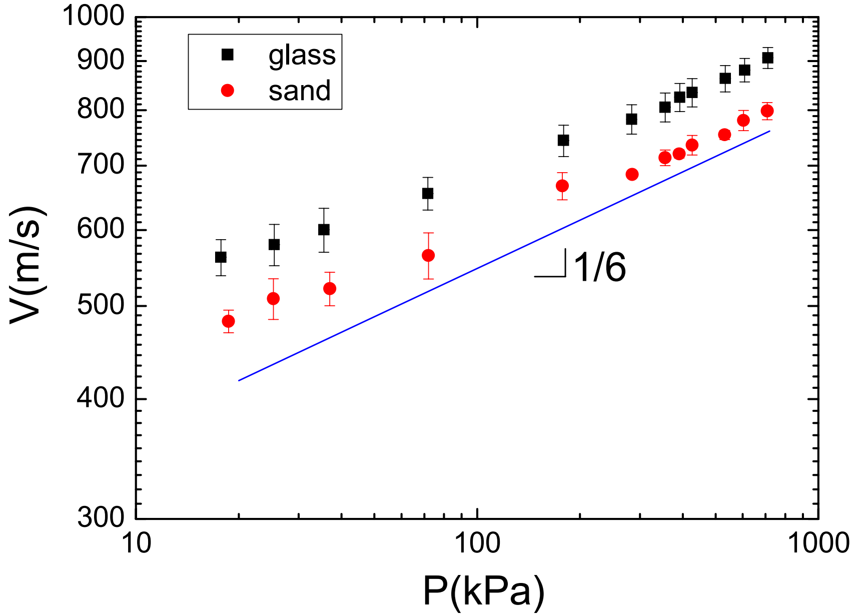
dependence on (macro):

pressure, volume fraction



Hertzian contact: $k \propto p^{1/3}$

$v_p \propto p^{1/6}$ $v_s \propto p^{1/6}$



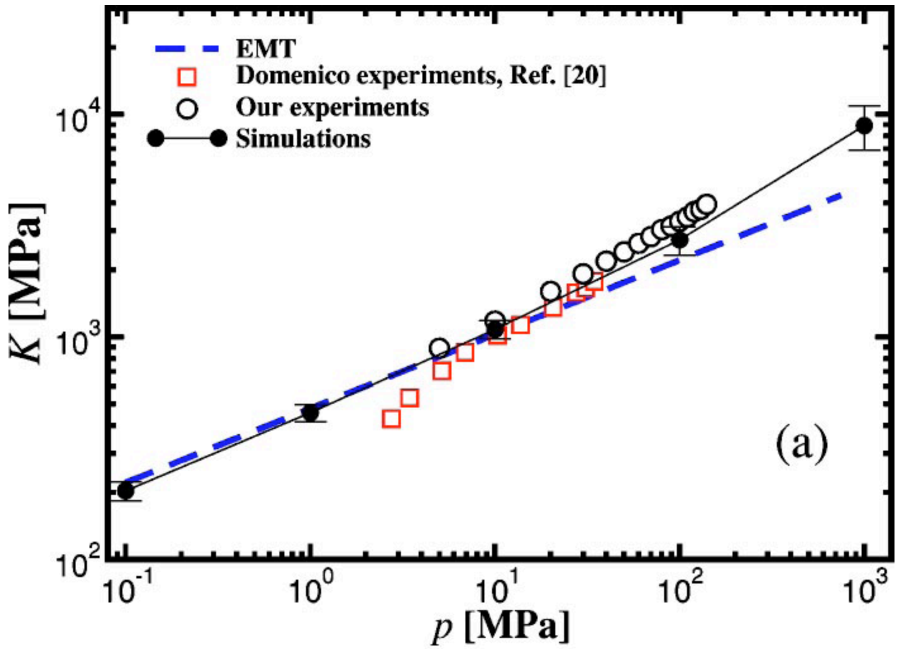
[Domenico (1977), Jia& Mills (2001), Wildenberg et al (2013),…]

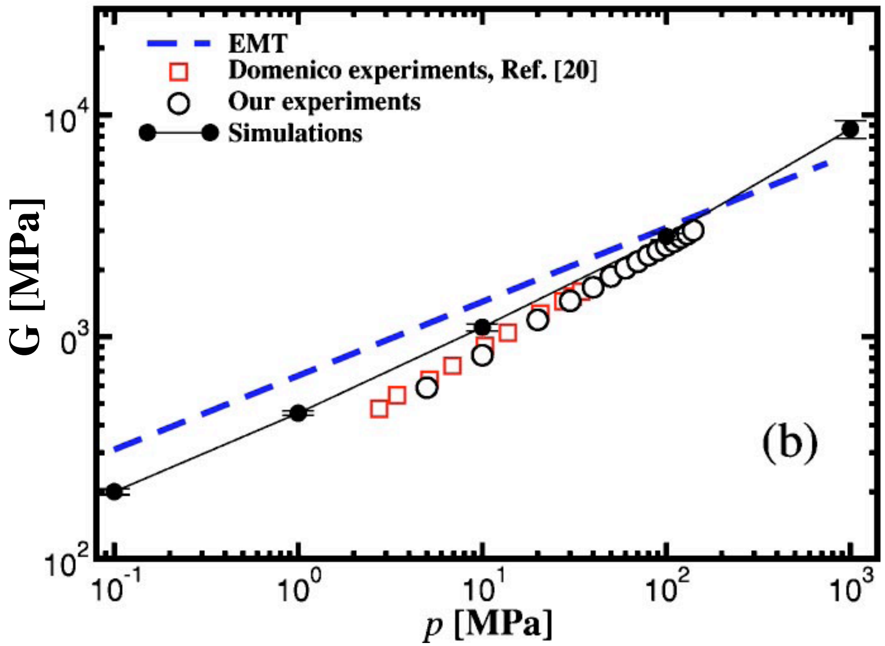
Small strain (elastic) stiffness

$$G_{bulk} \propto \frac{k}{R}$$

[Bathurst and Rothenburg, J. Appl. Mech. (1988)]

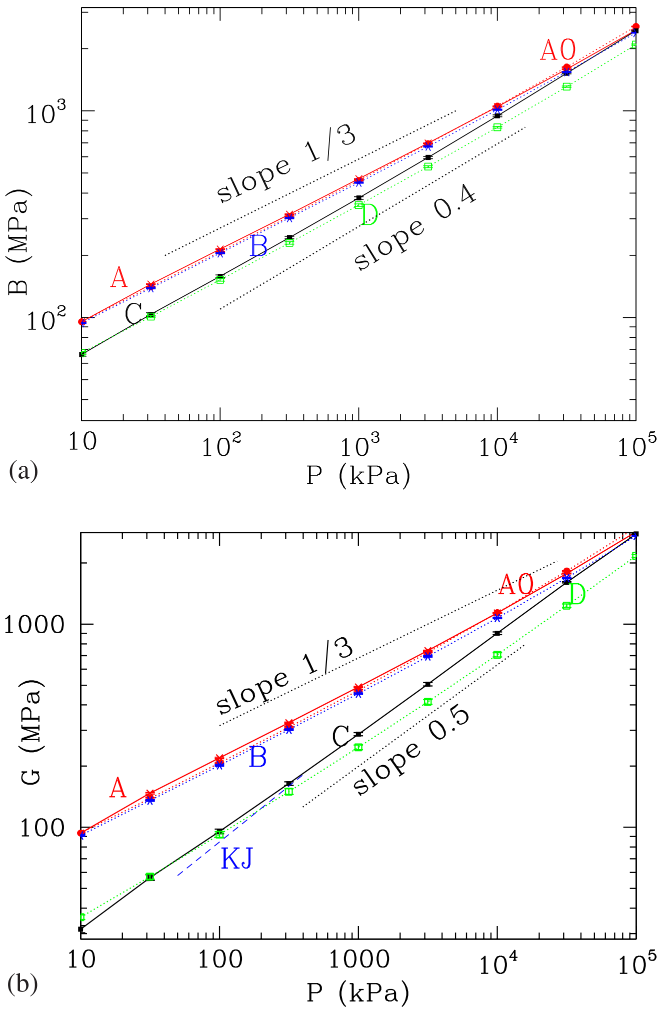
Because of Hertzian interaction we expect: $K(p) \propto G(p) \propto p^{1/3}$





[Gland et al., PRE (2005)]

Granular Elasticity



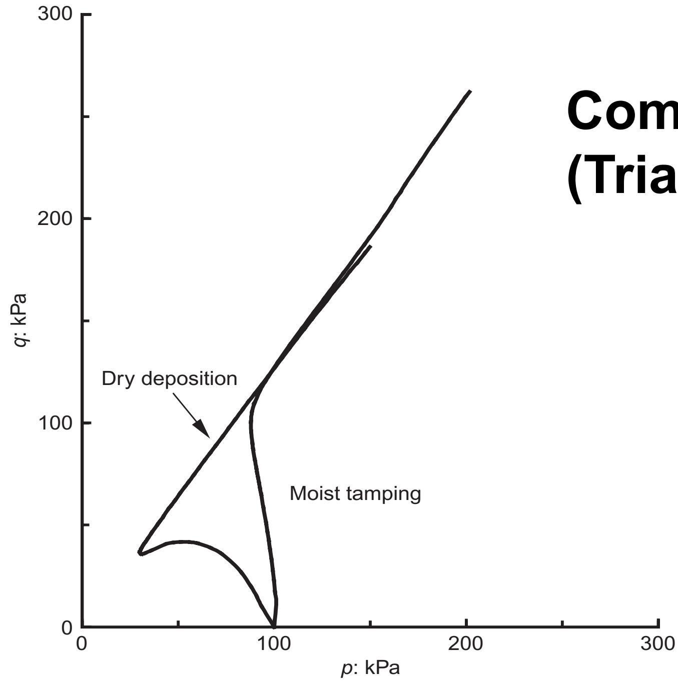
[Agnolin et al (2007)]

$v_p = v_p\,(\phi, \mathbf{p})$ $\quad v_s = v_s\,(\phi, \mathbf{p})$

Not enough!

Response depends on preparation

[Chen et al (1988), Agnolin et at (2005), Jia (2005)
Kuwano&Jardine (2002), Ezaoui et al. (2009), …]

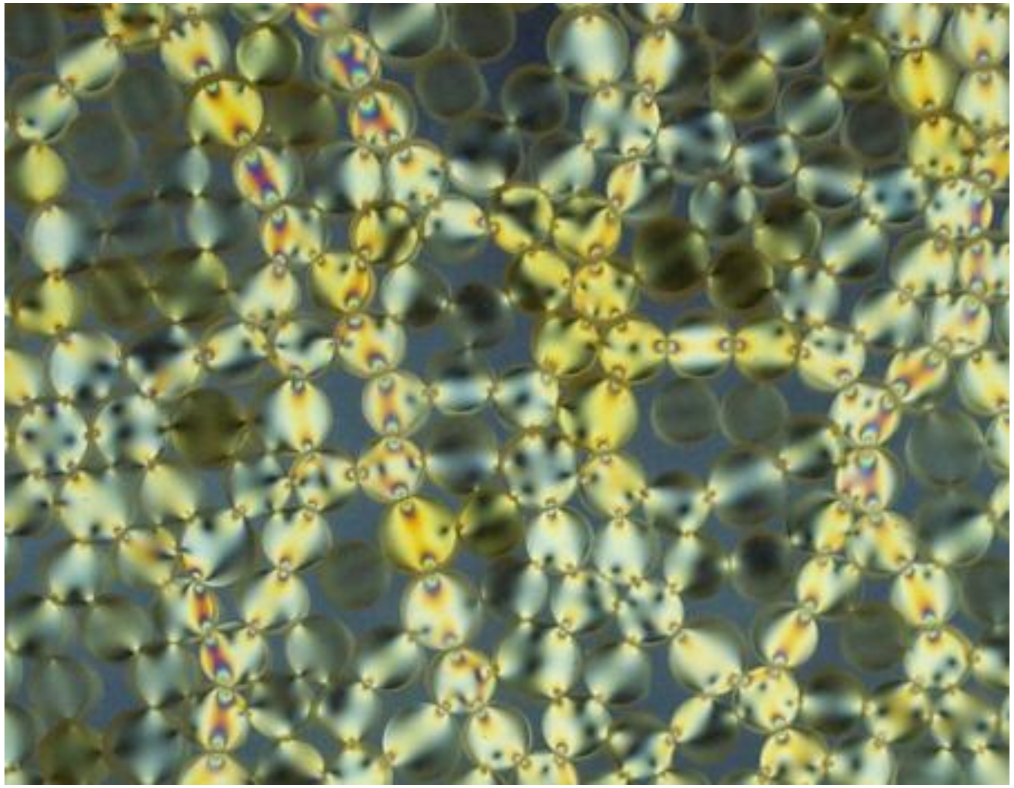


Compression (Triaxial) test

[Yoshimine (1988), Yang, Li and Yang (2008), …]

WHY?

microstructure matters



[Behringer (Duke)]

DEM simulations: wave propagation

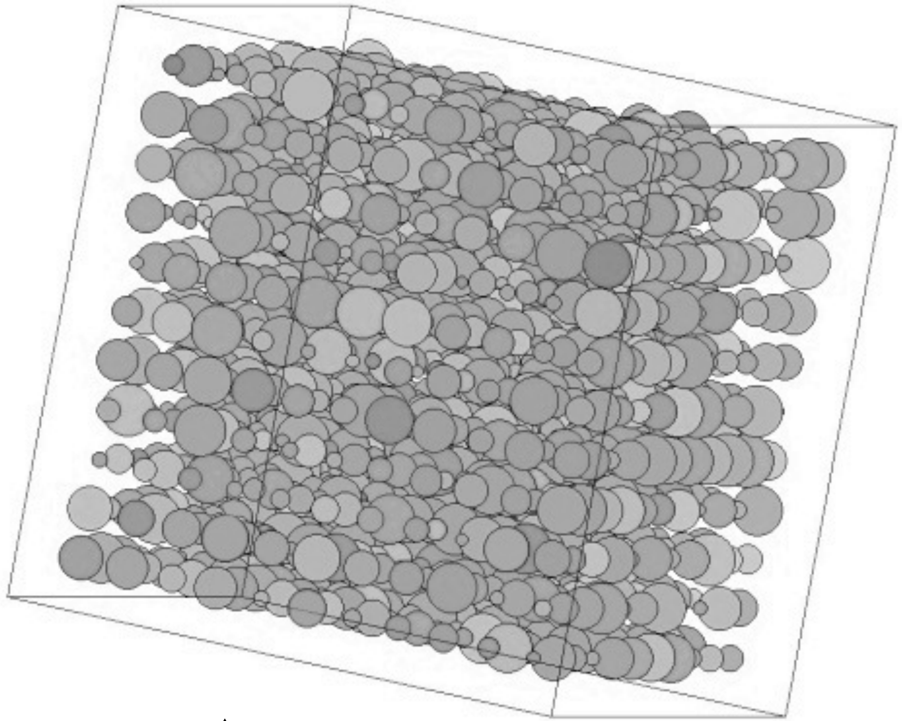
We can look closer: simulation of wave propagation by DEM

