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Anomalous energy cascades in dense granular materials yielding under simple shear deformations*

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* K. Saitoh and H. Mizuno, *Soft Matter* **12** (2016) 1360. *Communication*.

Granular materials

Ubiquitous in nature
Understanding of their flow properties is crucial to industry

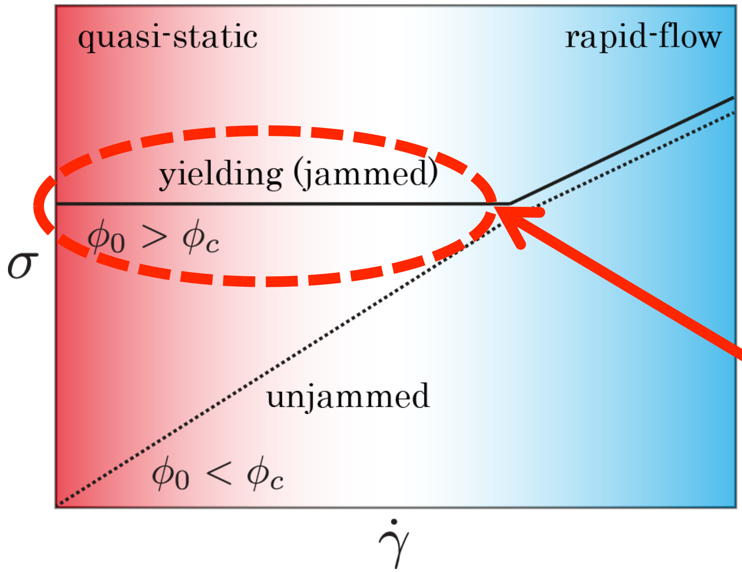



Different from usual fluids
Constituents are macroscopic particles (from few *mm* to *mm*)

- **Thermal fluctuations are negligibly small.**
- **Inelastic interactions cause energy dissipations.**
- **The microscopic (Coulomb's) friction is intrinsic.**

Granular rheology

Dependent not only on the *shear rate*, γ,
but also on the *fraction* of granular materials, ϕ_0



The microscopic insight is still unknown!

Unjammed state, $\phi_0 < \phi_c$

- **Bagnold's scaling, $\sigma \sim \gamma^2$, predicted by *kinetic theory***

Yielding state, $\phi_0 > \phi_c$

- **A finite yield stress in a *quasi-static limit*, $\gamma \to 0$**
- **The shear stress is *rate-independent* if $\gamma \ll 1$**

Molecular dynamics simulations

Rigid body dynamics (frictional contact model)

$m\mathbf{r}_i = \sum_{j\neq i} \mathbf{f}_{ij}$ $\quad \boldsymbol{\omega}_i = \sum_{j\neq i} \mathbf{f}_{ij}\times\mathbf{n}_{ij}$

$\mathbf{f}_{ij}$: **Linear spring-dashpot**
+ Coulomb's friction

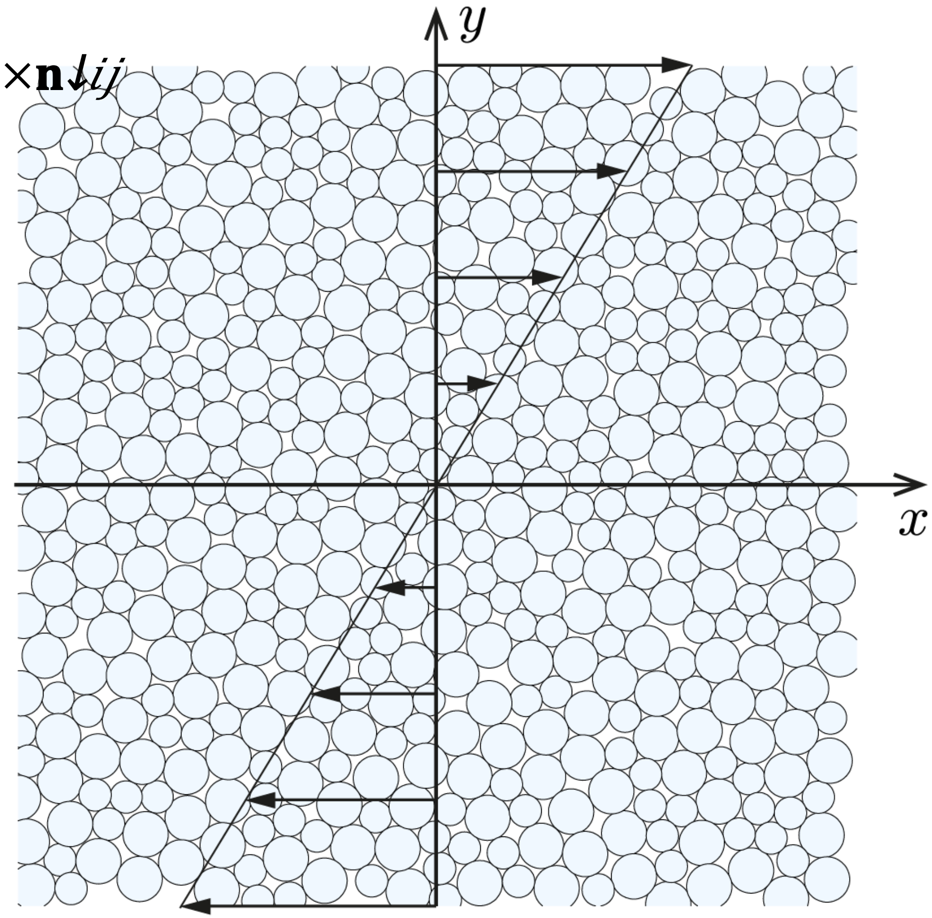
Simple shear deformations

$(x_i, y_i) \rightarrow (x_i + \Delta\gamma y_i, y_i)$

The Lees-Edwards boundaries

Steady states

The applied strain is $1<\gamma<2$.



