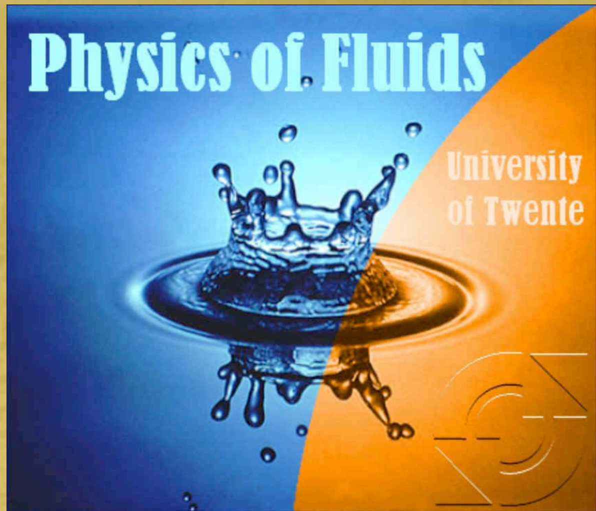
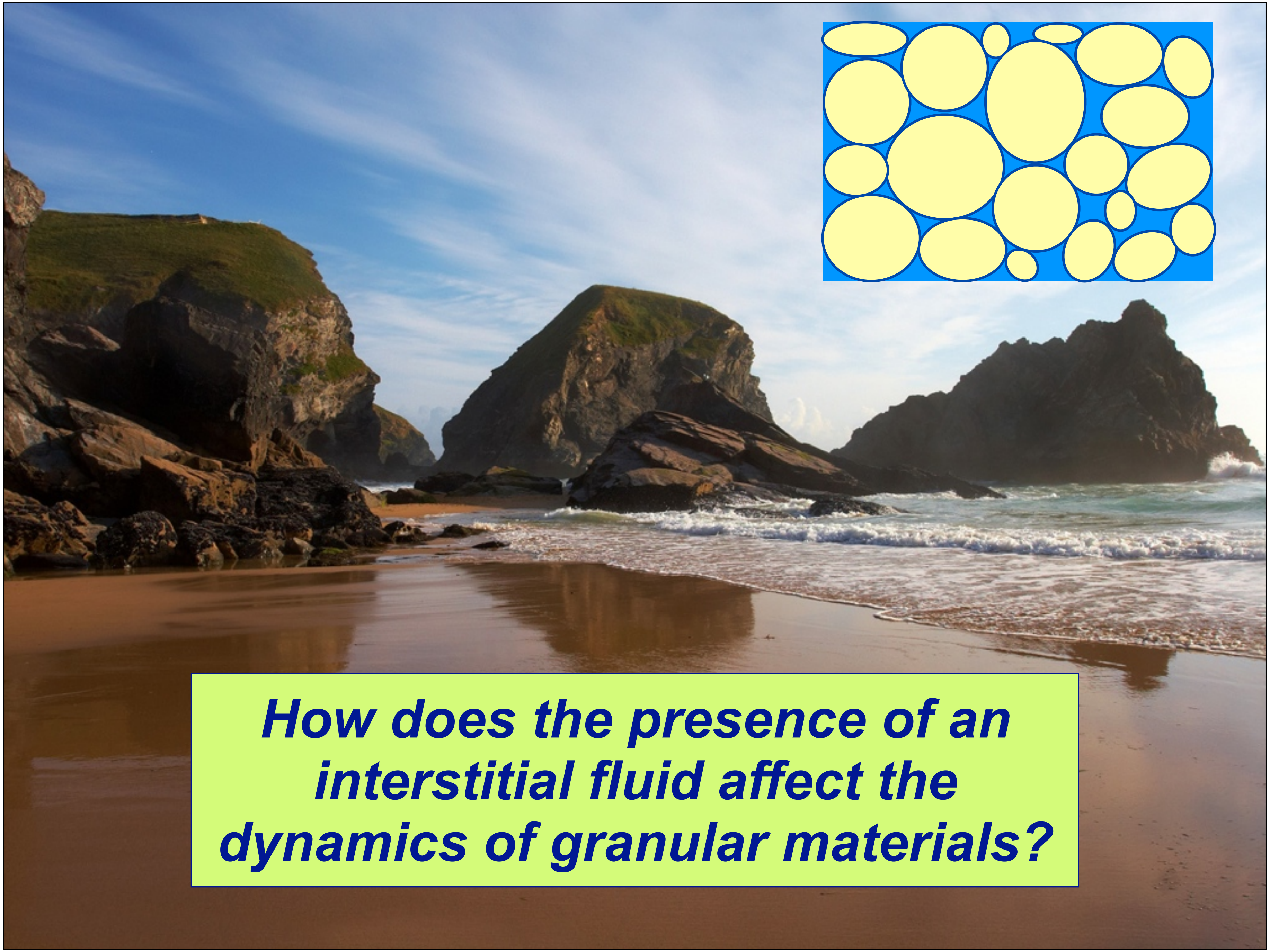
Granular matter and interstitial fluids

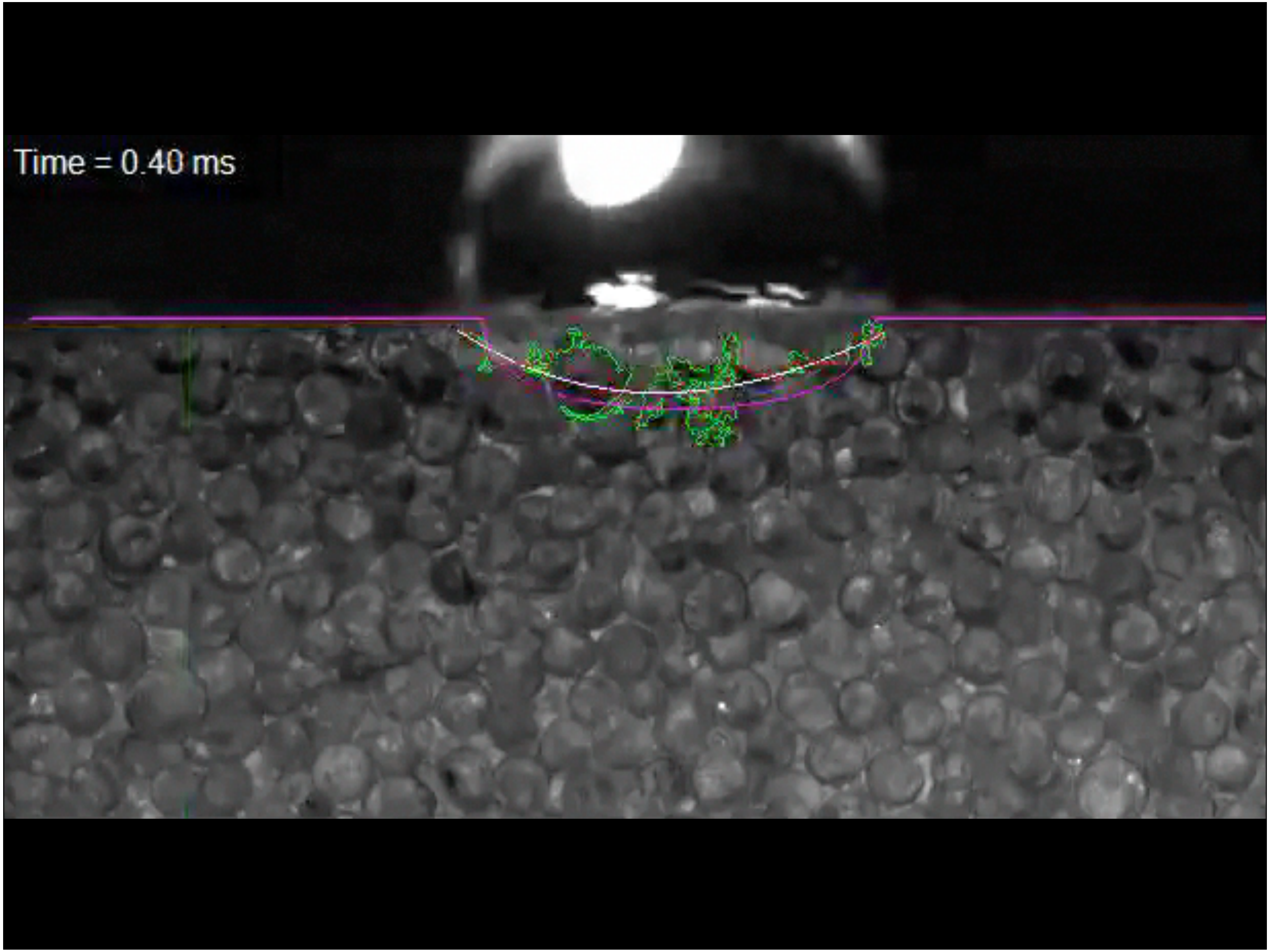
Devaraj van der Meer

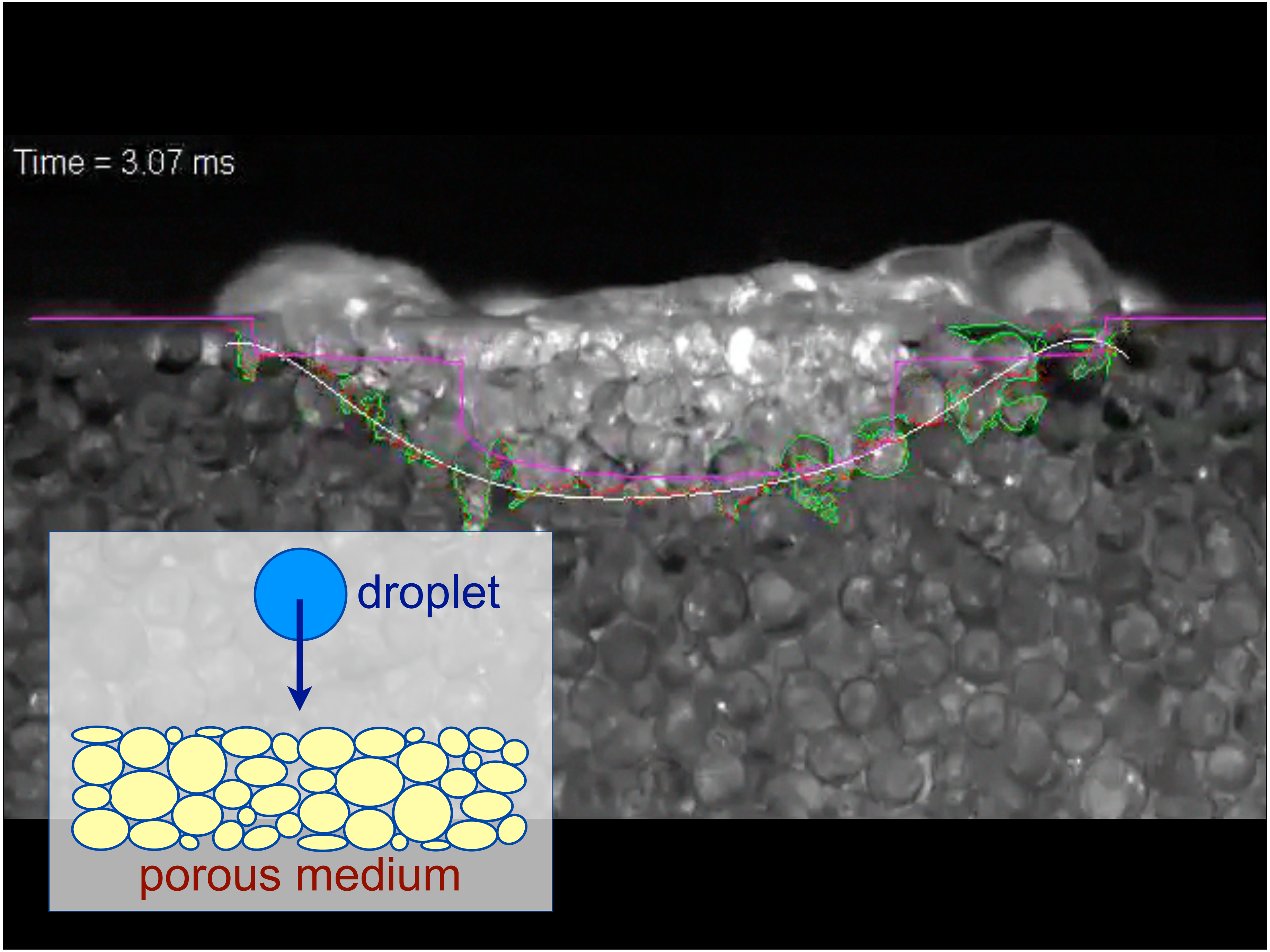


J.M. Burgerscentrum

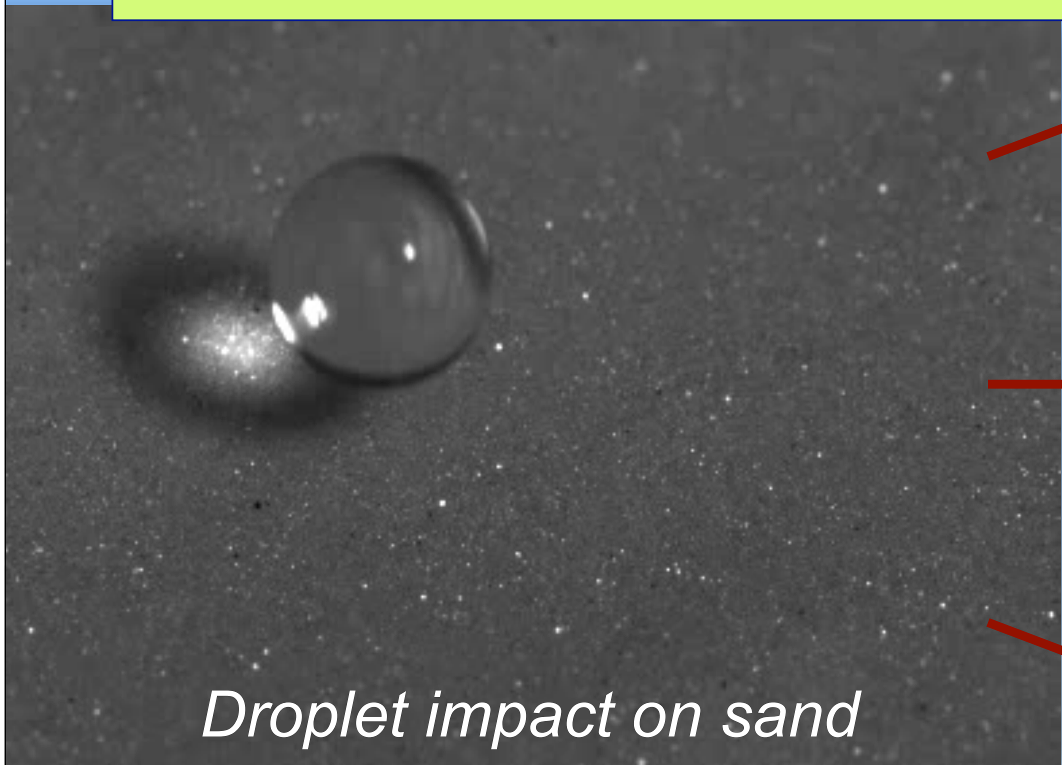
Physics of Fluids - University of Twente – The Netherlands


How does the presence of an interstitial fluid affect the dynamics of granular materials?

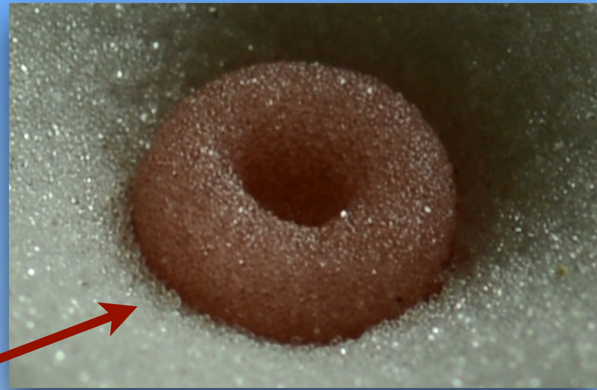

Time = 0.40 ms


Time = 3.07 ms
droplet
porous medium

residue shape depends on mixing rate

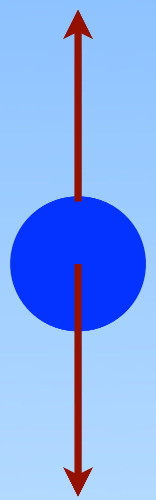


Droplet impact on sand



Role of intersititial fluid: single particle

$$F_{drag} = 3\pi\eta d V$$



$$F_g = \tfrac{1}{6}\pi d^3 \rho_p g$$

d = particle diameter
V = typical particle velocity
η = air viscosity ($2{\cdot}10^{-5}$ Pa·s)
ρ_p = part. density ($2.5{\cdot}10^3$ kg/m^3)
g = grav. acceleration (10 m/s^2)

$$\boxed{B \equiv \frac{F_{drag}}{F_g} = \frac{18\eta V}{\rho_p g d^2}}$$

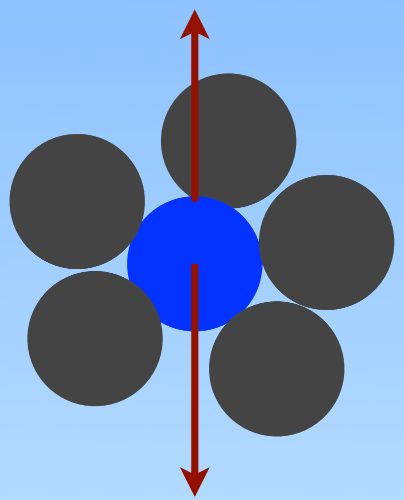
$$B \approx 1 \rightarrow d \approx \sqrt{\frac{18\eta V}{\rho_p g}}$$

$$V \approx 1 \text{ m/s} \rightarrow d \approx 120\ \mu\text{m}$$

$$V \approx \sqrt{2gd} \rightarrow d \approx 16\ \mu\text{m}$$

Role of interstitial fluid: packed particle

$$F_{f\rightarrow s} = 2k\frac{1-\varepsilon}{\varepsilon^3}F_{drag}$$



$$F_g = \tfrac{1}{6}\pi d^3\rho_p g$$

$\varepsilon = 1 - \varphi =$ porosity (≈ 0.5)
$k =$ Kozeny constant (≈ 5)

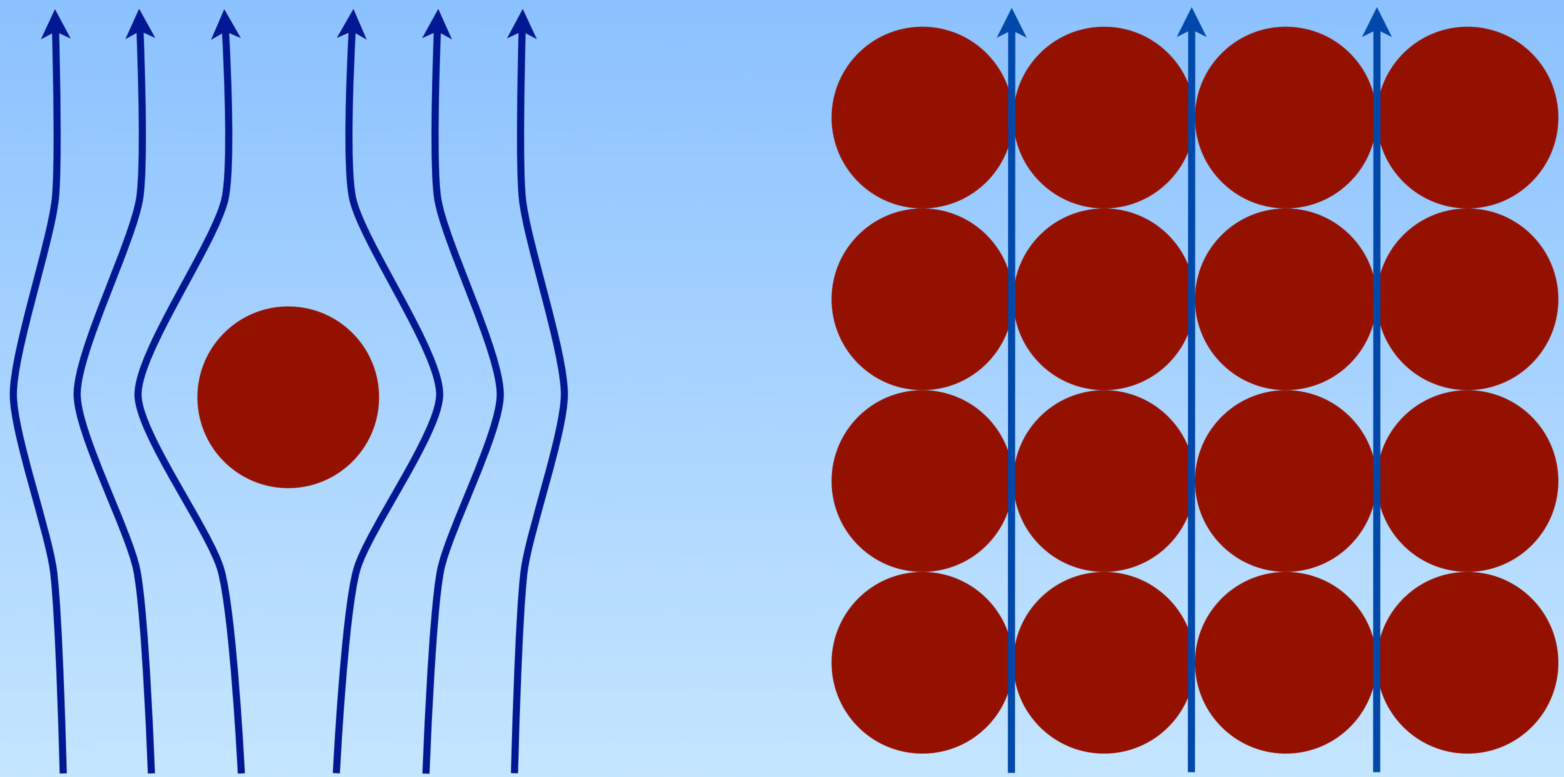
$$\boxed{B_p \equiv \frac{F_{f\rightarrow s}}{F_g} \approx 40\frac{18\eta V}{\rho_p g d^2}}$$

$$B_p \approx 1 \rightarrow d \approx \sqrt{40}\sqrt{\frac{18\eta V}{\rho_p g}}$$

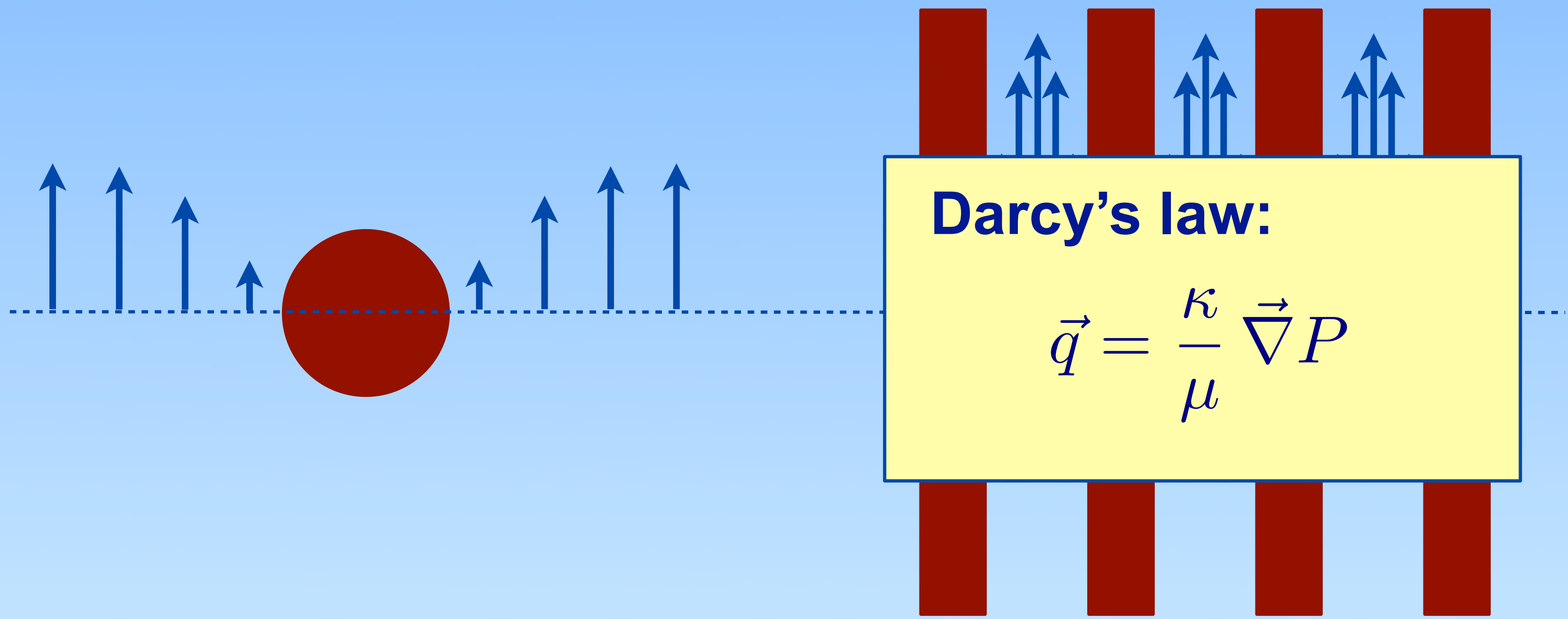
$$V \approx 1\ \mathrm{m/s} \rightarrow d \approx 760\ \mu\mathrm{m}$$

$$V \approx \sqrt{2gd} \rightarrow d \approx 190\ \mu\mathrm{m}$$

Why this difference?