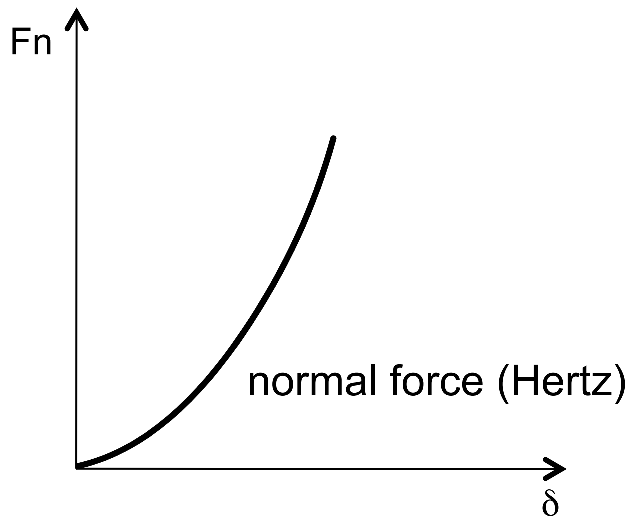
Material idealization – DEM simulations

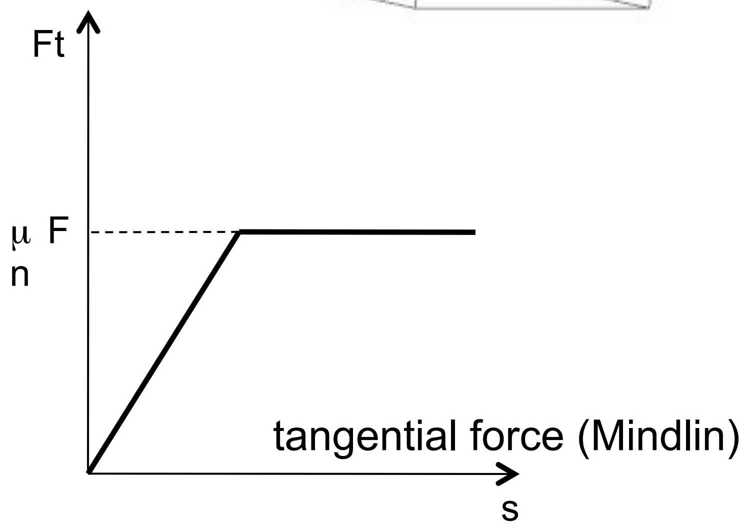
Real Material
Random aggregate
Elastic Frictional particles
Interact by mean of contact forces

Idealized Material
Random aggregate of
IDENTICAL
FRICTIONAL
SPHERES

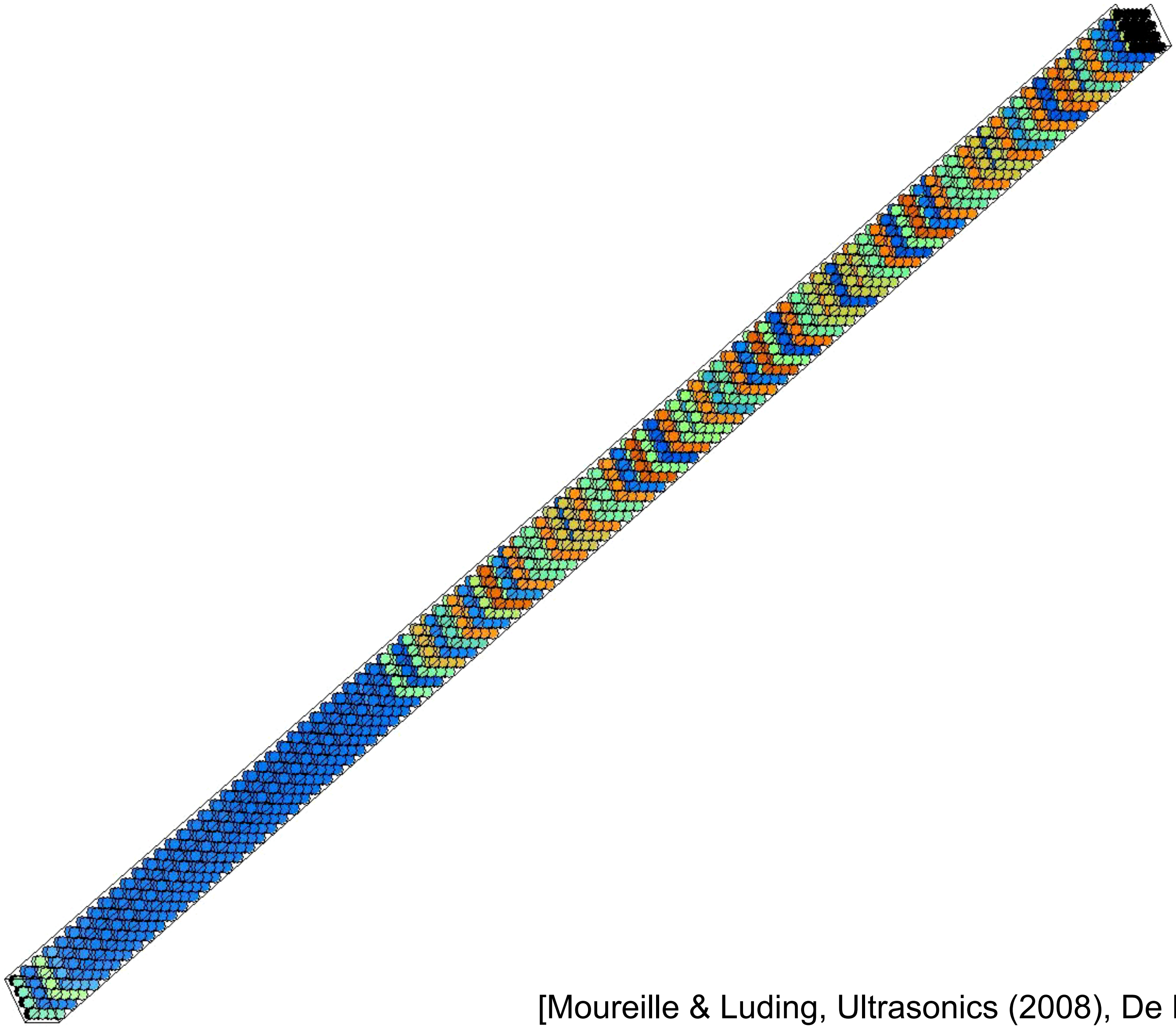
Hetz-Mindlin interaction





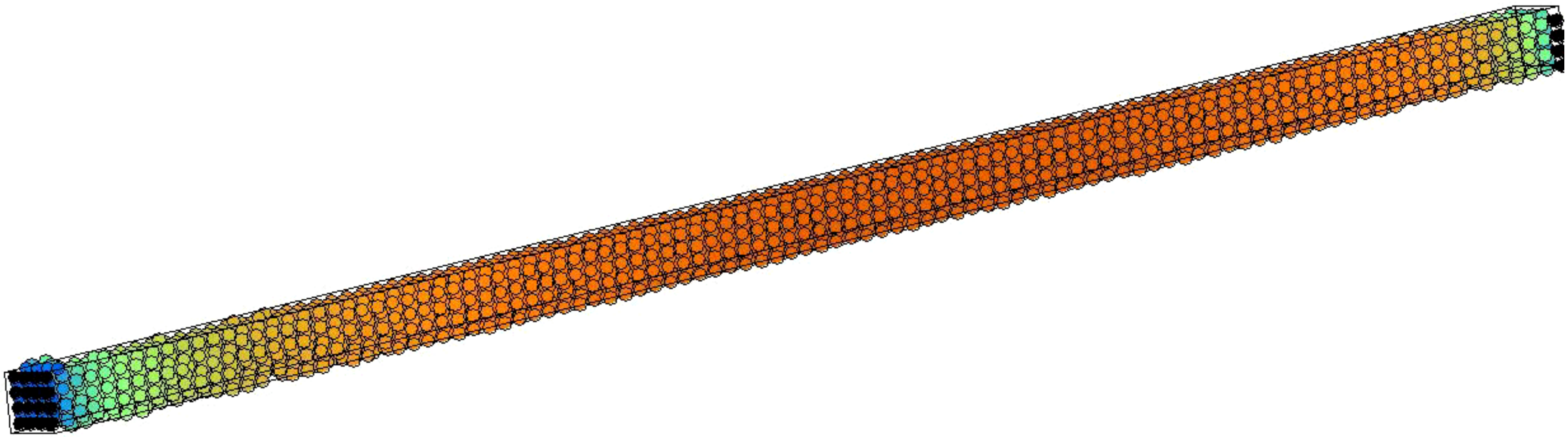


DEM simulations: wave propagation - lattice



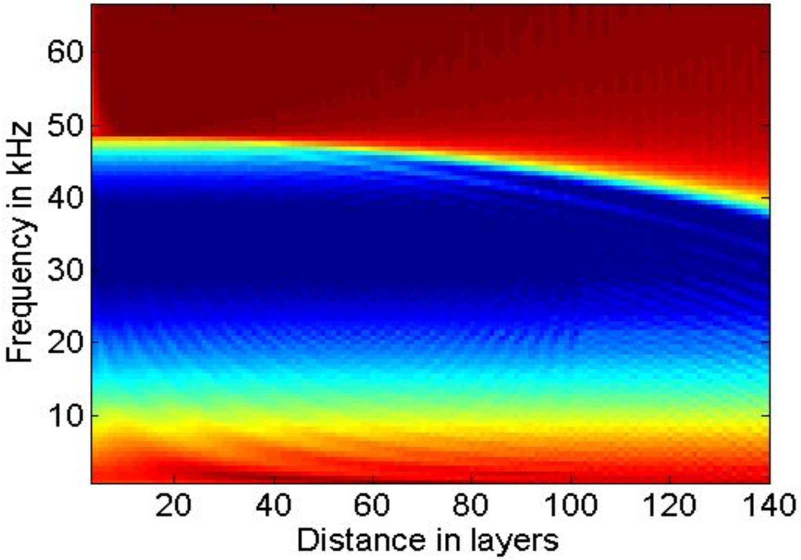
[Moureille & Luding, Ultrasonics (2008), De Mol, M.Sc thesis (2013)]

DEM simulations: wave propagation - random

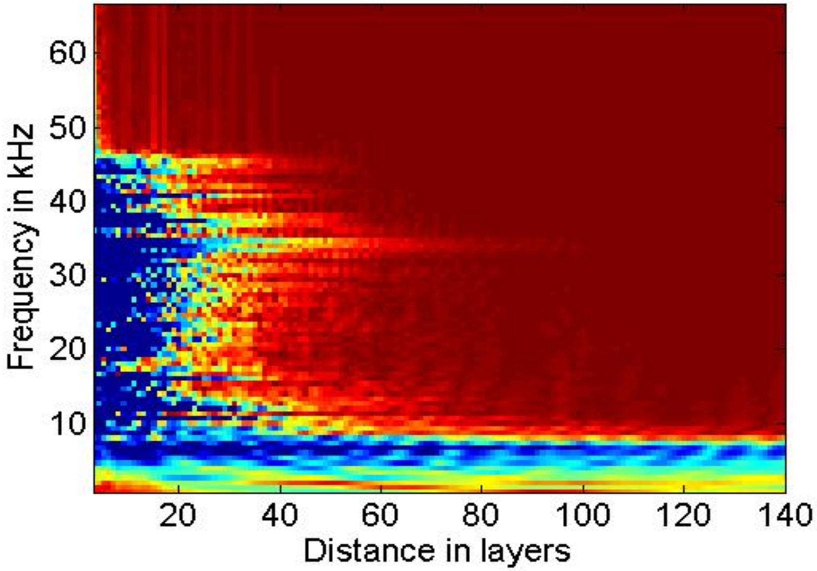


[Moureille & Luding, Ultrasonics (2008), De Mol, M.Sc thesis (2013)]

DEM simulations: wave propagation
- dispersion relation



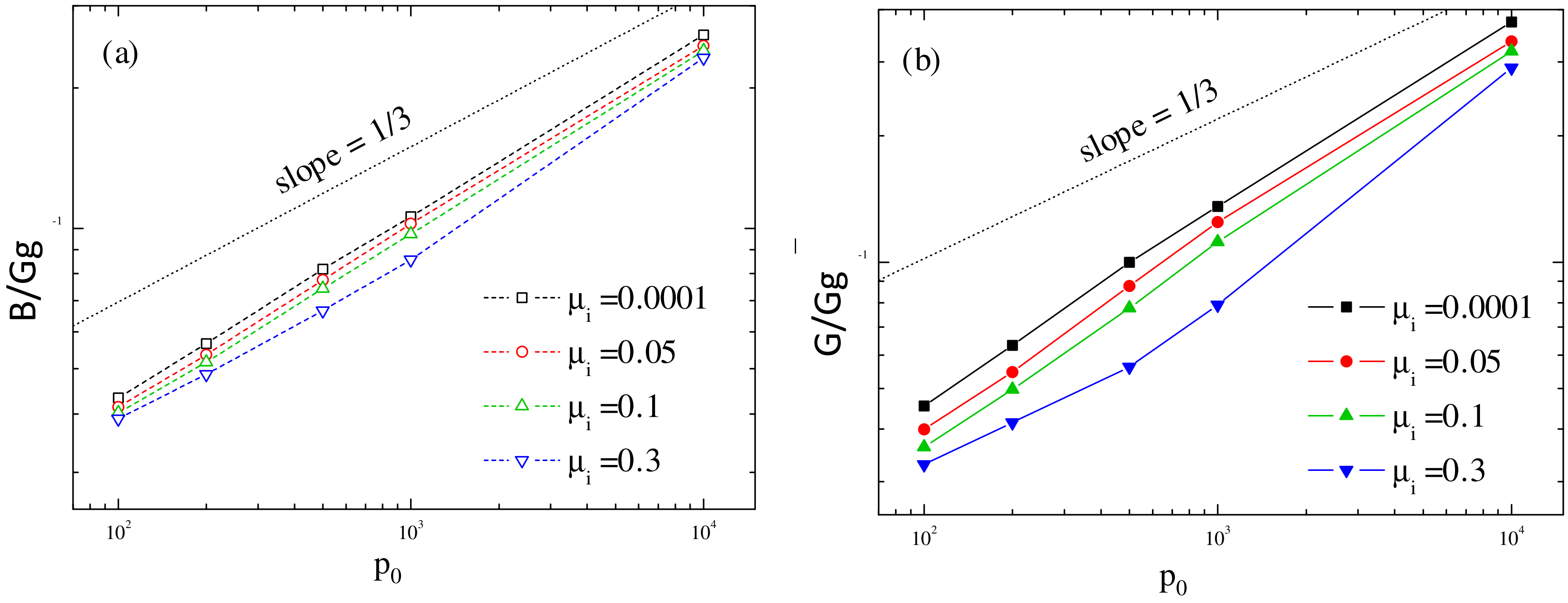
regular lattice



random system

[Moureille & Luding, Ultrasonics (2008), De Mol, M.Sc thesis (2013)]

Elastic moduli dependence on microstructure



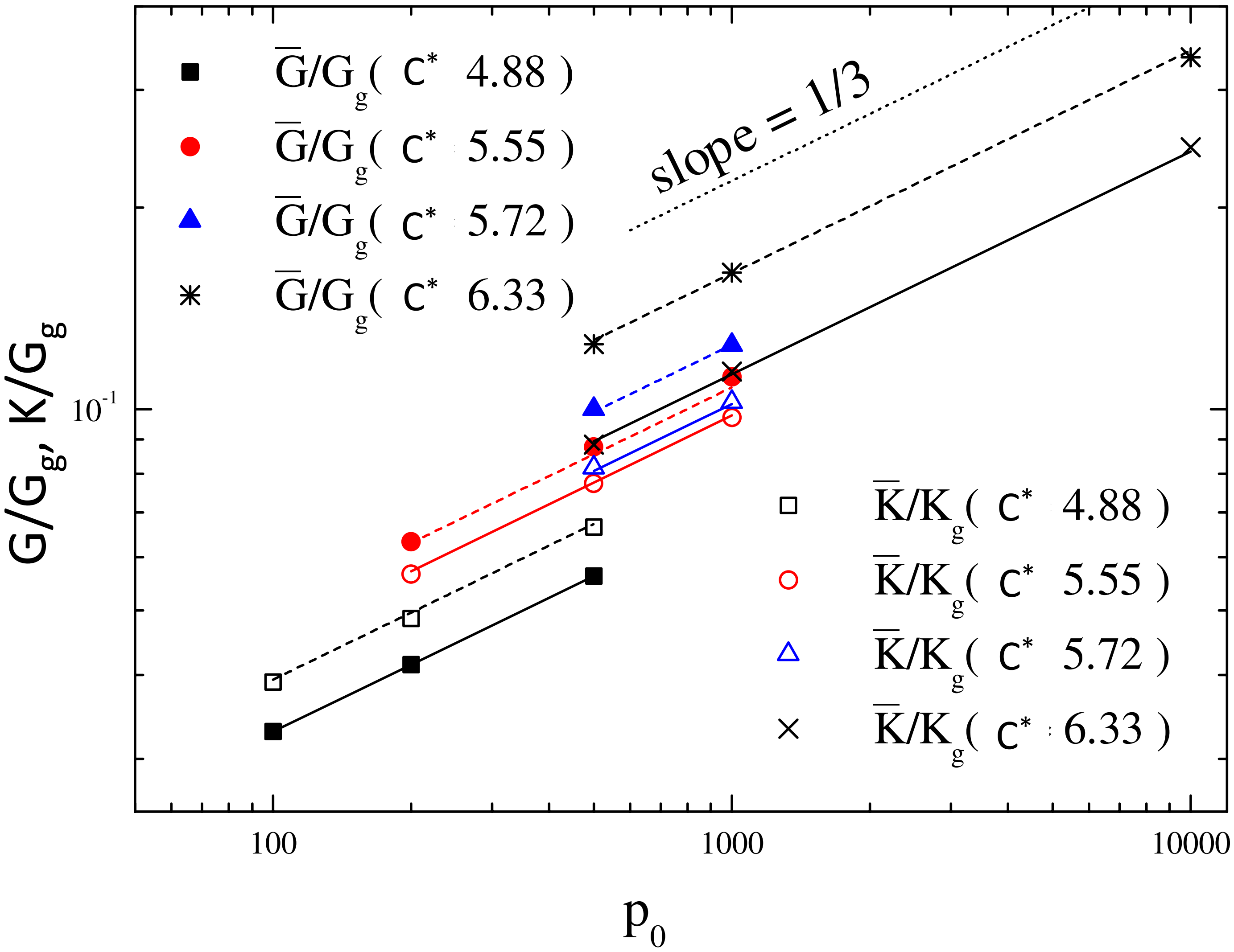
Let's collect packings with the same Z*…

