From particle to granular flow

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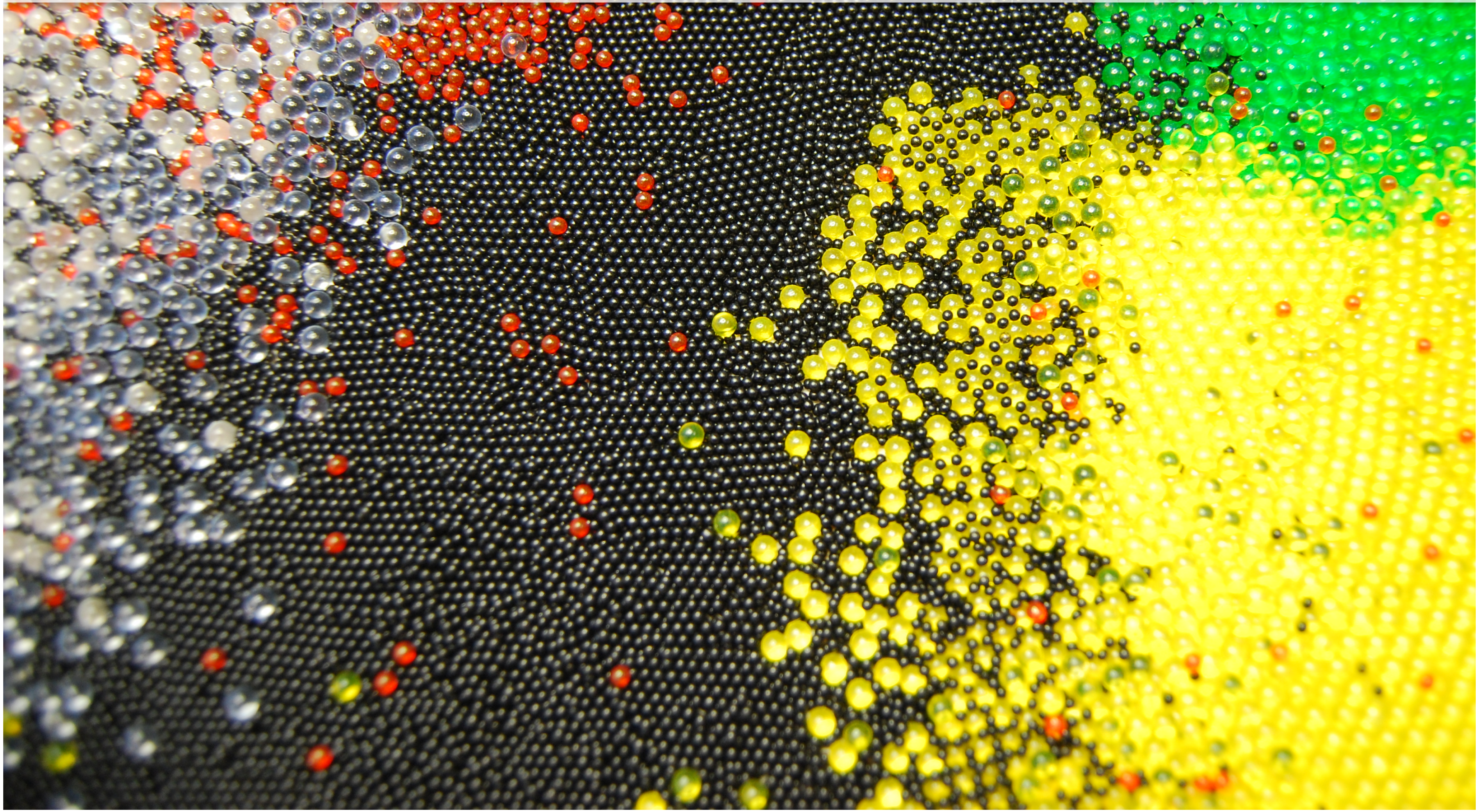
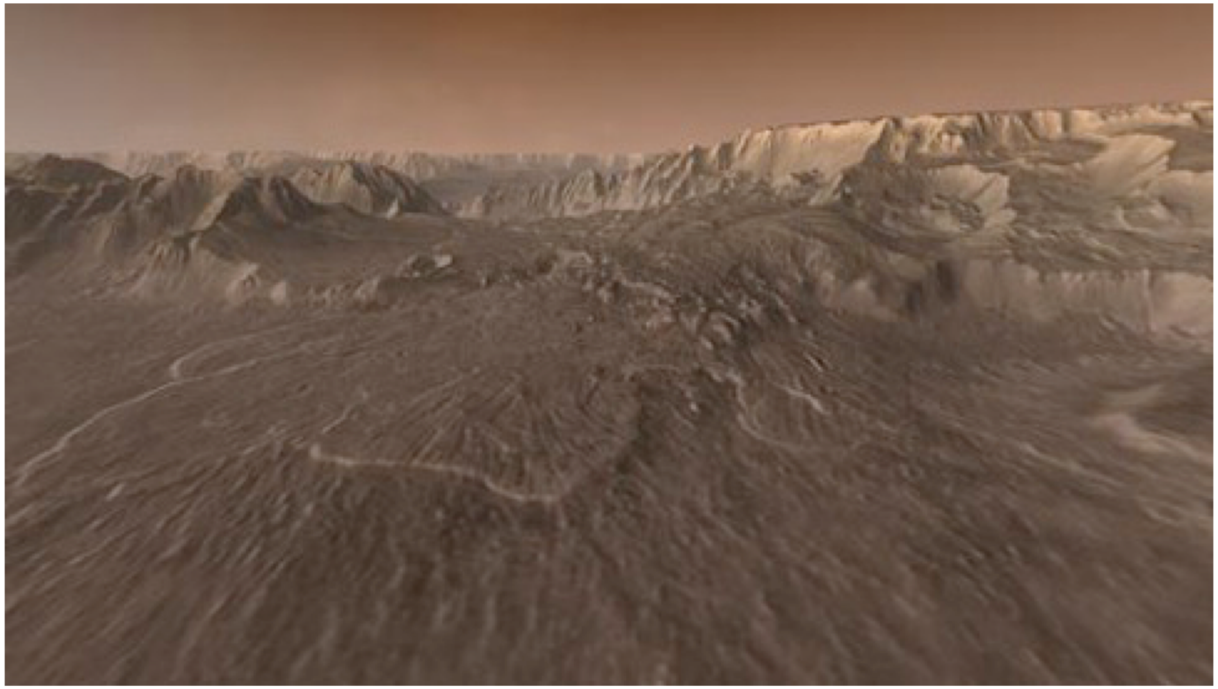
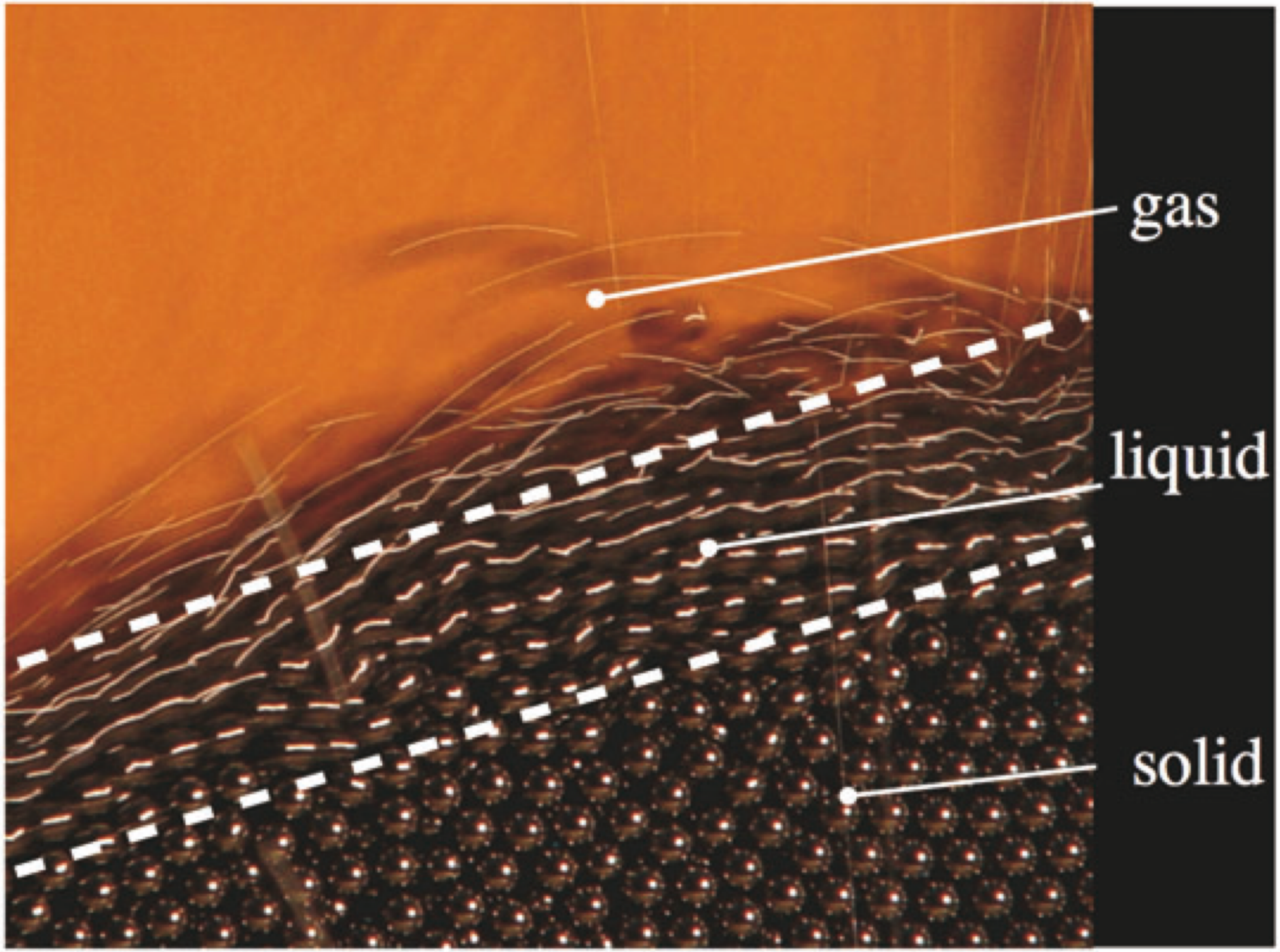