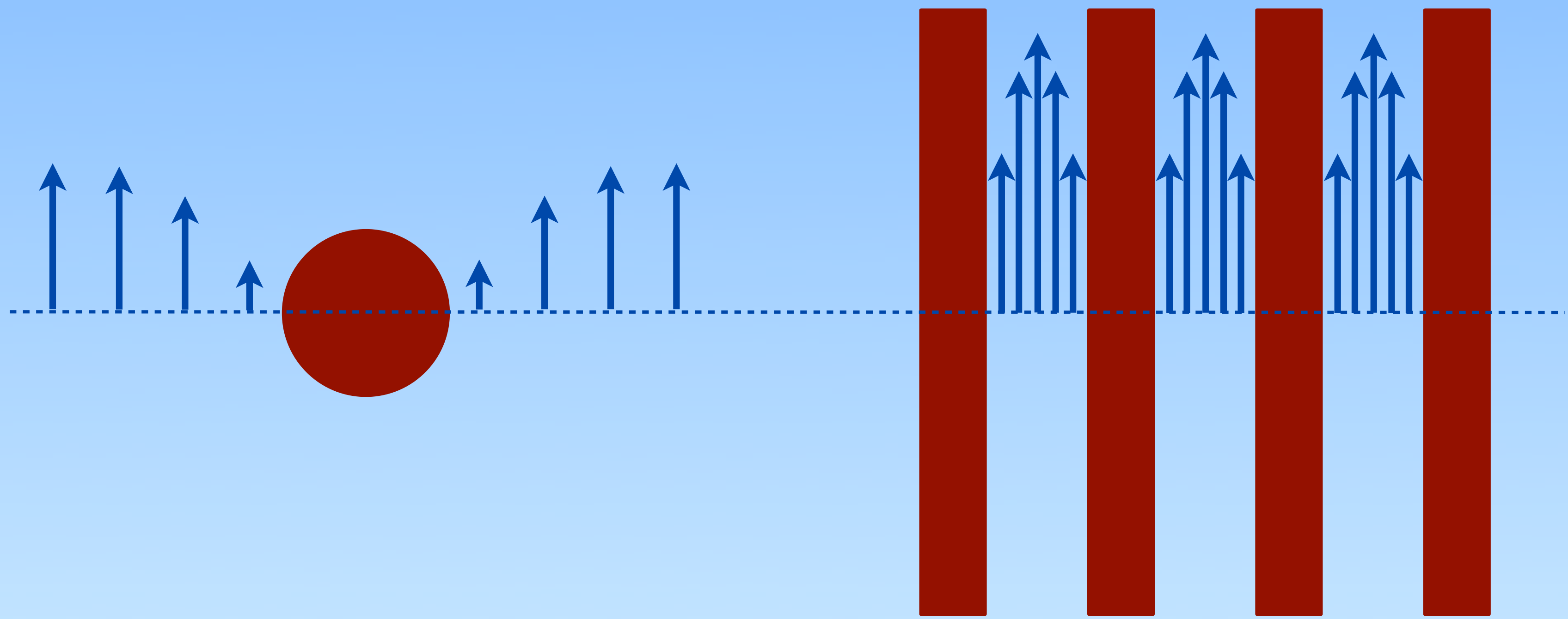
Why this difference?



- space around sphere
- moderate shear rate and shear stress

- narrow channels
- large shear rate and shear stress

Why this difference?



- space around sphere
- moderate shear rate and shear stress

- narrow channels
- large shear rate and shear stress

Air

Example 1

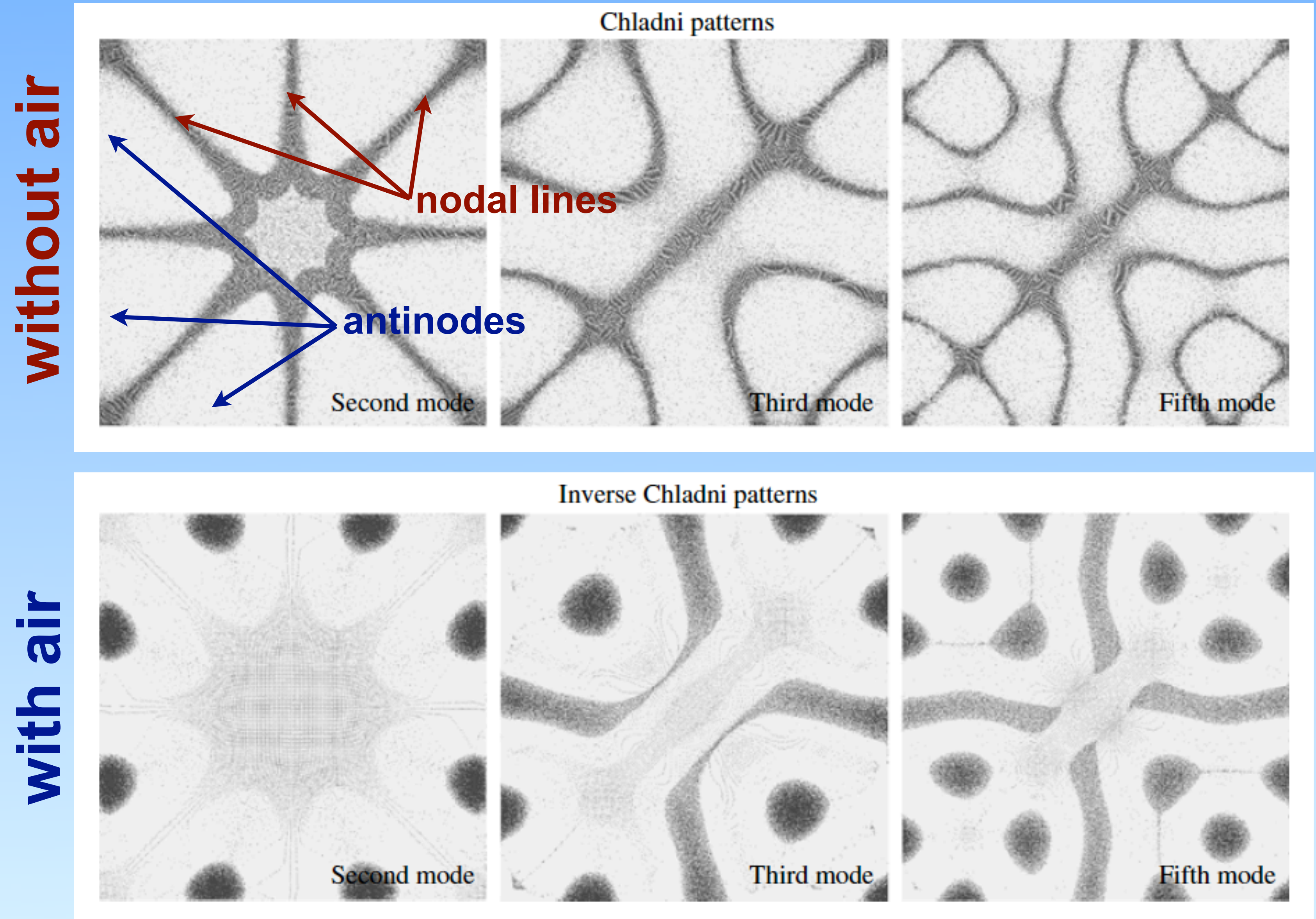
When air is forced through a granular layer

Faraday heaping

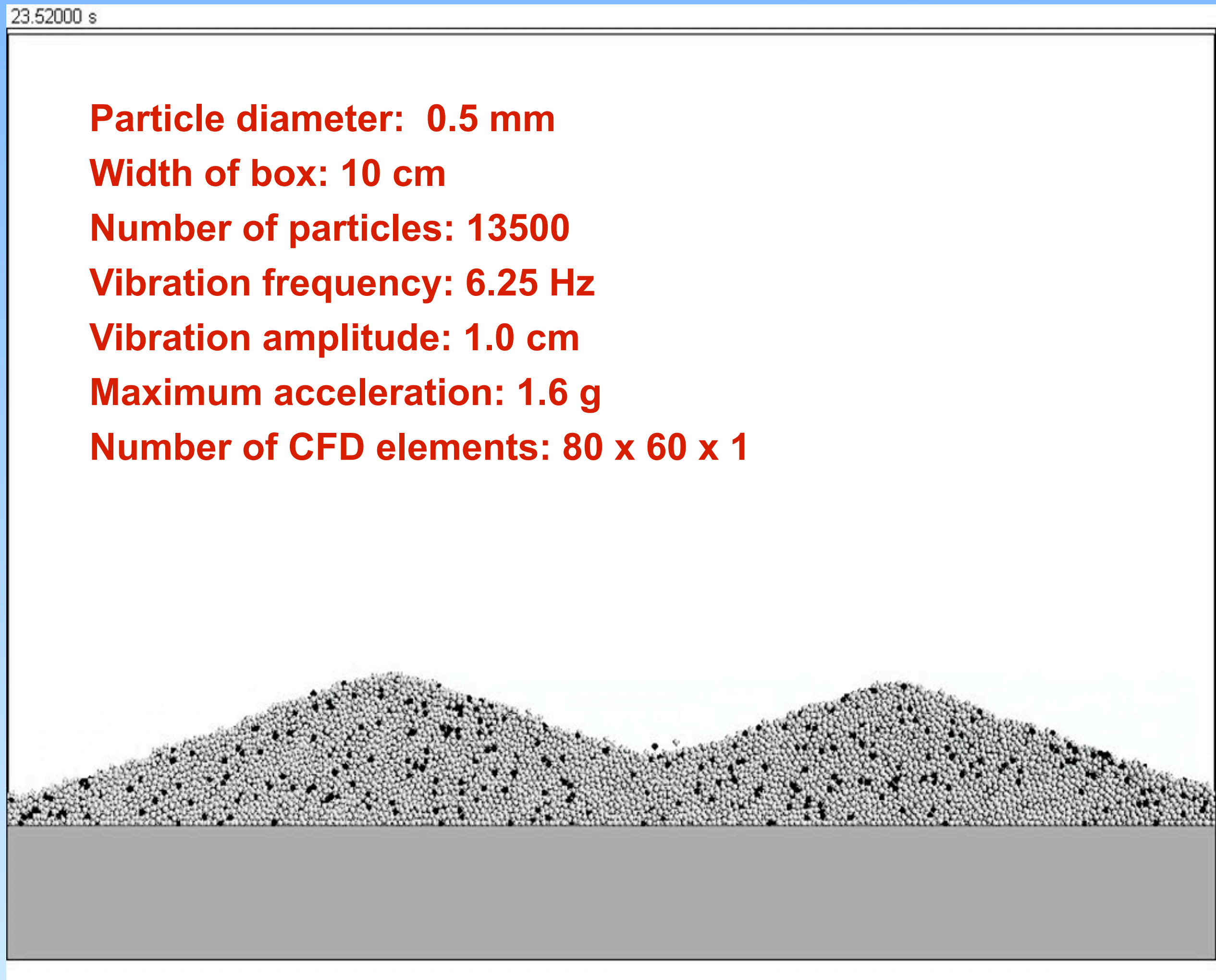
... goes back all the way to Michael Faraday

M. Faraday, *On a peculiar class of acoustical figures; and on certain forms assumed by groups of particles upon vibrating elastic surfaces*, Philos. Trans. R. Soc. London **52,** 299 (1831).

Not: Chladni patterns

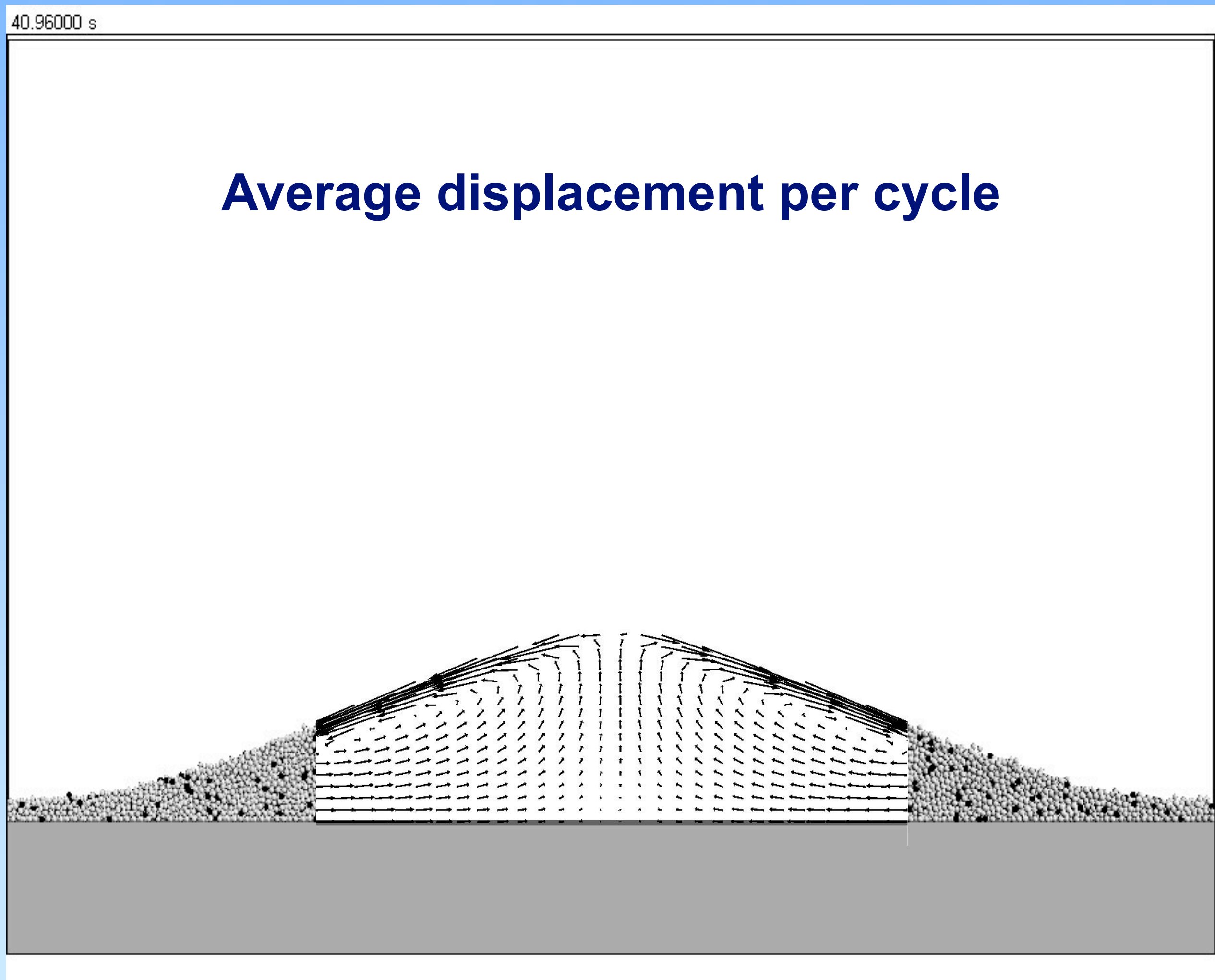


Faraday heaping



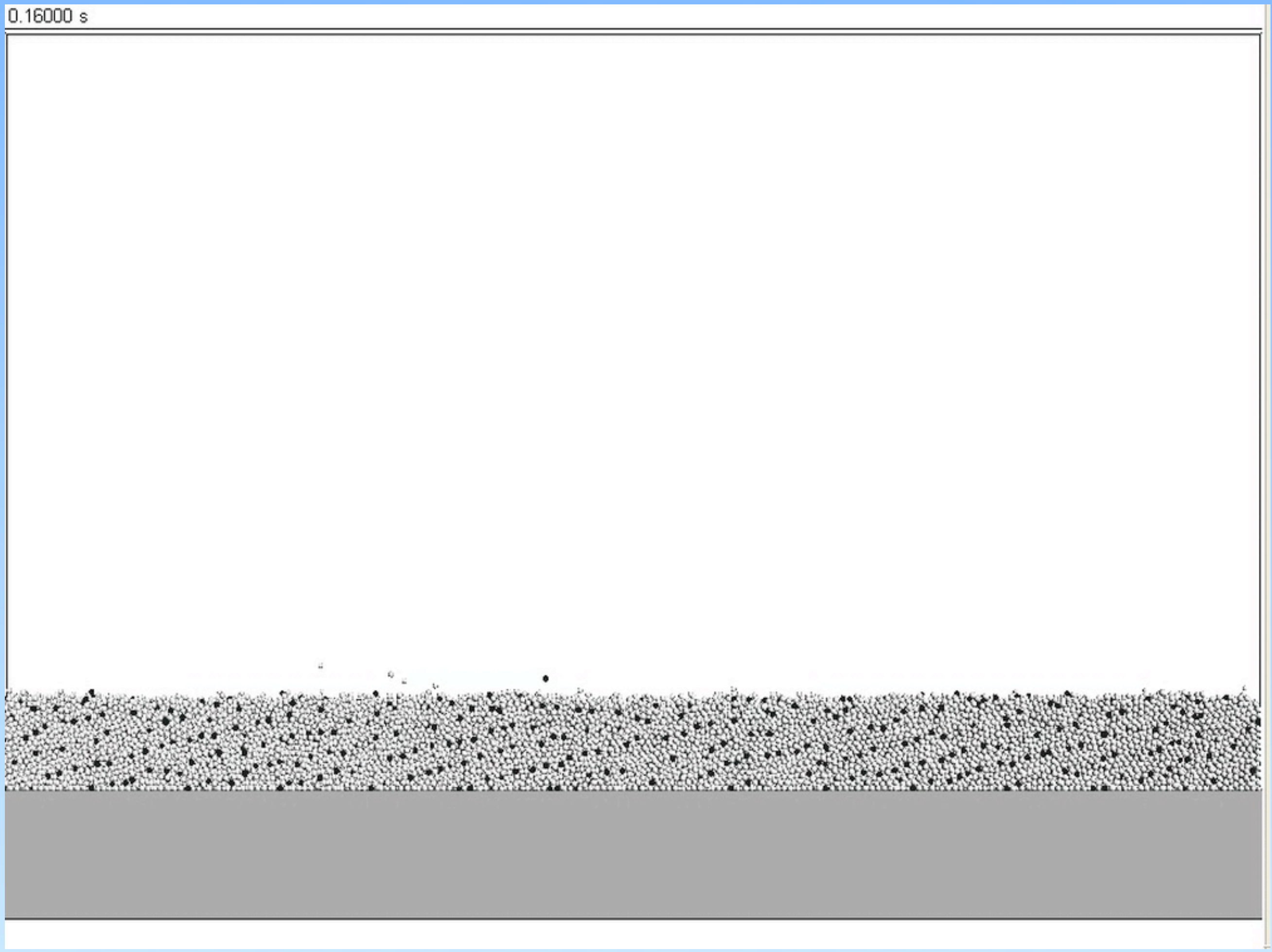
Numerical simulation of heaping with a hybrid GD-CFD code

Faraday heaping



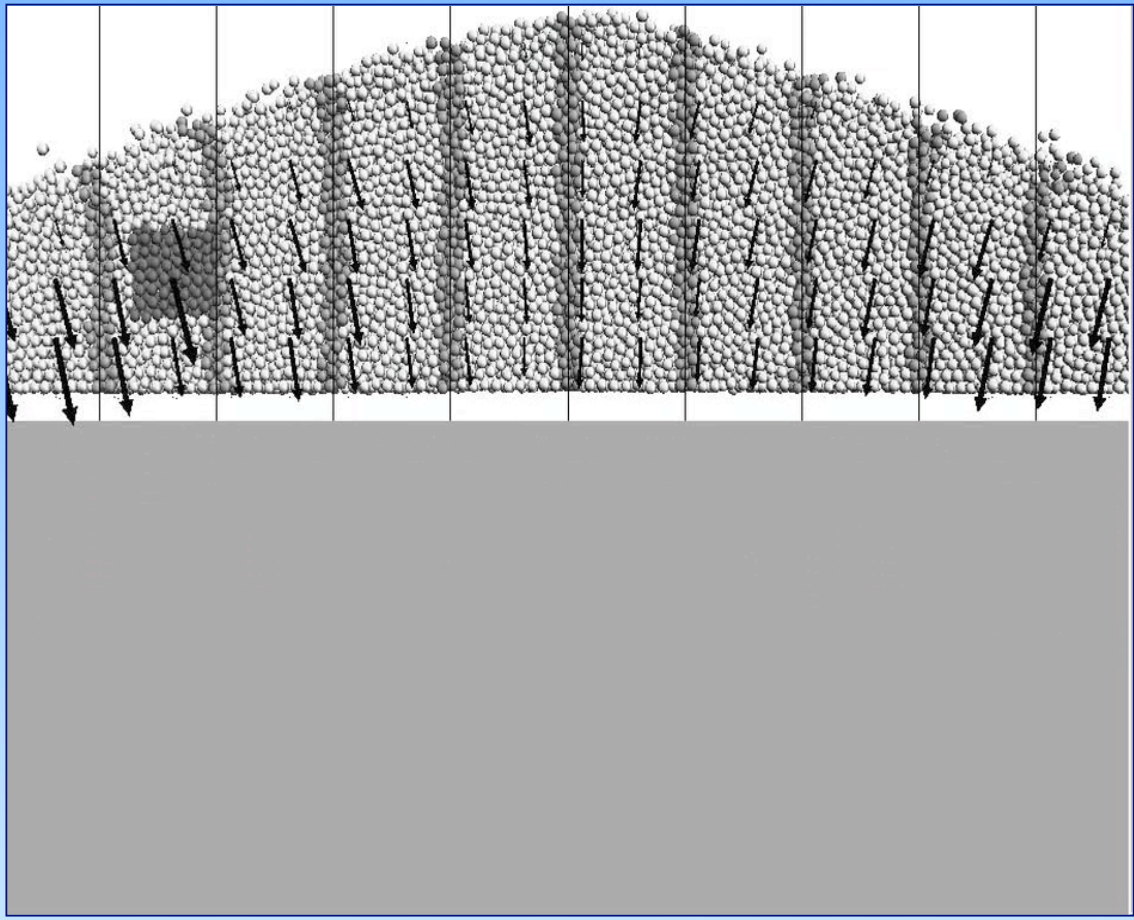
Numerical simulation of heaping with a hybrid GD-CFD code

Without air…



… there is no heap !