(Bi-dispersed granular particles in 2D)

Non-affine velocity fields

Mean velocity fields

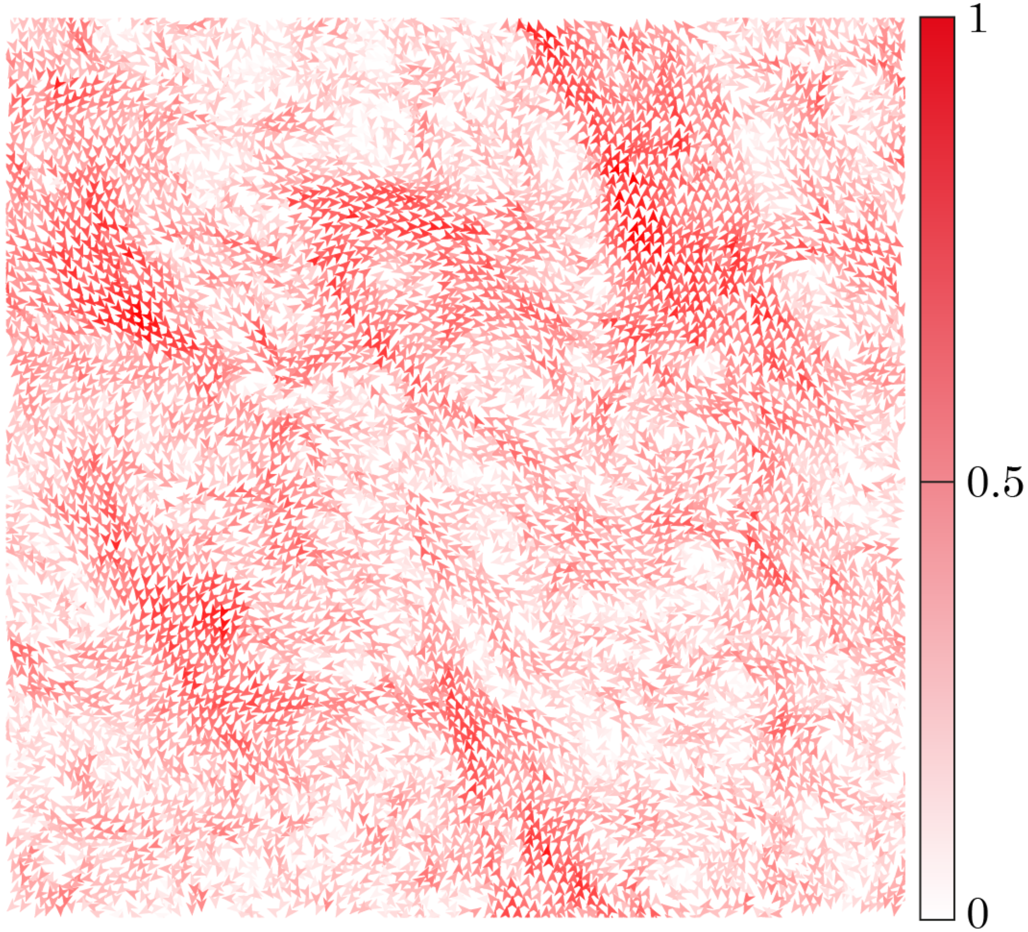
Affine deformations, $\gamma y_i \mathbf{e}_x$