Granular material regimes

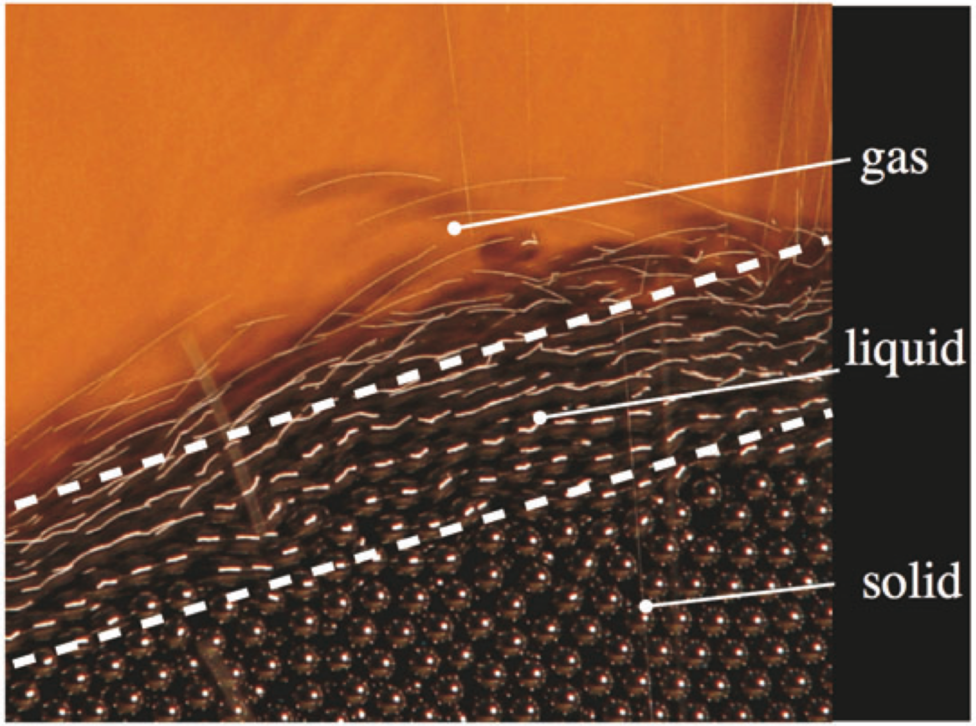


Solid+liquid+gas



[Forterre & Pouliquen, Annu. Rev. Fluid Mech. (2008)]

Solid+liquid+gas



Three regimes:

- **solid** – static
particles interact via frictional contacts

- **liquid** – dense, flow-like behavior
both collisions and friction

- **gas** – rapid dilute flow
particles interact via collisions

[Jaeger et al. (1996), Forterre & Pouliquen, Annu. Rev. Fluid Mech. (2008)]

Outline

- Introduction
- Internal force transmission
- Solid state
- Quasistatic regime and flow threshold
- Collisional and rapid granular flows
- Dense slow flows and inertial regime – (extended) rheology

References

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Granular material flows – An overview

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References

Flows of Dense Granular Media

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0066-4189/08/0115-0001$20.00

Key Words

granular flows, rheology, friction, shallow water, instability, visco-plasticity

Abstract

We review flows of dense cohesionless granular materials, with a special focus on the question of constitutive equations. We first discuss the existence of a dense flow regime characterized by enduring contacts. We then emphasize that dimensional analysis strongly constrains the relation between stresses and shear rates, and show that results from experiments and simulations in different configurations support a description in terms of a frictional visco-plastic constitutive law. We then discuss the successes and limitations of this empirical rheology in light of recent alternative theoretical approaches. Finally, we briefly present depth-averaged methods developed for free surface granular flows.