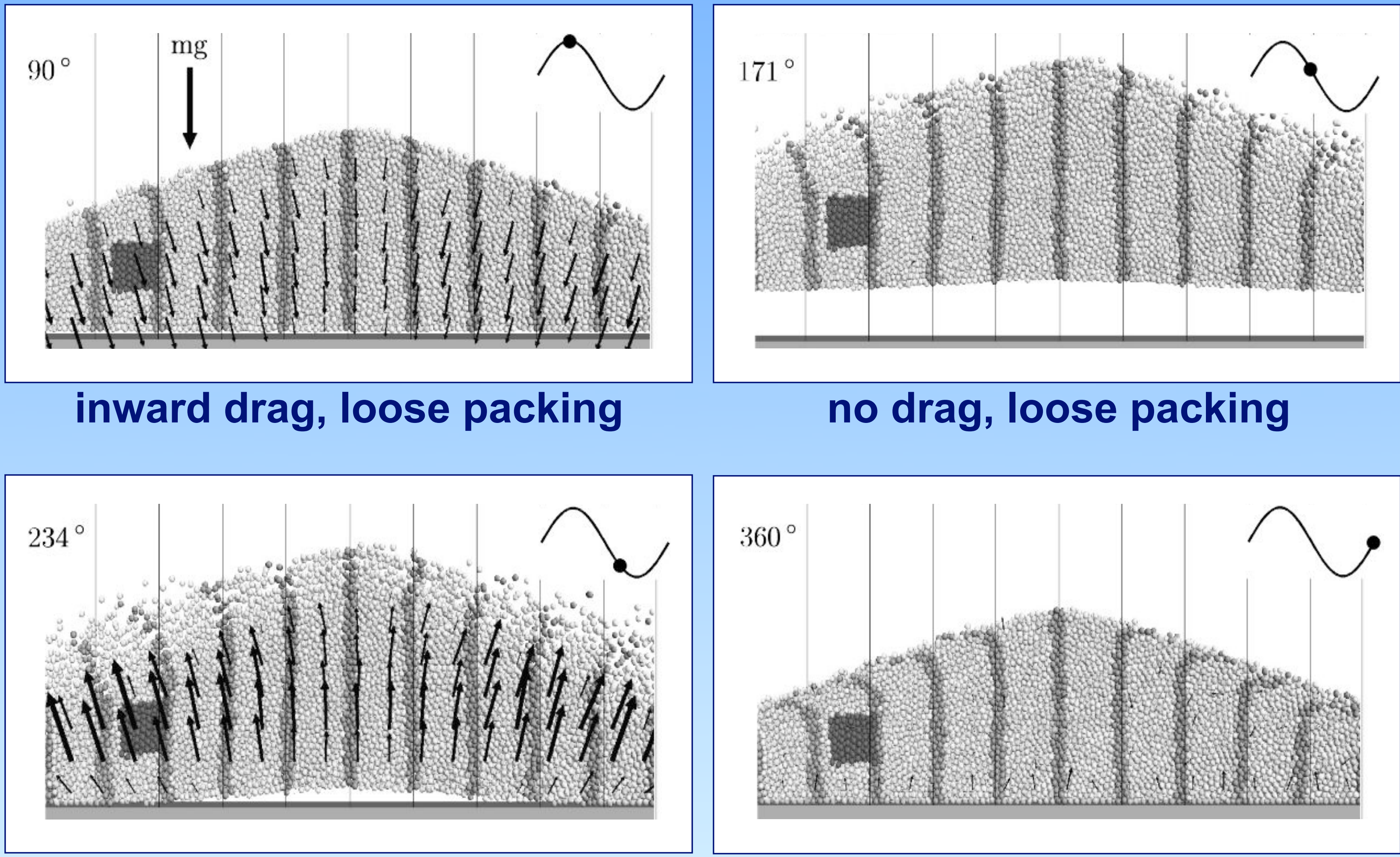
Steady state



arrows = air drag on particles

But why does the bulk only move inwards ?

4 snapshots



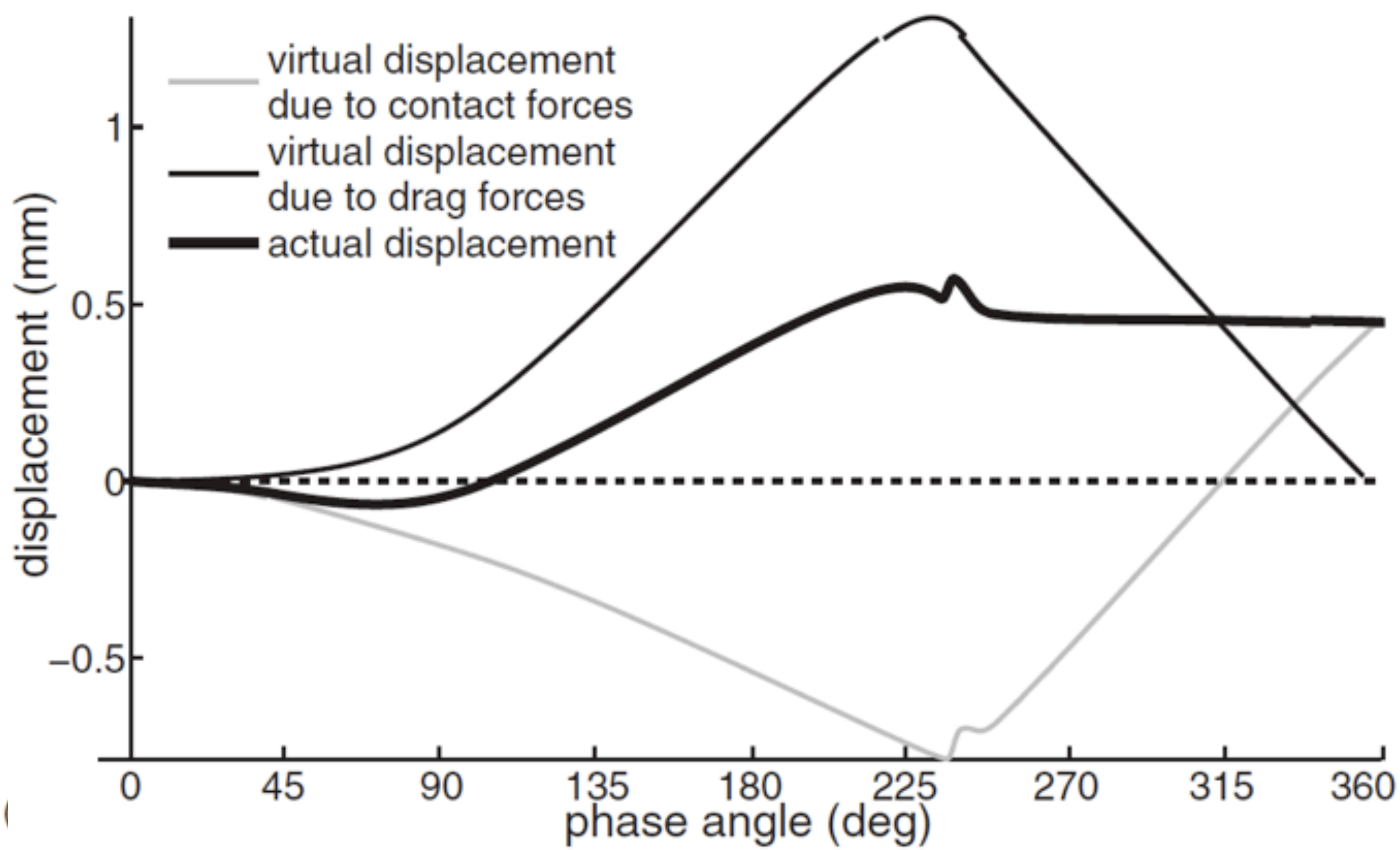
inward drag, loose packing

no drag, loose packing

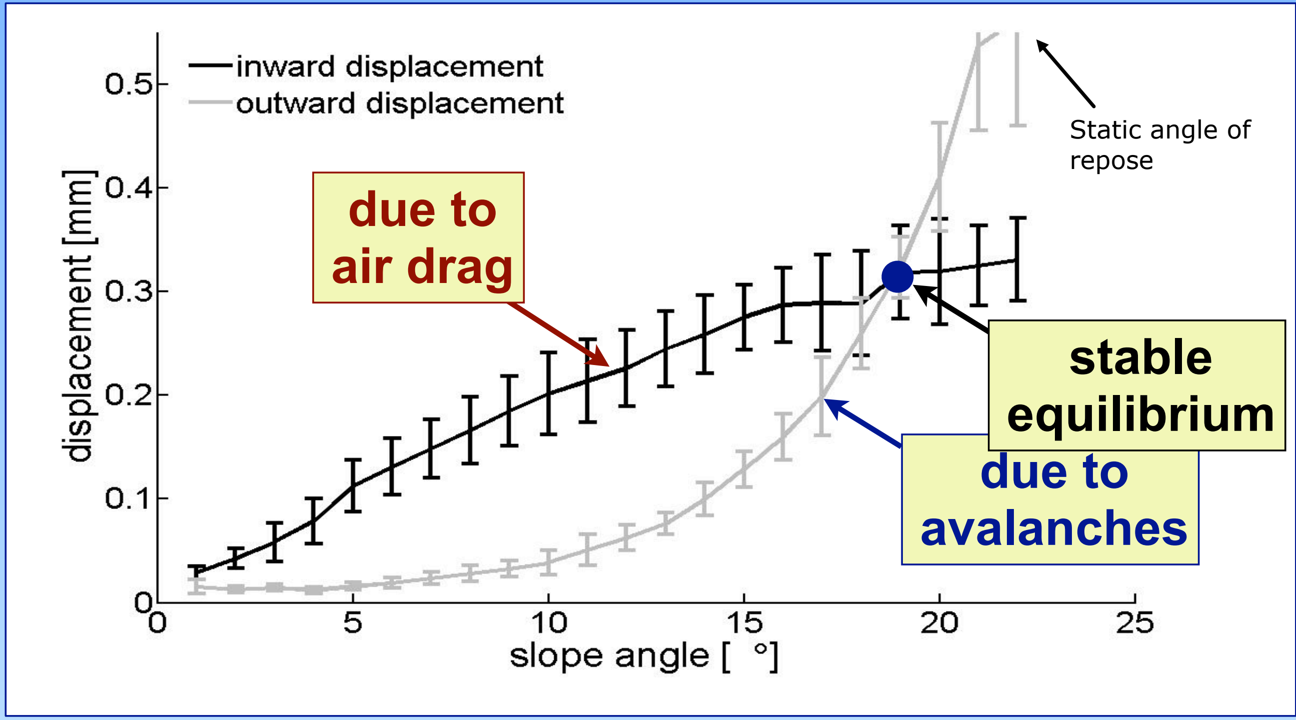
outward drag, dense packing

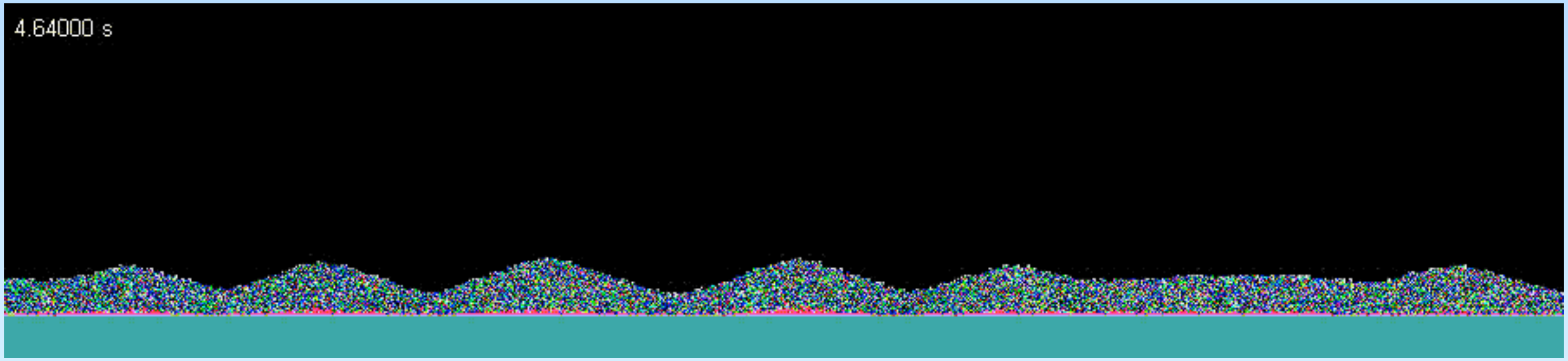
no drag, dense packing

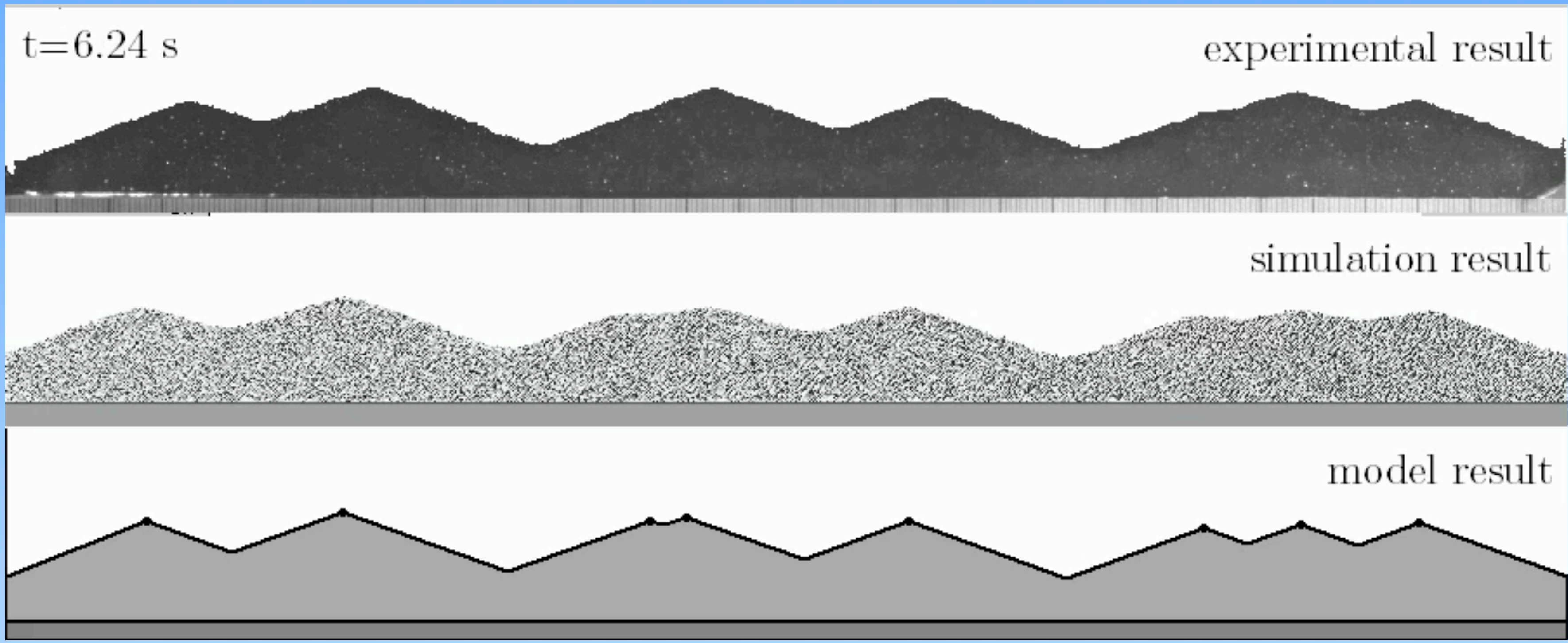
Force analysis (*x*-direction)

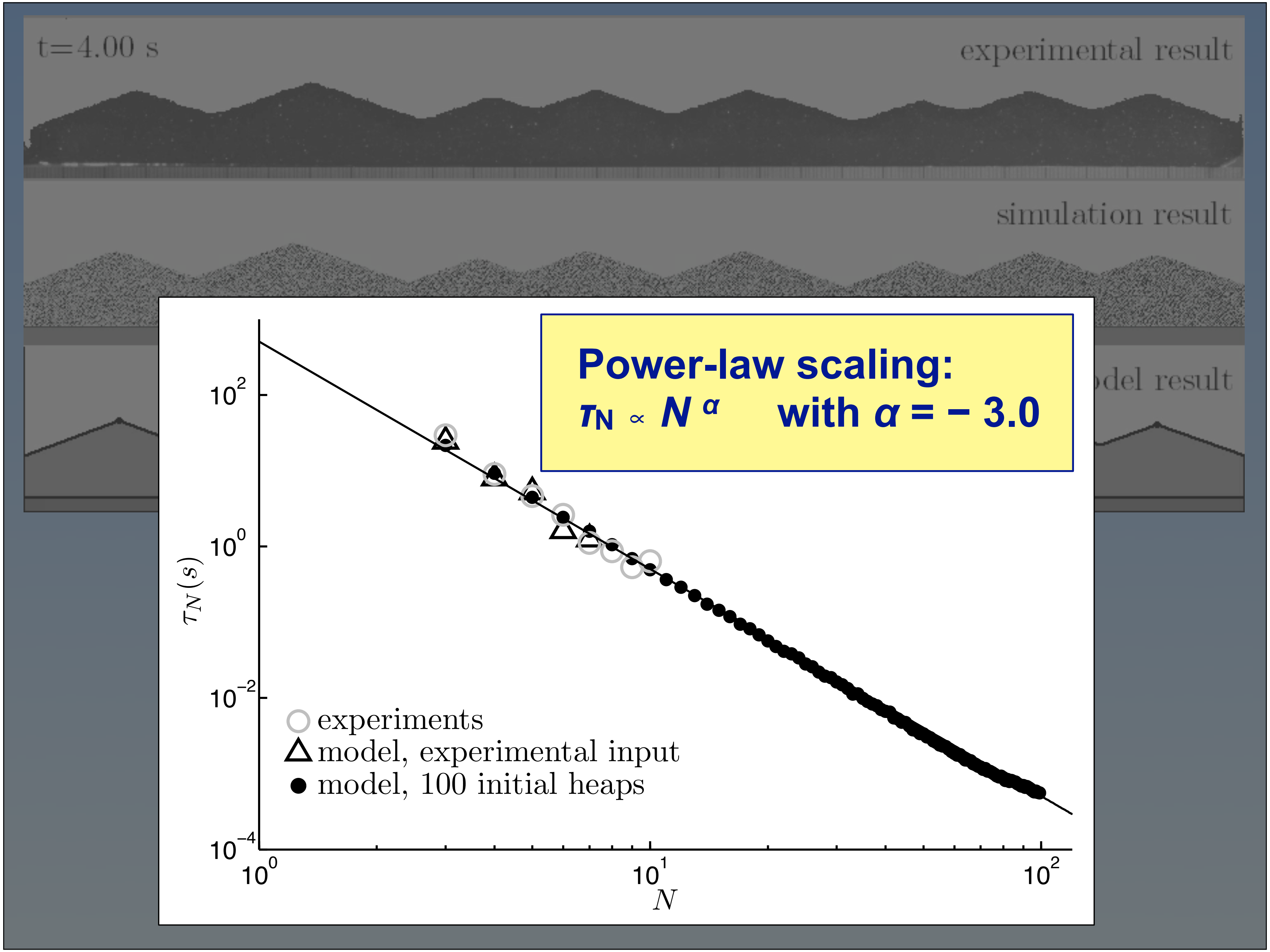


How does heaping start ?