Non-affine velocities

$$\delta \mathbf{u}_i \equiv \mathbf{u}_i - \gamma y_i \mathbf{e}_x$$

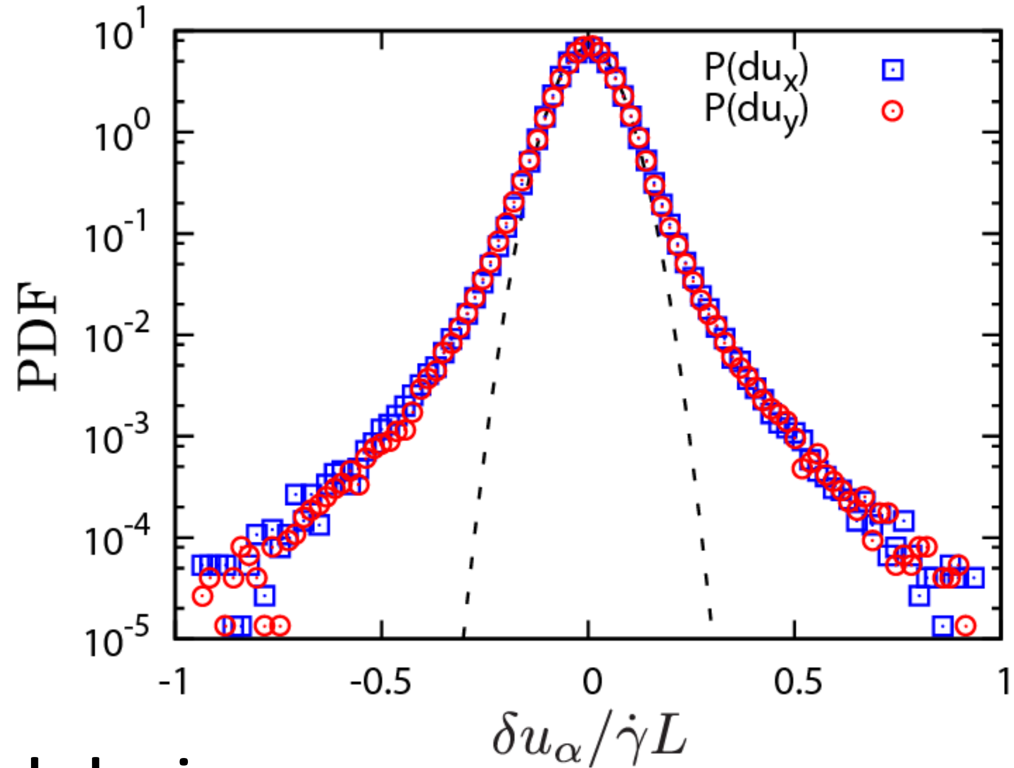
Rearrangements **around the mean velocity fields**

Turbulent-like structures



Non-affine velocities (the red arrows, N=8192) scaled by the maximum (the color coordinate means $|\delta \mathbf{u}_i| / |\delta \mathbf{u}_{max}|$).

Probability distribution functions (PDFs)



Non-Gaussian behavior

The PDFs of non-affine velocities are much wider than the Gaussian fit (dotted line), implying their *strong correlations*.

Isotropic distributions

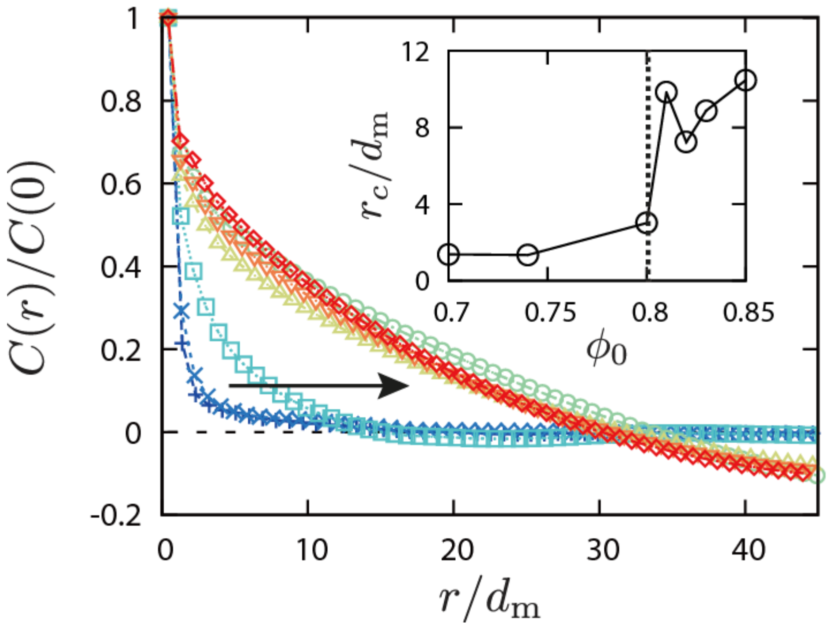
The PDFs are symmetric around zero and $P(\delta u_x)$ and $P(\delta u_y)$ well correspond with each other, i.e. non-affine velocities are *isotropic* in space.

Correlation functions (cf. *structure functions*)

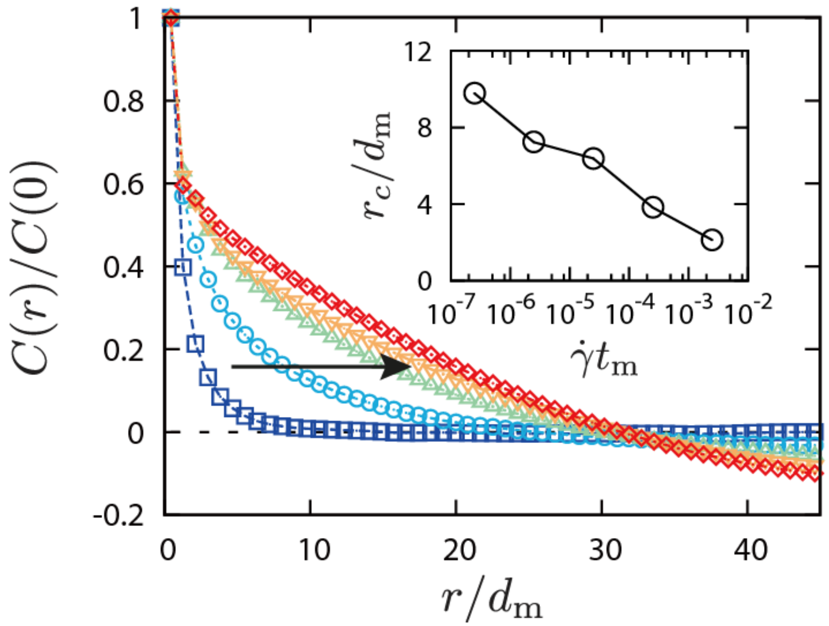
$C(\mathbf{r})=\langle \delta\mathbf{u}(\mathbf{r})\cdot\delta\mathbf{u}(\mathbf{0})\rangle$