Coordination number

Avarage number of contacts in the system

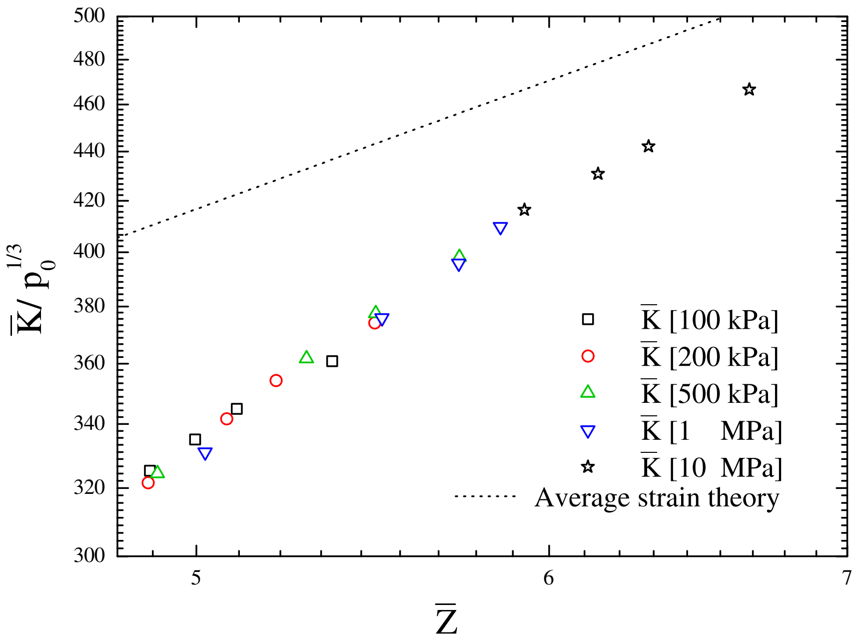
$$\overline{Z} = \frac{2Nc}{Np}$$

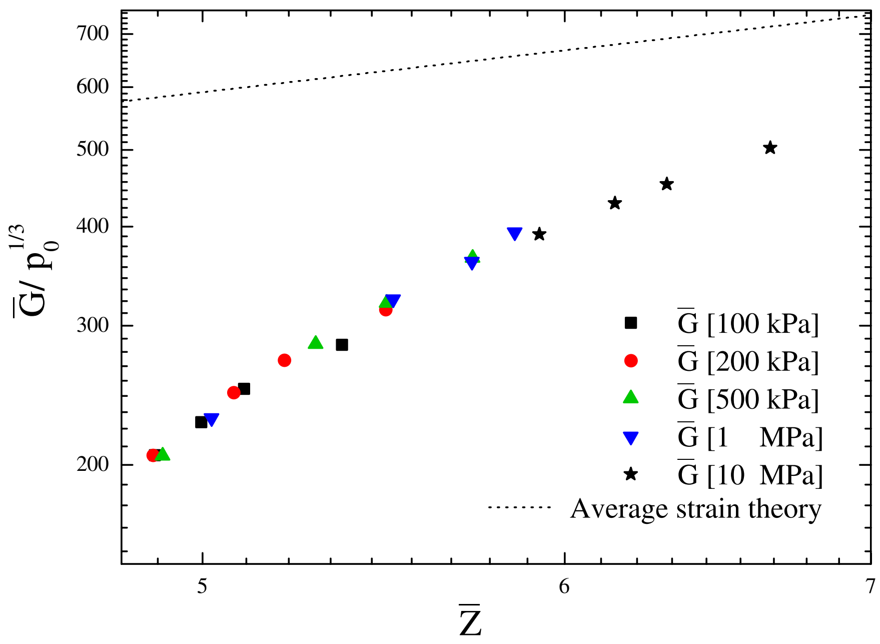
Elastic moduli vs Pressure



In the case of same C*: Hertzian scaling is recovered (also close to jamming)

Dependence on coordination number





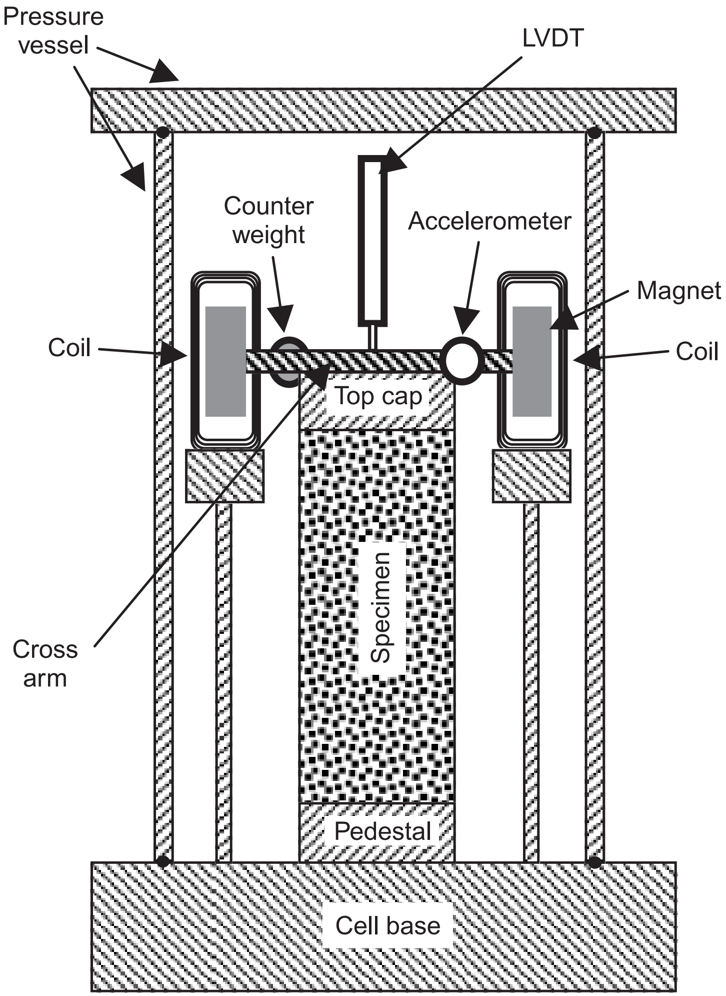
Coordination number

Avarage number of contacts in the system

$$\overline{Z} = \frac{2Nc}{Np}$$

[Magnanimo et al., EPL (2008)]

Resonant Column experiments at U. Bochum



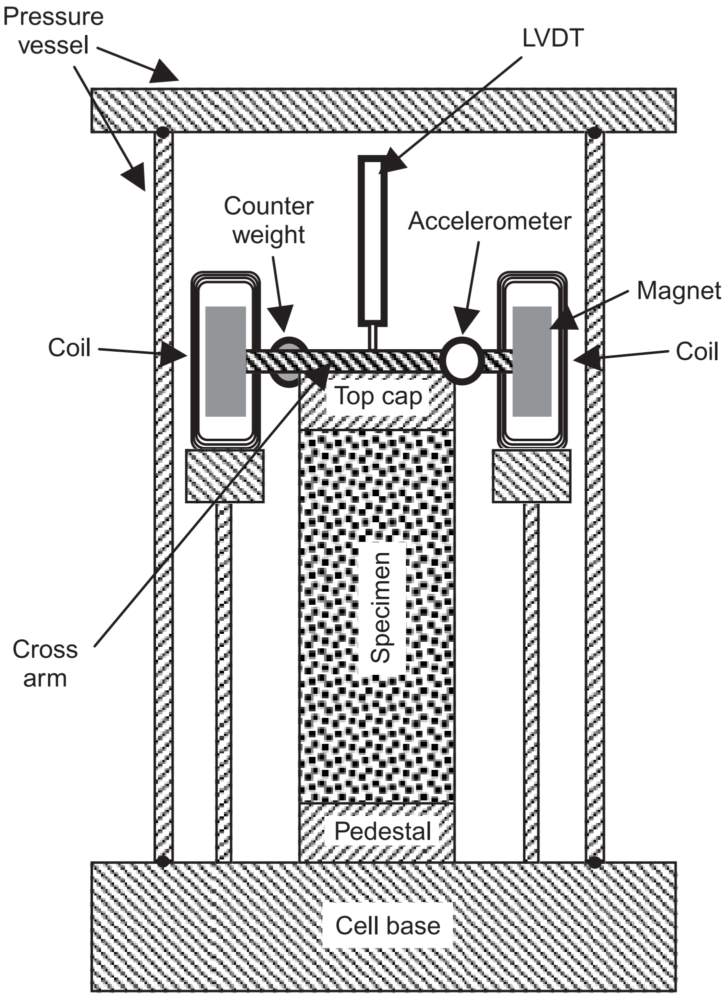
(a)


(b)

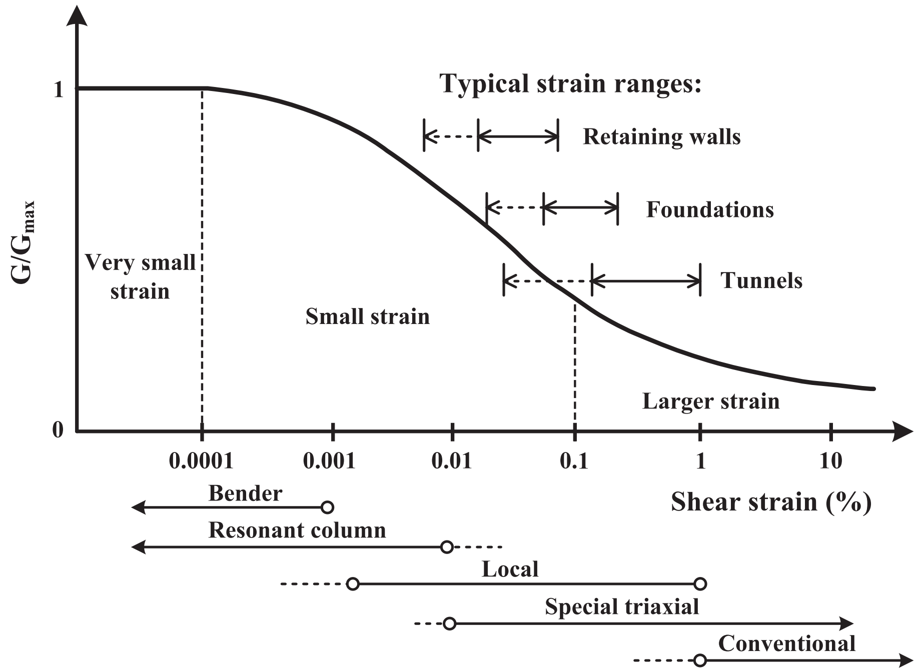
An electromagnet head, with four magnets attached to the specimen cap. Torsion is applied by running current through the four coils

[Clayton (2011)]

Resonant Column experiments at U. Bochum

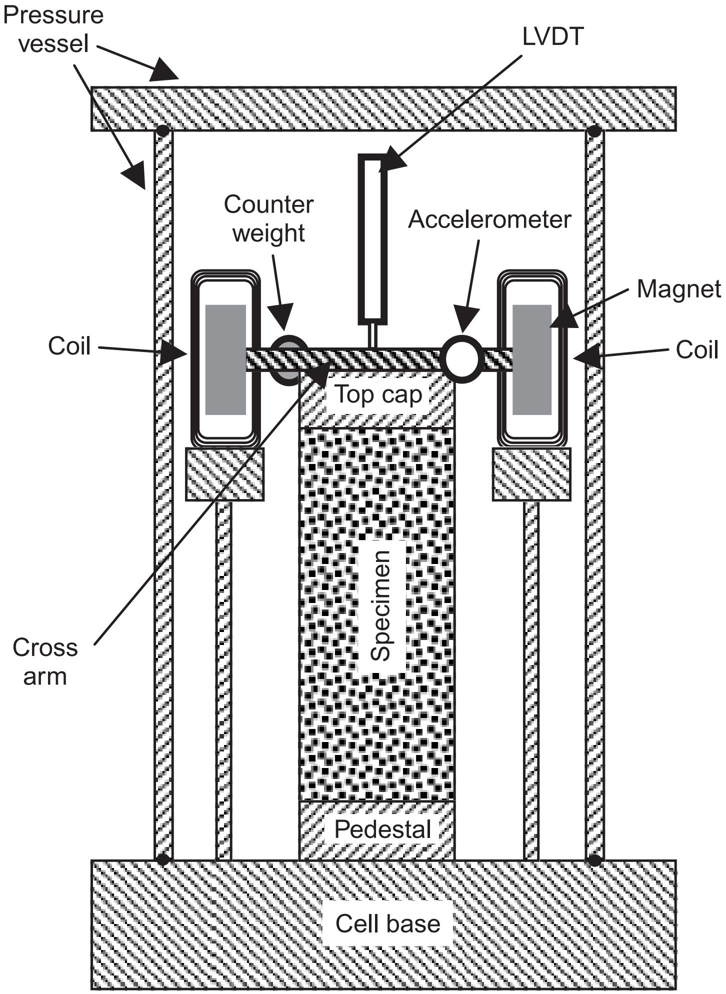


(a)

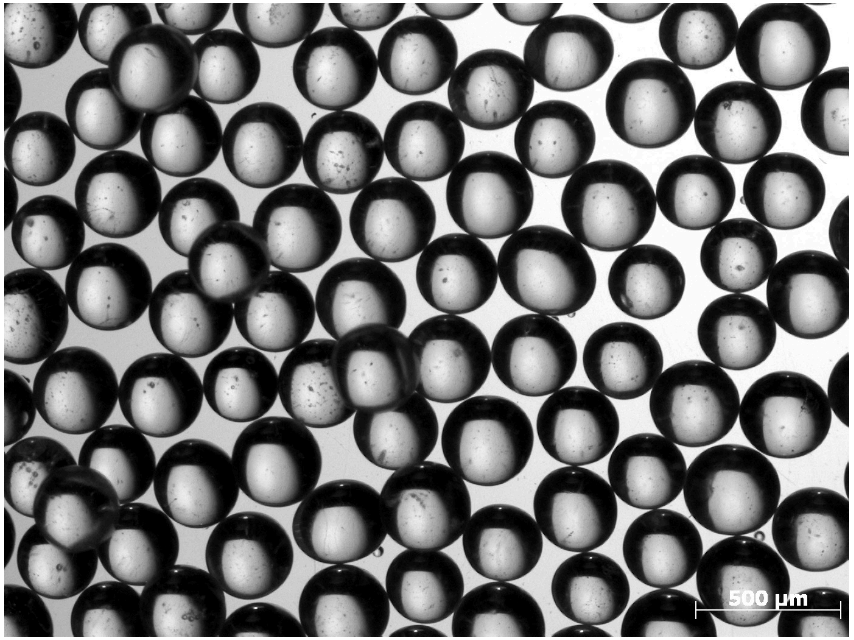


[Atkinson (1991), Mair (1993)]