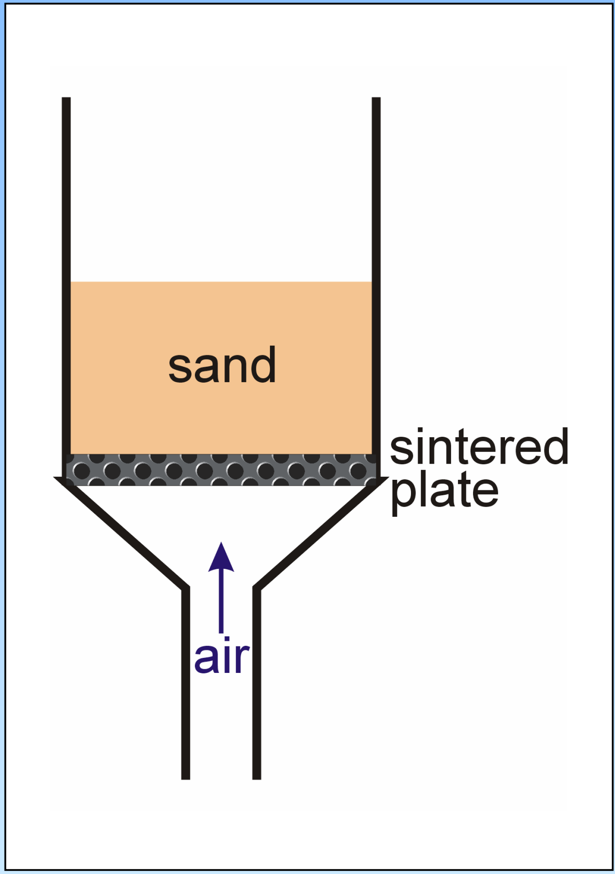

t=6.24 s
experimental result
simulation result
model result


t=4.00 s
experimental result
simulation result
odel result
Power-law scaling:
τN ∝ N α with α = − 3.0
τN(s)
N
10^2
10^0
10^-2
10^-4
10^0
10^1
10^2
experiments
model, experimental input
model, 100 initial heaps

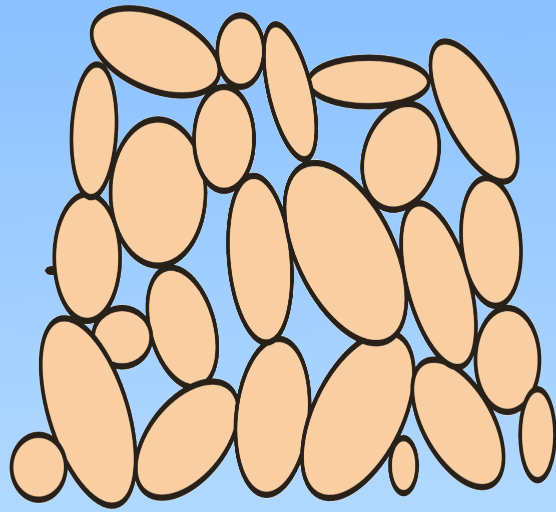
Example 2

The influence of air on drag in a granular bed

Preparing the sand



turn off air

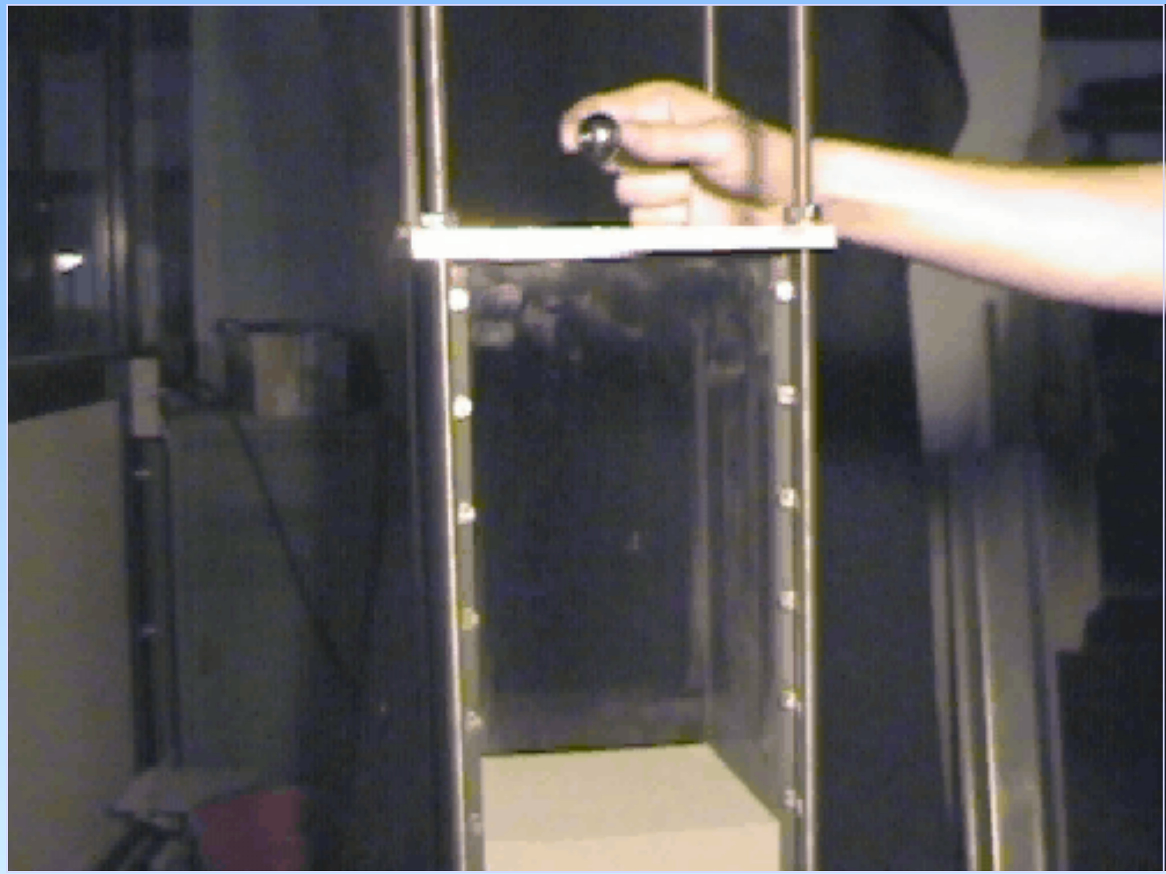


very loose packing:

solid fraction = 41 %

Controlled experiments

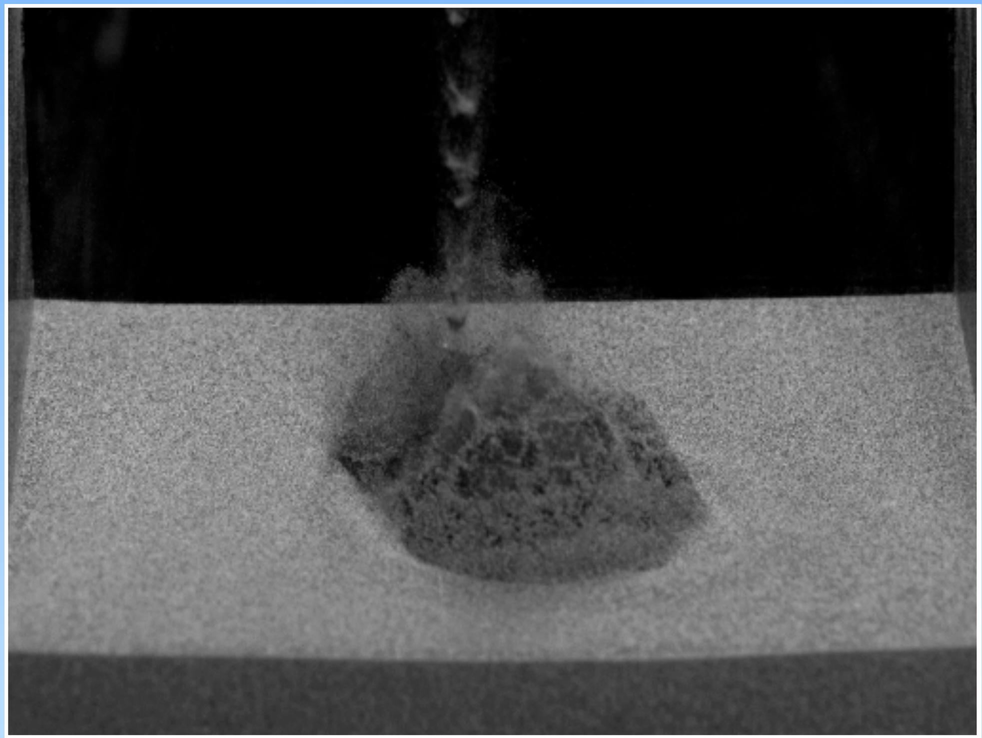
Ball dropped on **decompactified**, very fine sand



Controlled experiments

Ball dropped on **decompactified**, very fine sand

Ball impact on sand



3 events:

- Impact creates splash
- A jet is formed
- Granular eruption

Mechanism similar to disk impact on water

Disk impact on water (side view)

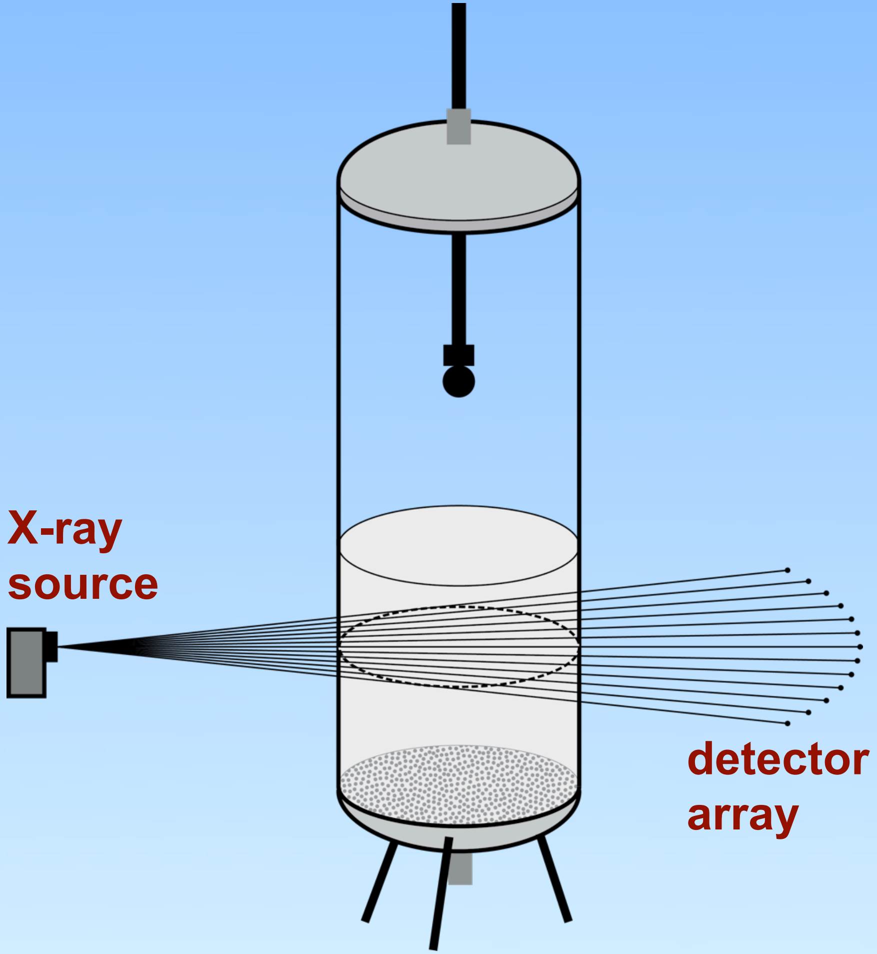
$v_{impact} \approx 1.0$ m/s

$R_{disk} = 0.03$ m

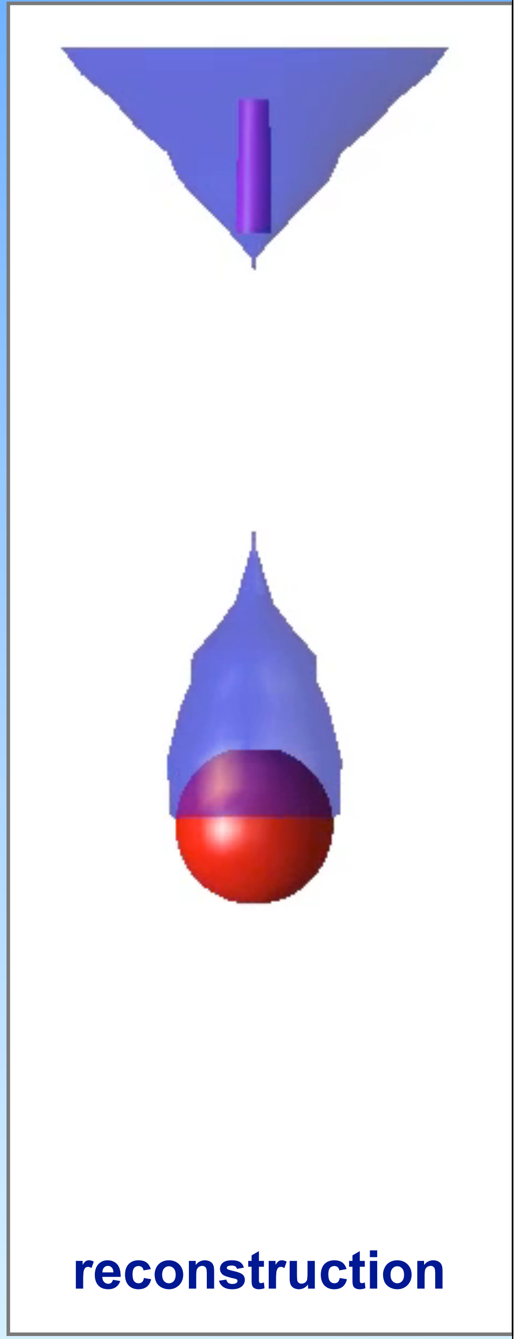
Bergmann, DvdM, Stijnman, Sandtke, Prosperetti, Lohse, Phys. Rev. Lett. **96**, 154505 (2006)

Gekle, Peters, Gordillo, DvdM, Lohse, Phys. Rev. Lett. **104**, 024501 (2010)

X-ray imaging: Measuring jet formation



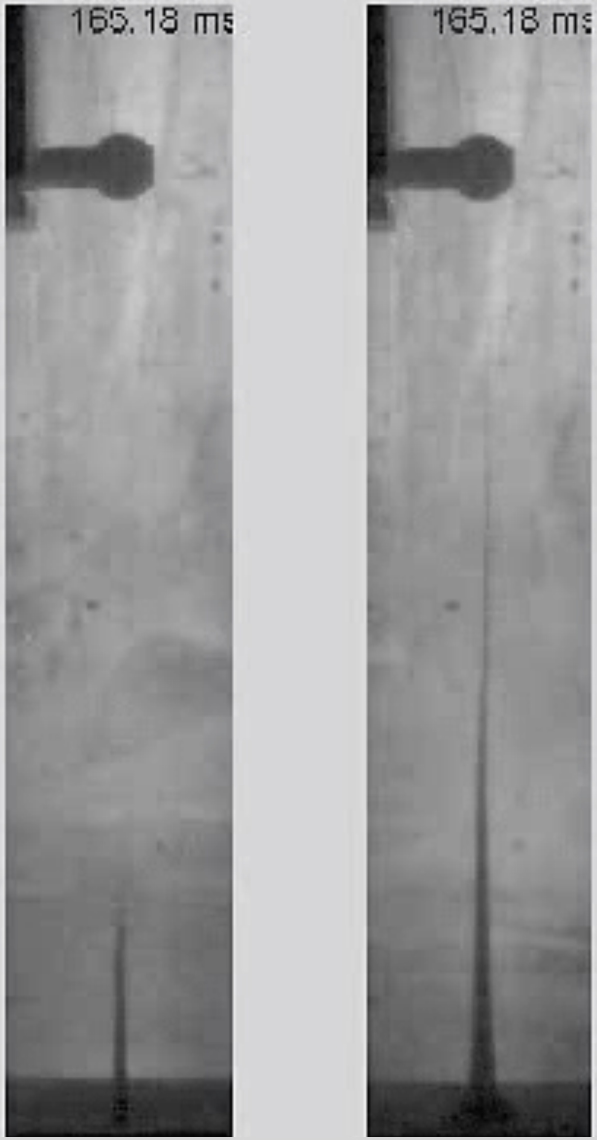
Rob Mudde, TUD



What is the role of air in granular jet formation ?

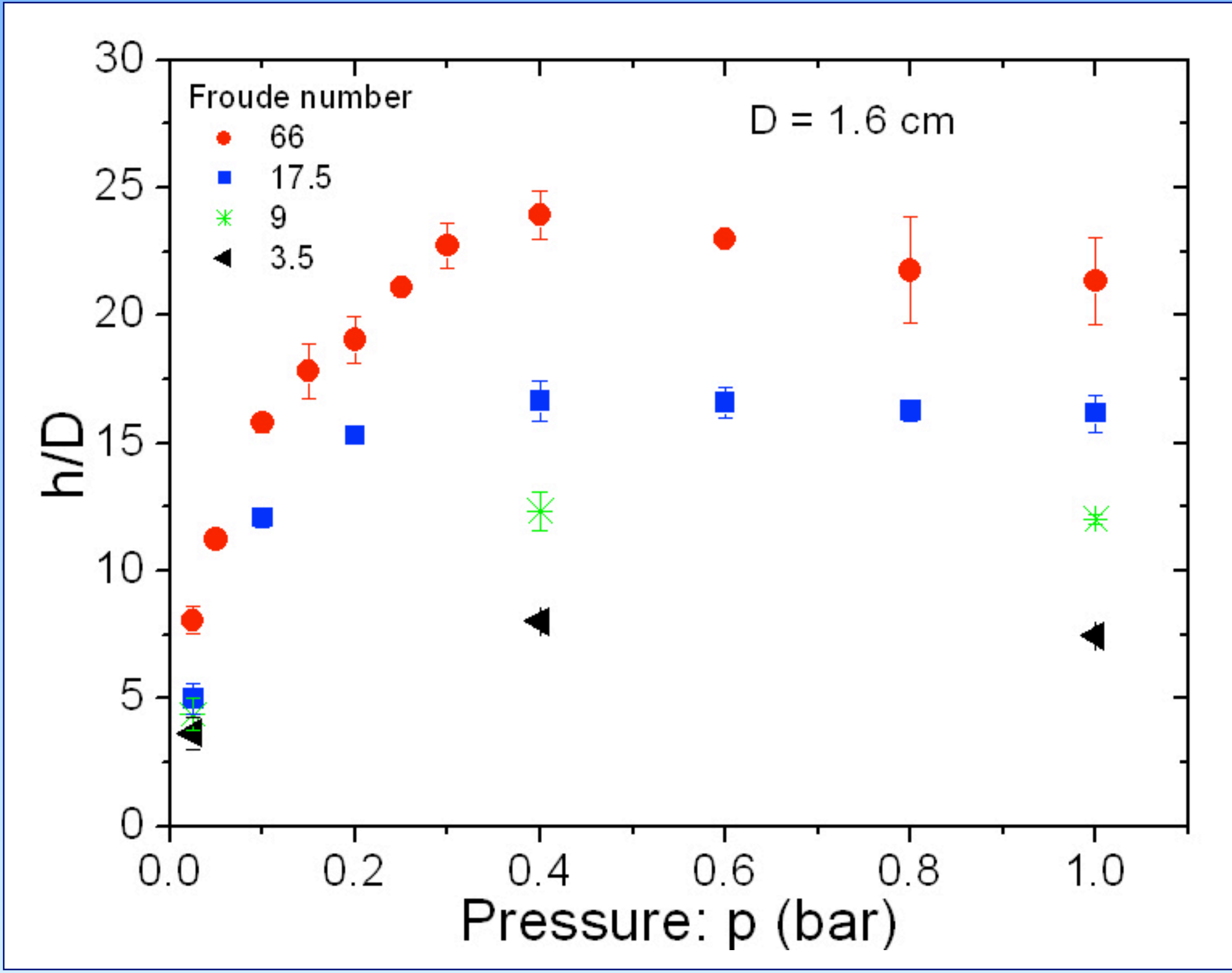
Royer, Corwin, Flior, Cordero, Rivers, Eng, and Jaeger, *Nature Physics* 1, 164 (2005).

Caballero-Robledo, Bergmann, DvdM, Prosperetti, and Lohse, *Phys. Rev. Lett.* 99, 018001 (2007)



25 mbar   1 bar

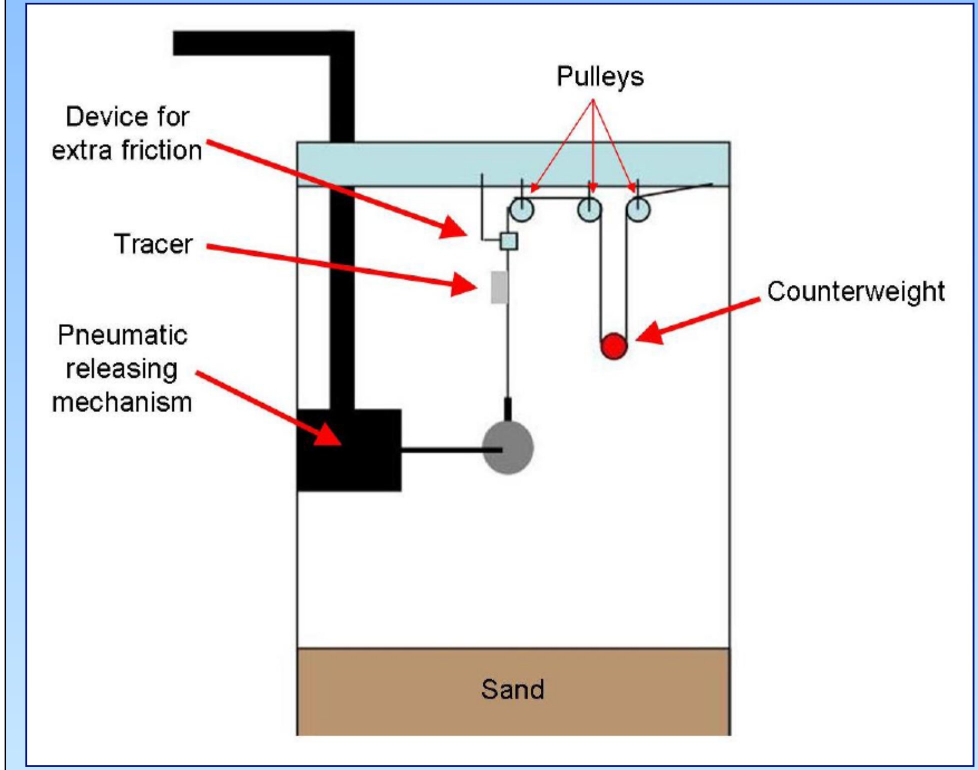
Jet height vs. pressure

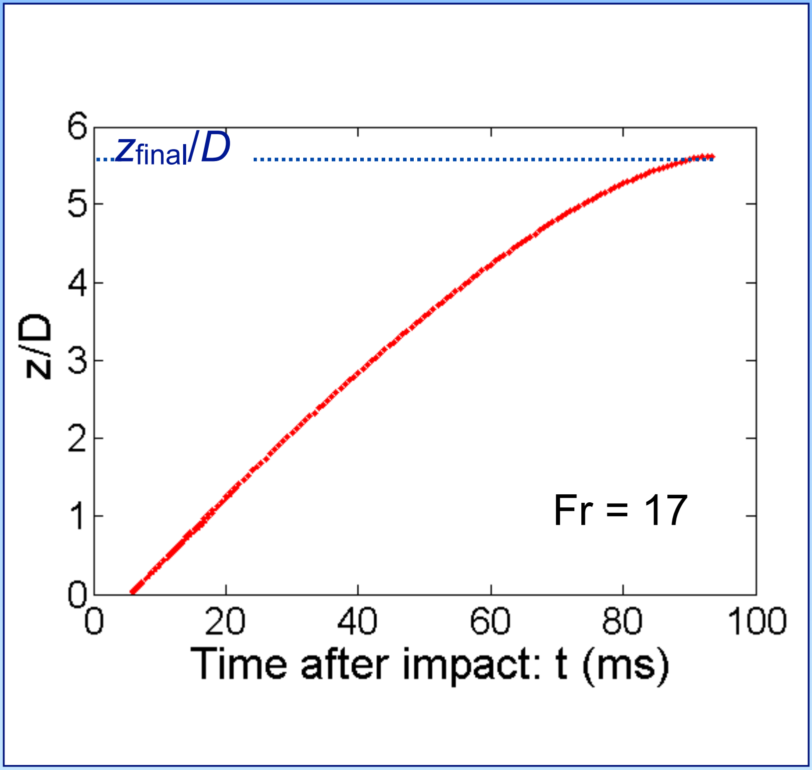


Froude number:

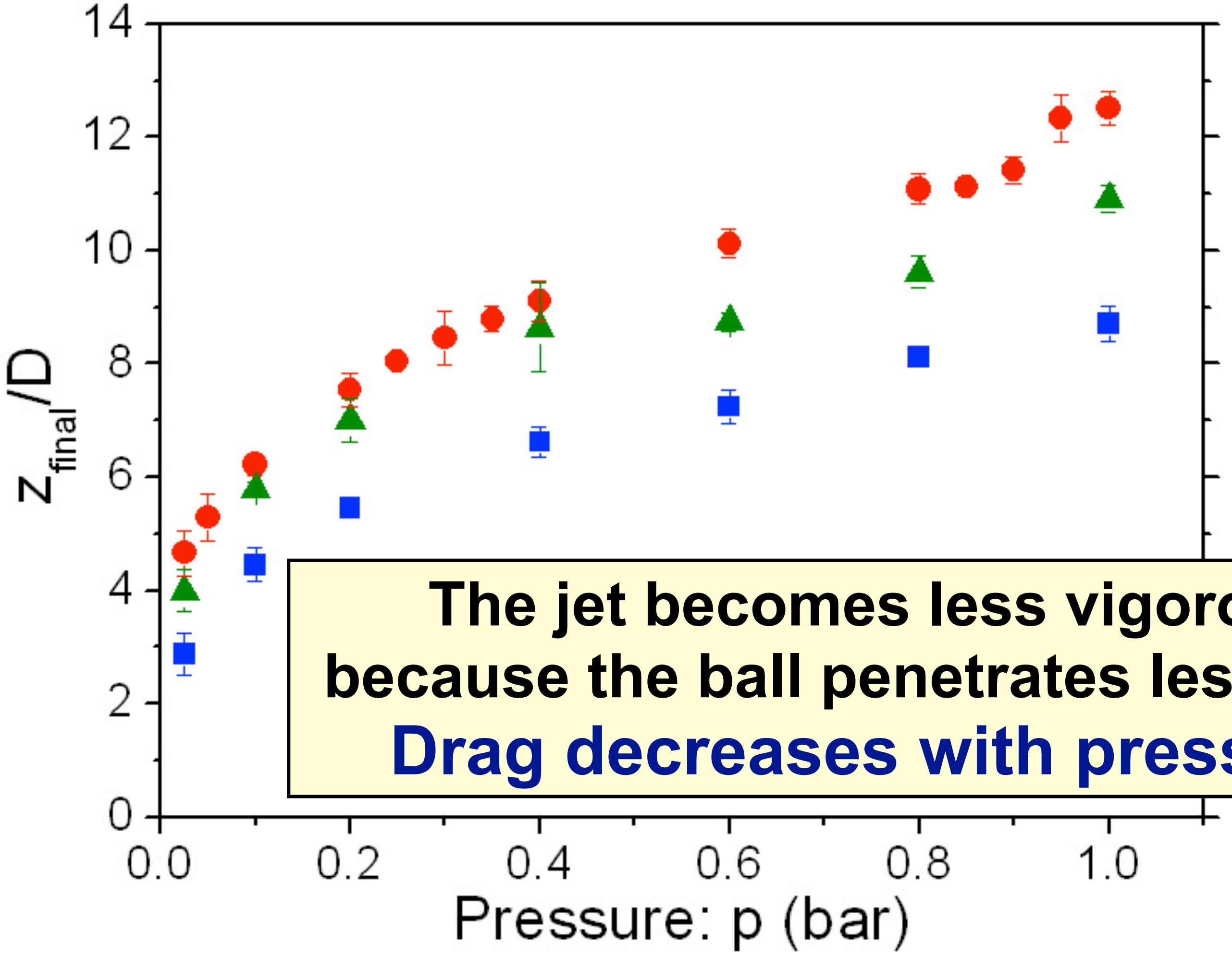
$$Fr \equiv \frac{U^2}{gD}$$

Trajectories in the sand





Final depth vs. pressure



The jet becomes less vigorous because the ball penetrates less deep

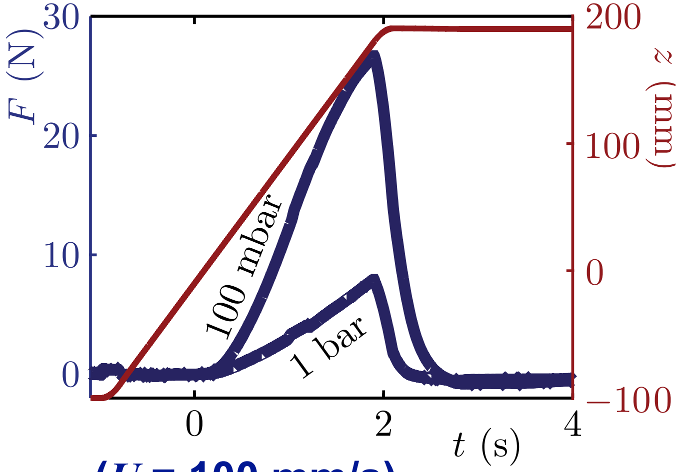
Drag decreases with pressure

Directly measuring drag:

Modified penetrometer

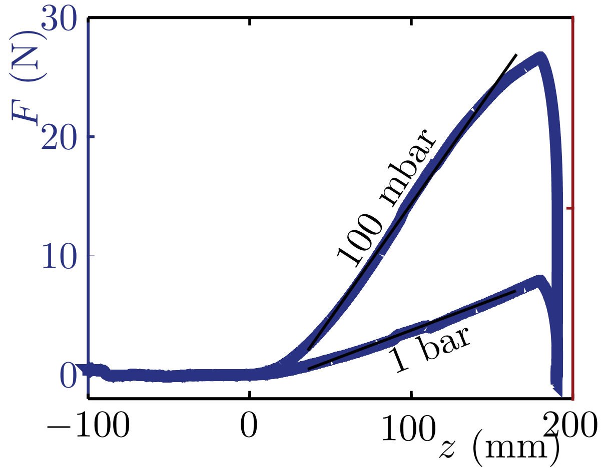
- *linear motor controls velocity U*
- *load cell measures drag F directly*
- *pump controls ambient pressure P_0*

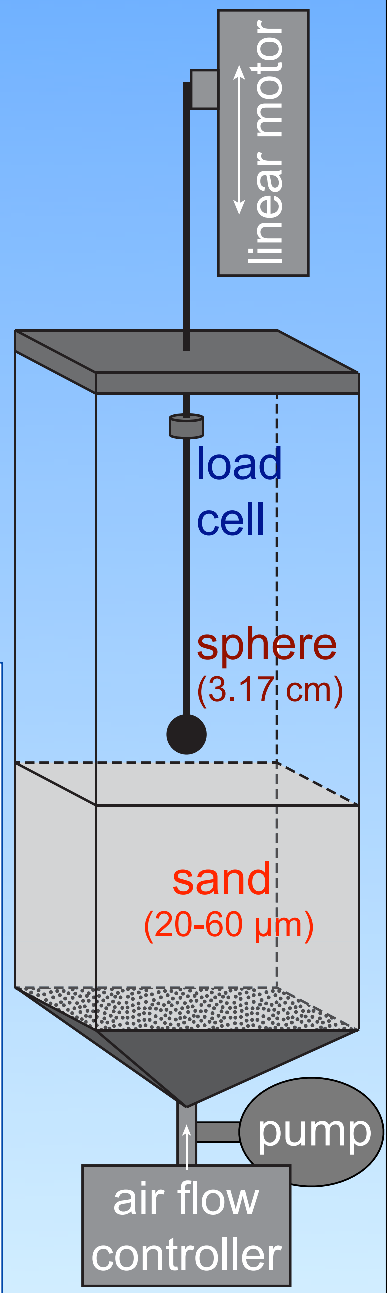
drag vs time



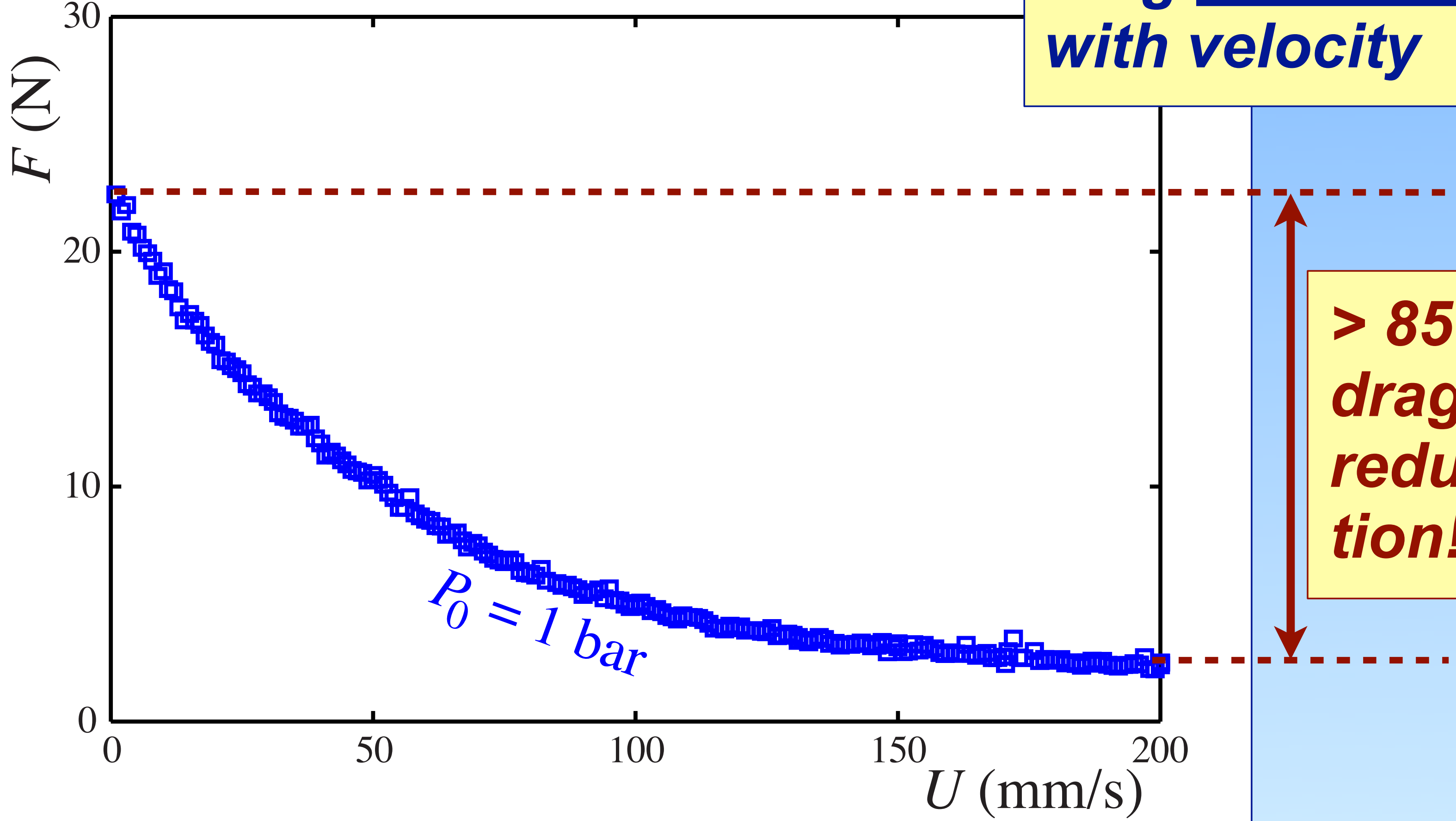
(U = 100 mm/s)

drag vs position





Drag F versus velocity U (P_0 = 1 bar)

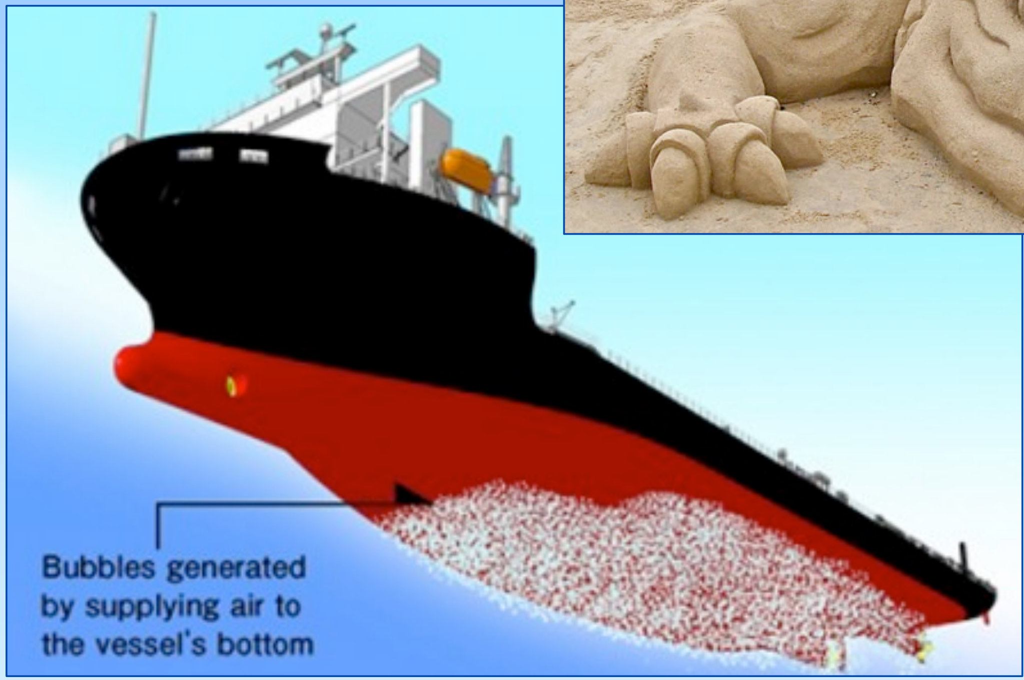


drag <u>decreases</u> with velocity

> 85 % drag reduc-tion!

(drag F measured at z = 10 cm below surface)

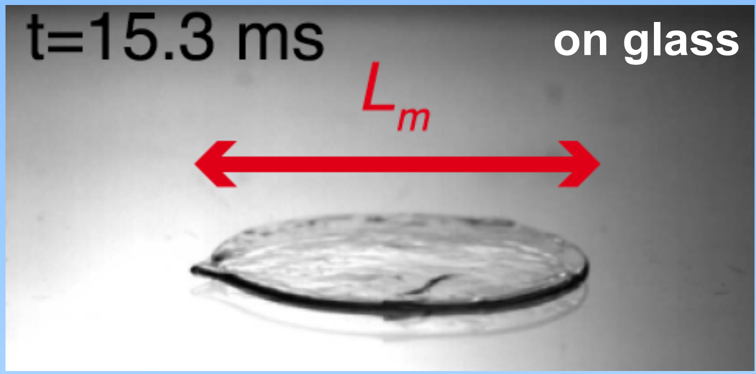
Drag reduction in literature

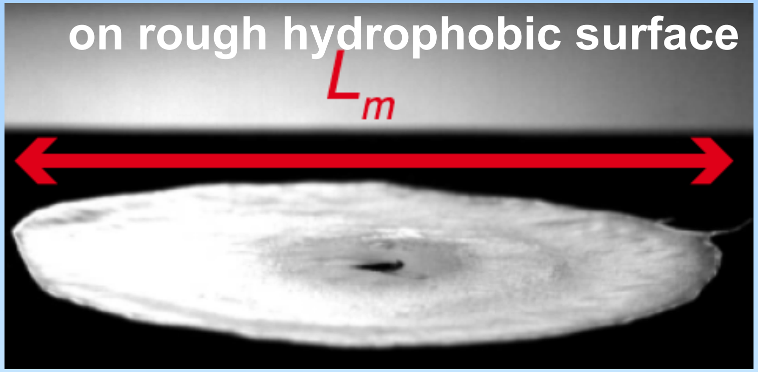


drag reduction
≈ 5 % to 30 %

Drag reduction in literature

Impact Carbopol droplet





drag reduction ≈ 85 %

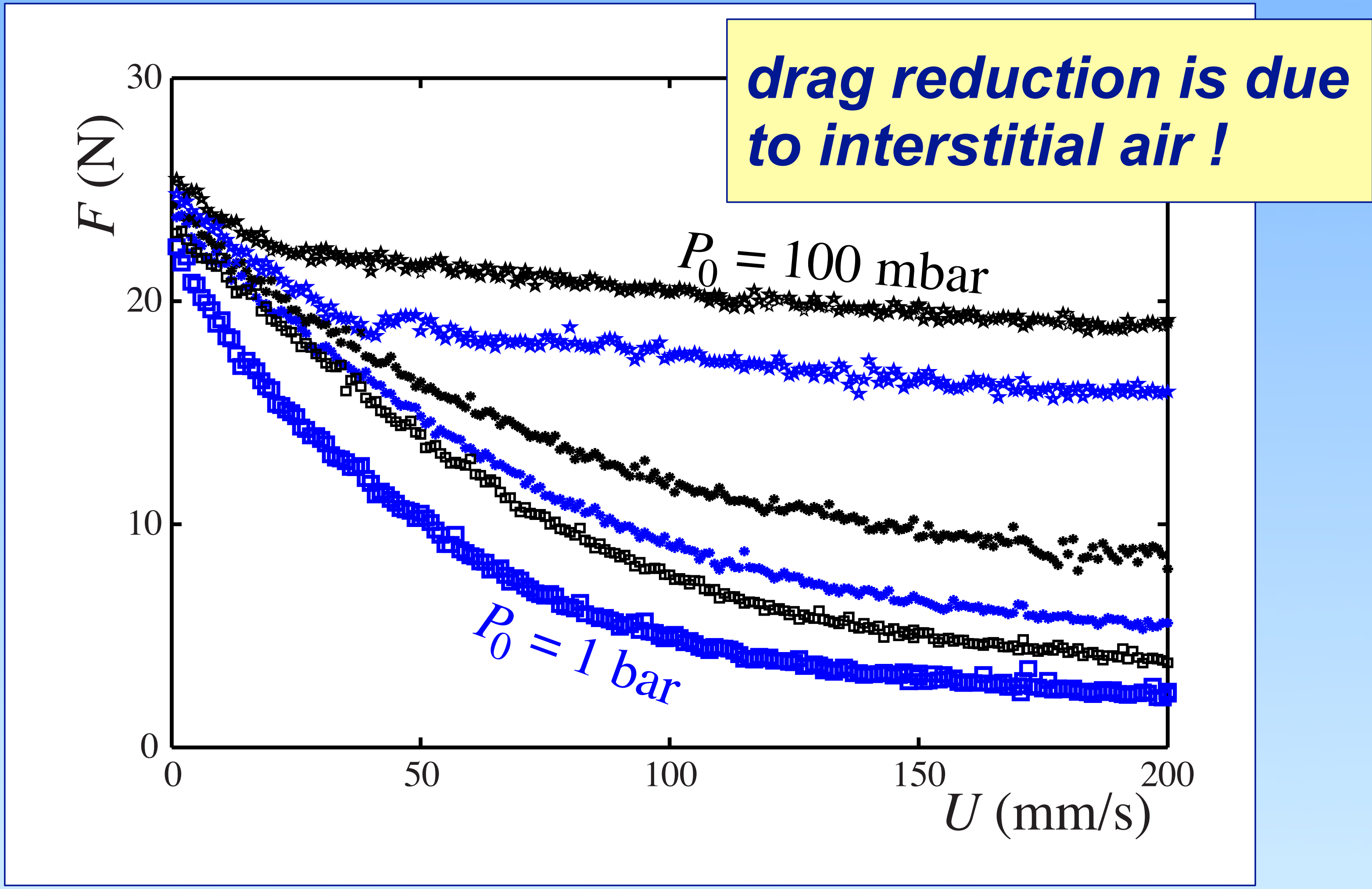
Luu, Forterre,
Phys. Rev. Lett. 110, 184501 (2013).

Impact heated sphere

drag reduction ≥ 85 %

Vakarelski, Marston, Chan, Thoroddsen,
Phys. Rev. Lett. 106, 214501 (2011).

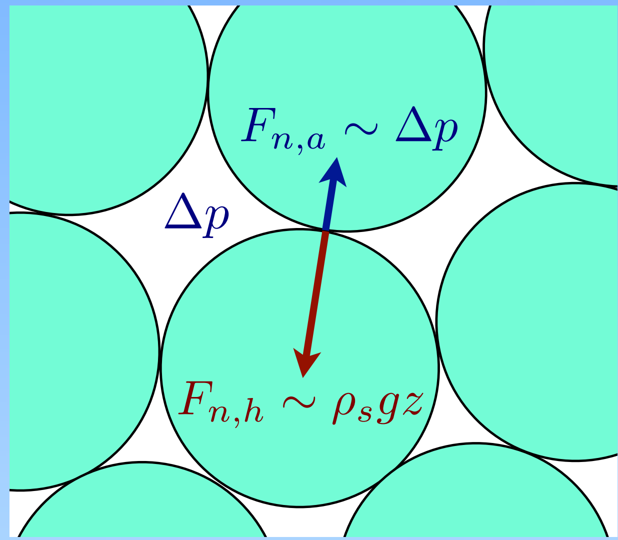
Drag *F* versus velocity *U* ($P_0 \leq 1$ bar)



(drag *F* measured at *z* = 10 cm below surface)

Model

assumption:
air modifies the contact forces between grains.



drag without air:
grains perform work against contact forces ~ hydrostatic pressure $\rho_s g z$

$$\frac{F}{\rho_s g z\, D^2} = f$$

drag with air:
excess air pressure ΔP works against hydrostatic pressure and therefore decreases the contact forces

$$\frac{F}{\rho_s g z\, D^2} = f\left(\frac{\Delta P}{\rho_s g z}\right)$$

Problem:
$f(\,)$ is unknown

What determines ΔP ?

