Overview of segregation:
From inclined planes to drums; via a volcano

A. R. Thornton 5th March 2015

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Segregation in long rotating cylinders

9 Conclusions

Granular segregation, hard or easy?

- Granular segregation is very easy to observe, preventing segregation is often the problem.
- Segregation in granular materials can occur for a number of reasons
  - Difference in size
  - Difference in size
  - Difference in density
  - Difference in contact properties
  - Difference in angle of repose
  - Differential forcing (air drag etc...)
  - plus many others ....

Extract from a 1978 paper : Particle segregation ... and what to do about it

Do's and don'ts of handling segregating materials

1. Do use mass-flow bins with tall cylindrical sections.
2. Do use a mixing device for charging a multiple-outlet bin at the center to cause uniform symmetric segregation.
3. Do proportion and mix badly segregating materials just before process usage, with as little surge capacity as possible.
4. Do use a tangential entry for pneumatic conveying of fluidizable solids whenever possible, or provide a deflector plate.
5. Don't split material into various bins from a belt conveyor, since the material may be segregated on the belt.
6. Don't design a nonsymmetric multiple-outlet bin.
7. Don't use free-fall chutes to transfer materials having differing friction angles unless there is a mixing device downstream.
8. Don't charge a mixture of fine fluidizable powder and nonfluidizable coarse particles from a pneumatic conveying line with a

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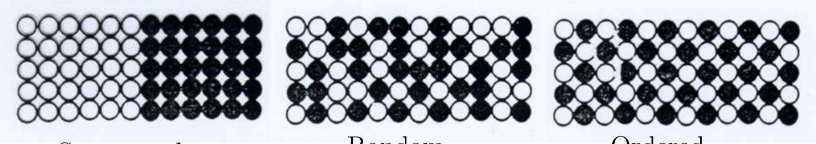
What is a mixture?

Type of mixtures



Segregated Random Ordered

What is a mixture?

The true composition of a mixture p is often not know, but by sampling N times, each with value y_i, we can obtain an estimate, $\bar{y}$

$$\text{mean}: \bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

$$\text{standard deviation}: \sigma = \frac{\sum_{i=1}^{N}(y_i - \bar{y})}{N-1}$$

$$\text{random case}\ : \sigma_r^2 = \frac{p(1-p)}{n}$$

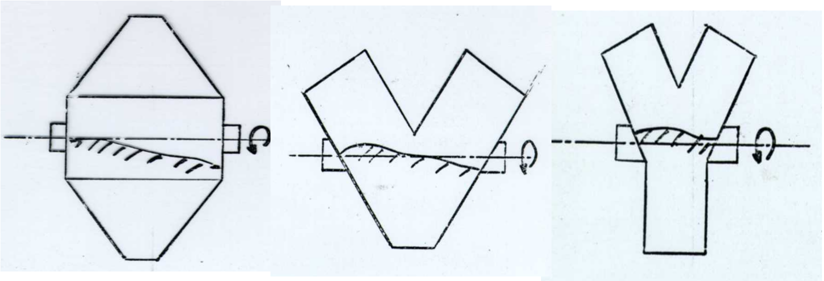
where n is the number of particles in the samples.

$$\text{segregated case}: \sigma_o^2 = p(1-p)$$

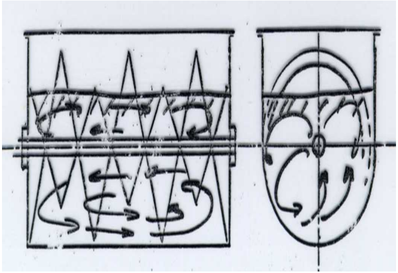
Mixing indices

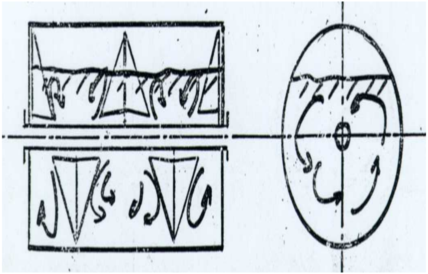
- Lacey mixing index $M_L = \frac{\sigma_0^2 - \sigma^2}{\sigma_0^2 - \sigma_r^2}$
- Problem with M_L is practical values only lie in range $0.75 - 1.0$
- Poole, Taylor & Wall mixing index $M_P = \frac{\sigma_r}{\sigma}$
- This gives better discrimination
- Many, many other indexes exist
- Note σ measured by sampling may not be the true mixture σ. This brings us to the topic of confidence intervals which will not be discussed here.

Tumbling mixer

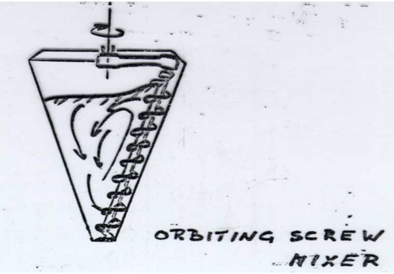


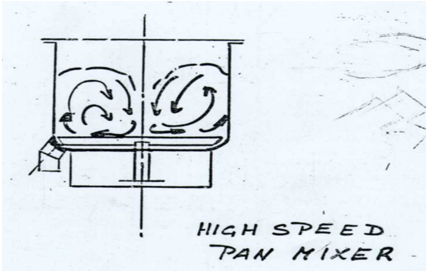
Ribbon blade mixers



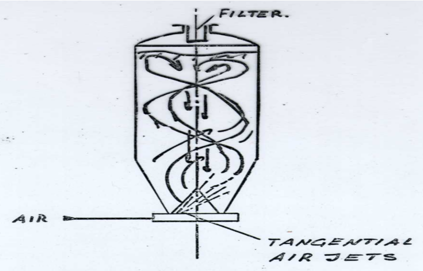


Rotating mixers

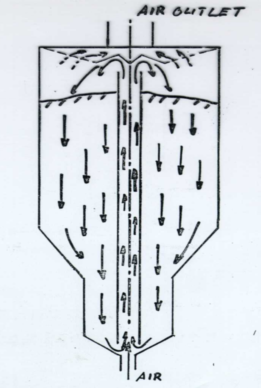




Air-jet mixer



Pneumatic mixer



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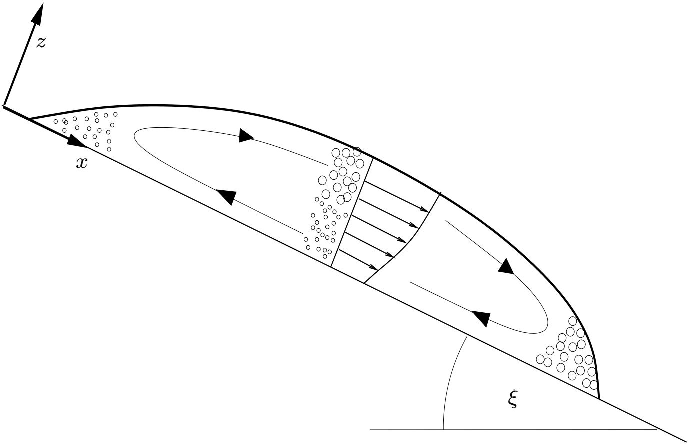
Segregation in long rotating cylinders

Motivation

Co-ordinate setup



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Segregation in long rotating cylinders

Basic concepts

- Follow structure of Savage & Lun (1988)
- Two constituents mixture theory
  - Small particles, s
  - Large particles, b
- With volume fractions

$$0 \leq \phi^{\mu} \leq 1, \quad \mu = s, b$$

and

$$\phi^s + \phi^b = 1$$

[Gray & Thornton(2005), Proc. Royal Soc.]