$\delta\mathbf{u}(\mathbf{r})\equiv\sum_i \delta\mathbf{u}_i\,\delta(\mathbf{r}-\mathbf{r}_i)$

(Different fraction, ϕ_0)



(Different shear rate, $\dot{\gamma}$)



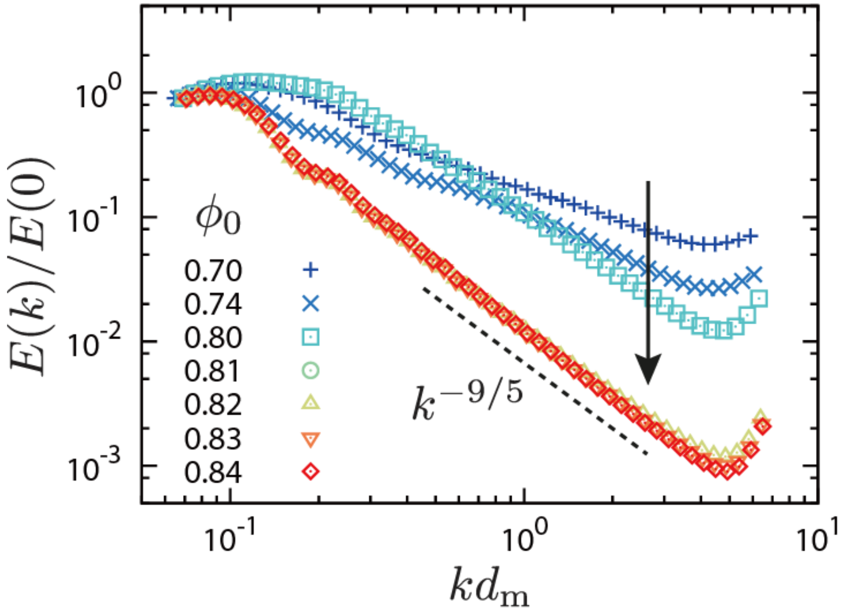
Strong correlations **in granular materials** ***yielding***, $\phi_0 > \phi_c$,
under ***quasi-static deformations***, $\gamma \ll 1$.

The correlation length, $C(r_c)/C(0) \equiv e^{-1}$. **See the symbols in the next.**

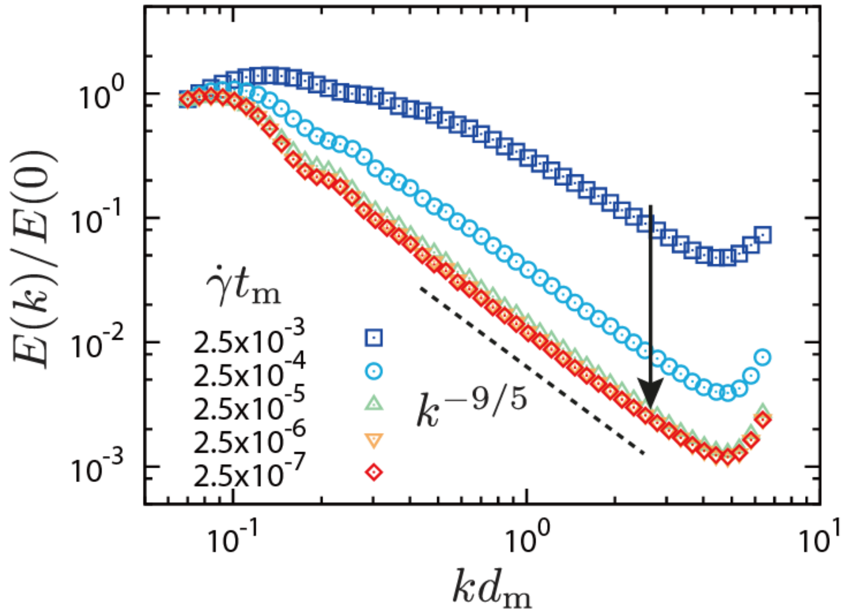
Energy spectra

$$E(\mathbf{k}) = \frac{\rho_0}{2} \langle |\delta\mathbf{u}(\mathbf{k})|^2 \rangle \qquad \delta\mathbf{u}(\mathbf{k}) \equiv \sum_i \delta\mathbf{u}_i e^{-i\mathbf{k}\cdot\mathbf{r}_i}$$

(Different fraction, ϕ_0)



(Different shear rate, $\dot{\gamma}$)



Anomalous energy cascades **in** ***yielding states***, $\phi_0 > \phi_c$, **and** ***quasi-static regime***, $\dot{\gamma} \ll 1$ **(cf. $E(k) \sim k^{-3}$ in 2D turbulence).**

Note that the spectrum is the Fourier transform of the correlation function.