Internal forces transmission

Contact forces

- Unique feature of granular material arises from internal force transmission

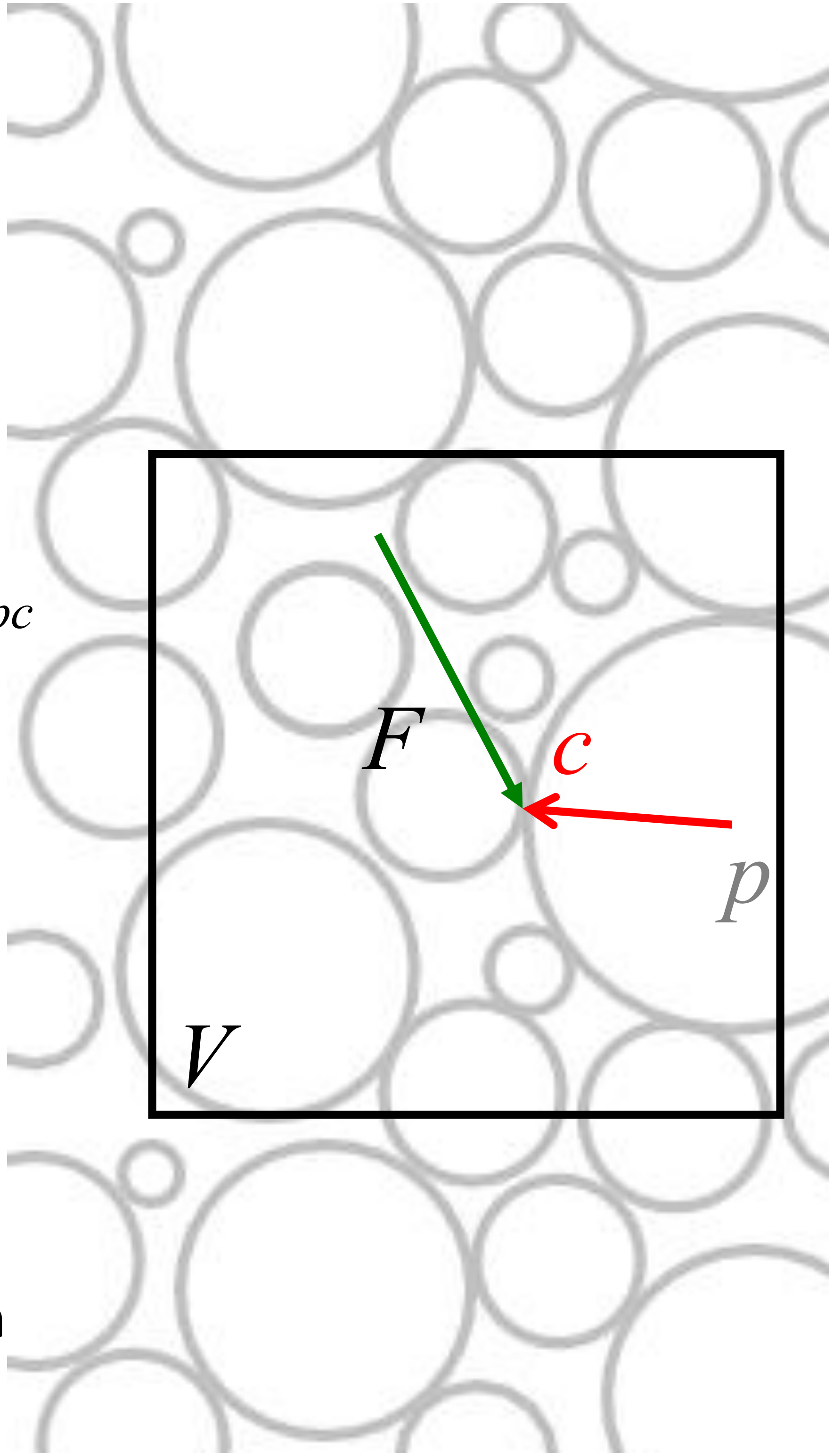
- Most fundamental microscopic property of granular materials: irreversible energy dissipation in the course of interaction collision between particles.