Mixture theory - basic postulate

The basic mixture postulate

States that every point in the mixture is 'occupied simultaneously by all constituents'

- Mixture theory deals with **partial** variables defined per unit mixture volume.
- Whereas **intrinsic** variables are defined by unit constituent volume.
- So each constituent we can define a local volume fraction ϕ^ν and clearly

$$\sum_{\nu=1}^{n} \phi^\nu = 1$$

- Hence the sum across all constituents of an intrinsic

Mixture theory

- Mass balance

$$\frac{\partial \rho^\nu}{\partial t} + \nabla \cdot (\rho^\nu \boldsymbol{u}^\nu) = 0,$$

- Momentum balance

$$\rho^\nu \frac{D\boldsymbol{u}^\nu}{Dt} = -\nabla p^\nu + \rho^\nu \boldsymbol{g} + \boldsymbol{\beta}^\nu.$$

where
$\rho^\nu \boldsymbol{g}$ is the gravitational acceleration
$\boldsymbol{\beta}^\nu$ is the interaction drag
ρ^ν, p^ν and $\boldsymbol{u}^\nu$ are partial variables defined per unit **mixture** volume

Mixture theory - key relations

- The internal drags must sum to zero

$$\Sigma_\nu \beta^\nu = 0$$

- The partial and intrinsic density are related by simple linear volume fraction scaling

$$\rho^\nu = \phi^\nu \rho^{\nu *}$$

- The partial and intrinsic velocities are the same

$$\boldsymbol{u}^\nu = \boldsymbol{u}^{\nu *}$$

- The pressures are related by an unknown function normally take to be the volume fraction

$$p^\nu = f^\nu(\phi^\nu) p^{\nu *}$$

Assumptions

- Bulk flow incompressible
- Normal acceleration terms are negligible
- Interaction drag is Darcy type
- Kinetic sieving process
  - Modelled by a non-linear pressure
  - Different forms suggested

[Gray & Thornton(2005), Proc. Royal Soc.]

Pressure scalings

Gray & Thornton : $f^\nu = \phi^\nu - B\phi^\nu(1-\phi^\nu)$

Marks, Rognon & Einav : $f^\nu = \phi^\nu \frac{s^\nu}{\sum s^\nu \phi^\nu}$

Tunuguntla, *et al.* : $f^\nu = \phi^\nu \frac{(s^\nu)^\gamma}{\sum (s^\nu)^\gamma \phi^\nu}$

γ determines how pressure scales:

$\gamma = 1$ size,

$\gamma = 2$ area, or,

$\gamma = 3$ volume.

[Gray & Thornton(2005), JFM]
[Marks *et al.*(2012)Marks, Rognon & Einav, JFM]
[Tunuguntla *et al.*(2014)Tunuguntla, Bokhove & Thornton, JFM]

The binary segregation equation

$$\frac{\partial \phi}{\partial t}+\frac{\partial}{\partial x}(\phi u)+\frac{\partial}{\partial y}(\phi v)+\frac{\partial}{\partial z}(\phi w)-S_r\frac{\partial}{\partial z}(F[\phi])=\frac{\partial}{\partial z}\left(D_r\frac{\partial \phi}{\partial z}\right)$$

where ϕ : is the volume fraction of small particles

u, v, w : down slope/cross slope/normal velocity components

S_r : is a dimensionless segregation rate

and D_r : is a dimensionless diffusion rate.

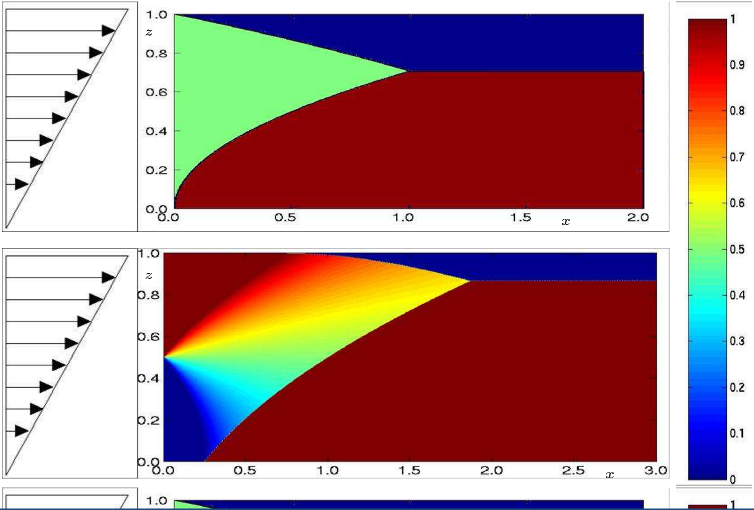
G & T : $F[\phi]=(\phi(1-\phi))$

T, B & T : $F[\phi]=(\hat{s}^{\gamma}-\hat{\rho})\left[\dfrac{\phi(1-\phi)}{\phi+(1-\phi)\hat{s}^{\gamma}}\right]$ $\quad \hat{s}=\dfrac{s^2}{s^1}, \hat{\rho}=\dfrac{\rho^{2*}}{\rho^{1*}}.$

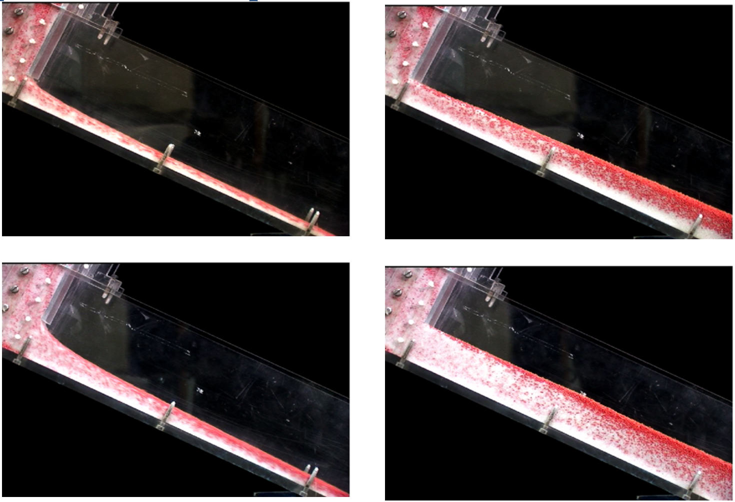
Note : Experiments and simulations show $D_r/S_r \approx 1/20$.