Savage's continuum theory

Hydrodynamic equations

$$D\phi/Dt = -\phi\nabla_\alpha u_\alpha$$
(mass)

$$\phi Du_\alpha/Dt = \nabla_\beta \sigma_{\alpha\beta}$$
(momentum)

$$\phi D\theta/Dt = \sigma_{\alpha\beta}\nabla_\alpha u_\beta - \nabla_\alpha q_\alpha - \chi$$
(energy)

Constitutive model

$$\sigma_{\alpha\beta} = \eta(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha) + \delta_{\alpha\beta}[(\xi-\eta)\nabla_l u_l - p]$$

$$p = p_{kin} + p_{con}$$

Kinetic part $p_{kin} \propto \phi\theta$

Contact part $p_{con} \propto \log\left(\frac{\phi_\infty - \phi_c}{\phi_\infty - \phi}\right)$

The contact contribution, p_{con}, is *rate-independent.*
***Transport coefficients* and *dissipation rate* also consist of both parts.**

A theoretical expression of the spectrum

Fluctuations

$$\phi = \phi_0 + \delta\phi(\mathbf{r}, t)$$

$$\theta = \theta_0 + \delta\theta(\mathbf{r}, t)$$

$$\mathbf{u} = \epsilon y \mathbf{e}_x + \delta\mathbf{u}(\mathbf{r}, t)$$

Non-affine velocity fields

Linearized hydrodynamics

$$\mathcal{L}\psi = -I\omega\psi$$

ψ : ***Hydrodynamic modes***
(i.e. Fourier components of the fluctuations)

Perturbation theory

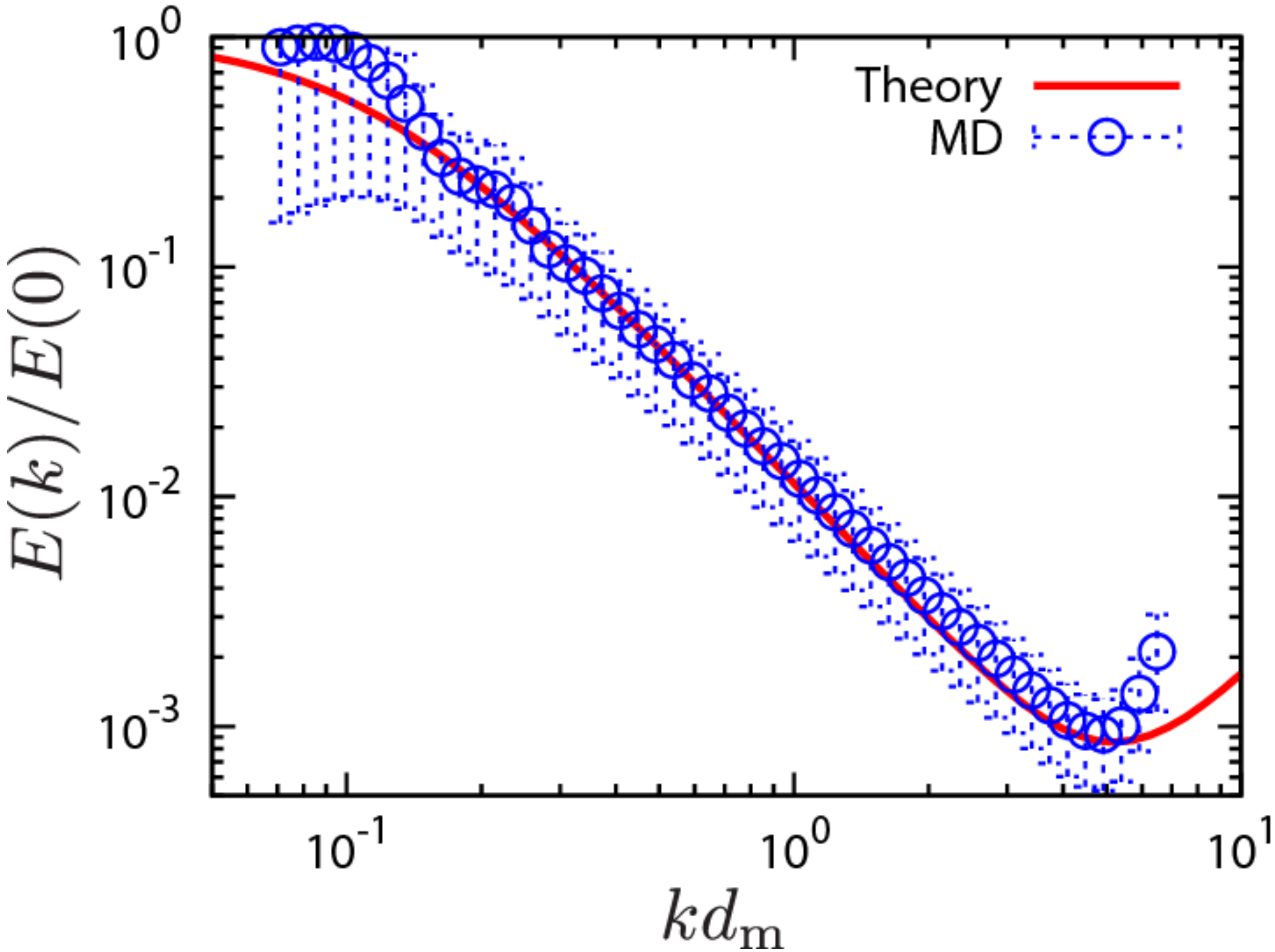
$$\epsilon \equiv \gamma t_m \ll 1$$

$$\therefore E(k)/E(0) = \delta u_x^2 + \delta u_y^2 = a_1^{(1)2} + \epsilon^2 C^2$$