Time evolution of the excess pressure:

$$\frac{d\Delta P}{dt} = \boxed{\alpha \frac{P_0 U}{D}} - \boxed{\beta \frac{P_0}{\eta} \Delta P}$$

$$\eta = \frac{\mu D^2}{\kappa} \approx 4.5 \cdot 10^4 \mathrm{Pa\,s}$$

increase due to compressional compaction

decrease due to Darcy flow inside the sand

For constant U this linear ODE is directly solved ($t = z/U$):

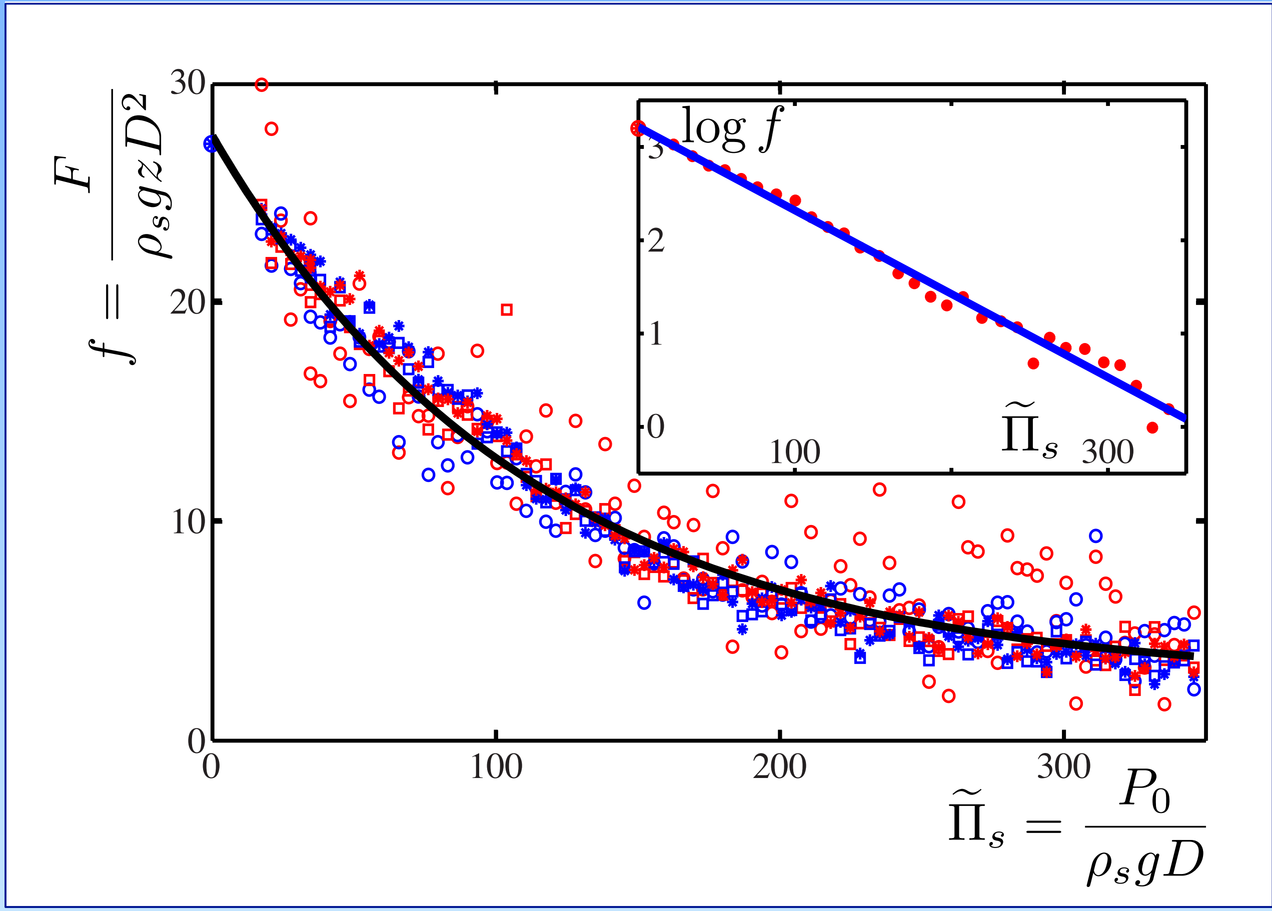
$$\widetilde{\Pi} = \frac{\Delta P}{\alpha \rho_s g z} = \frac{1}{\beta} \frac{\eta U}{\rho_s g D z} \left[1 - \exp\left(-\beta \frac{P_0}{\eta U} z \right) \right]$$

which in the limit of large U becomes constant (z-independent):

$$\widetilde{\Pi} \to \widetilde{\Pi}_s = \frac{P_0}{\rho_s g D}$$

Can we use this to determine $f(\,)$?

Drag F versus pressure P_0 (U=200 mm/s)



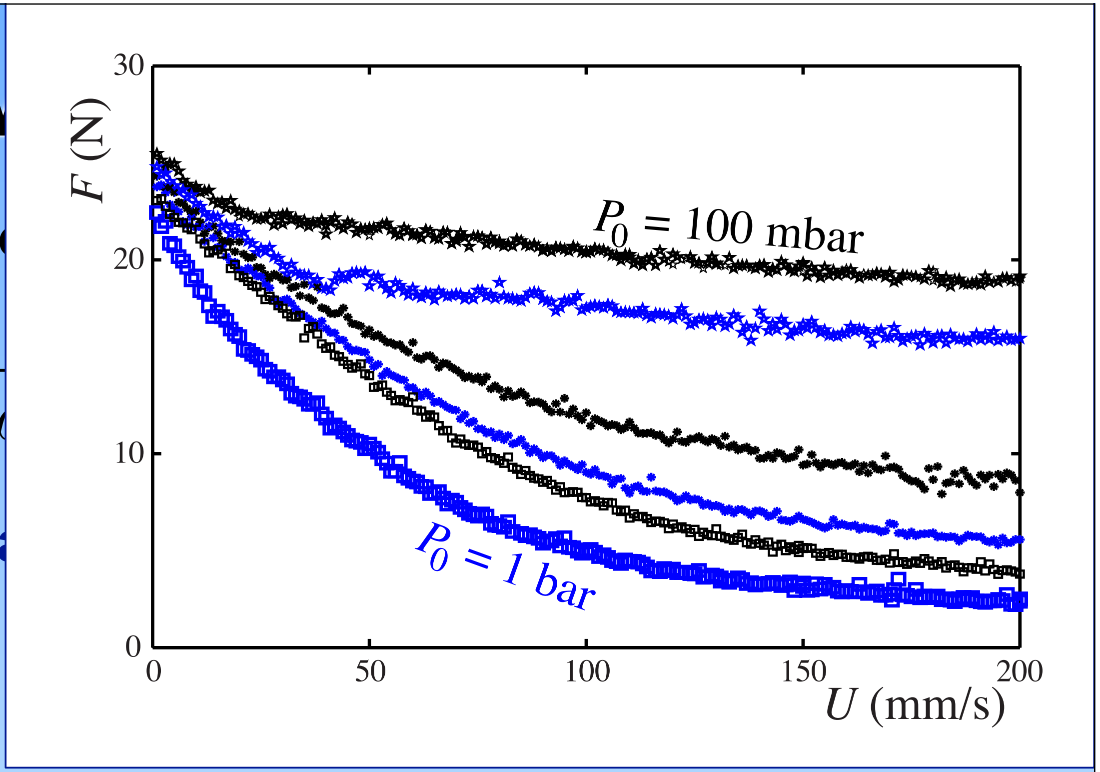
Calculate $\widetilde{\Pi}_s$ from P_0 and f from F in this limit of large U

Fit to functional form: $f(\widetilde{\Pi}) = f_s + f_0 \exp(-\widetilde{\Pi}/\widetilde{\Pi}_0)$

model v

Turn back to the time ev

$$\widetilde{\Pi} = \frac{\Delta P}{\alpha \rho_s g z} = \frac{1}{\beta} \cdots$$



For all of our $F(z,U,P_0)$ da

$$\zeta = \frac{P_0 z}{\eta U}$$

$$\widetilde{\Pi}^* = \frac{\rho_s g D}{P_0} \widetilde{\Pi}$$

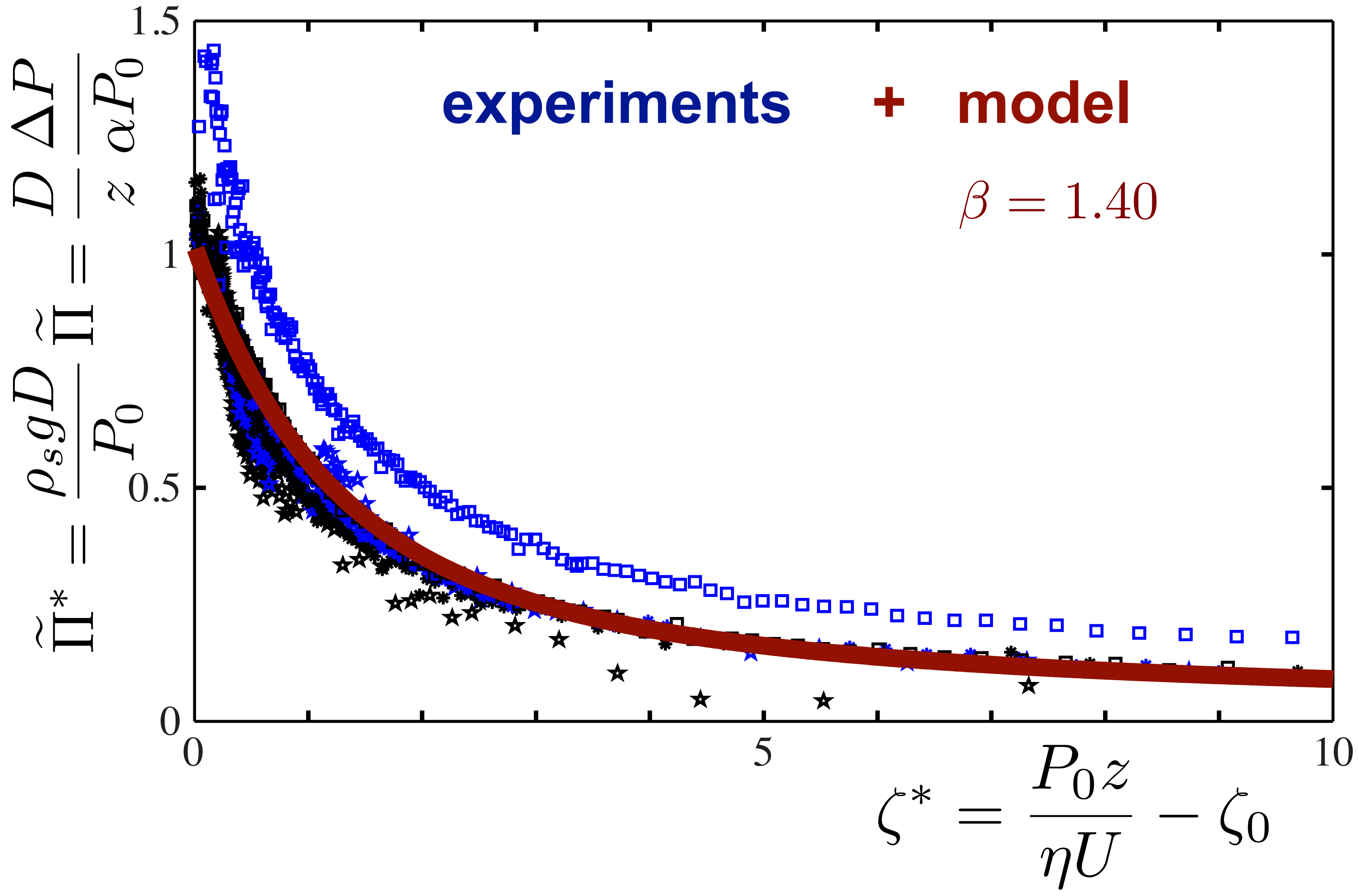
with $\widetilde{\Pi}$ determined from the dimensionless drag force f by inverting $f(\widetilde{\Pi})$.

which turns the equation into:

$$\widetilde{\Pi}^* = \frac{[1 - \exp(-\beta\zeta)]}{\beta\zeta}$$

Does this single parameter equation fit the data?

comparison


experiments + model
$\beta = 1.40$
$\widetilde{\Pi}^* = \frac{\rho_s g D}{P_0}\widetilde{\Pi} = \frac{D}{z}\frac{\Delta P}{\alpha P_0}$
$\zeta^* = \frac{P_0 z}{\eta U} - \zeta_0$
0
0.5
1
1.5
0
5
10

WATER

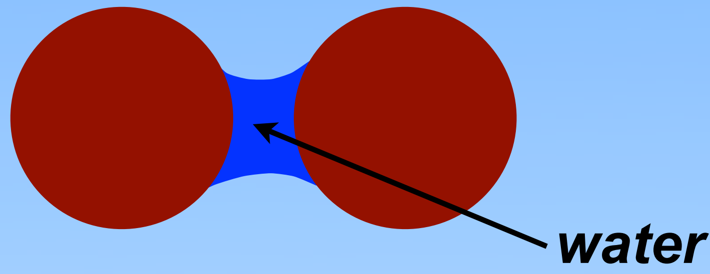
Granular matter and water

Wet granular matter

little liquid,
much particles

liquid consolidates granular material

capillary bridges



$$F_{cap} \sim F_g$$

$$\sigma \, \pi d \sim \tfrac{1}{6} \pi d^3 \, \rho \, g$$

$$\boxed{d \sim \sqrt{\frac{6\sigma}{\rho g}} \approx 4 \text{ mm}}$$

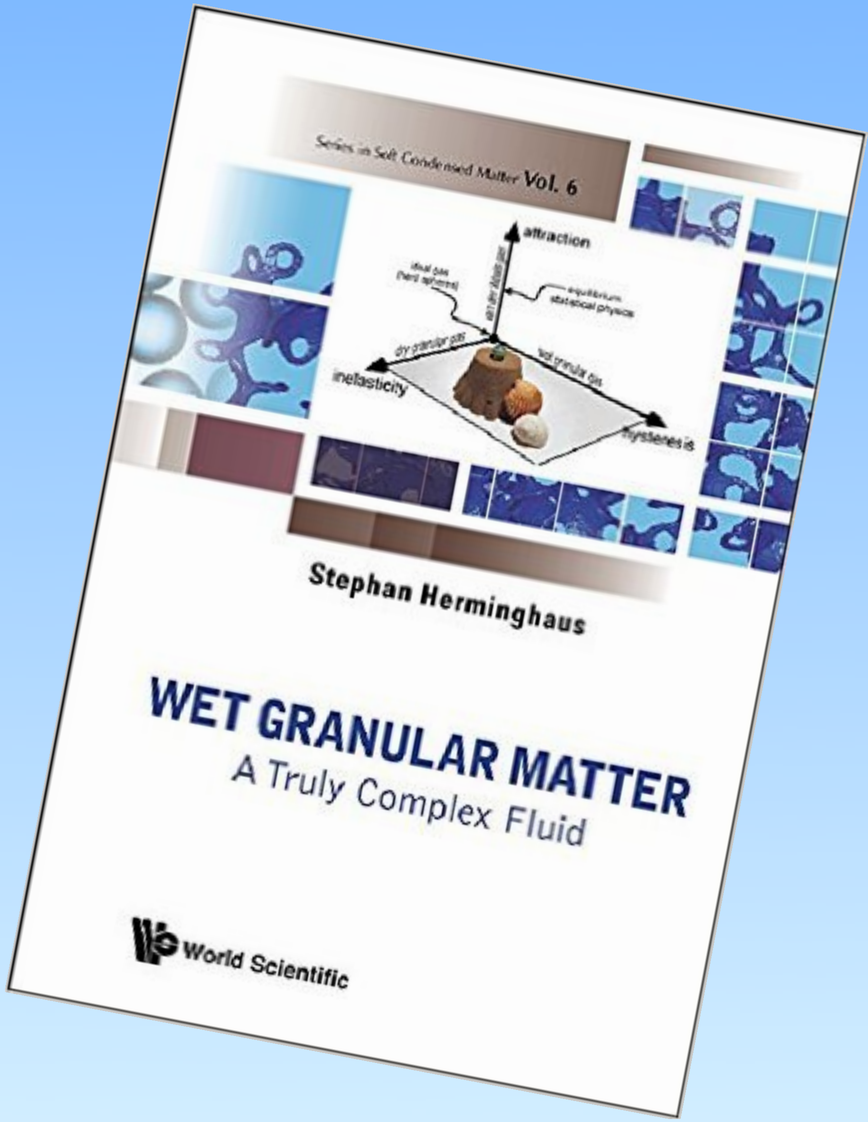
$$\frac{F_{cap}}{F_g} \sim \frac{1}{d^2}$$

Granular matter and water

Wet granular matter

little liquid,
much particles

liquid consolidates
granular material



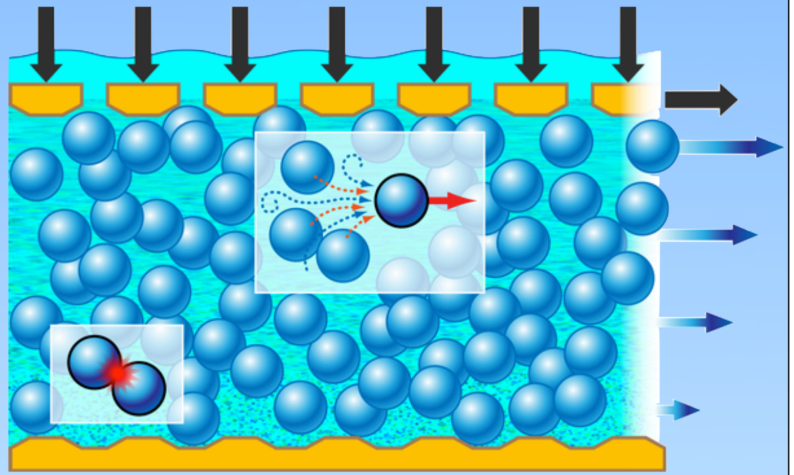
Granular matter and water

Wet granular matter

little liquid,
much particles

liquid consolidates
granular material

Suspension

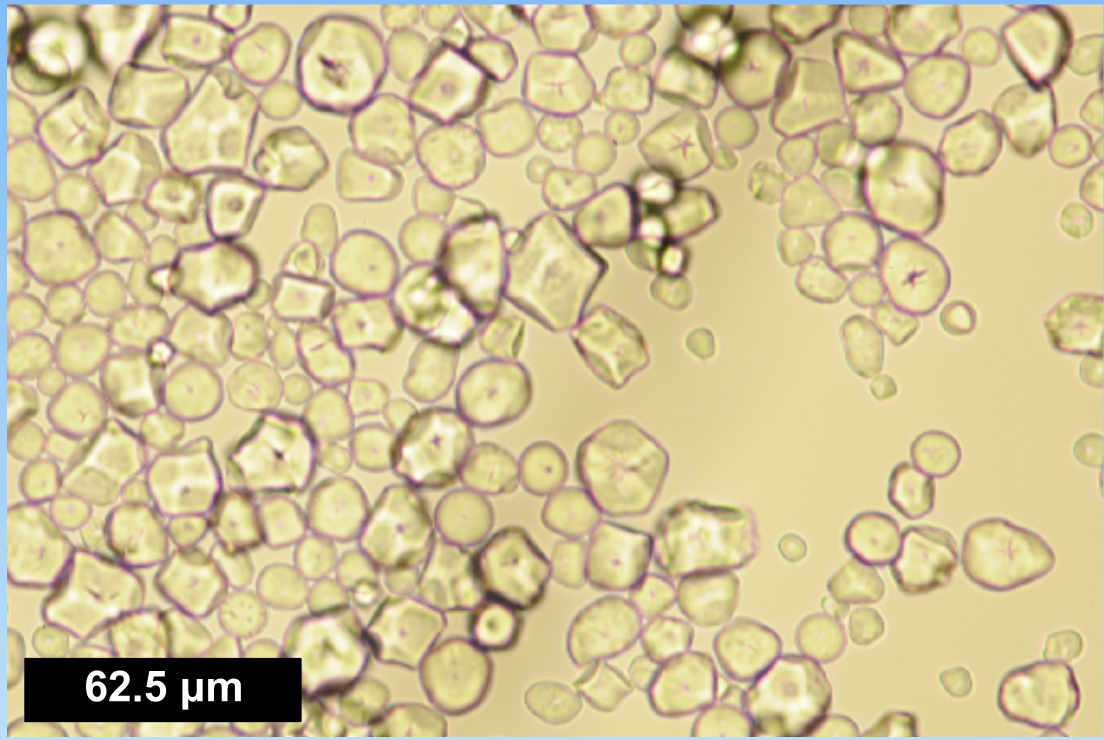


much liquid,
few particles

particles determine
suspension rheology

equal amount of liquid and grains

Cornstarch



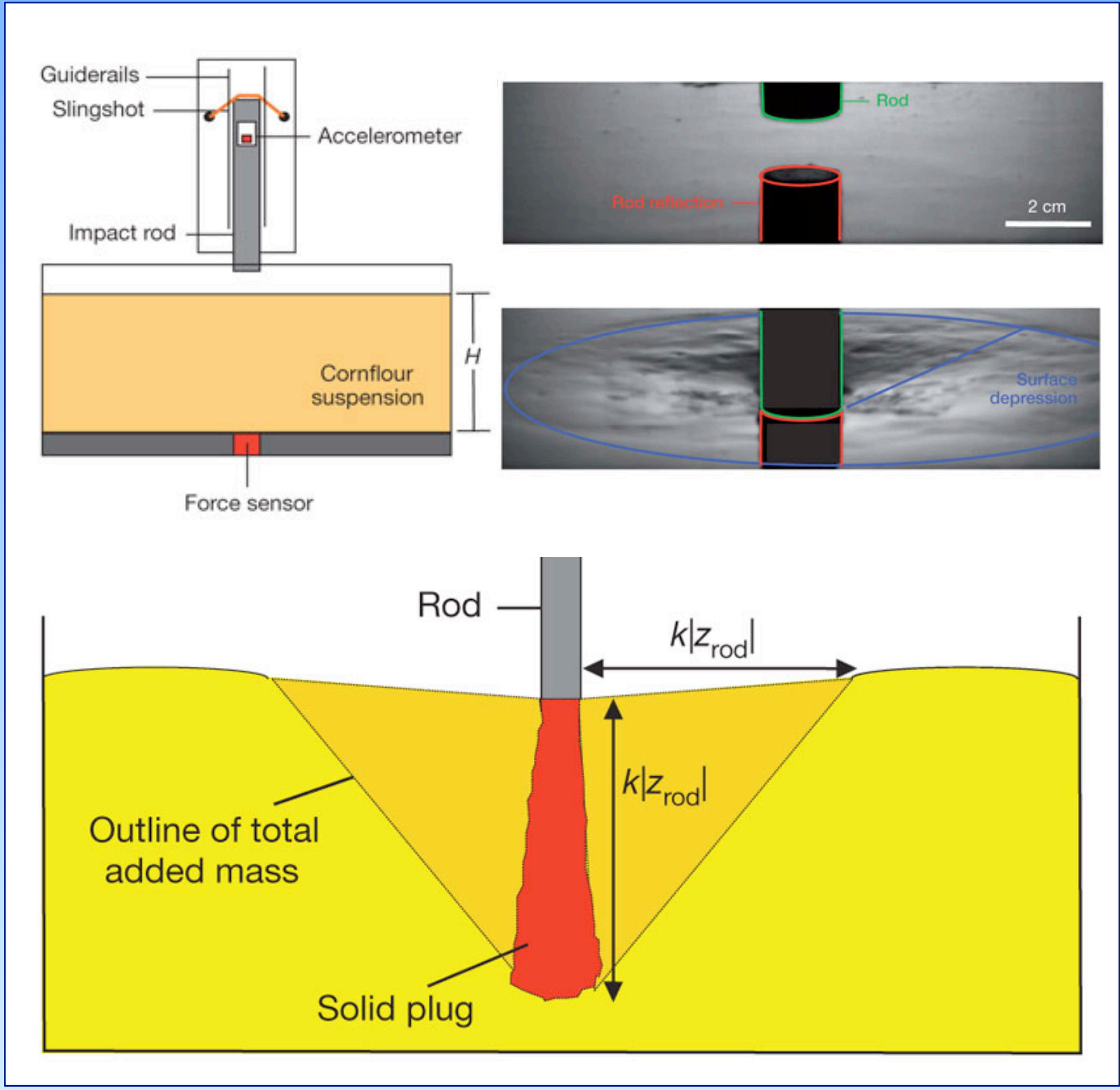
diameter: 5-20 µm,
flat distribution of sizes (numbers)
Irregular shapes
ρ = 1.5 g/cm^3

“shear thickening suspension”

Cornstarch on a shaker

Walking on cornstarch

How is this possible?