[Gray & Thornton(2005), Proc. Royal Soc.]
[Tunuguntla *et al.*(2014)Tunuguntla, Bokhove & Thornton, JFM]

Exact Solutions



Experimental comparison



Wave breaking numerical results

Click to start movie

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Segregation in long rotating cylinders

The binary segregation equation

$$\frac{\partial \phi}{\partial t}+\frac{\partial}{\partial x}(\phi u)+\frac{\partial}{\partial y}(\phi v)+\frac{\partial}{\partial z}(\phi w)-S_r\frac{\partial}{\partial z}(F[\phi])=\frac{\partial}{\partial z}\left(D_r\frac{\partial \phi}{\partial z}\right)$$

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[Gray & Thornton(2005), Proc. Royal Soc.]
[Tunuguntla *et al.*(2014)Tunuguntla, Bokhove & Thornton, JFM]

The force model

- Discrete particle model governed by Newtonian mechanics:

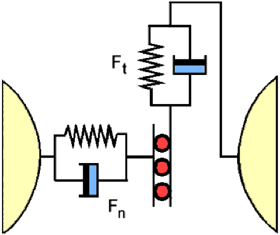
$$m_i \frac{\mathrm{d}^2 \vec{x}_i}{\mathrm{d}t^2} = \vec{f}_i$$

- Contact forces and body forces:

$$\vec{f}_i = \sum_j \vec{f}_{ij} + \vec{b}_i,$$

- Contact force model:

$$\vec{f}_{ij} = f_{ij}^n \vec{n} + f_{ij}^t \vec{t},$$



$$f_{ij}^n = k\delta_{ij} + \gamma v_{ij}^n, \quad f_{ij}^t = -min(\mu f_{ij}^n, k^t \delta_{ij}^t + \gamma^t v_{ij}^t)$$

[Luding(2008), Enviro. and Civil. Eng.]

Contact Properties

Can relate these properties to a restitution coefficient r and contact time t_c

$$r = e^{\frac{-\pi\gamma}{\sqrt{4km_{ij}-\gamma^2}}}$$

$$t_c = \frac{2m_{ij}\pi}{\sqrt{4km_{ij} - \gamma^2}}$$

We define γ and k for each pair of particle-interactions such that r and t_c are the same.

[Luding(2008), Enviro. and Civil. Eng.]

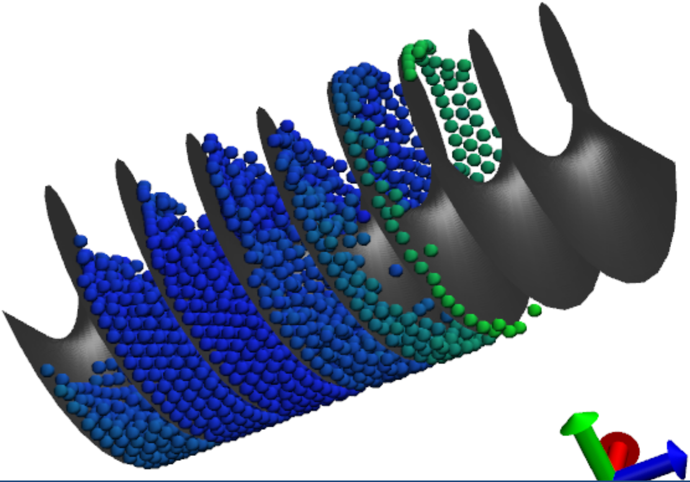
Introduction to MercuryDPM

The simulations presented are done in MercuryDPM, our in-house code. Features :

- Hierarchical Grid contact detection algorithm
- Built-in coarse-graining statistical package
- Access to continuum fields in real time
- Contact laws for granular materials
- Simple C++ implementation
- Complex walls

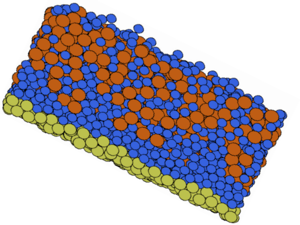
Currently available as a beta version from
`http://MercuryDPM.org`

Screw Feeder



Discrete particle simulation of a periodic chute

- Periodic box inclined at 26 degrees
- The box is filled with equal volume fraction of each type
- Also with equal total depth of flow in mean particle size
- The base of the box is rough
- Simulations undertaken in MercuryDPM



DPM of size segregation

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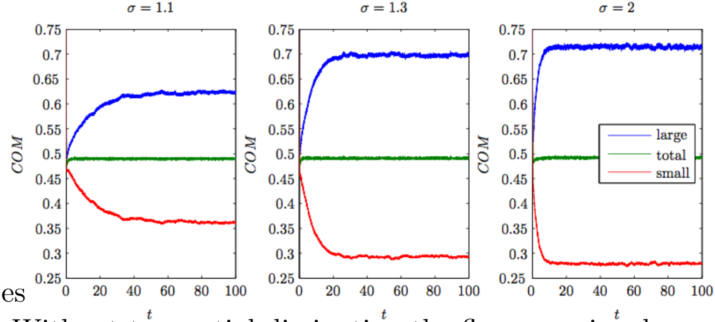
Centre of mass during segregation

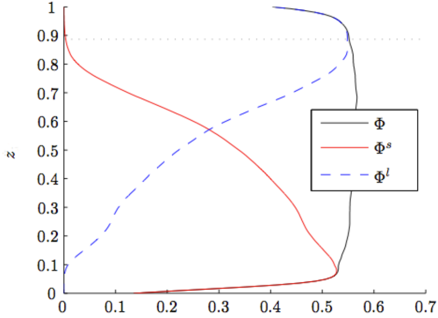


Notes

- Without tangential dissipation the flow occasional spontaneous compacted
- Added tangential dissipation removed this effect
- Also adding tangential elastic forces lead to steadier states

[Thornton *et al.*(2012*b*)Thornton, Weinhart, Luding & Bokhove, Mod. Phys. C.]

Where is the base and free surface?