Resonant Column experiments at U. Bochum

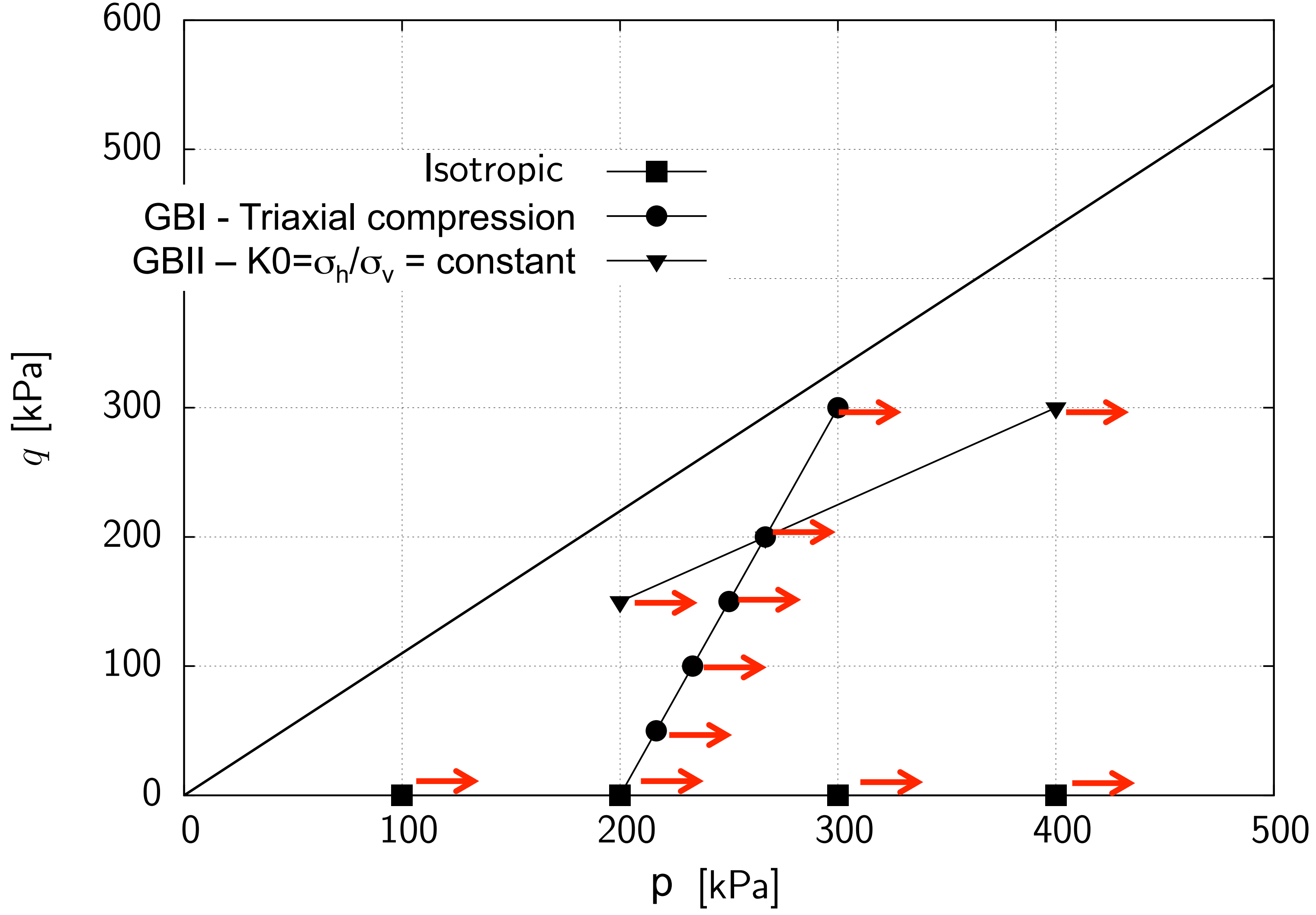


(a)