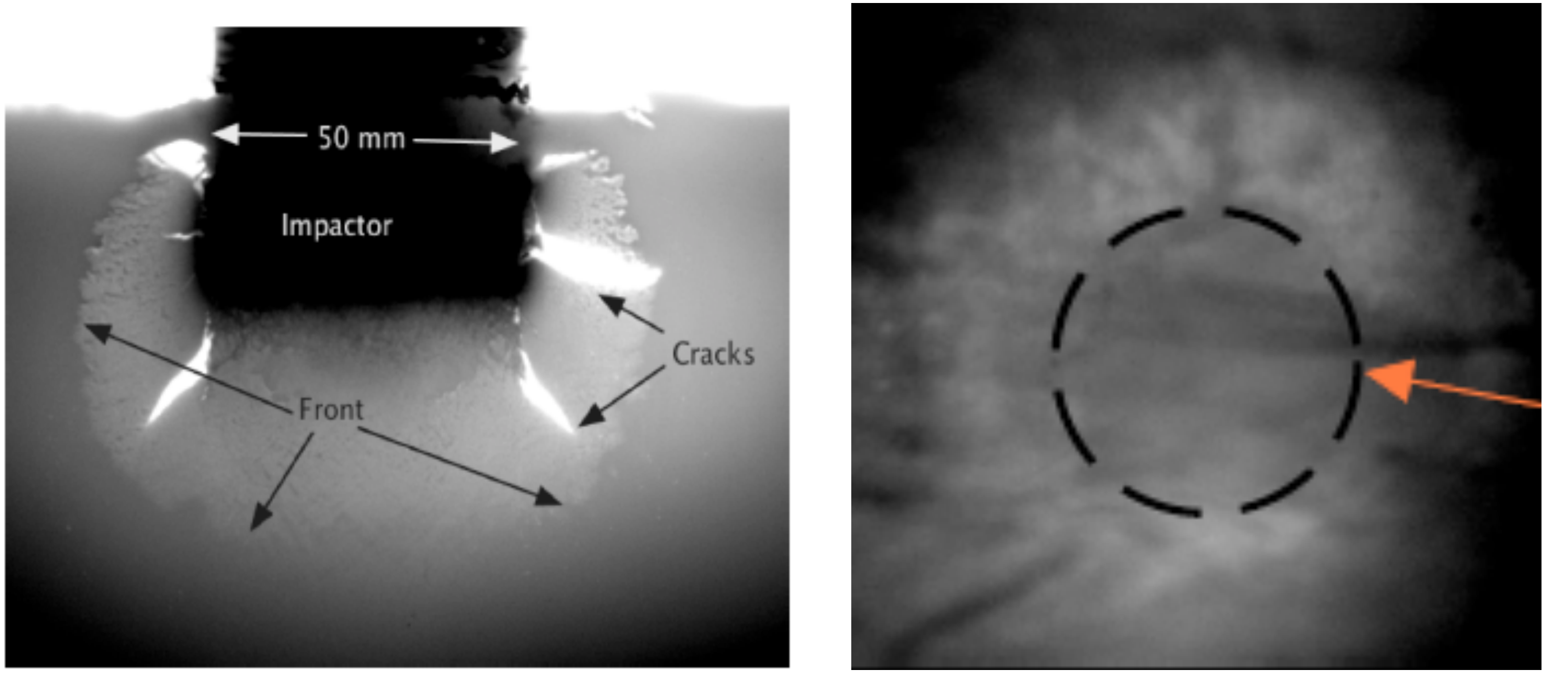
added mass provides force

Waitukaitis & Jaeger, *Nature* (2012)

Is this force large enough?

side view

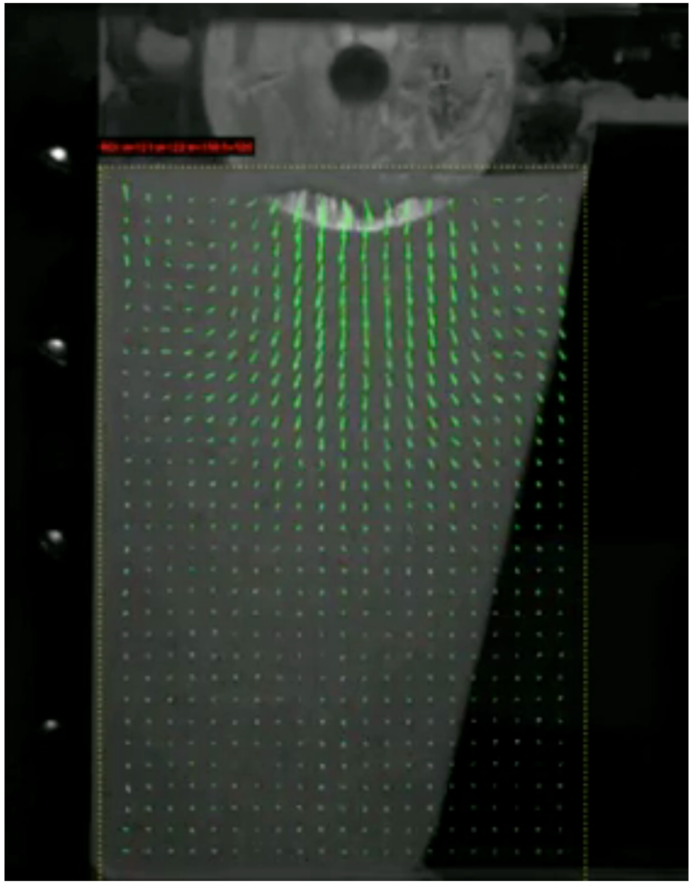
bottom view

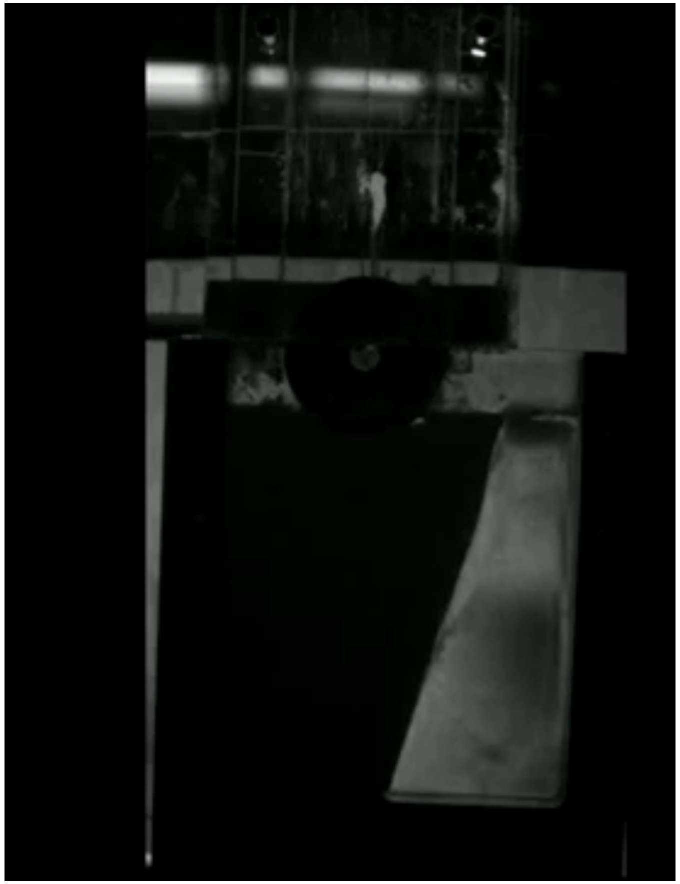


above a critical impact velocity a solid-like (jammed) front moves towards bottom and provides the force

Mukhopadhyay, Allen & Brown, *cond-mat* (2014)

How fast is the shock wave?





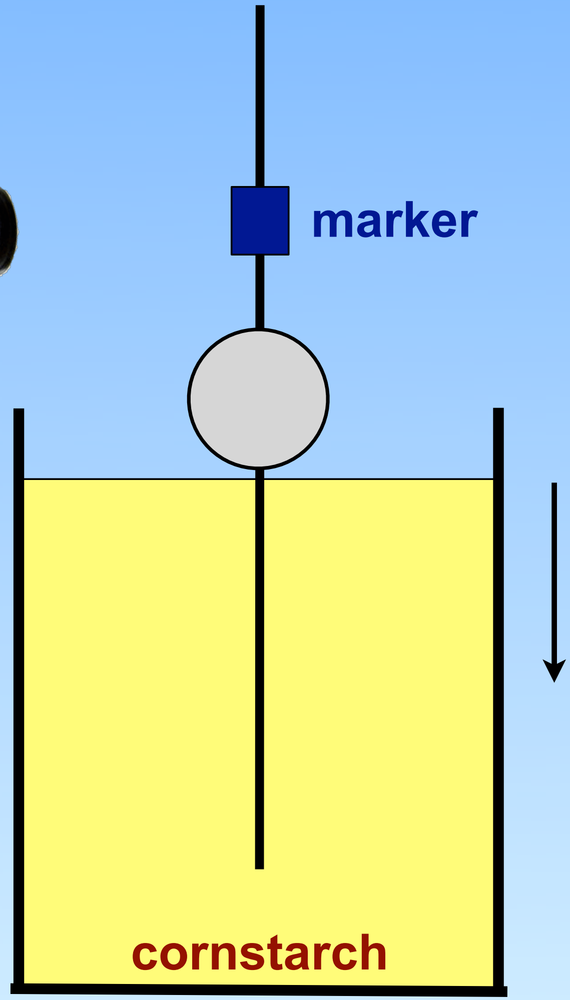
shock wave speed > 2,000 m/s (!)

Lim, Barés & Behringer, *youtube* (2016)