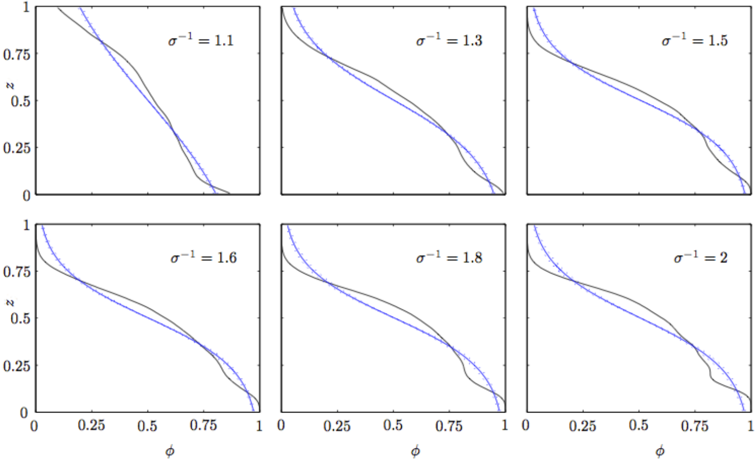


Considered

- The point where the bulk density decreases
- The point where the downwards normal stress goes to zero

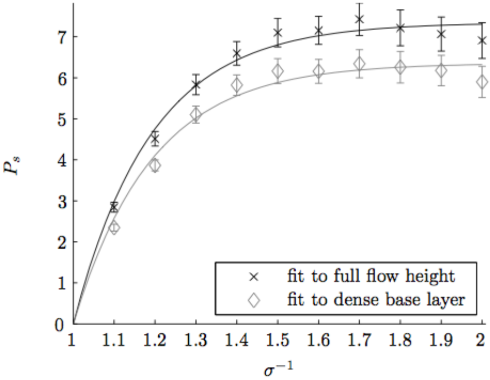
[Thornton *et al.*(2012*b*)Thornton, Weinhart, Luding & Bokhove, Mod. Phys. C.]

Comparison with theory



[Thornton et al.(2012b)Thornton, Weinhart, Luding & Bokhove, Mod. Phys.

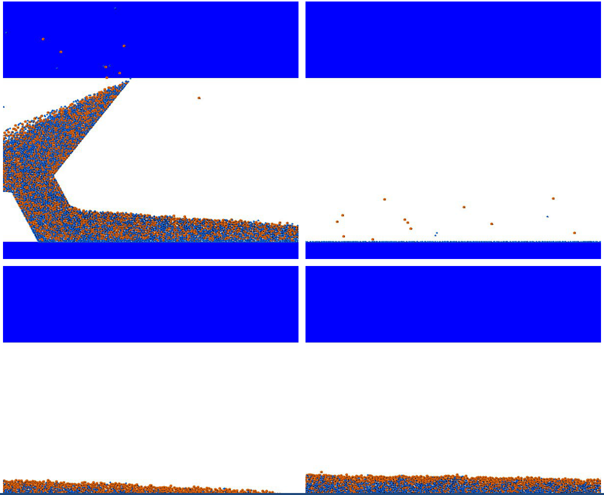
Size ratio v



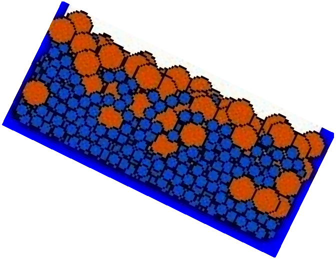
Fit to $P_s = P_{max}(1 - e^{-k(\sigma^{-1}-1)})$ with $P_{max} = 7.35$ and $k = 5.21$.

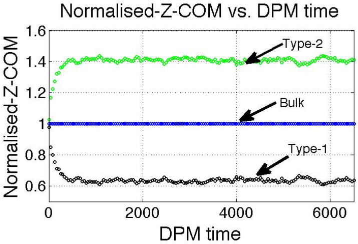
[Thornton *et al.*(2012*b*)Thornton, Weinhart, Luding & Bokhove, Mod. Phys. C.]

MD long chute



Size Only Segregation

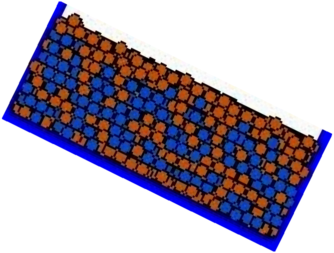


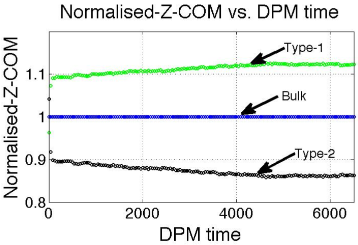


Purely size segregation $\hat{s} = 2$ and $\hat{\rho} = 1$.
Type-1: small particles and Type-2: large particles.

[Tunuguntla *et al.*(2014)Tunuguntla, Bokhove & Thornton, JFM]

Density Only Segregation

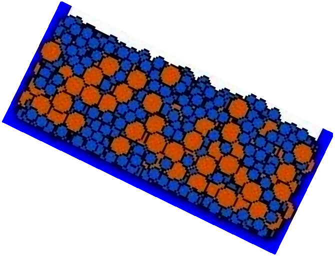


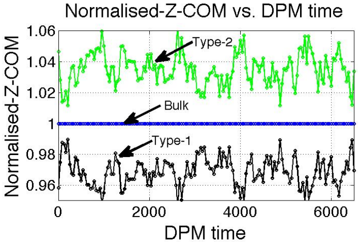


Purely density-based segregation $\hat{s} = 1$ and $\hat{\rho} = 2$.
Type-1: light particles and Type-2: heavy particles.

[Tunuguntla *et al.*(2014)Tunuguntla, Bokhove & Thornton, JFM]

Density Balance Size



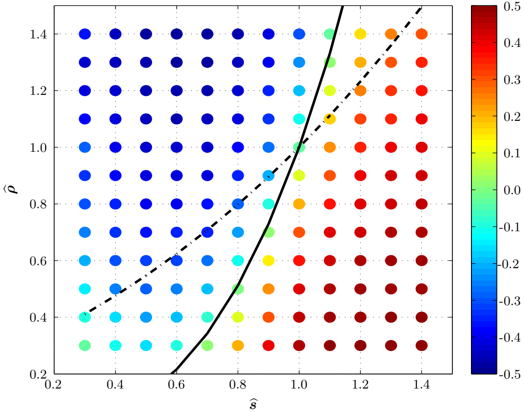


size- and density-based segregation $\hat{s} = 1.1$ and $\hat{\rho} = 1.3$.
Type-1: small and light and Type-2: large and heavy.

[Tunuguntla *et al.*(2014)Tunuguntla, Bokhove & Thornton, JFM]

Phase Space Segregation Strength

- We define $D_{com} = (COM2 - COMB)/COMB$.
- Solid black line $\gamma = 3$.
- Dotted line from Kinetic theory of Jenkins and Yoon.



[Tunuguntla *et al.*(2014)Tunuguntla, Bokhove & Thornton, JFM]
[Jenkins & Yoon(2002), PRL]

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Segregation in long rotating cylinders

Mount Ruaprhu avalanche

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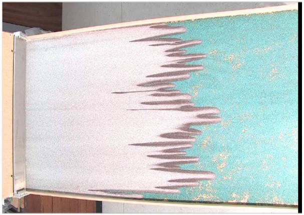
Experimental results



Savage-Hutter Assumption

- Assumes a flowing granular material is fluidised and incompressible
- The equations are depth integrated and averages are defined i.e.

$$\bar{f} = \int_b^s f \, \mathrm{d}z$$

- Exploits that in real avalanches $\epsilon = H/L \ll 1$
- Using Mohr-Coulomb yield criterion by model the ratio of $K = \bar{\sigma}_{xx}/\bar{\sigma}_{zz}$
- Here we will use a simpler inviscid fluid model which assumes $K = 1$
- Also assume $\overline{u^2} = \alpha \bar{u}^2$; often taken $\alpha = 1$.