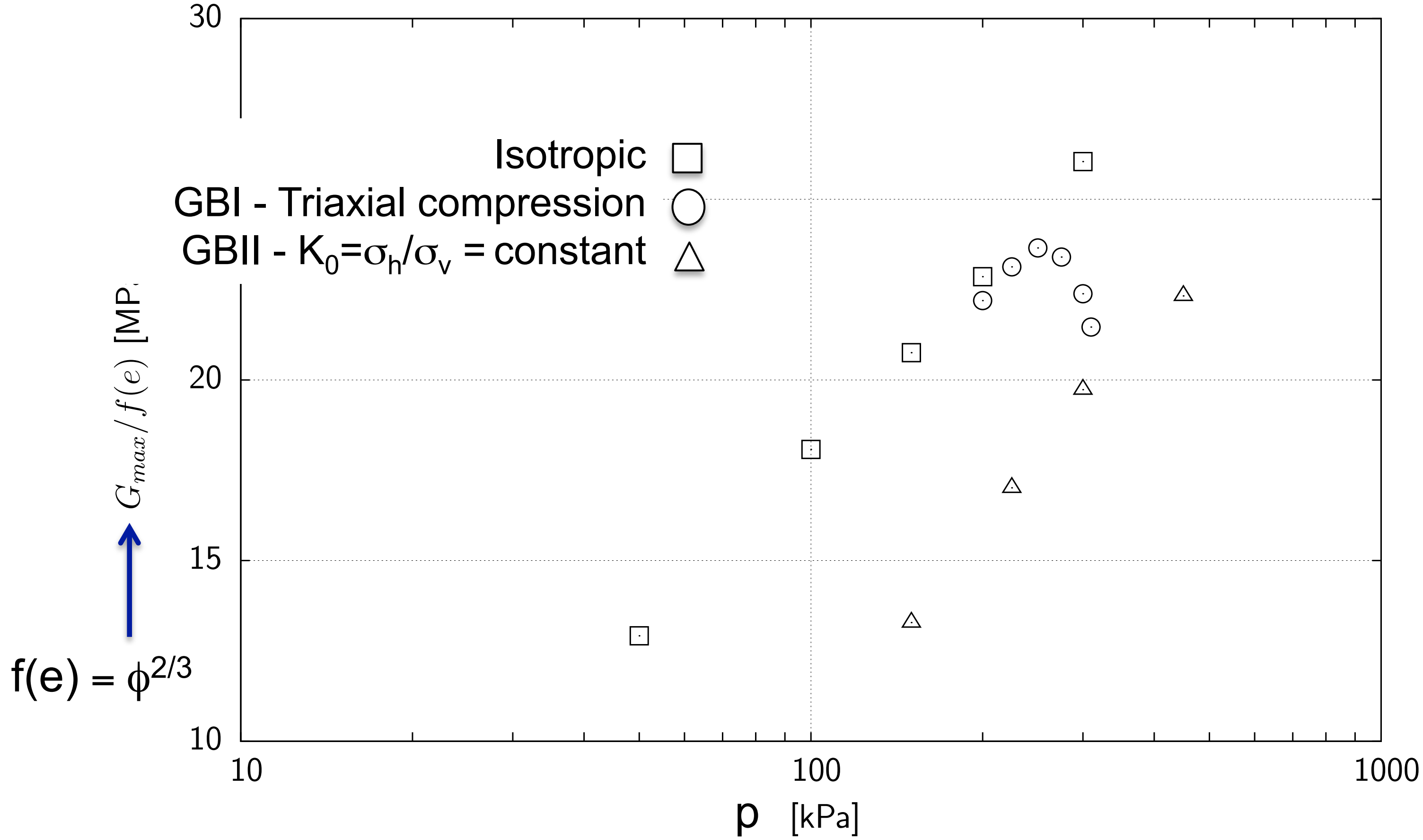


Glass beads d= 1.25mm
Sample: 10cm by 20 cm

Resonant Column Tests

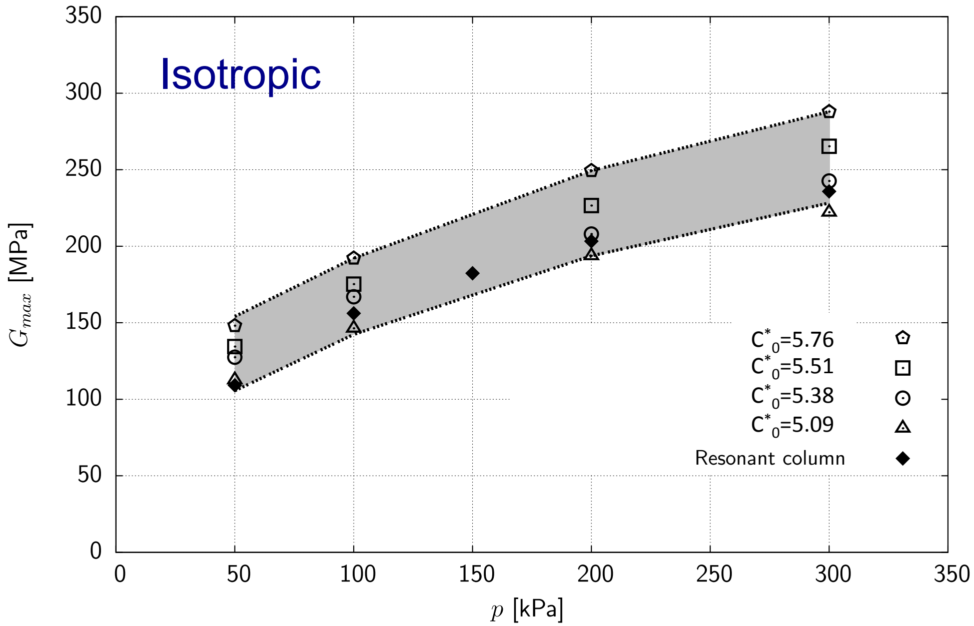


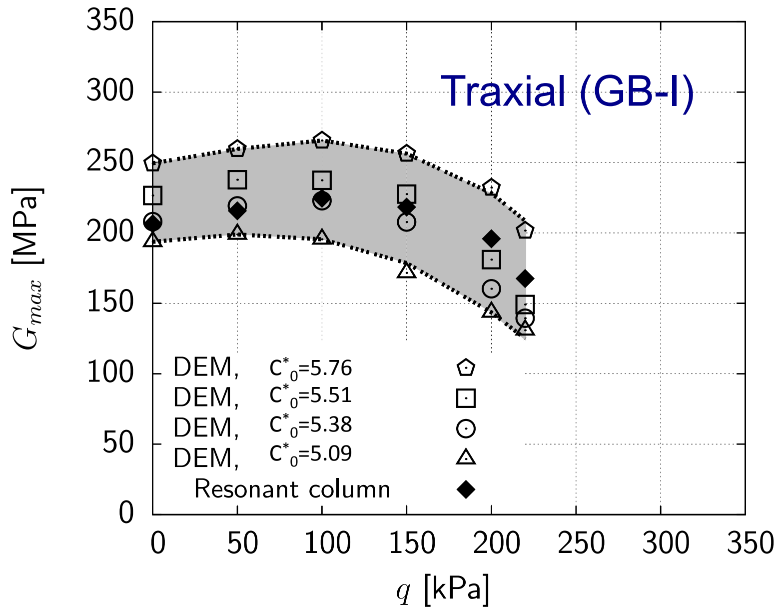
Resonant Column Tests

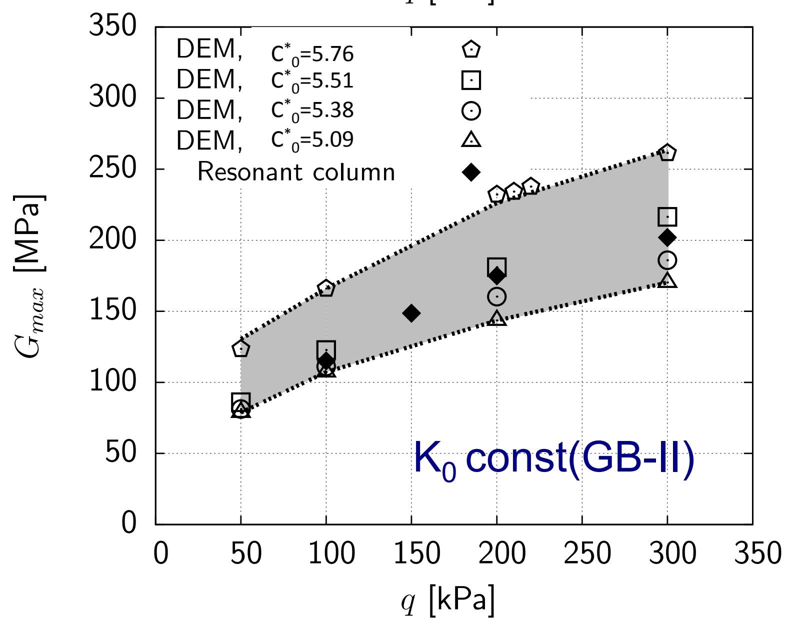


The three stress paths can never be described by the dependence on p and ϕ alone

Reproducing Resonant Column data





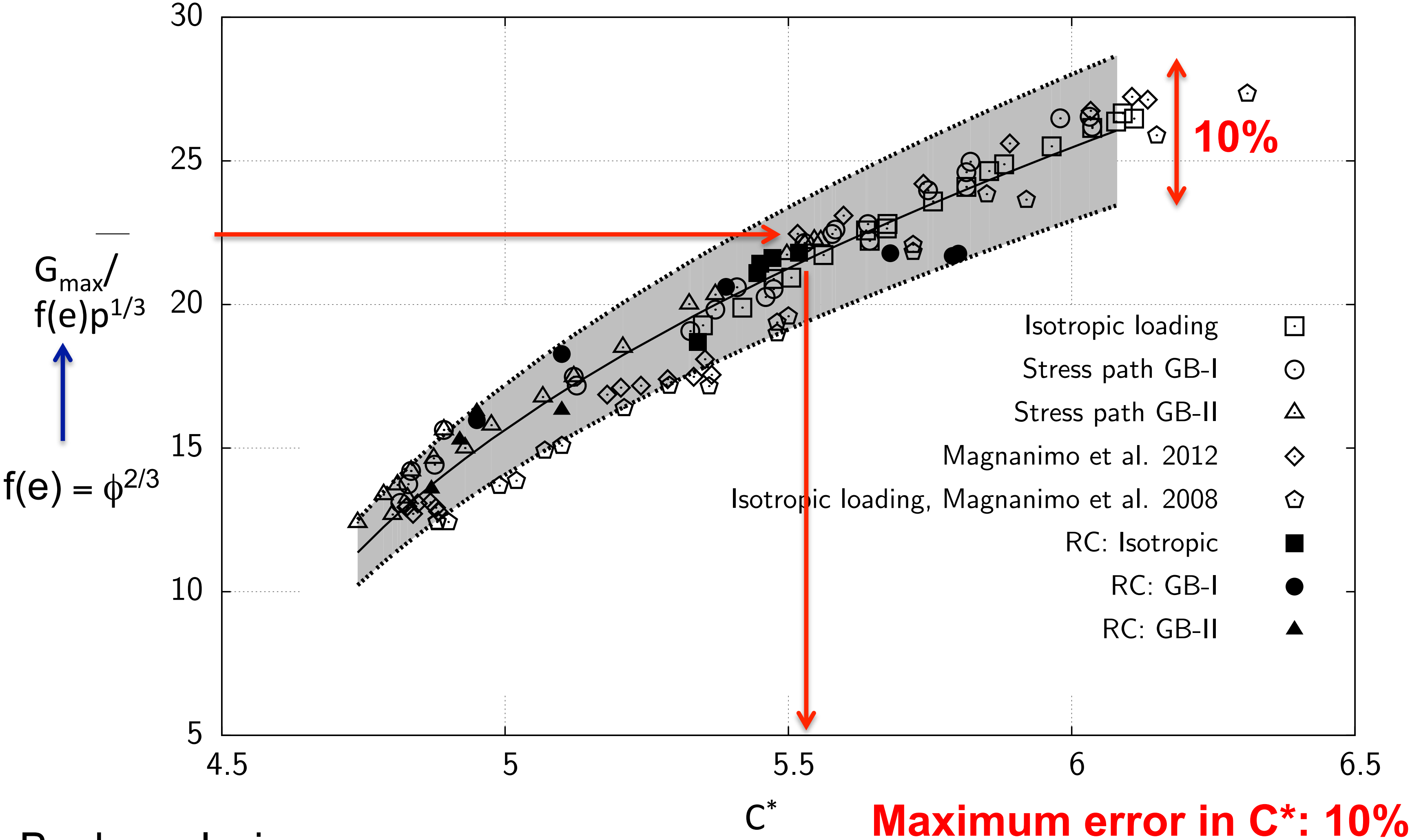


- DEM (same particle characteristics): Isotropic, Triaxial and K_0constant

- We calculate G_{max} along the paths
- **Only calibration parameter C^*_0**

RC tests lie in between DEM data

Unique scaling law

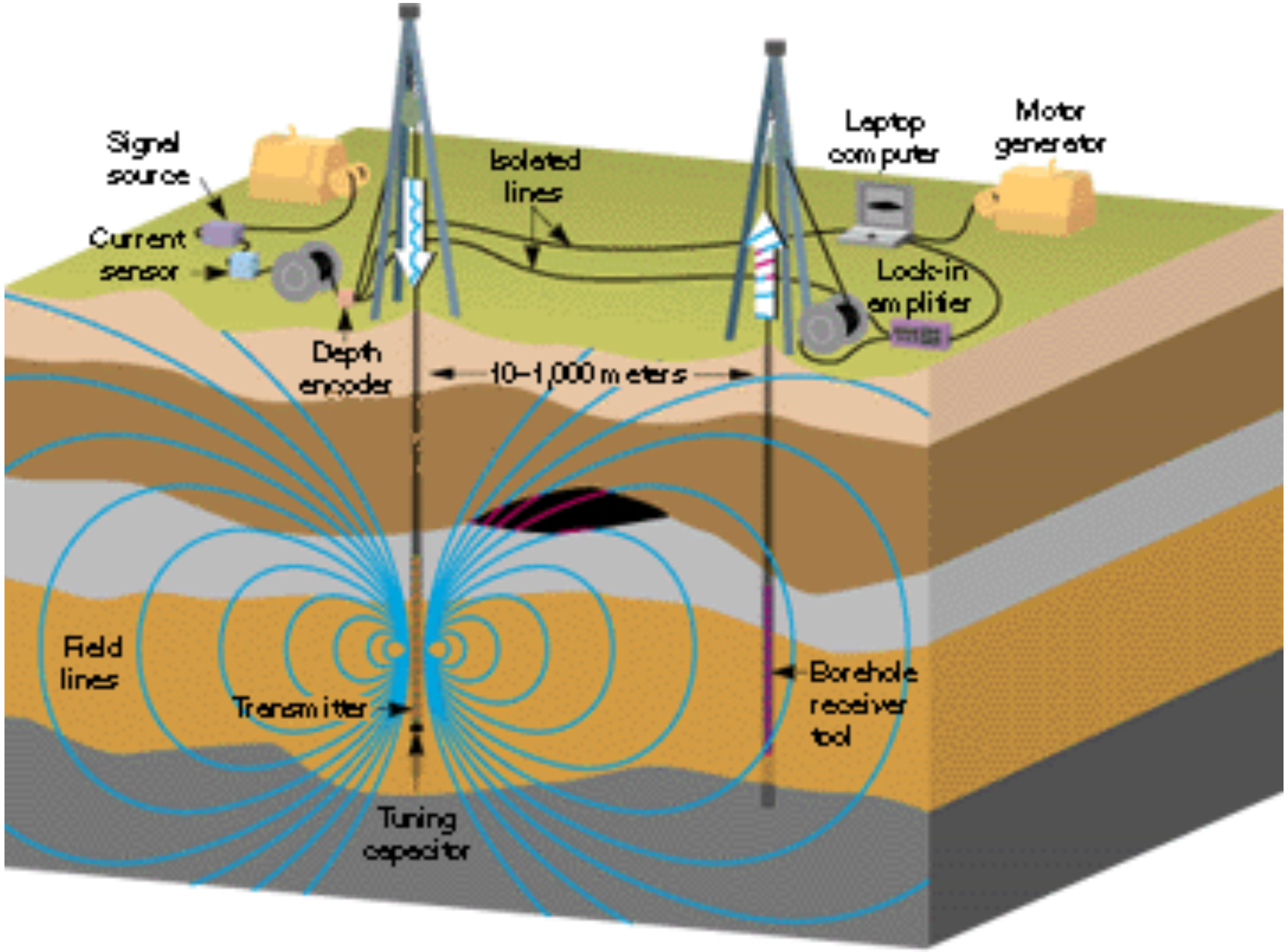


Back analysis:

we can infer the microstructure from G_{max} experiments

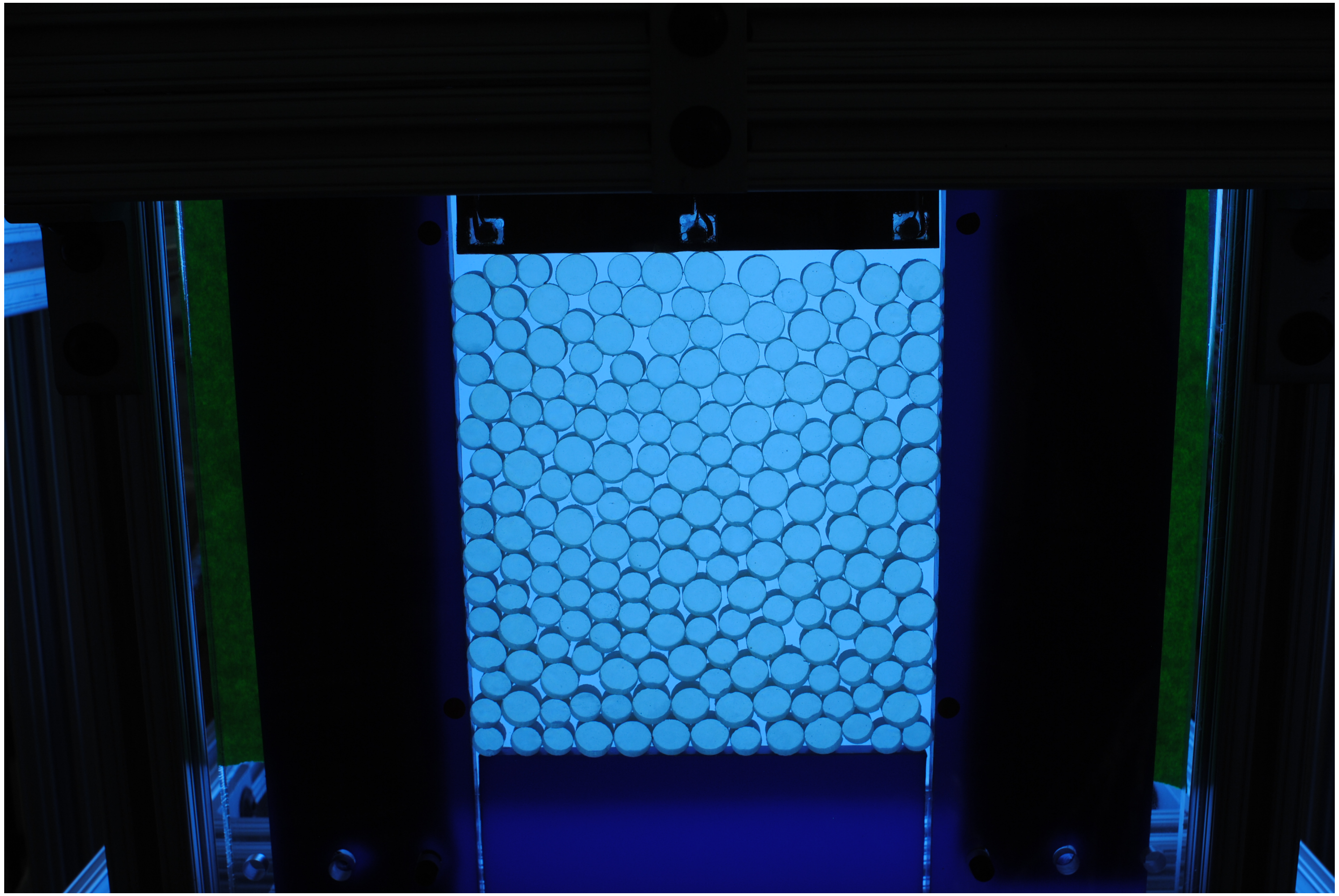
[Goudarzy, Magnanimo, Schanz (2016) submitted to Géotecnique]

Application: oil recovery



Photoelastic analysis : wave propagation

We can look closer: as an alternative/complementary study, we can use photoelastic discs to analyze the path of the wave in the bulk.



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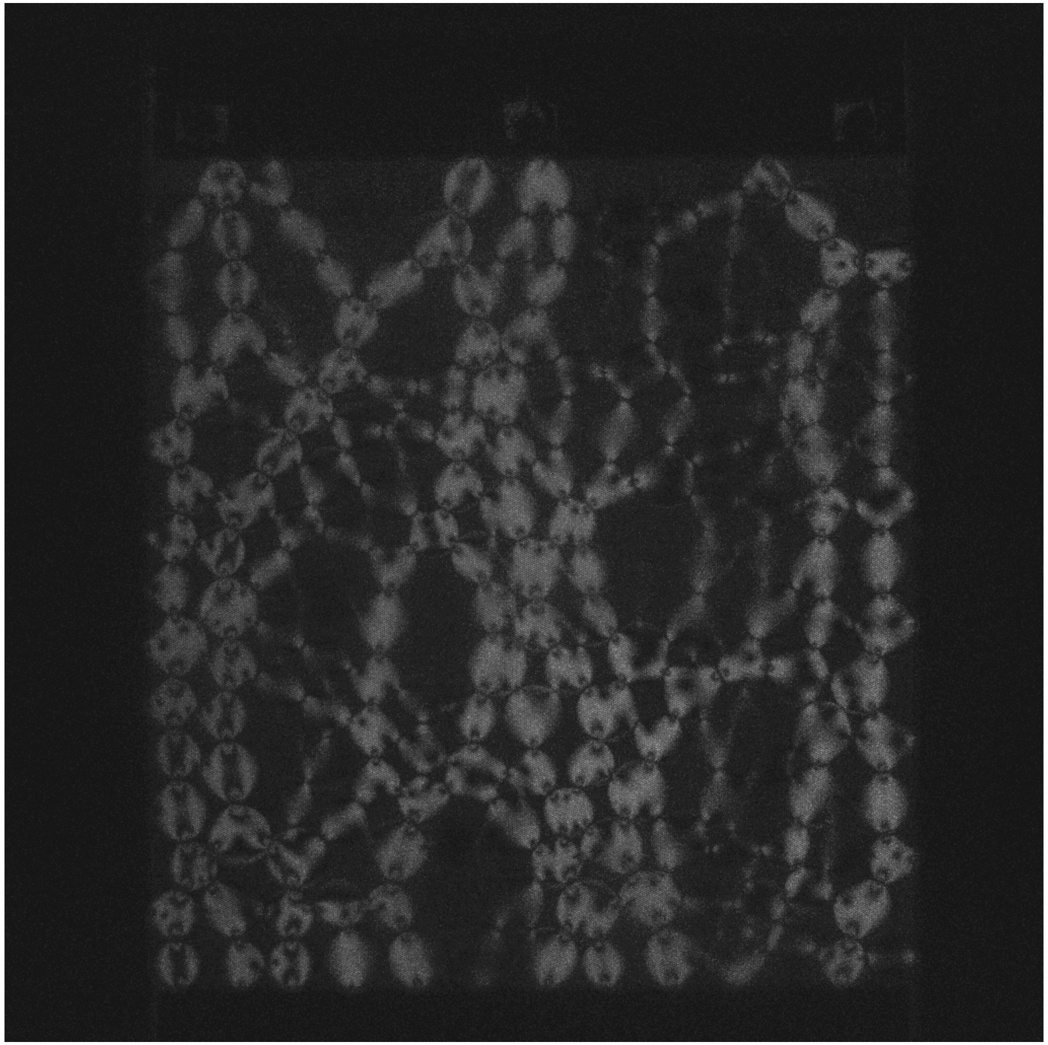
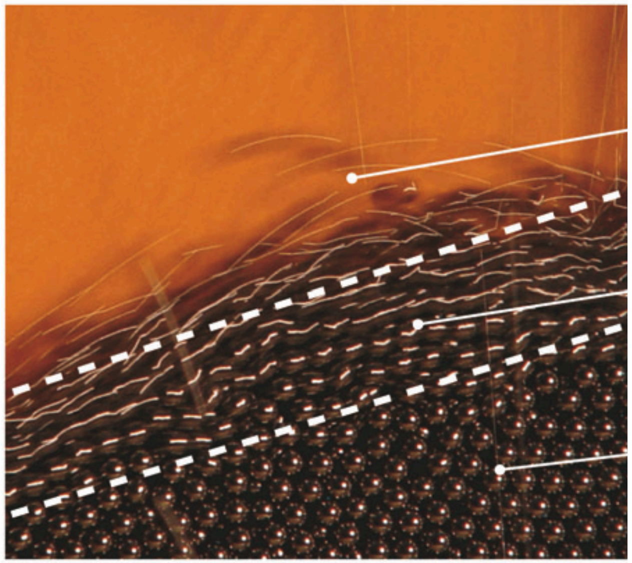


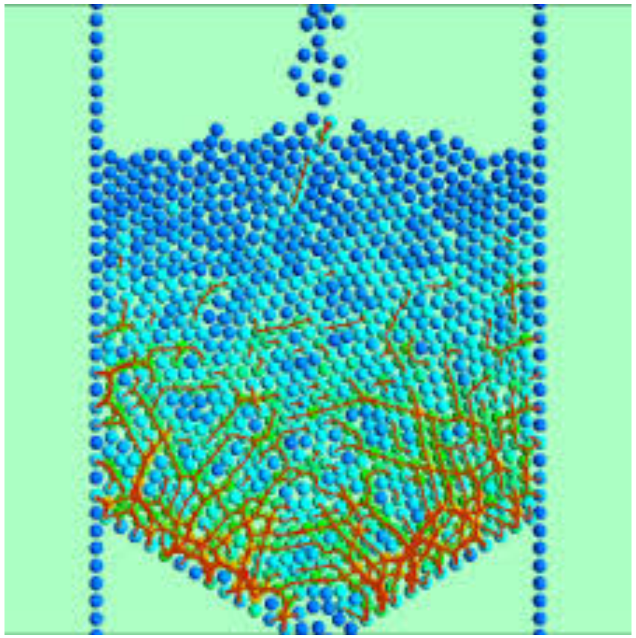
WORK IN PROGRESS!