$k \sim \epsilon$ and $\chi \sim \epsilon^2$

(See **our paper**)

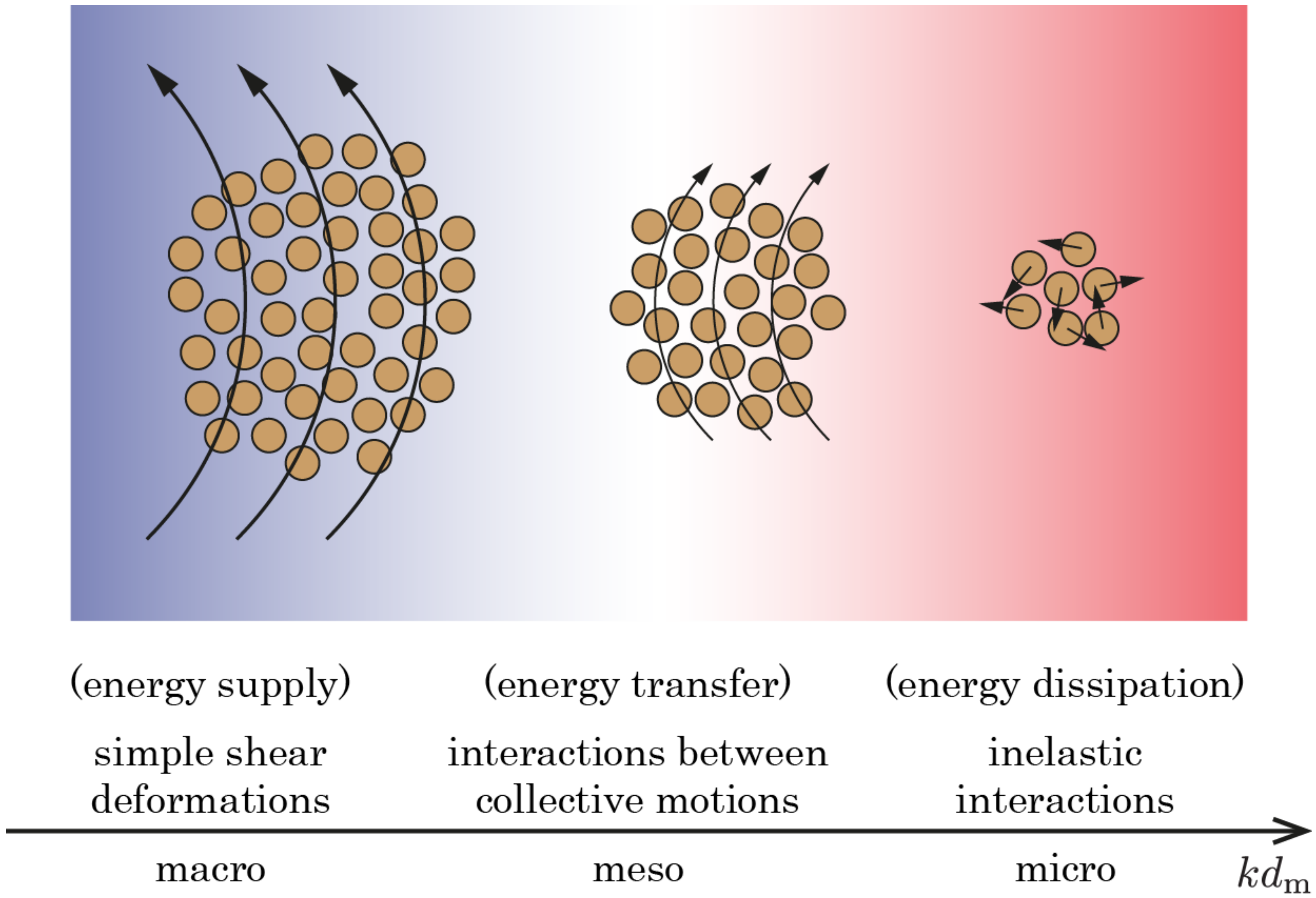
Theory (hydrodynamics) vs. MD



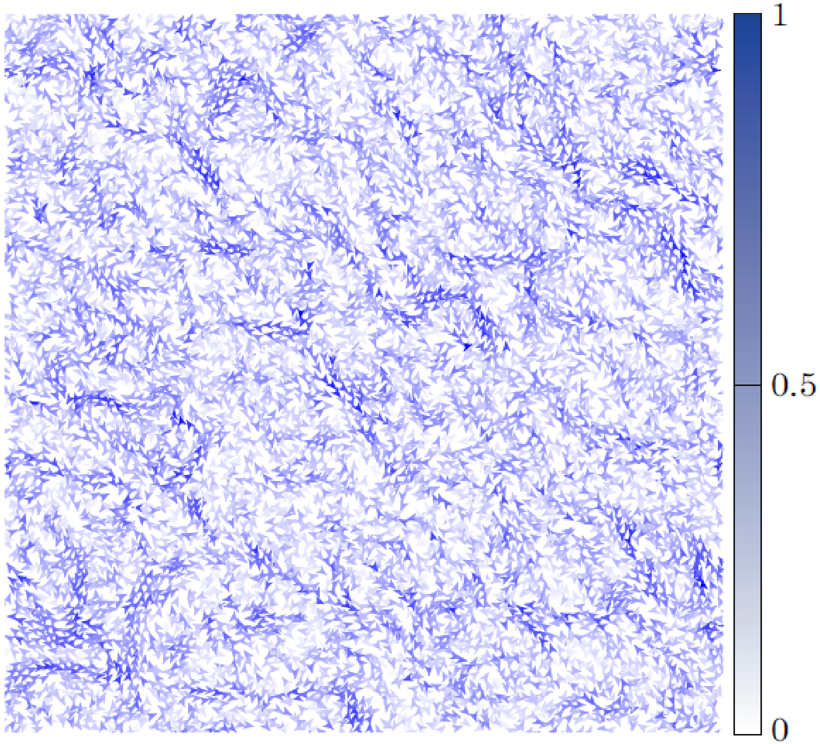