Shallow water like theories

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h\bar{u}) + \frac{\partial}{\partial y}(h\bar{v}) = 0,$$

$$\frac{\partial}{\partial t}(h\bar{u}) + \frac{\partial}{\partial x}\left(h\bar{u}^2\right) + \frac{\partial}{\partial y}(h\bar{u}\bar{v}) + \frac{1}{2}\frac{\partial}{\partial x}\left(gh^2\cos\theta\right) = gh\left(\sin\theta - \mu\frac{\bar{u}}{\sqrt{\bar{u}^2+\bar{v}^2}}\cos\theta\right)$$

$$\frac{\partial}{\partial t}(h\bar{v}) + \frac{\partial}{\partial x}(h\bar{u}\bar{v}) + \frac{\partial}{\partial y}\left(h\bar{v}^2\right) + \frac{1}{2}\frac{\partial}{\partial y}\left(gh^2\cos\theta\right) = gh\left(-\mu\frac{\bar{v}}{\sqrt{\bar{u}^2+\bar{v}^2}}\cos\theta\right)$$

[Savage & Hutter(1989), JFM]

The Pouliquen friction law

- The law

$$\mu(h, \boldsymbol{u}) = \tan\delta_1 + [\tan\delta_2 - \tan\delta_1] \exp\left\{\frac{-\sqrt{g}\beta h^{3/2}}{L\,\|\boldsymbol{u}\|}\right\}$$

- δ_1 is minimum angle for the material to flow
- δ_2 is the maximum angle at which steady uniform flows can be observed
- L is a characteristic length scale

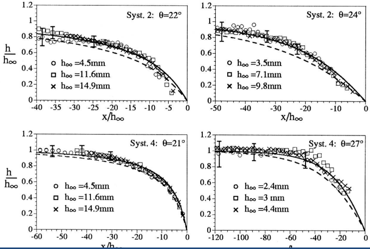
[Pouliquen(1999), Phys. Fluids 11 (3)]

The Pouliquen finger shape

By Coupling the Pouliquen friction law with the shallow-water type equations can show there is a travelling wave solution given by the simple first order o.d.e.

$$\tan\theta - \mu(h_\infty h_0(x), U_\infty) = \frac{dh_0}{dx}$$

$$h_0(0) = 0$$



Pouliquen's Analysis

- Introduce a depth average concentration $C(x, y, t)$ of small particles.
- Choose $\mu(h, \|u\|, C)$ to be a simple linear scaling i.e.

$$\mu(h, \|u\|, C) = (1 - C)\mu^l(h, \|u\|) + C\mu^s(h, \|u\|).$$

- Assume $C = \frac{1}{2}\left(1 + \tanh\left(\frac{x+L^l}{D}\right)\right)$
- Performed a stability analysis and concluded mono-dispersed fronts were stable.
- Bi-dispersed fronts are unstable if $\delta_1^l, \delta_2^l, L^l, \delta_1^s, \delta_2^s$ and L^s are chosen such that $\mu^S(1, 1) < \mu^l(1, 1)$.

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Segregation in long rotating cylinders

9 Conclusions

3D segregation equation

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x}(\phi u) + \frac{\partial}{\partial y}(\phi v) + \frac{\partial}{\partial z}(\phi w) - \frac{\partial}{\partial z}(S_r \phi (1-\phi)) = 0$$

where

ϕ : is the volume fraction of small particles

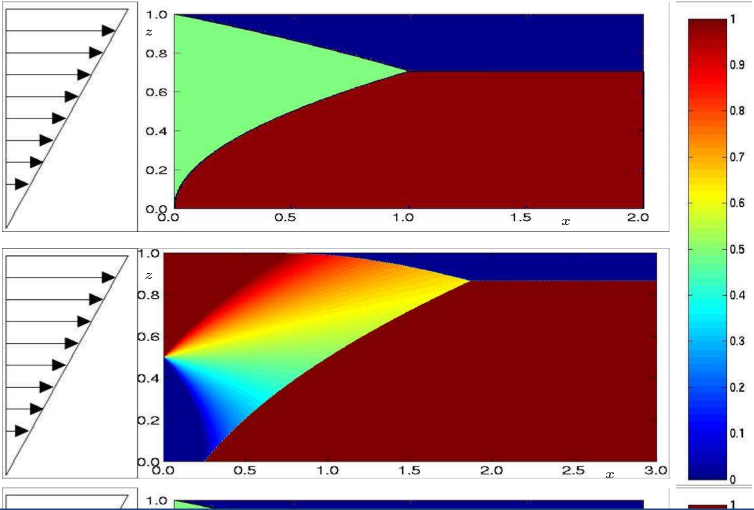
u, v, w : down slope/cross slope/normal velocity components

S_r : is a dimensionless segregation rate

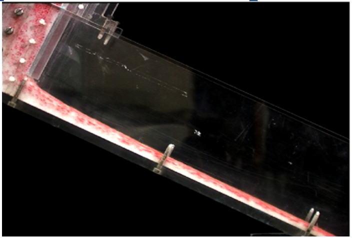
[Gray & Thornton(2005), Proc. Royal. Soc.]

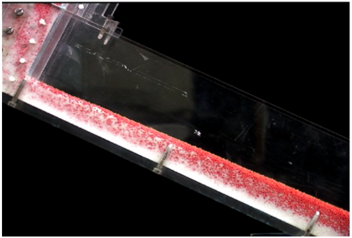
[Thornton *et al.*(2006)Thornton, Gray & Hogg, JFM]

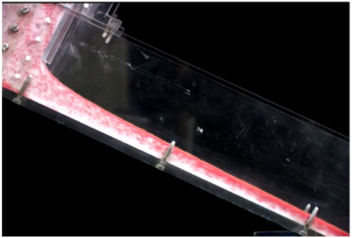
Exact Solutions

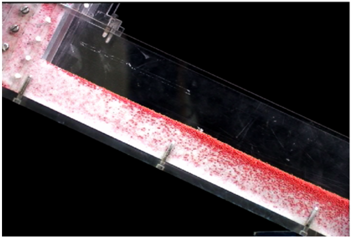


Experimental comparison









Adding in segregation dynamics

- Depth integrate the 3D segregation equation
- Introduce the averages

$$C = \bar{\phi} = \frac{1}{h}\int_0^1 \phi \, \mathrm{d}z$$

- Assume segregation is instantaneous i.e. take the limit $S_r \to \infty$ and that the velocity profile is $u = \bar{u}\left(\alpha + 2(1-\alpha)\left(\frac{z-b}{h}\right)\right)$.
- Leads to

$$\frac{\partial}{\partial t}(hC) + \frac{\partial}{\partial x}(h\bar{u}C) + \frac{\partial}{\partial y}(h\bar{v}C) = (1-\alpha)\left(\frac{\partial}{\partial x}(h\bar{u}(C-C^2)) + \frac{\partial}{\partial y}(h\,\bar{v}(C-C^2))\right) = 0. \quad (2)$$