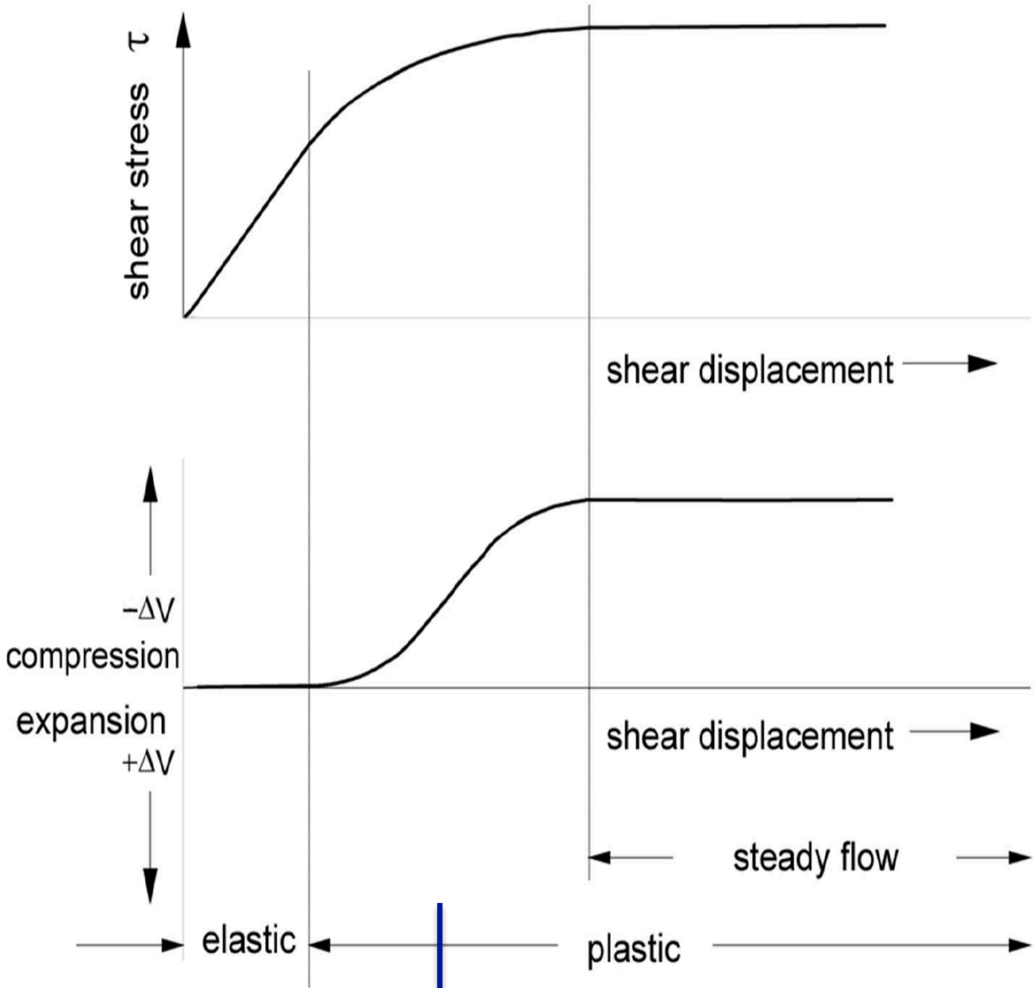
[G. Oliveri M.Sc, MSM/NCU (2016 ?)]

Quasitatic behavior and flow threshold

Shearing

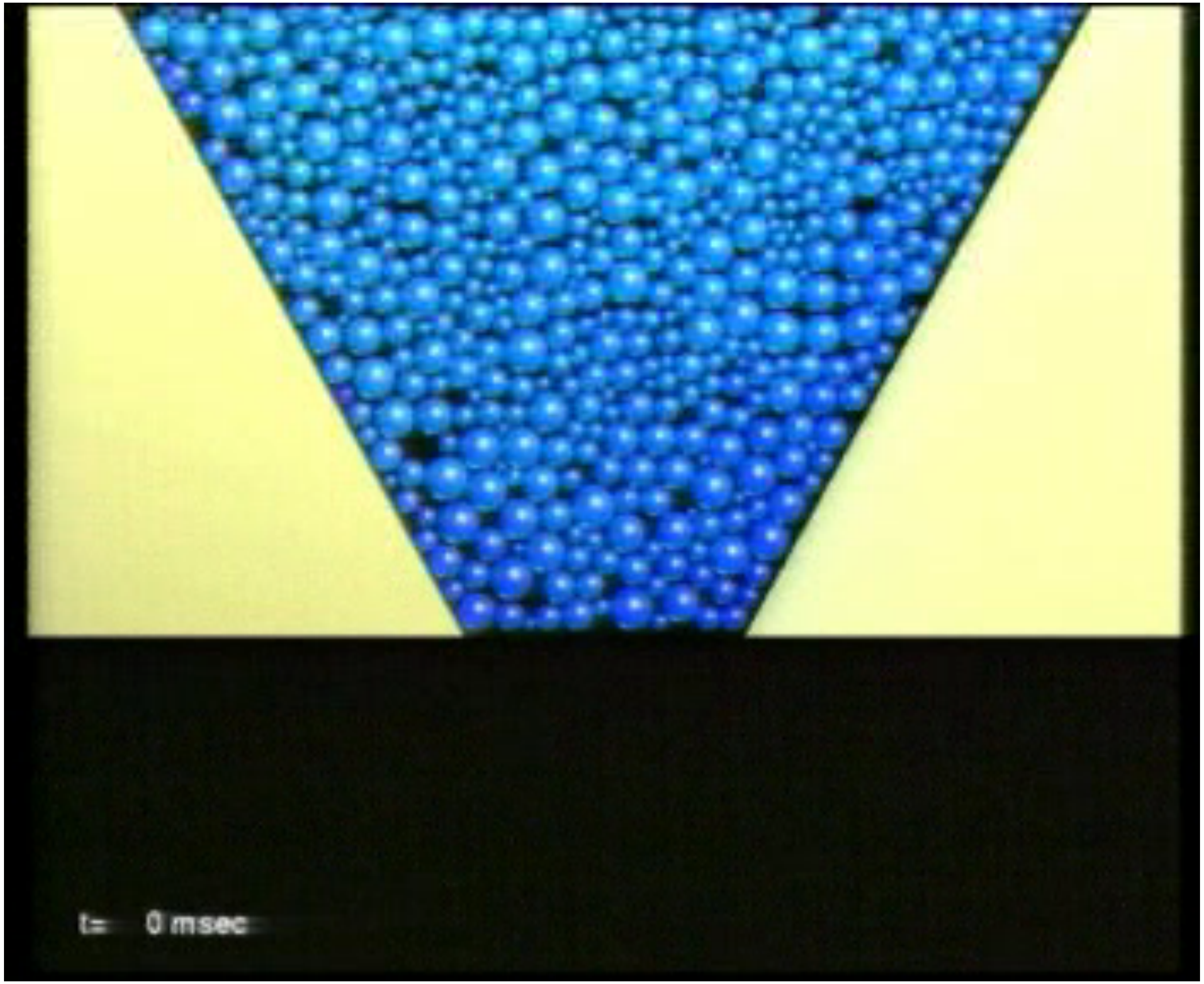






elasto-plasticity
hypoplasticity
…

Granular material in a silo



Quasistatic behavior

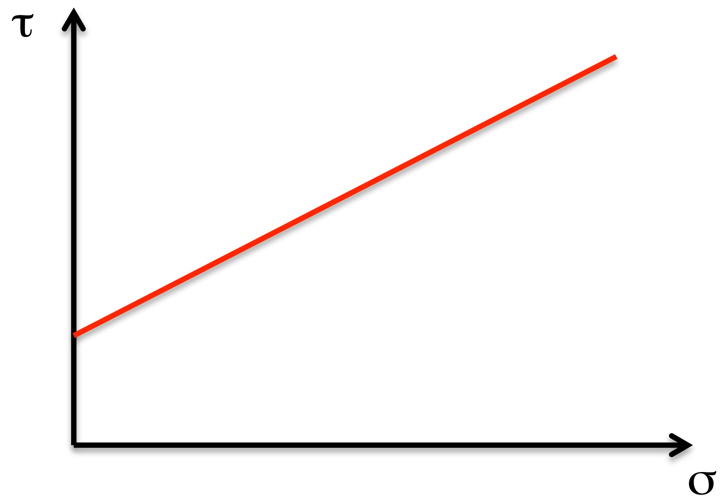
Coulomb (1773)
Yielding of granular material as frictional process
Interested in prediction of soil failures for Civil Engineering

$$\tau < c + \sigma \tan\phi$$

When $\tau = c + \sigma \tan\phi$ the material yields and starts to flow

c = cohesion
ϕ = friction angle

ϕ and c are material **constant**



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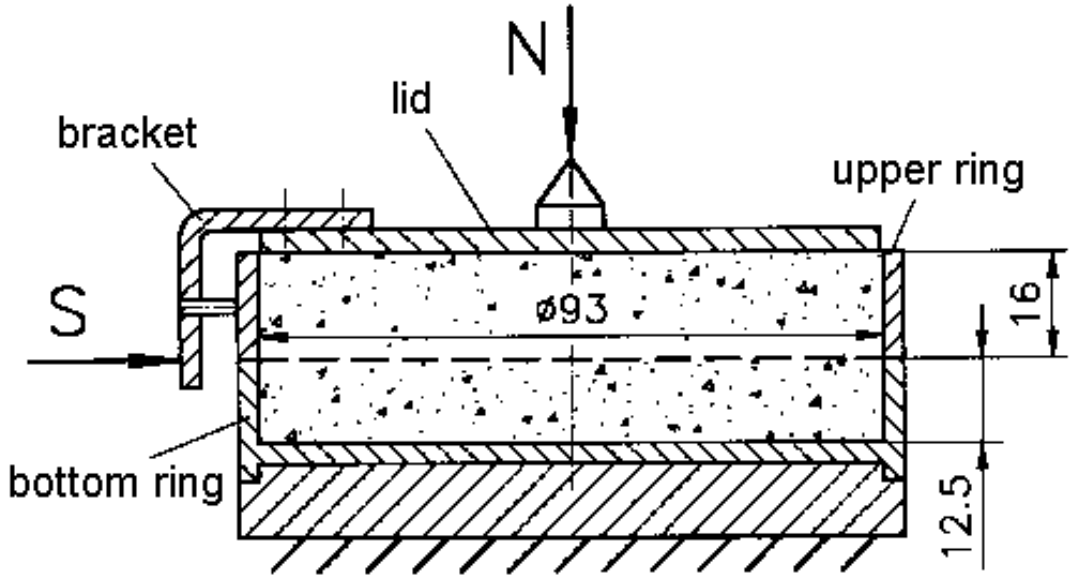
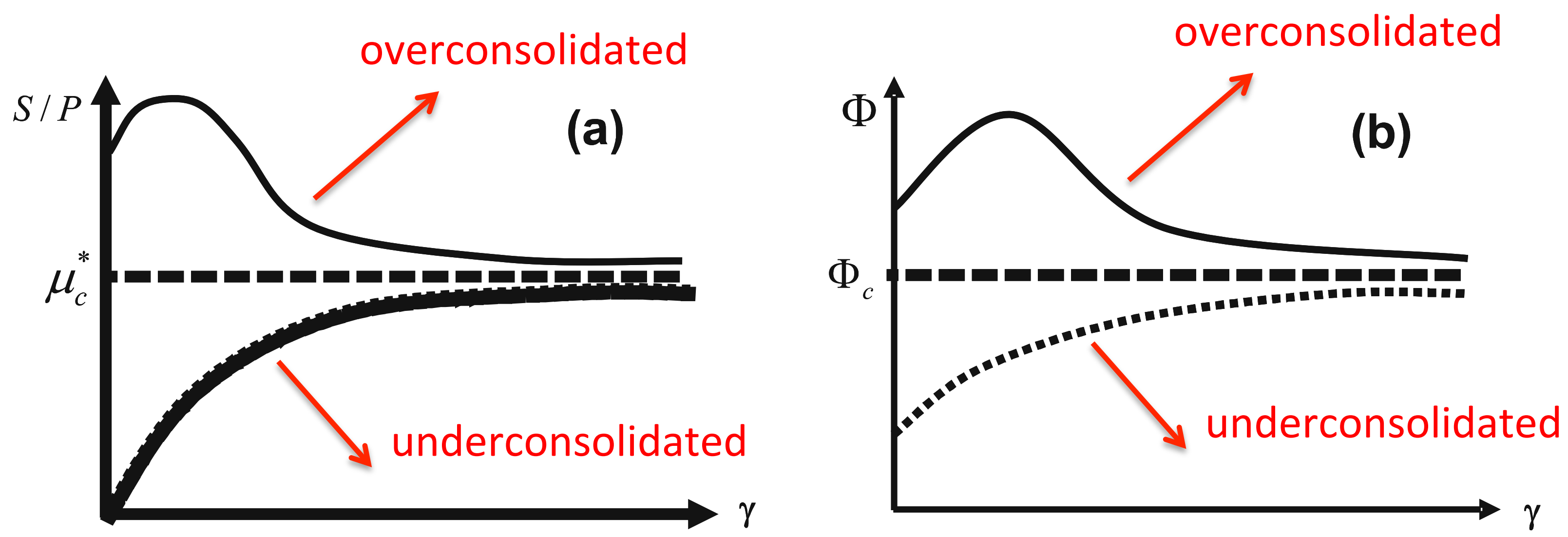


Critical state

A shearing granular material will ALWAYS approach a **critical** concentration
This is the **ONSET OF FLOW**



ϕ_c is again a material **constant**

The granular material **DILATES**

Critical state

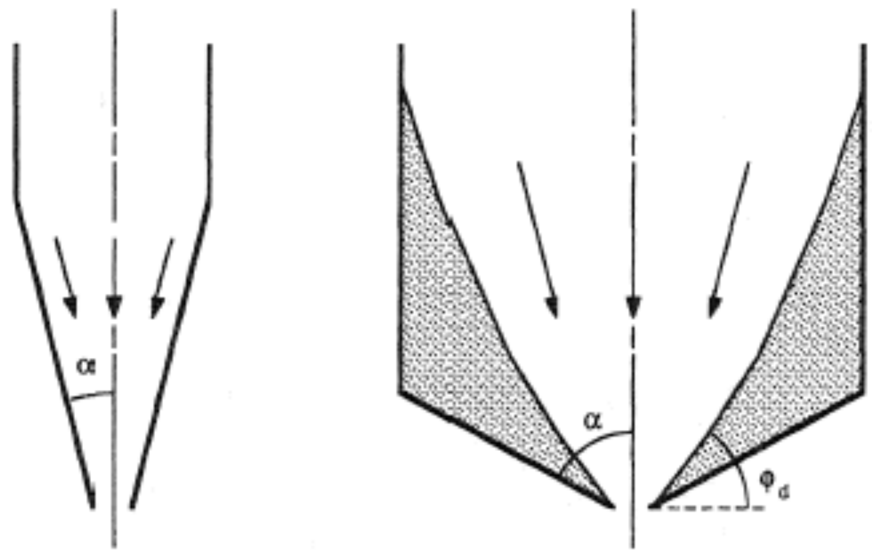
Soil mechanics: widely used

Particle Technology: flow behavior from silo

- when the material starts flowing is **always** yielding **everywhere** in the hopper (mass flow) or in a region (core flow)

$$\tau = c + \sigma \tan\phi$$

- the material is **always** at the critical concentration and it is **incompressible**.



Critical state

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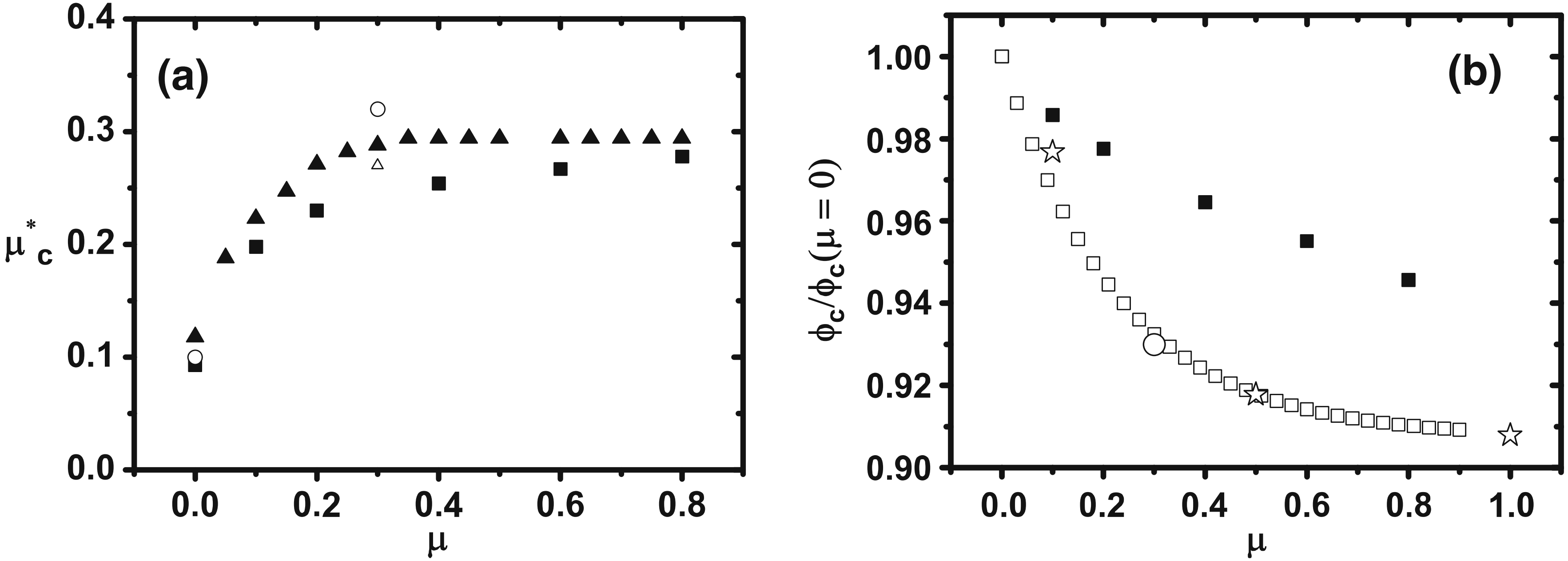
N.B.!!