Example 3

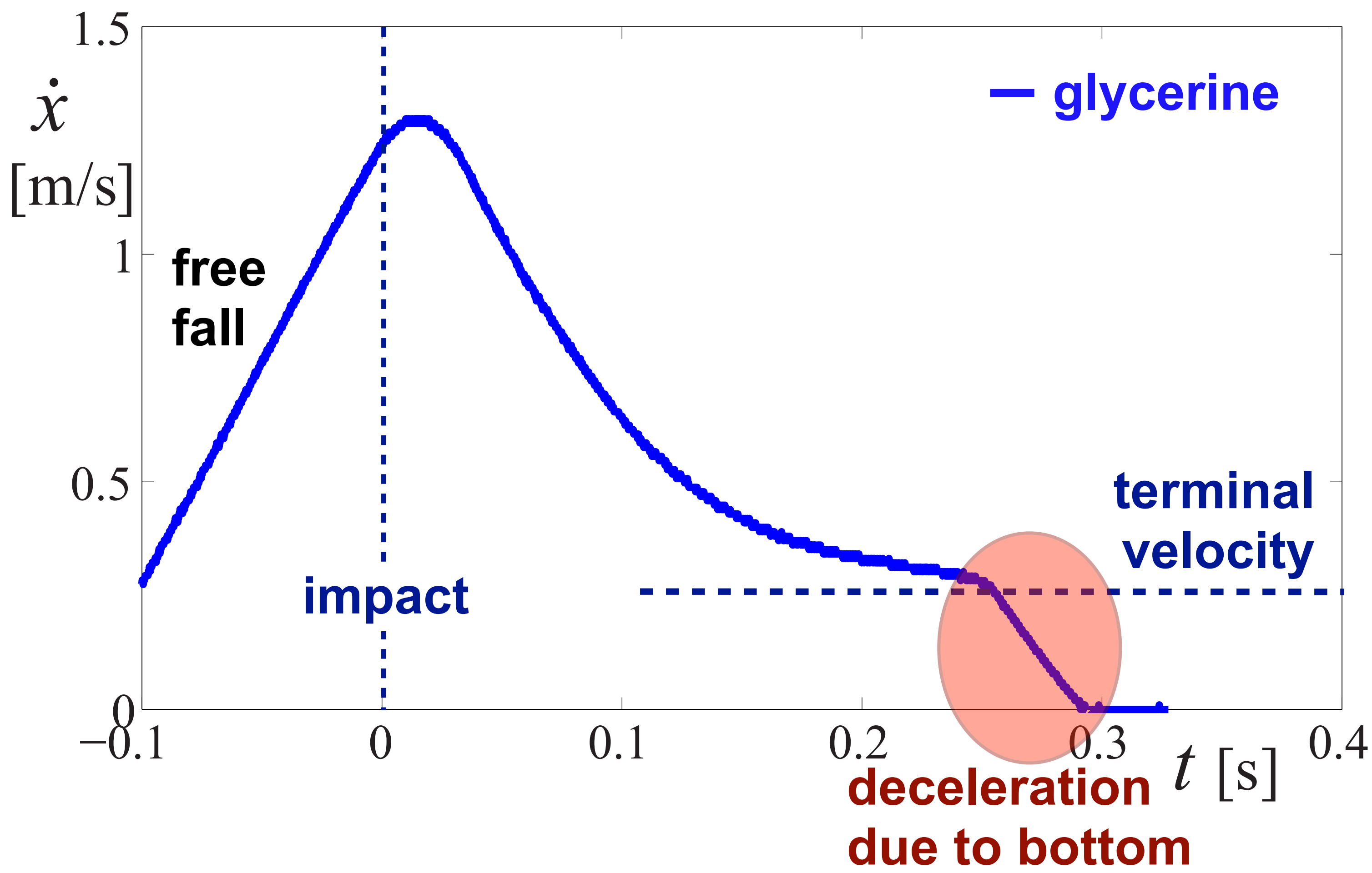
Settling in a cornstarch suspension

Experimental setup

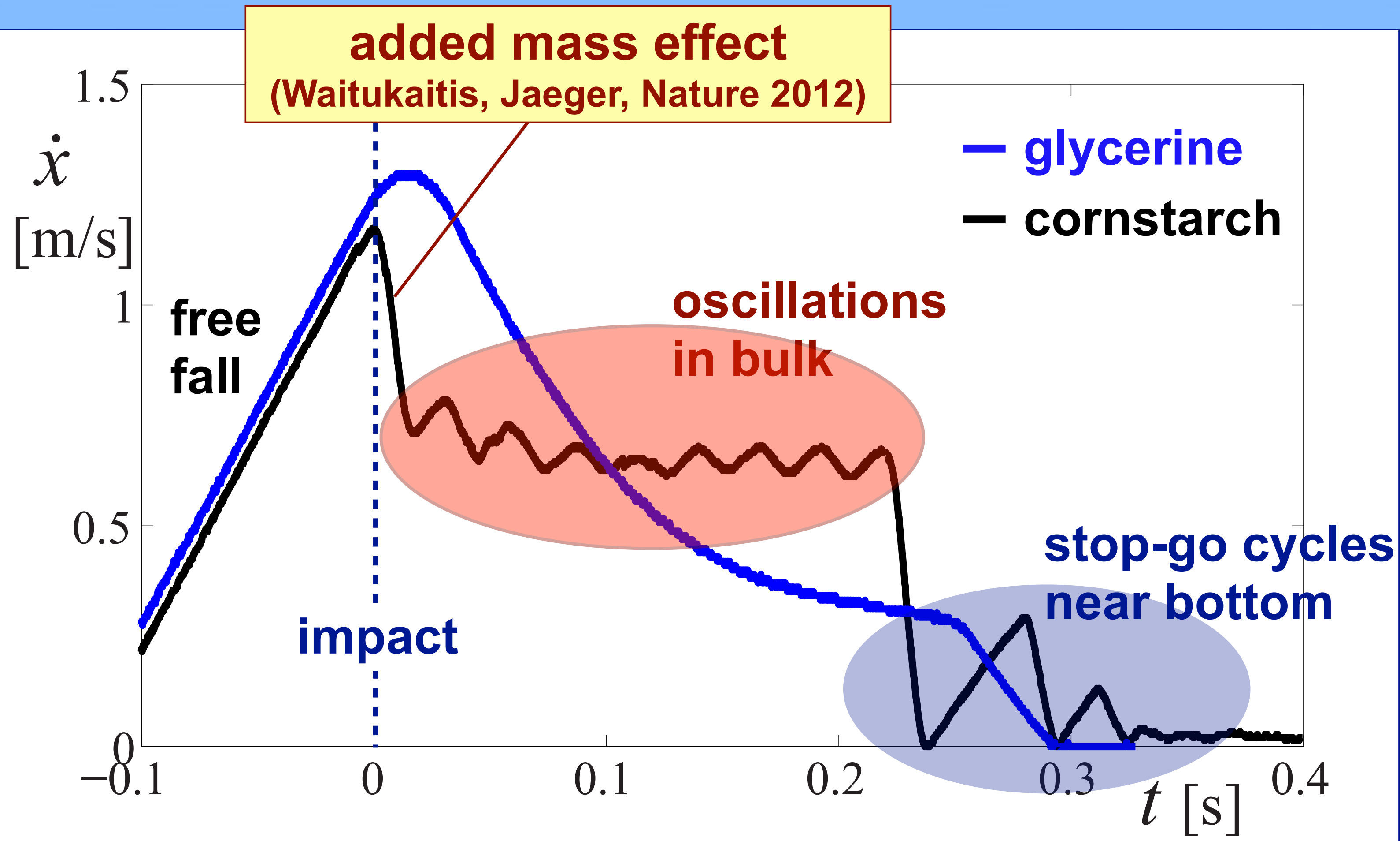


Control parameters:
- packing fraction φ
- object mass

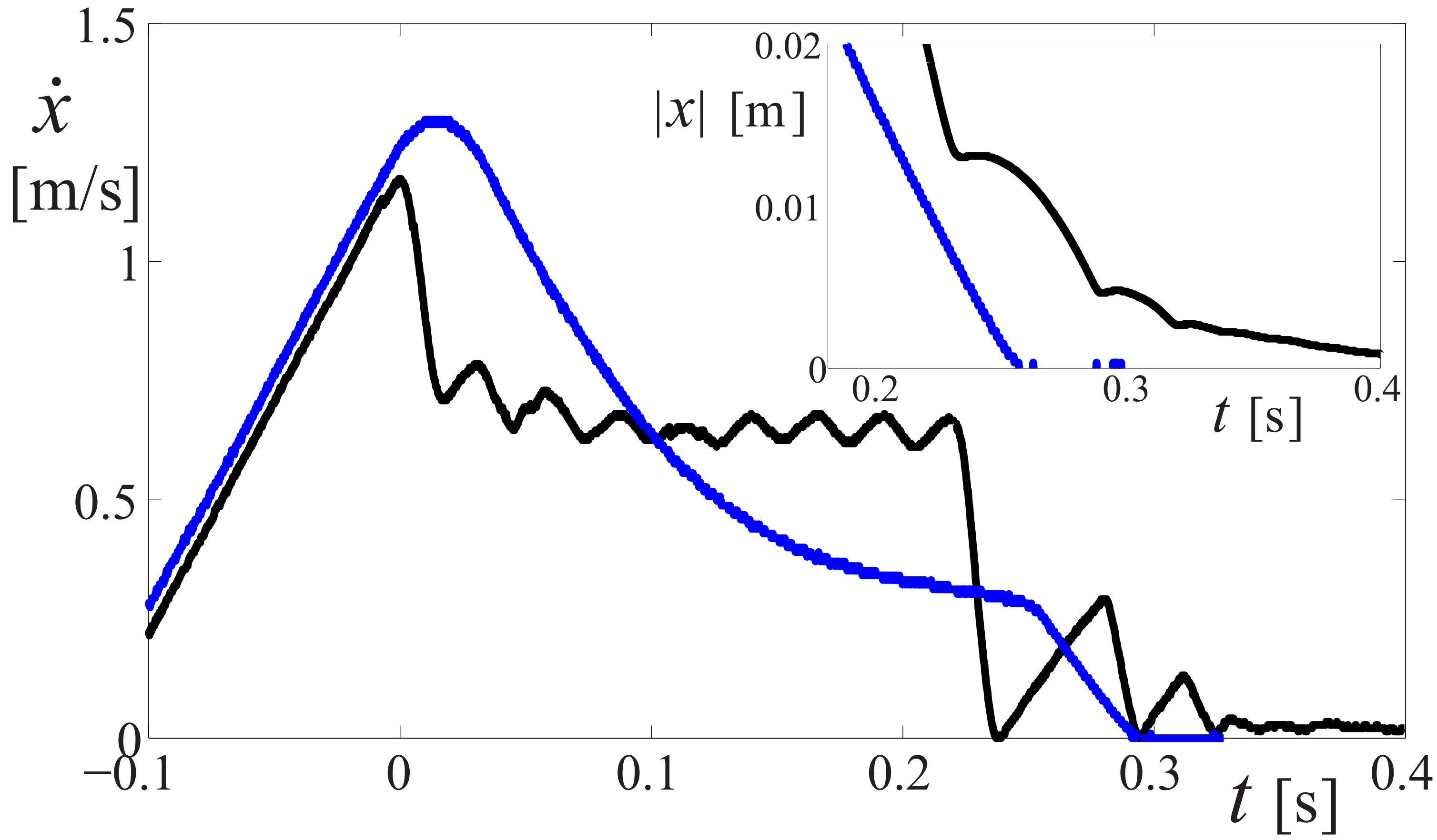
Liquid vs suspension



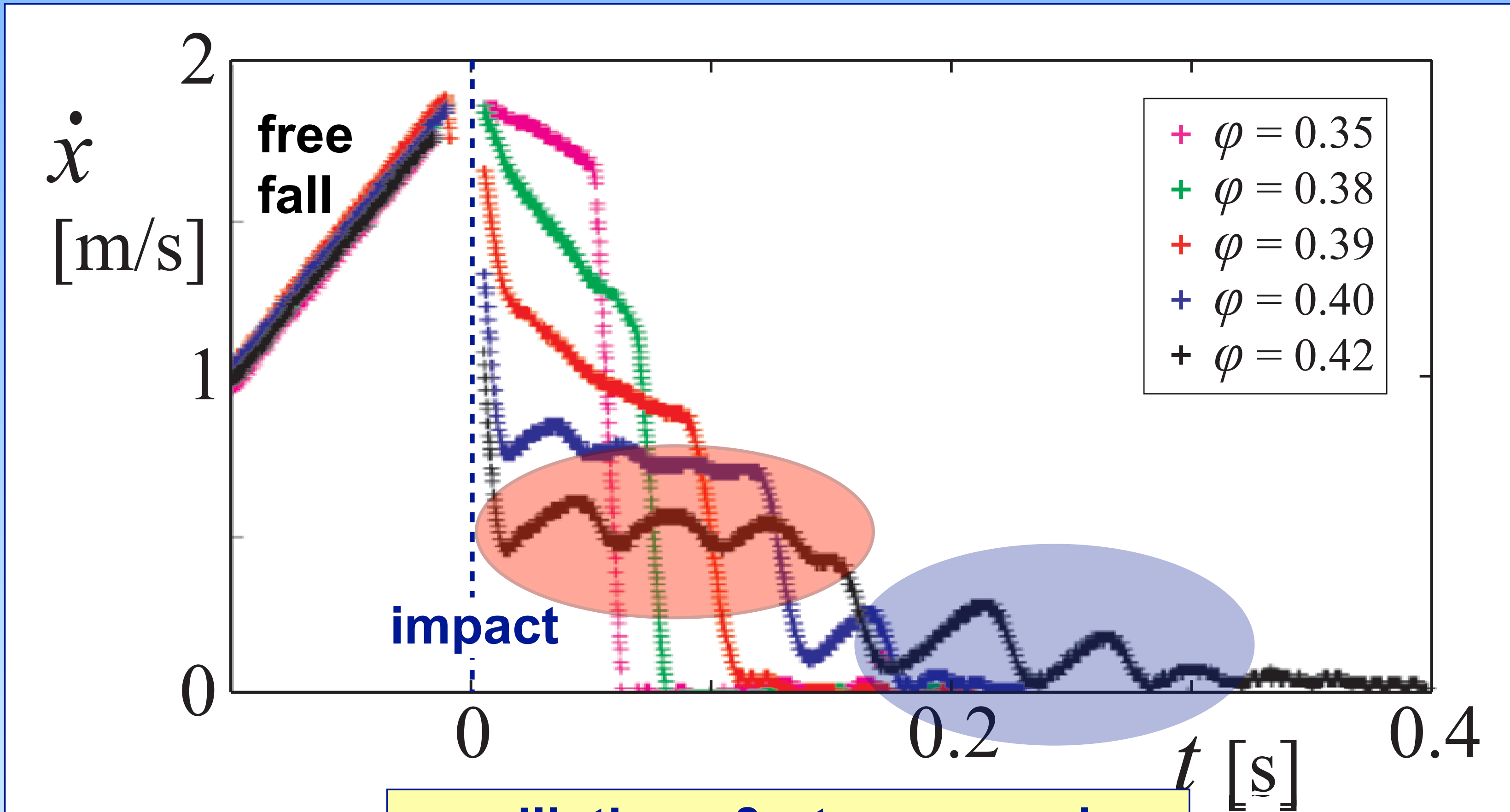
Liquid vs suspension



Liquid vs suspension



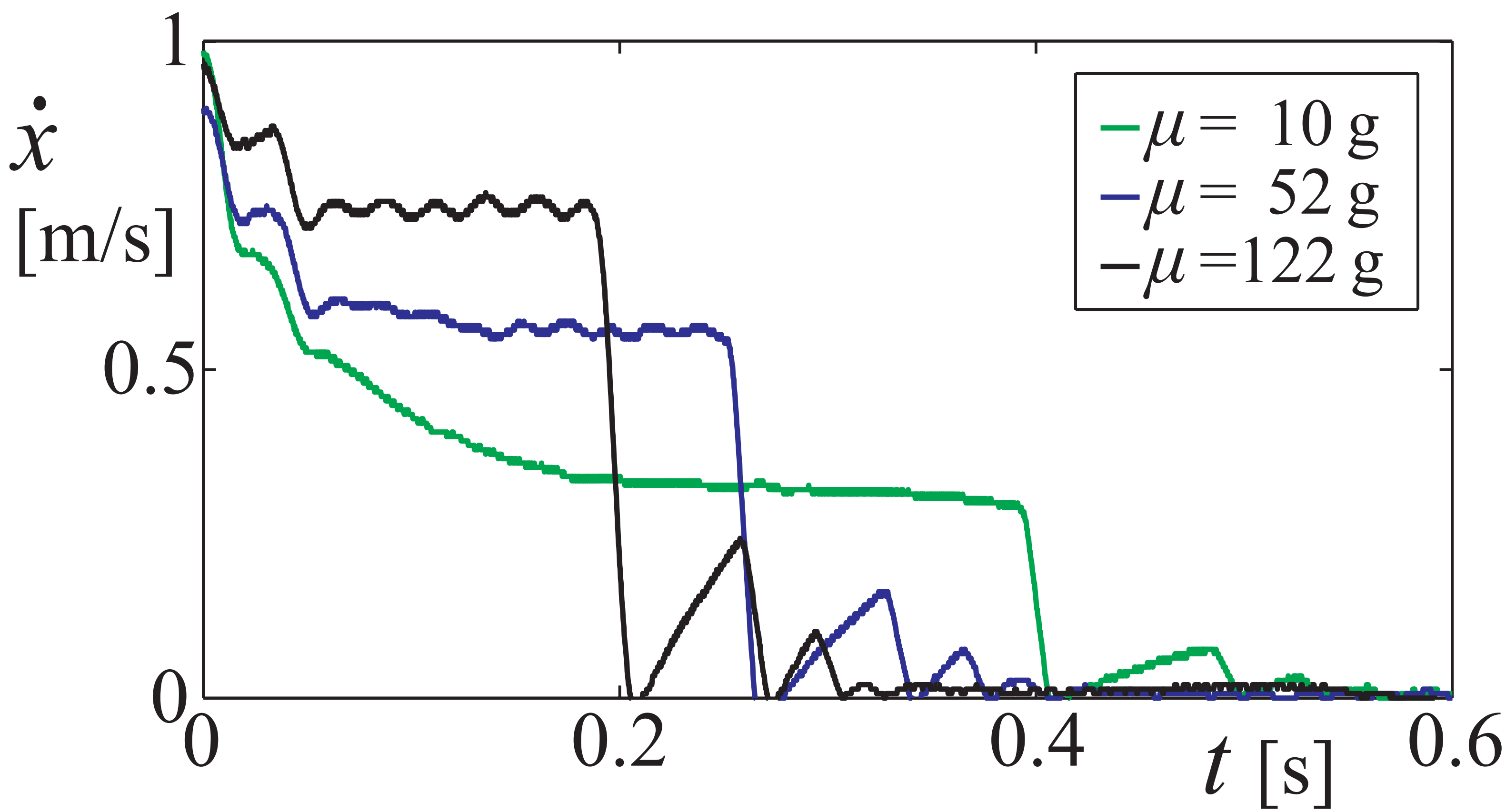
Increasing packing fraction φ



oscillations & stop-go cycles appear for φ > 0.39

Increasing sphere mass μ

φ = 0.44



Equation of motion

$$\boxed{m\ddot{x} = \mu g + D}$$

Added mass corrected mass:

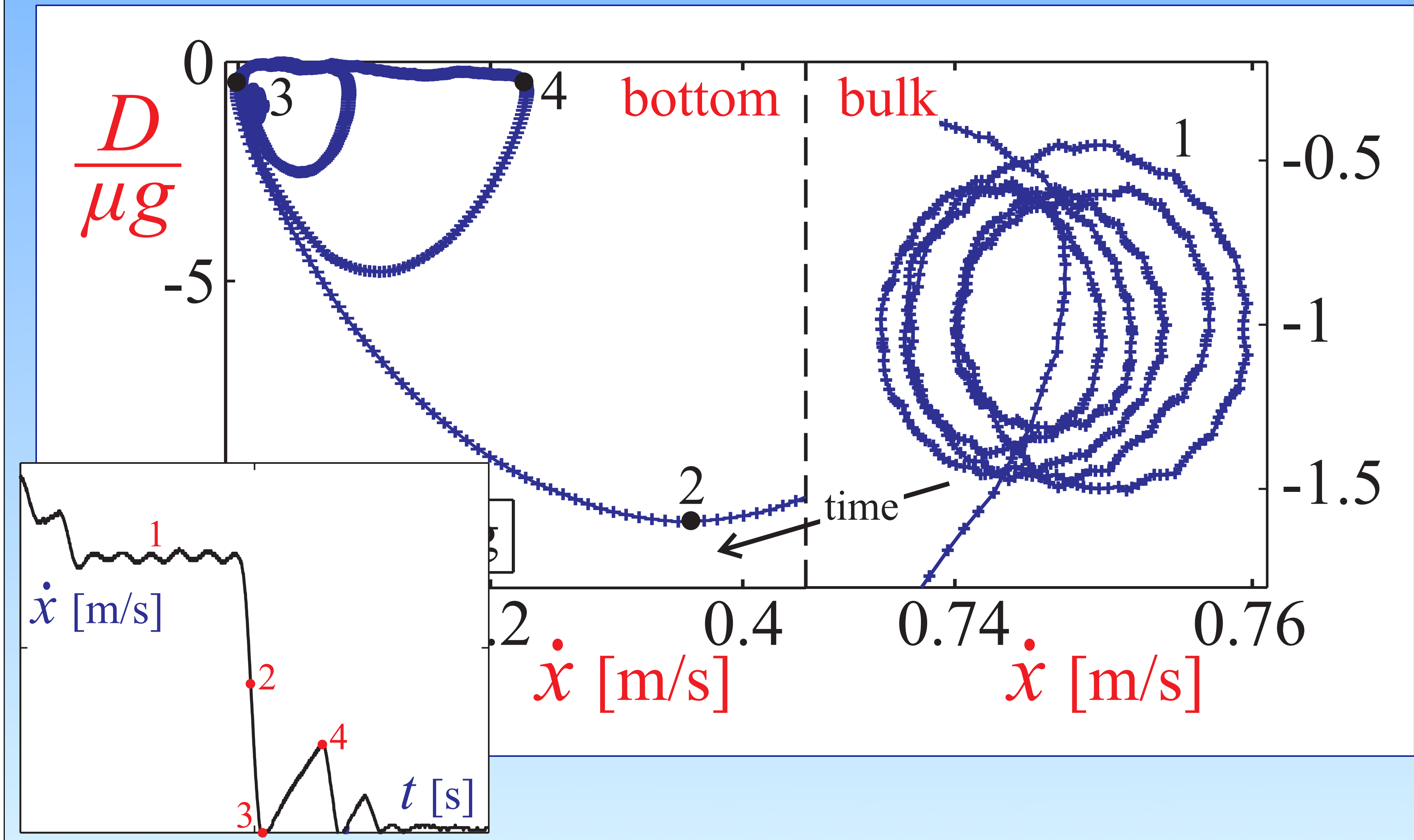
$$m = m_{sphere} + m_{added} = m_{sphere} + \frac{1}{12}\pi d^3 \rho_{susp}$$

Buoyancy corrected mass:

$$\mu = m_{sphere} - m_{buoy} = m_{sphere} - \frac{1}{6}\pi d^3 \rho_{susp}$$

use this equation to calculate drag D vs velocity $\dot{x}$

Drag D vs velocity $\dot{x}$



Bulk oscillations

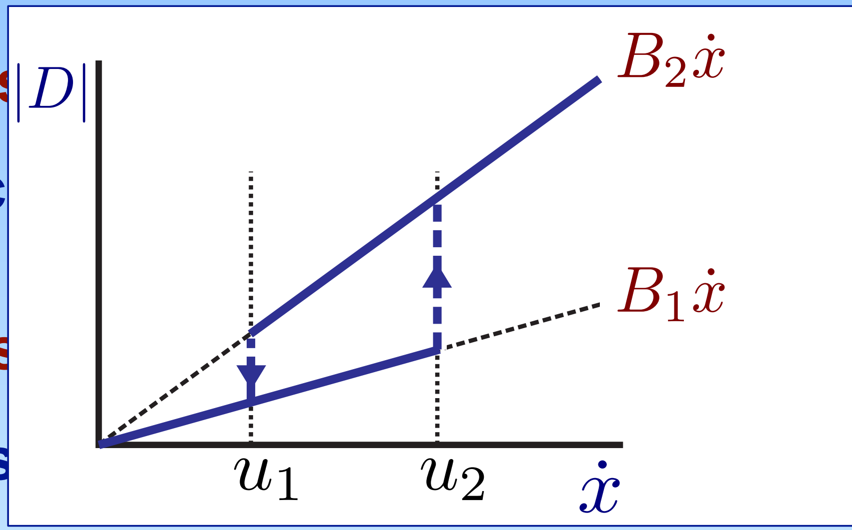
What type of model could describe the bulk oscillations?

Shear thickening or other s

No. Leads to monotonic

Visco-elastic liquid models

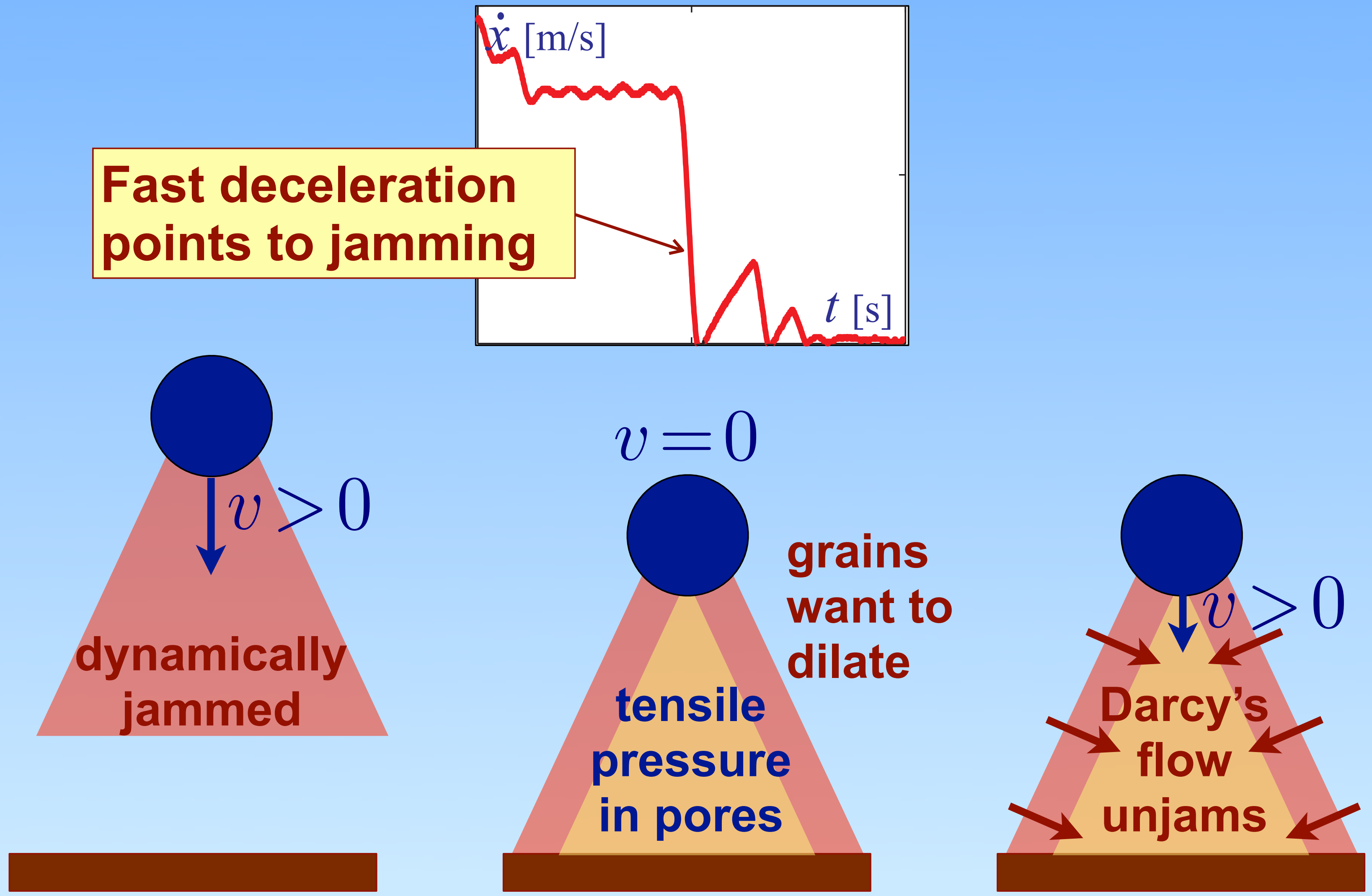
No. Leads to damped os



Hysteretic drag model? [R.D. Deegan, *Phys. Rev. E* 81, 036319 (2010).]

Works reasonably well

Stop-go cycles at the bottom



similar to: S. Mukhopadhyay, B. Allen, E. Brown, arXiv:140719 (2014).

A minimal model

$$\begin{cases} m\ddot{x} = \mu g + D & \text{when } \phi < \phi_{cr} \\ \dot{x} = 0 & \text{when } \phi \geq \phi_{cr} \end{cases}$$

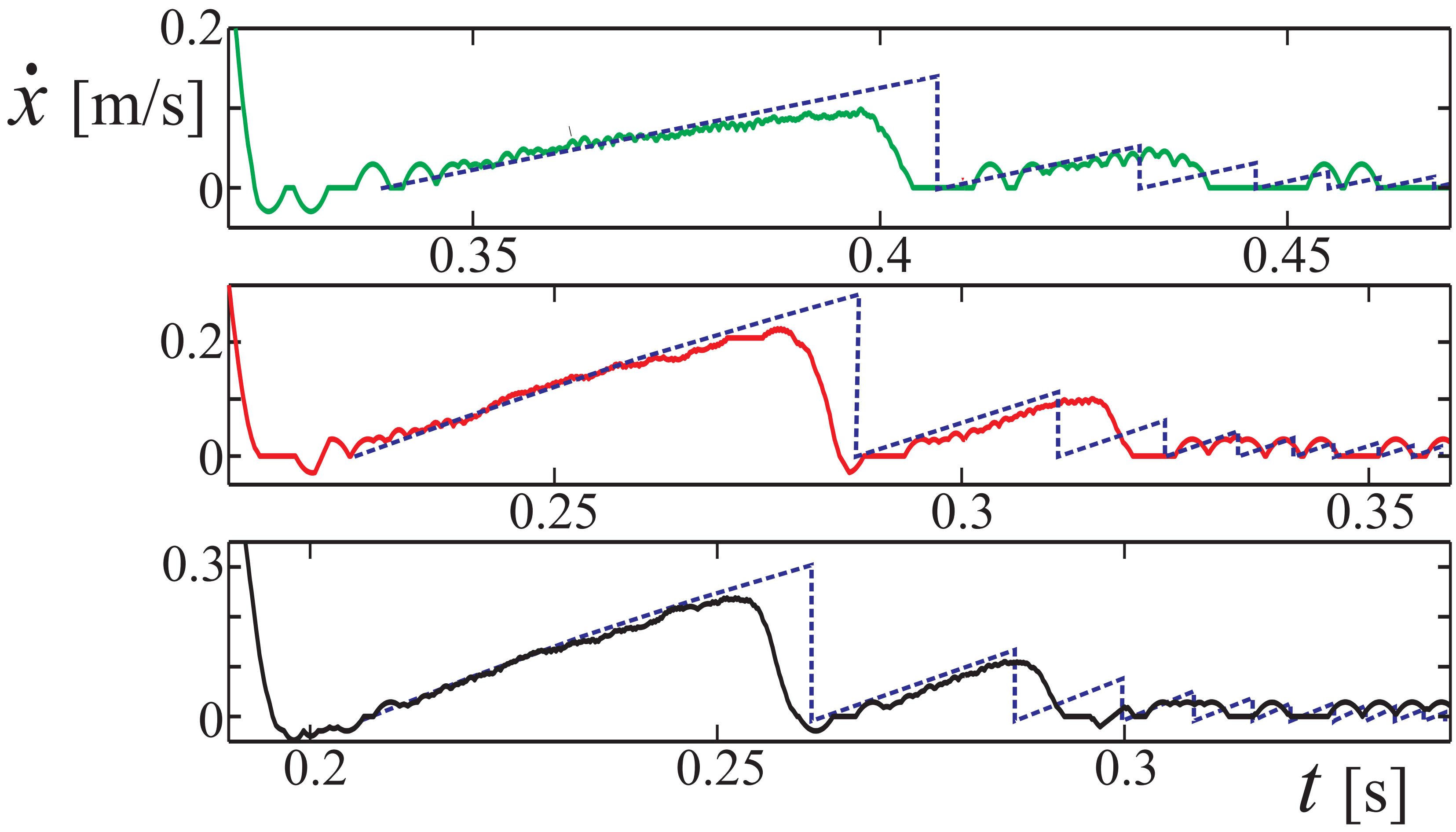
$$\dot{\phi} = -c\frac{\dot{x}}{x} - \kappa(\phi - \phi_{eq})$$

increases ϕ due to compression

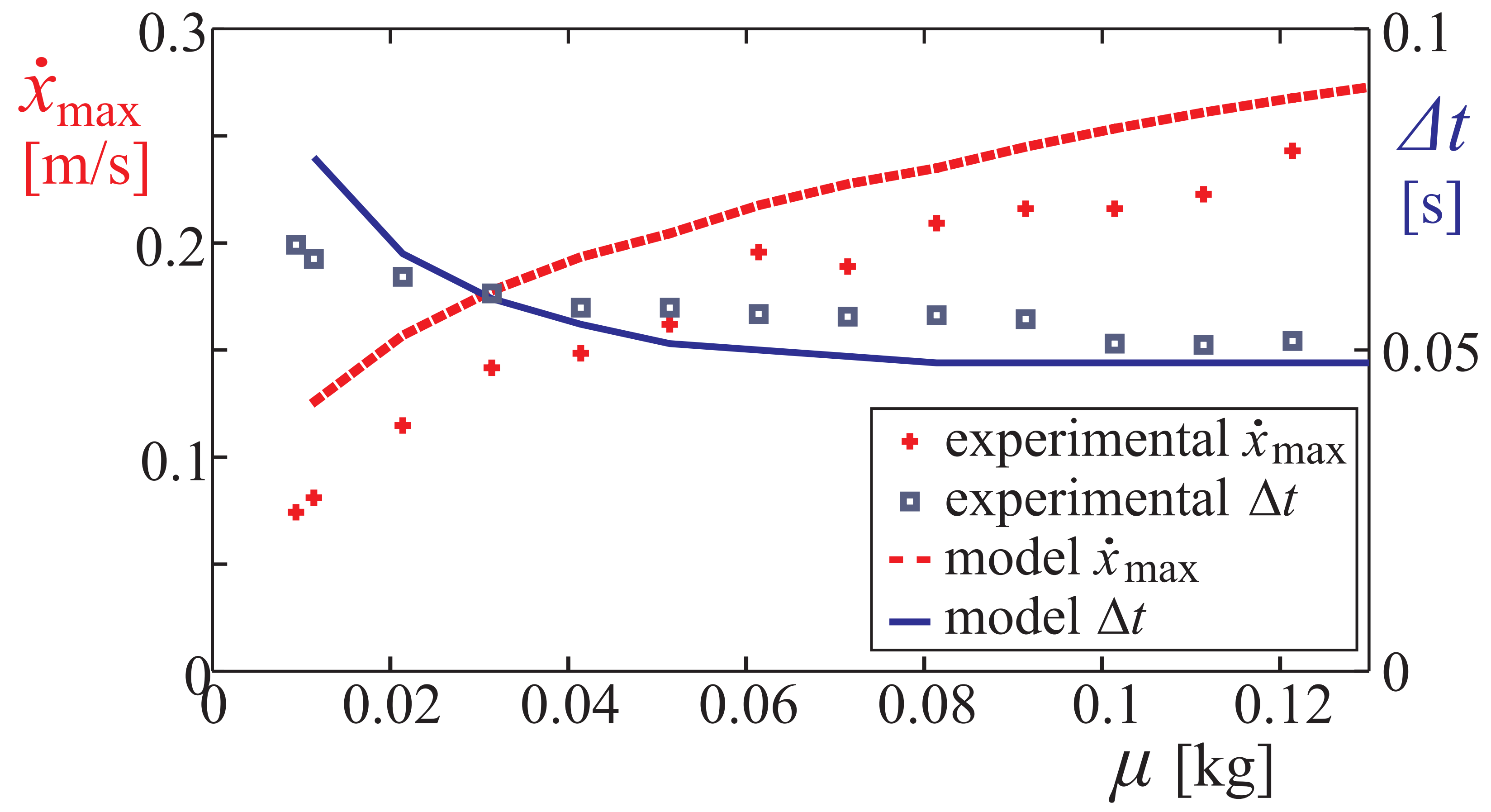
($-\dot{x}/x$ = compression rate)

decreases ϕ due to relaxation

Comparing to experiment



Comparing to experiment




THANK YOU !