The fully coupled system

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0,$$

$$\frac{\partial}{\partial t}(hu) + \frac{\partial}{\partial x}\left(hu^2\right) + \frac{\partial}{\partial y}(huv) + \frac{1}{2}\frac{\partial}{\partial x}\left(gh^2\cos\theta\right) = gh\left(\sin\theta - \mu\frac{u}{\sqrt{u^2+v^2}}\cos\theta\right)$$

$$\frac{\partial}{\partial t}(hv) + \frac{\partial}{\partial x}(huv) + \frac{\partial}{\partial y}\left(hv^2\right) + \frac{1}{2}\frac{\partial}{\partial y}\left(gh^2\cos\theta\right) = hg\left(-\mu\frac{v}{\sqrt{u^2+v^2}}\cos\theta\right)$$

One-dimensional travelling wave solution

We will seek one-dimenstional travelling solution. Hence making the transformation

$$\hat{x} = x - u_f t, \quad \frac{\partial}{\partial y} = 0. \quad \hat{t} = t$$

It can be shown that the equation for $\bar{u}$ can be reduced to the following o.d.e.

$$\frac{d\bar{u}}{d\hat{x}} = \frac{s}{\left((1 - \bar{u}_f) - \epsilon \cos\theta \frac{(1-u_f)}{(\bar{u}-u_f)^2}\right)},$$

where

$$s = \mu \frac{u}{\sqrt{u^2 + v^2}} \cos\theta$$

Relationships with $\bar{u}$ and h

Once you have solved the o.d.e for $\bar{u}$, both h and C are similar given by the following algebraic equations

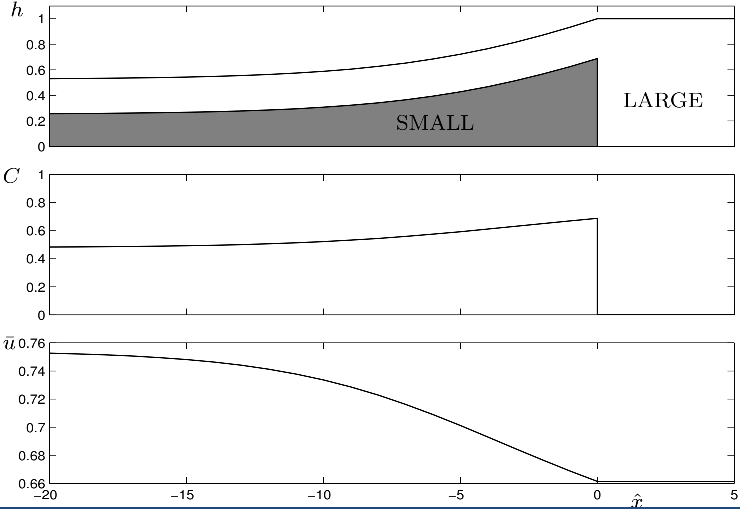
$$h = \frac{1 - u_f}{\bar{u} - u_f}.$$

$$C^2 + c_1 C + c_0 = 0$$

where

$$c_1 = \frac{\alpha \bar{u} - u_f}{(1-\alpha)\,\bar{u}} \quad \text{and} \quad c_0 = \frac{\bar{u} - u_f}{\bar{u}} \left(\frac{C_0\,(1 - C_0)}{1 - u_f} - \frac{C_0}{1-\alpha} \right).$$

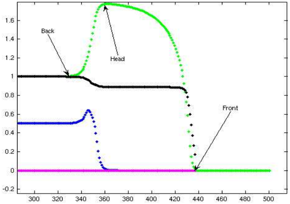
Solution


h
1
0.8
0.6
0.4
0.2
0
SMALL
LARGE
C
1
0.8
0.6
0.4
0.2
0
$\bar{u}$
0.76
0.74
0.72
0.7
0.68
0.66
–20
–15
–10
–5
0
5
$\hat{x}$

The bulbous head solution

ϵ	0.1	ϕ_{inflow}	0.9
δ_1^s	20°	δ_2^s	30°
δ_1^l	27°	δ_2^l	37°
α	0.0	$L_l = L_s$	1.0
x-length	500	y-length	20
no. points x	500	no. points y	500

The bulbous head solution



- By considering mass balance we can show

$$U_{front} = U_{inflow}\left(1 - \alpha\phi_0 + \phi_0^2 - \alpha\phi_0^2\right)$$

$$U_{back} = U_{inflow}\left(\alpha + (1-\alpha)\,\phi_0\right)$$

- Since the front consists of a pure phase of large particles its shape is given by Pouliquen's finger solutions. Hence

The fully coupled system

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0,$$

$$\frac{\partial}{\partial t}(hu) + \frac{\partial}{\partial x}\left(hu^2\right) + \frac{\partial}{\partial y}(huv) + \frac{1}{2}\frac{\partial}{\partial x}\left(gh^2\cos\theta\right) = gh\left(\sin\theta - \mu\frac{u}{\sqrt{u^2+v^2}}\cos\theta\right)$$

$$\frac{\partial}{\partial t}(hv) + \frac{\partial}{\partial x}(huv) + \frac{\partial}{\partial y}\left(hv^2\right) + \frac{1}{2}\frac{\partial}{\partial y}\left(gh^2\cos\theta\right) = hg\left(-\mu\frac{v}{\sqrt{u^2+v^2}}\cos\theta\right)$$

The Pouliquen friction law

- The law

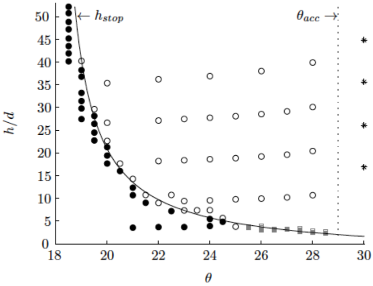
$$\mu(h, \boldsymbol{u}) = \tan\delta_1 + [\tan\delta_2 - \tan\delta_1]\exp\left\{\frac{-\sqrt{g}\beta h^{3/2}}{L\,\|\boldsymbol{u}\|}\right\}$$

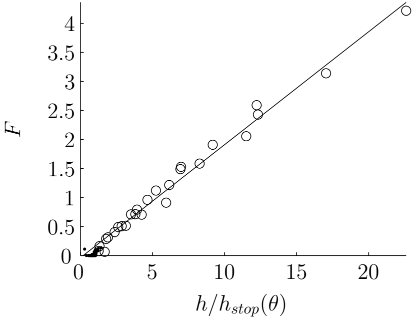
- Empirical law determined by measuring the minimum height for flow at various different inclination angles $h_{stop}(\theta)$
- Experiments show

$$\frac{u}{\sqrt{gh}} = \beta\frac{h}{h_{stop}}$$

  where $\beta = 0.136$ is a universal constant
- δ_1 is minimum angle for the material to flow
- δ_2 is the maximum angle at which steady uniform flows can

Comparison to Pouliquen friction law





- Fit is to $h_{stop}(\theta) = Ad\dfrac{\tan(\theta_2) - \tan(\theta)}{\tan(\theta) - \tan(\theta_1)}$.