Application of Critical State theory on is based **on Janssen theory:**
the pressure at bottom of the silo is independent of bed height

→ the whole bulk material is in the critical state.

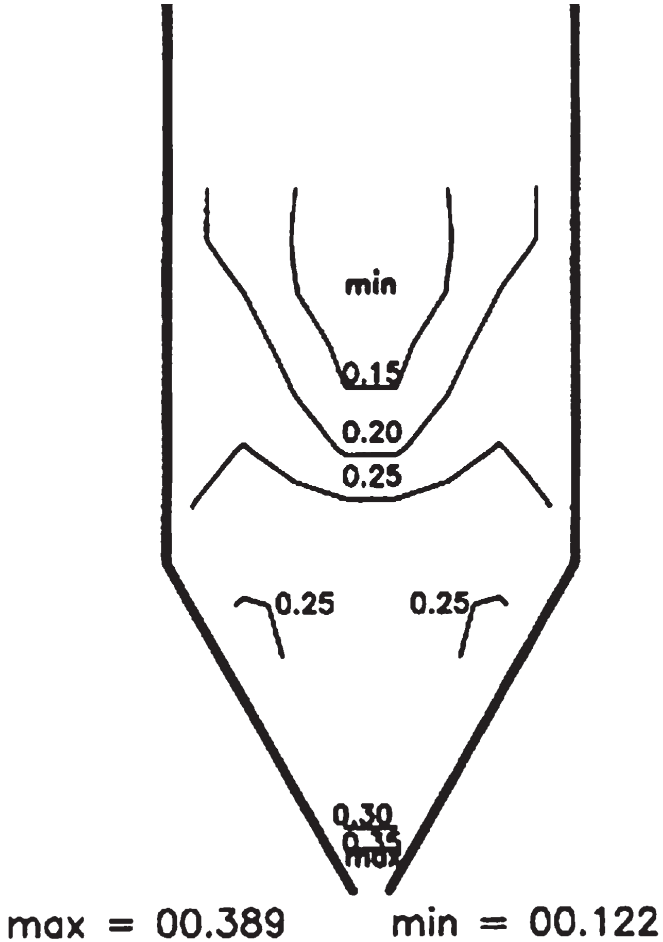
Dependence on microscopic properties



[Lamaitre et al., Rheol. Acta (2009)]

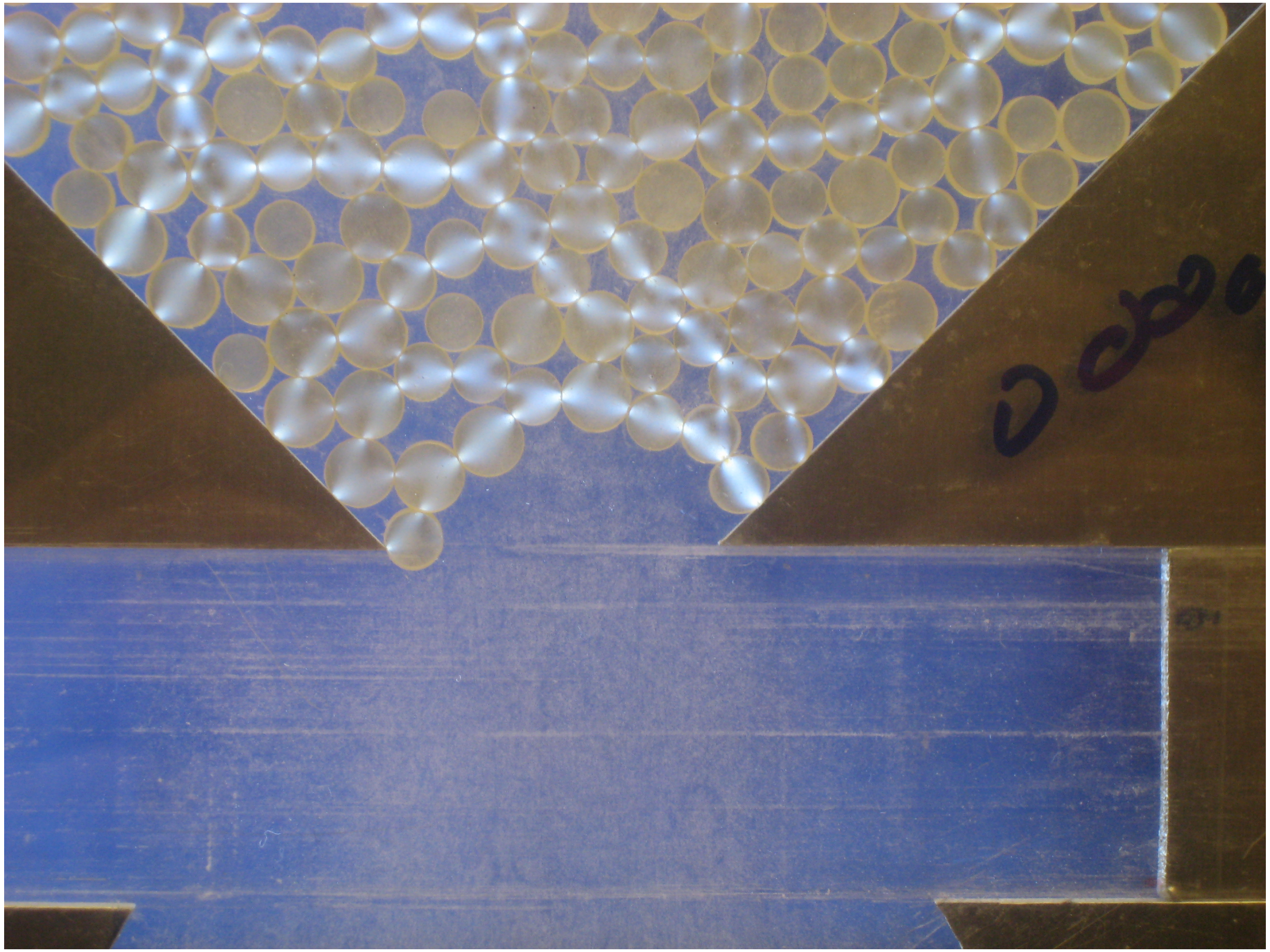
Critical state for silos - problems

ϕ is not constant in the silo



Friction and dilatancy laws

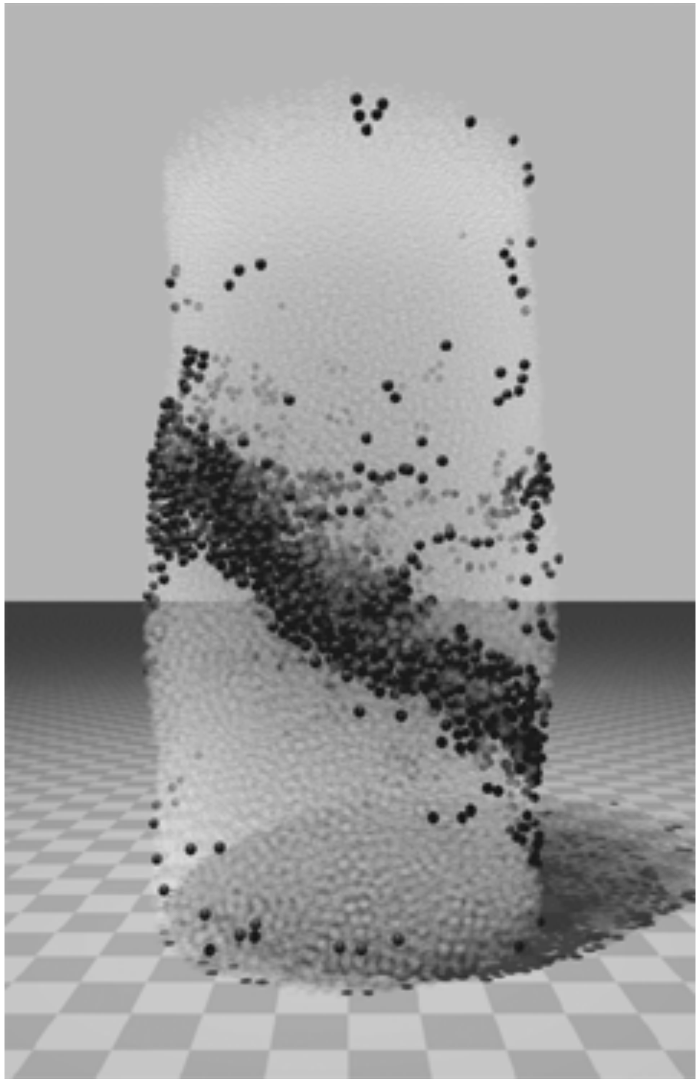
In solid and quasistatic flow, forces are transmitted through **force chains**



Shear bands and dilatant zones

Frictional Behavior = Mohr-Coulomb failure

Granular materials fail along narrow but **finite** zones: **SHEAR BANDS**



Shear bands and dilatant zones

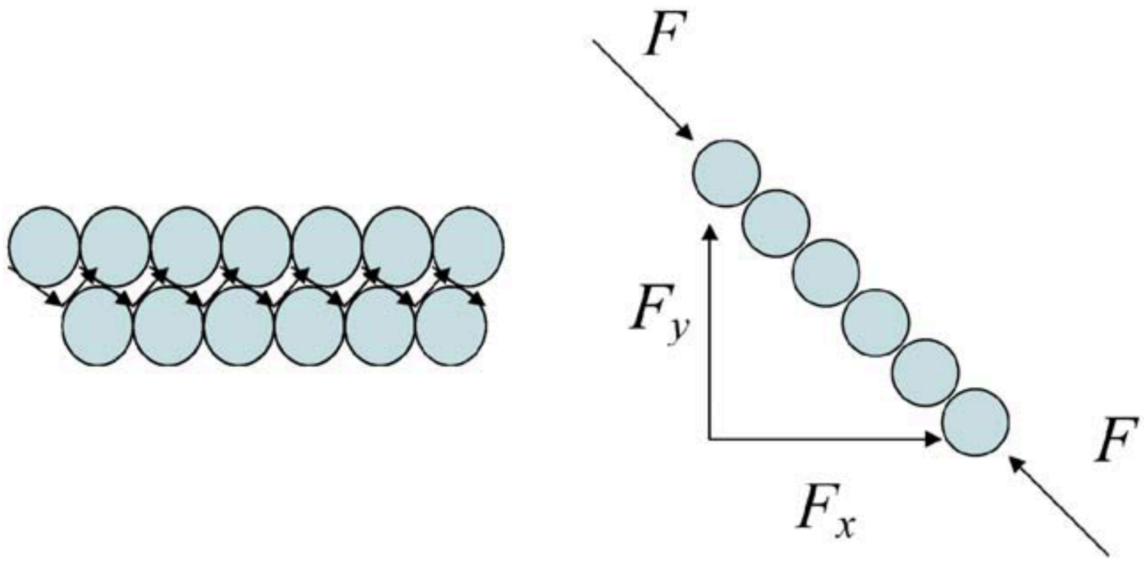
Frictional Behavior = Mohr-Coulomb failure

Granular materials fail along narrow but **finite** zones: **SHEAR BANDS**

GLOBALLY → frictional behavior

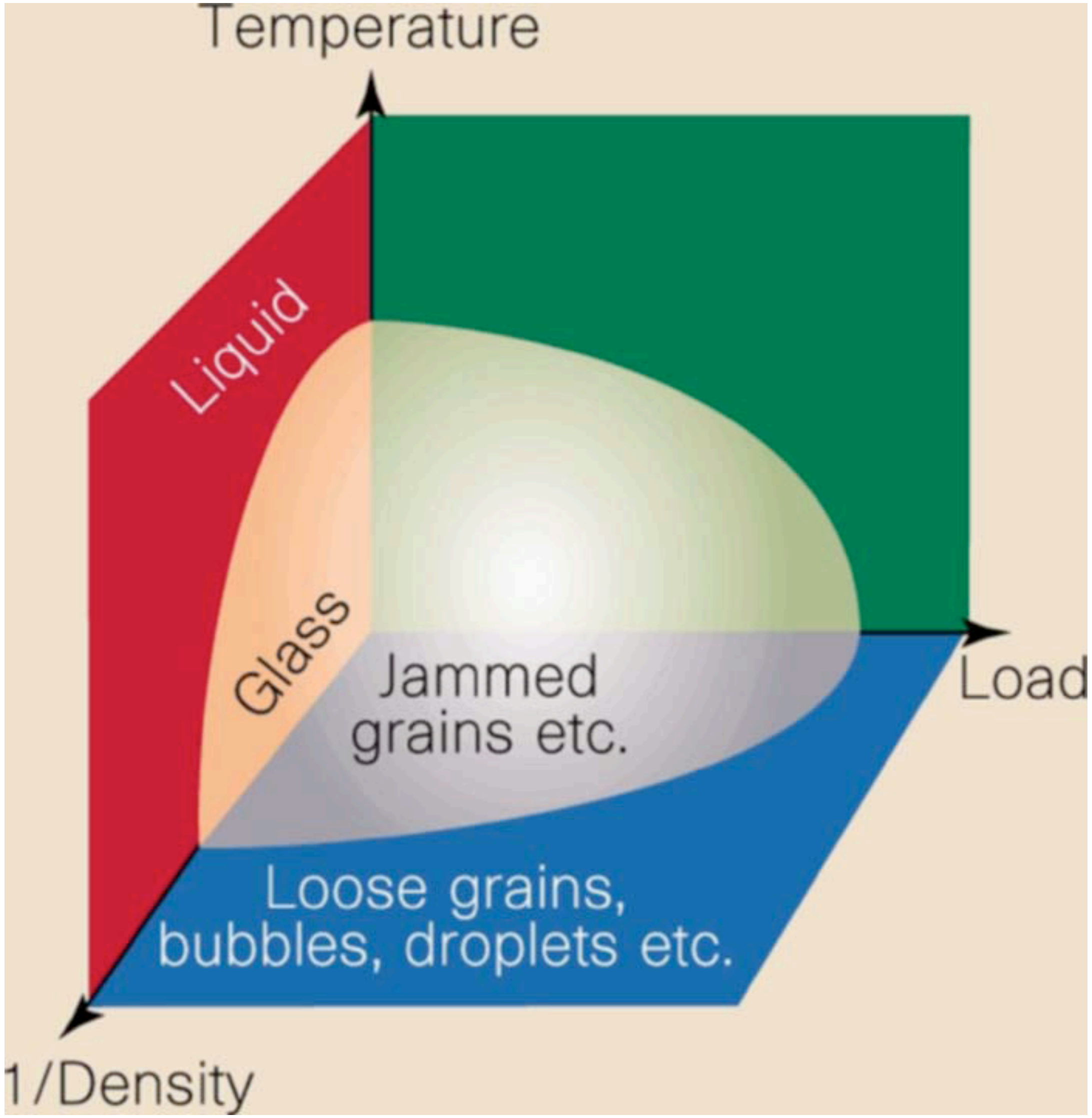
LOCALLY → force chains

$$\frac{\tau_{xy}}{\tau_{yy}} = \frac{\langle F_x l_y \rangle}{\langle F_y l_y \rangle} = const$$