A good agreement in *macro-* and *mesoscopic scales,* $kd_m < 1$.

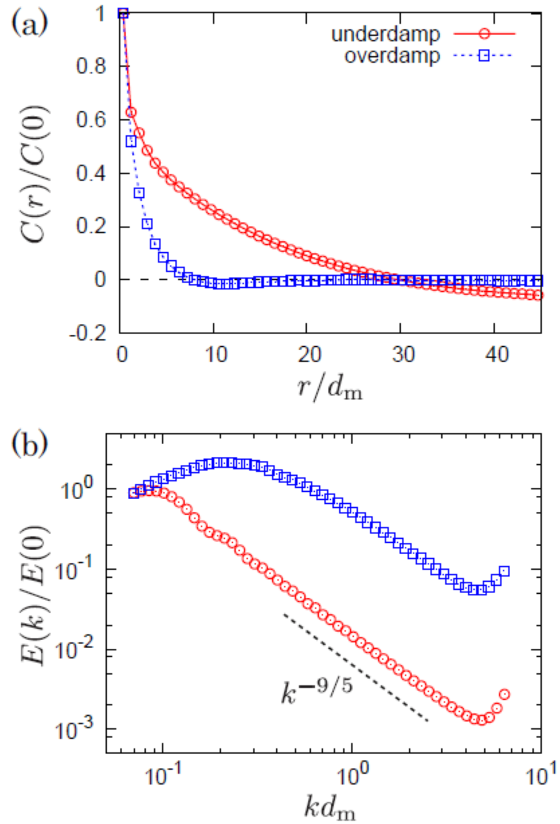
A qualitatively good agreement in *microscopic scale,* $kd_m > 1$.