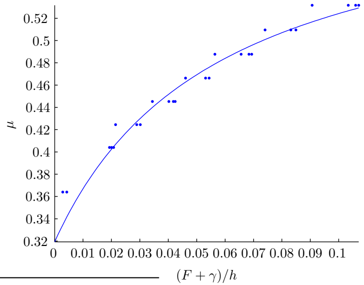
[Weinhart *et al.*(2012)Weinhart, Thornton, Luding & Bokhove, Granular Matter]

[Thornton *et al.*(2012*a*)Thornton, Weinhart, Luding & Bokhove, EPJ E]

Closure for the bed friction $\mu(h, \bar{u})$

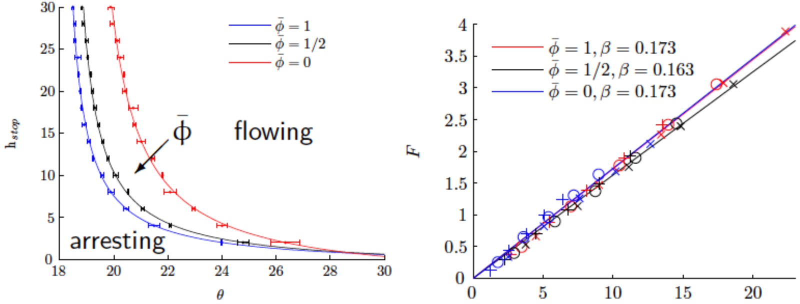
Substituting $\mu = \tan(\theta)$ into $F = \beta \frac{h}{h_{stop}(\theta)} - \gamma$ yields the closure

$$\mu(h, \bar{u}) = \tan(\theta_1) + (\tan(\theta_2) - \tan(\theta_1)) \left(\frac{\beta}{Ad} \frac{h}{F + \gamma} + 1 \right)^{-1}.$$

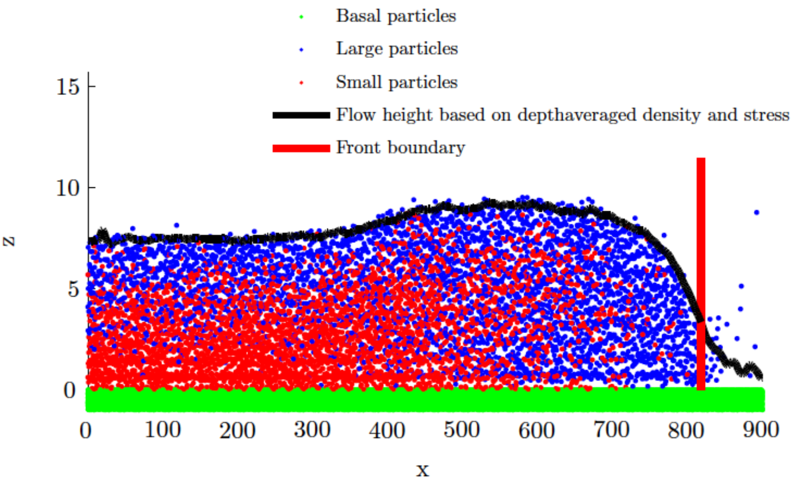


[Weinhart *et al.*(2012)Weinhart, Thornton, Luding & Bokhove, Granular Matter]

Comparison to Pouliquen friction law



- Fit is to $h_{stop}(\theta) = Ad\dfrac{\tan(\theta_2) - \tan(\theta)}{\tan(\theta) - \tan(\theta_1)}$.
- $F = \beta h / h_{stop}$
- That is the Pouliquen flow does holds for bidispersed.


Basal particles
Large particles
Small particles
Flow height based on depthaveraged density and stress
Front boundary
15
10
5
0
z
0
100
200
300
400
500
600
700
800
900
x

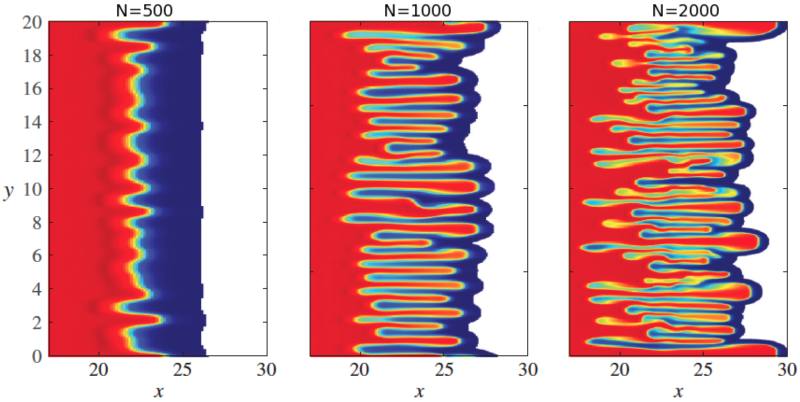
Lowering the angle

Click here to start movie

[Woodhouse *et al.*(2012)Woodhouse, Thornton, Johnson, Kokelaar & Gray, JFM]

Grid dependence

So problem solved, well no.



[Woodhouse *et al.*(2012)Woodhouse, Thornton, Johnson, Kokelaar & Gray, JFM]

Results

Asymptotic results for high k_x show

- That for $\bar{u_0} \neq u_s$ to leading order eigenvalues are purely imaginary for $k_x >> 1$.
- However, on the curve $\bar{u_0} = u_s \quad \sigma \approx k^{1/2}$ for $k_x >> 1$.
- Linear stability analysis of a constant solution shows system is ill posed on a single curve.
- Both fingering and propagating head solutions can be formed
- The number of fingers produced is grid **dependent**
- However, it is linear unstable at high wave numbers
- Shallow layer of fluid on an incline has a similar problem
- System can be stabilised by adding viscous and diffusion terms