(a) (b)

Jamming phase diagram

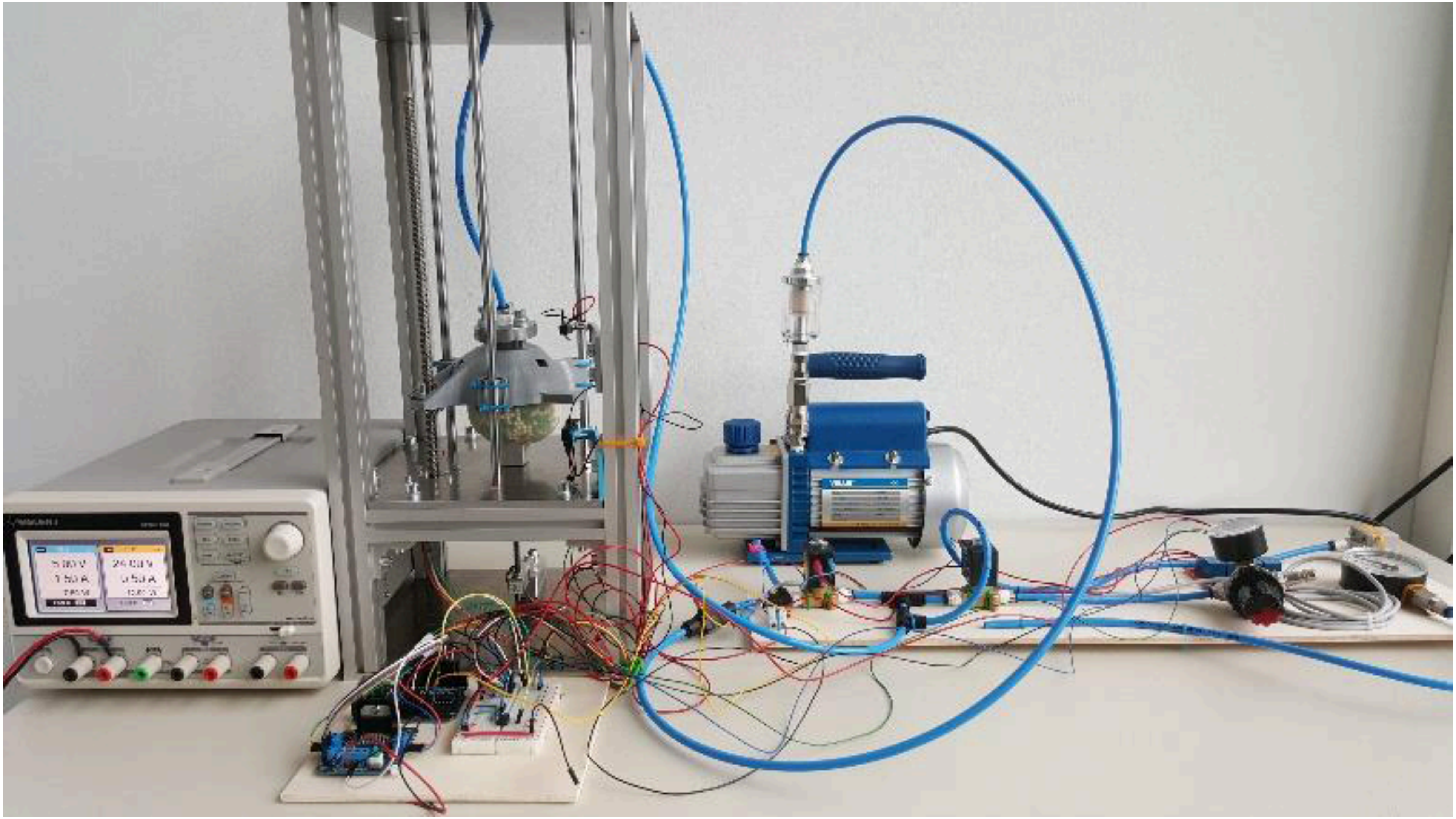


[Liu and Nagel., Nature (1998)]

Jamming gripper

[Brown et al., PNAS (2010)]

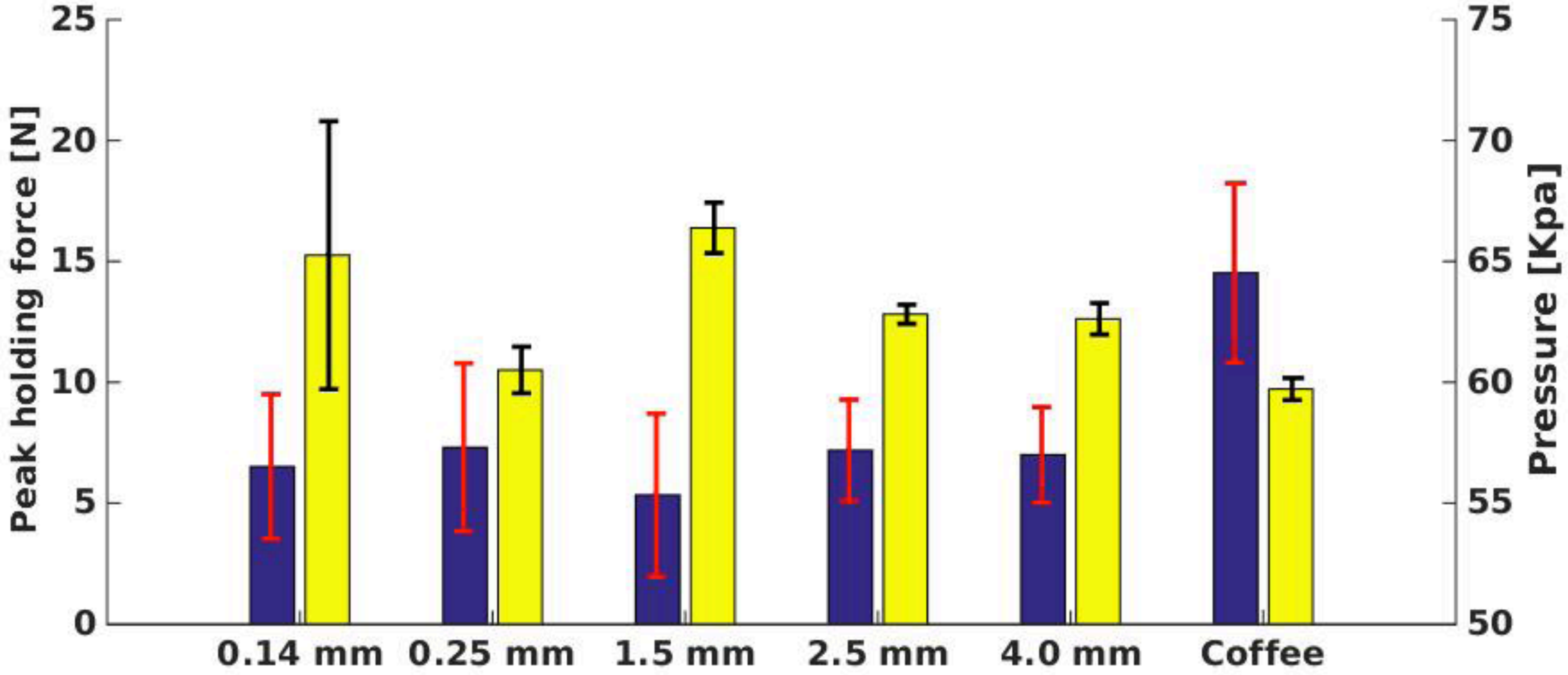
Optimization of jamming gripper



[ITO project UT (2015, 2016)]

Optimization of jamming gripper

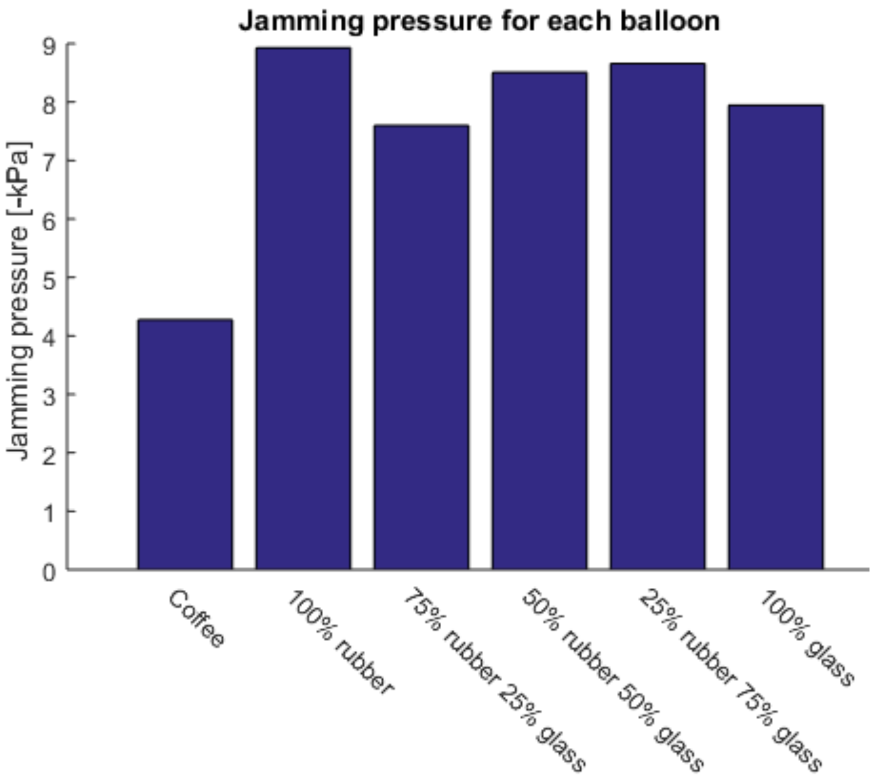
Filler: different sizes glass beads (homogeneaous)

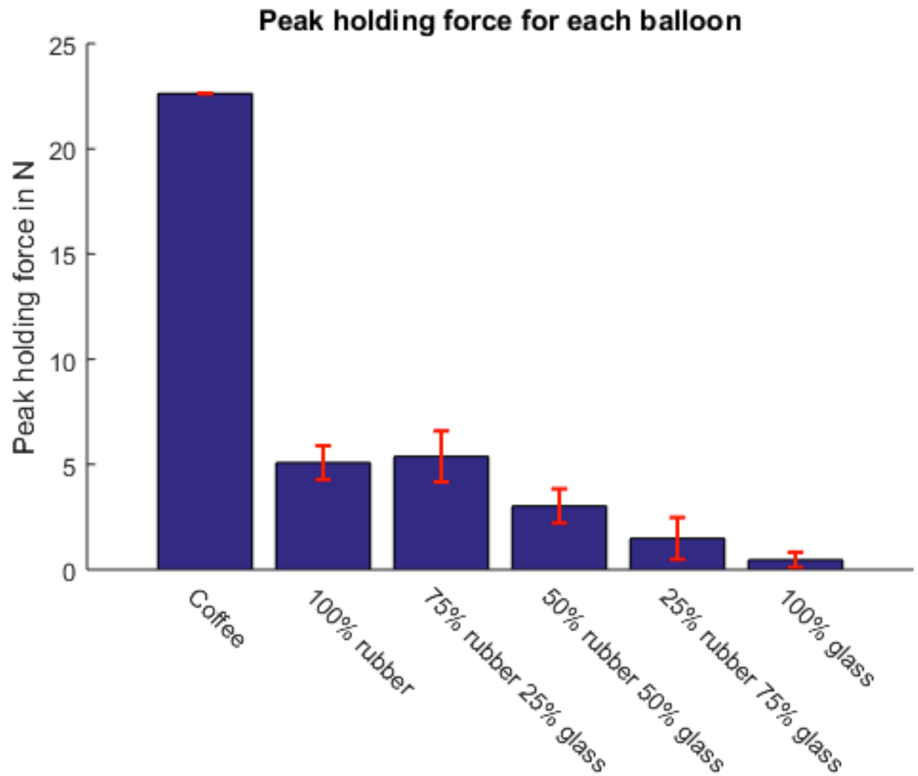


[ITO project UT (2015, 2016)]

Optimization of jamming gripper

Filler: mixtures of glass beads and rubber beads

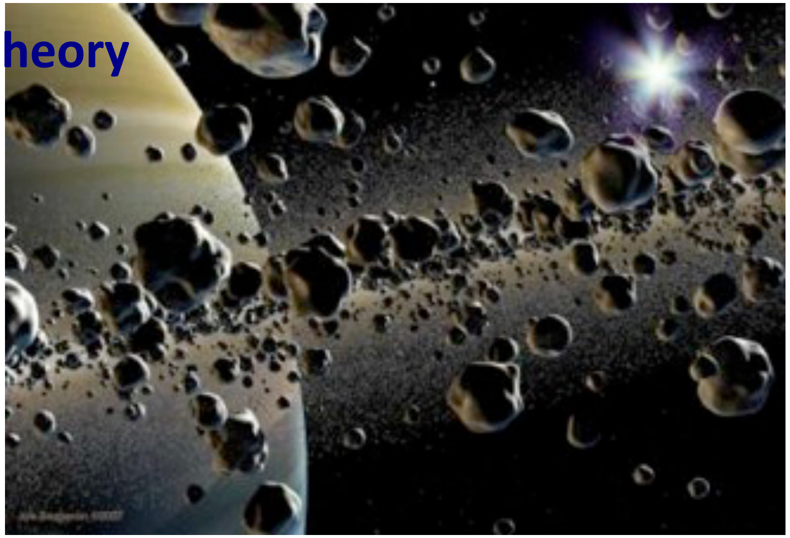




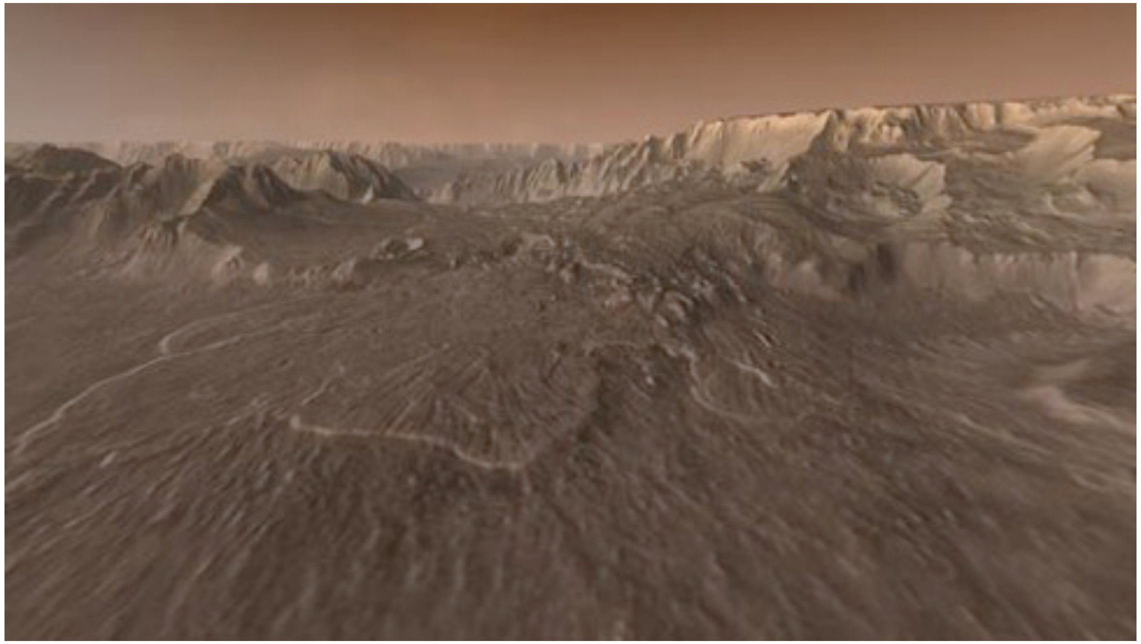
[ITO project UT (2015, 2016)]

Granular material: continuum approach

Gas: kinetic theory



Solid: soil mechanics



Liquid ??

Dense (slow) flows
and inertial regime