The basic picture behind the energy cascade

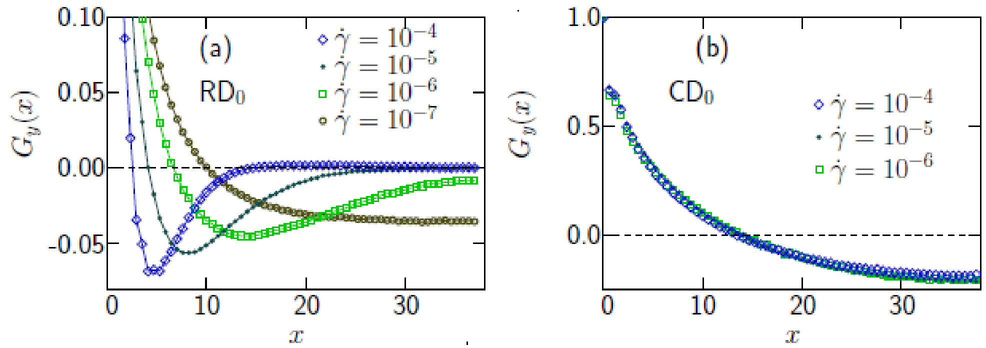


Overdamped dynamics

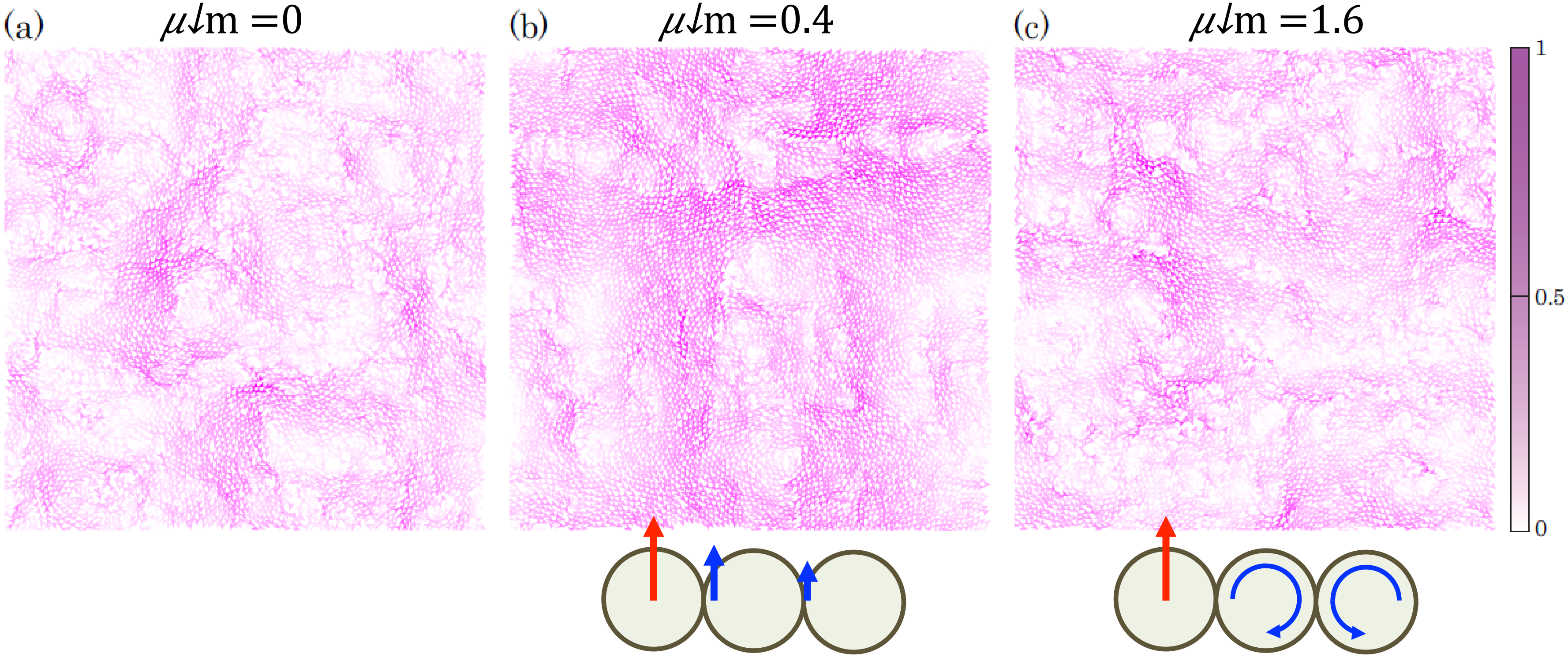




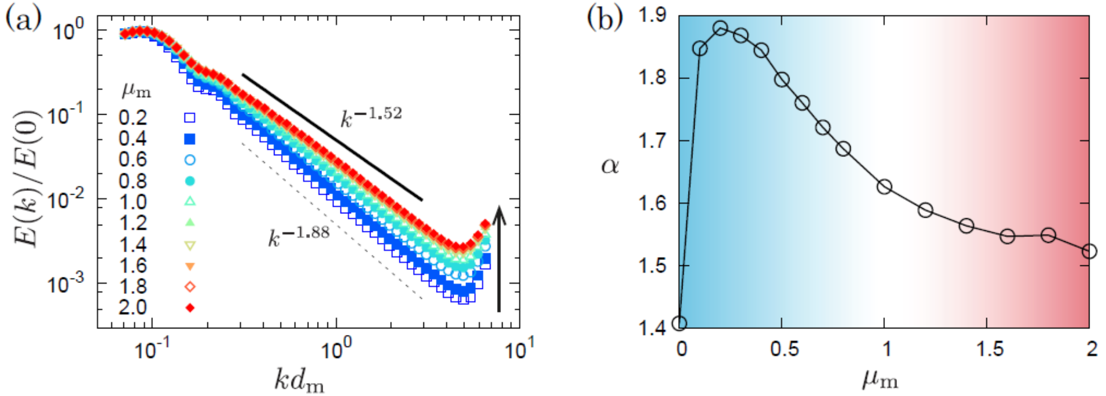
D. Vagberg, P. Olsson, and S. Teitel, PRL 113 (2014) 148002. Suppl.

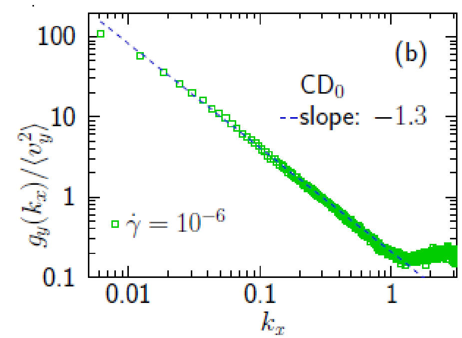


The microscopic friction



D. Vagberg et al., PRL (2014). Suppl.





μ↓m =0

Summary

- **Anomalous statistics of non-affine velocities (non-Gaussian behavior and strong correlations) and the power-law decay of energy spectrum are specific to dense granular materials yielding under quasi-static deformations.**
- **Dense granular rheology is well described by Savage's continuum theory, where the constitutive model includes both the kinetic and contact contributions (see Reference).**
- **We have derived a theoretical expression of the energy spectrum and have confirmed a good agreement with MD simulations over the wide range of length scales.**

Reference

K. Saitoh and H. Mizuno, "*Anomalous energy cascades in dense granular materials yielding under simple shear deformations*", Soft Matter **12** (2016) 1360.