[Woodhouse *et al.*(2012)Woodhouse, Thornton, Johnson, Kokelaar & Gray,

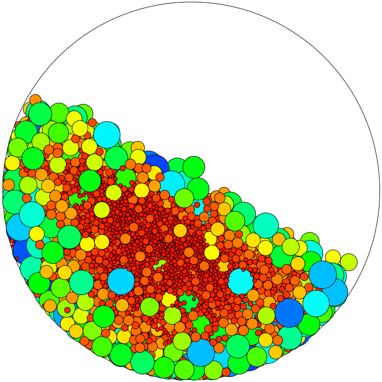
Outline - Next Section I

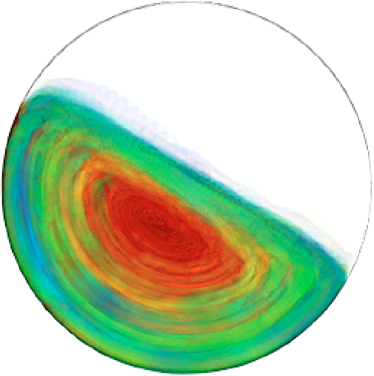
Outline - Next Section II

Segregation in long rotating cylinders

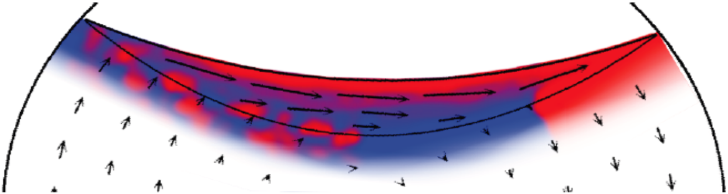
9 Conclusions

DPM of segregation in rotating drum



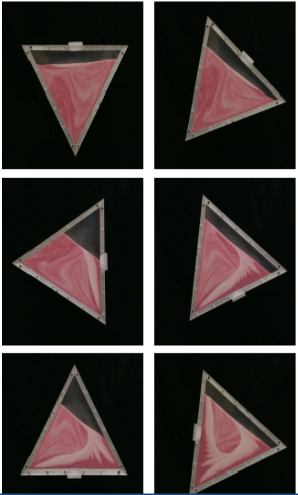


Schematic of segregating in rotating drum

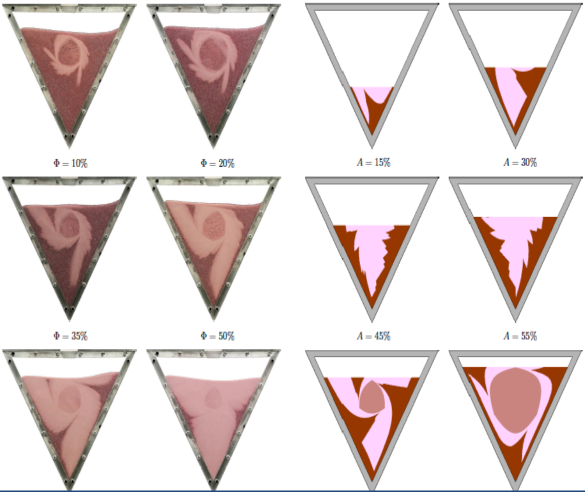


Large particles in red
Small particles in blue

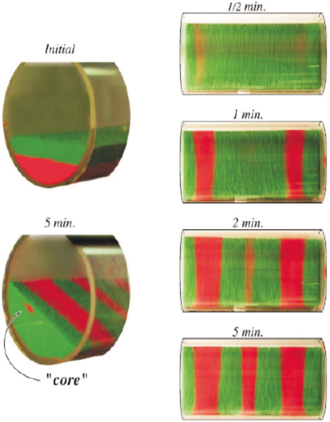
Segregating in a Rotating Triangle

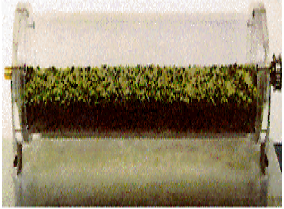


Final Patterns in Rotating Triangle



Segregating in Rotating Cylinder





Initial Mixture of Uncooked Rice and Split Peas

After Rotation About Horizontal Axis at 15 rpm for 2 hours

Outline - Next Section I

Outline - Next Section II

Segregation in long rotating cylinders

9 Conclusions

- Discussed definition of mixed state
- Showing different industrial mixers
- Showed a family of models for granular segregation
- Showed how to use DPM to calibrate and validate such models
- Coupled segregation and bulk flow models
- Showed how a reduced version of this model can be applied to rotating drums
- Consider axial patterns in long rotating cylinders
- Coupled the segregation model with shallow water equations to consider geophysical problems

- Offer corse in:
  - A Practical Introduction to C++
  - The Fundamentals of Discrete Particle Simulations
- 3 day course starts from 497.50 euros (722.50 with accommodation).
- Next given $20^{th} - 24^{th}$ July
- `http://MercuryLab.org`

GRAY, J. M. N. T. & THORNTON, A. R. 2005 A theory for particle size segregation in shallow granular free-surface flows. *Proc. Royal Soc. A* **461**, 1447–1473.

JENKINS, J. T. & YOON, D. K. 2002 Segregation in binary mixtures under gravity. *Phys. Rev. Lett.* **88** (19), 1.

LUDING, S. 2008 Introduction to discrete element methods DEM: Basics of contact force models and how to perform the micro-macro transition to continuum theory. *Euro. J. of Enviro. Civ. Eng.* **12** (7-8), 785–826.

MARKS, BENJY, ROGNON, PIERRE & EINAV, ITAI 2012 Grainsize dynamics of polydisperse granular segregation down inclined planes. *Journal of Fluid Mechanics* **690**, 499–511.

- POULIQUEN, O. 1999 Scaling laws in granular flows down rough inclined planes. *Phys. Fluids* **11** (3), 542–548.
- SAVAGE, S. B. & HUTTER, K. 1989 The motion of a finite mass of material down a rough incline. *Journal Fluid. Mech.* **199**, 177–215.
- THORNTON, A. R., GRAY, J. M. N. T. & HOGG, A. J. 2006 A three phase model of segregation in shallow granular free-surface flows. *J. Fluid Mech.* **550**, 1–25.
- THORNTON, A. R., WEINHART, T., LUDING, S. & BOKHOVE, O. 2012*a* Friction dependence of shallow granular flows from discrete particle simulations. *EPJ E* **35:127.**

THORNTON, A. R., WEINHART, T., LUDING, S. & BOKHOVE, O. 2012*b* Modelling of particle size segregation: Calibration using the discrete particle method. *Int. J. Mod. Phy. C.* **23** (1240014).

TUNUGUNTLA, D., BOKHOVE, O. & THORNTON, A. R. 2014 A mixture theory for size and density segregation in shallow granular free-surface flows. *JFM* **749**, 99–112.

WEINHART, T., THORNTON, A.R., LUDING, S. & BOKHOVE, O. 2012 Closure relations for shallow granular flows from particle simulations. *Granular Matter* **14**, 531–552.

WOODHOUSE, M., THORNTON, A. R., JOHNSON, C., KOKELAAR, P. & GRAY, J. N. M. T. 2012 Segregation induced fingering instabilities in granular avalanches. *J. Fluid Mech.* **709**, 543–580.