Micro-macro transition

Stress tensor

$$Q = \frac{1}{V} \sum_c \left(w_V^p \right) \boldsymbol{l}^{pc} \boldsymbol{F}^c$$

$$Q = \underline{\underline{\mathbf{F}}} = \frac{1}{V} \sum_{p \in V} w_V^p V^p \sum_c \boldsymbol{n}^{pc} \boldsymbol{n}^{pc}$$



Any quantity:

- Scalar
- Vector
- Tensor: Stress

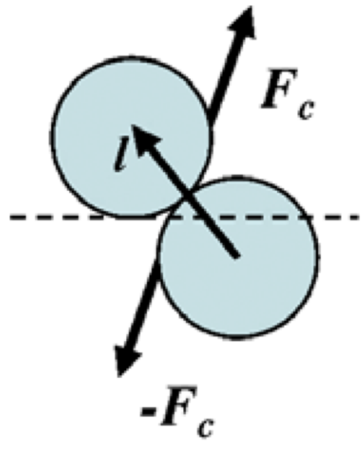
Overview of more complex formulations in [Weinhart et al. (2010)]

Stress tensor

a. Contact stress tensor

Due to the force transmission across interparticle forces

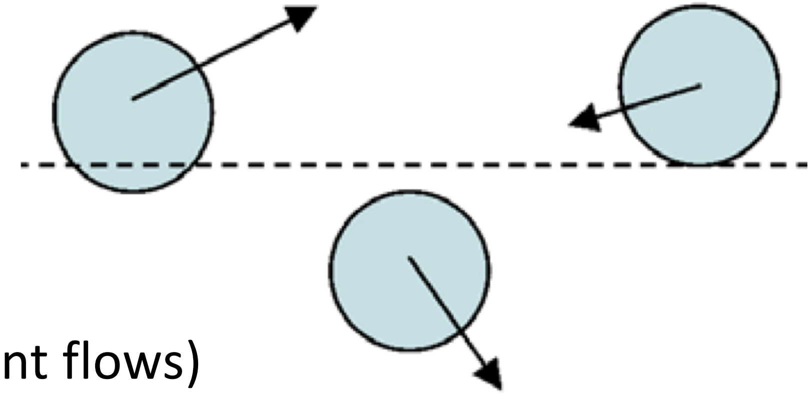
$$\sigma_{ij}^{c} = \frac{1}{V}\sum_{C=1}^{Nc} F_i^C l_j$$



b. Streaming stress tensor

Due to the motion of a particle relative to the bulk material (Reynolds stress tensor in turbolent flows)

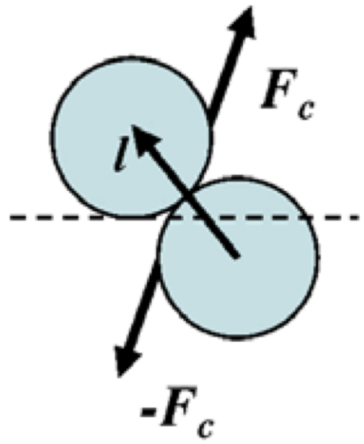
$$\sigma_{ij}^{s} = \frac{\rho_p \phi}{V}\sum_{p=1}^{Np} u'_i u'_j$$



Stress tensor

a. Contact stress tensor

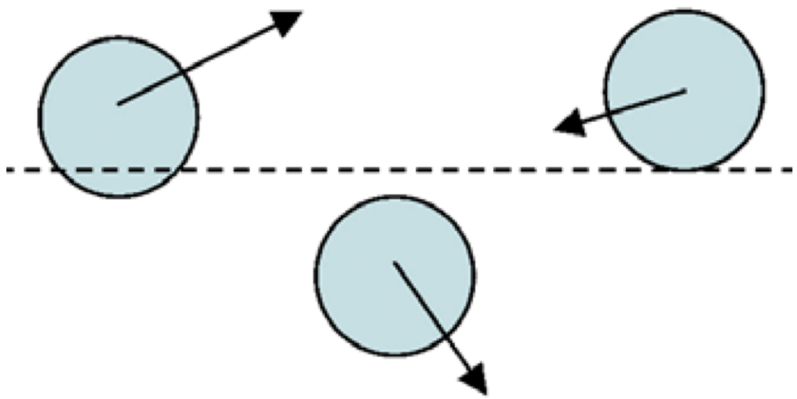
In hoppers, chutes, landslides: $\phi > \mathbf{50\%}$
is usually dominant in common granular flows



(a)

b. Streaming stress tensor

in loose flows or granular gases



(b)

Hertzian contact law

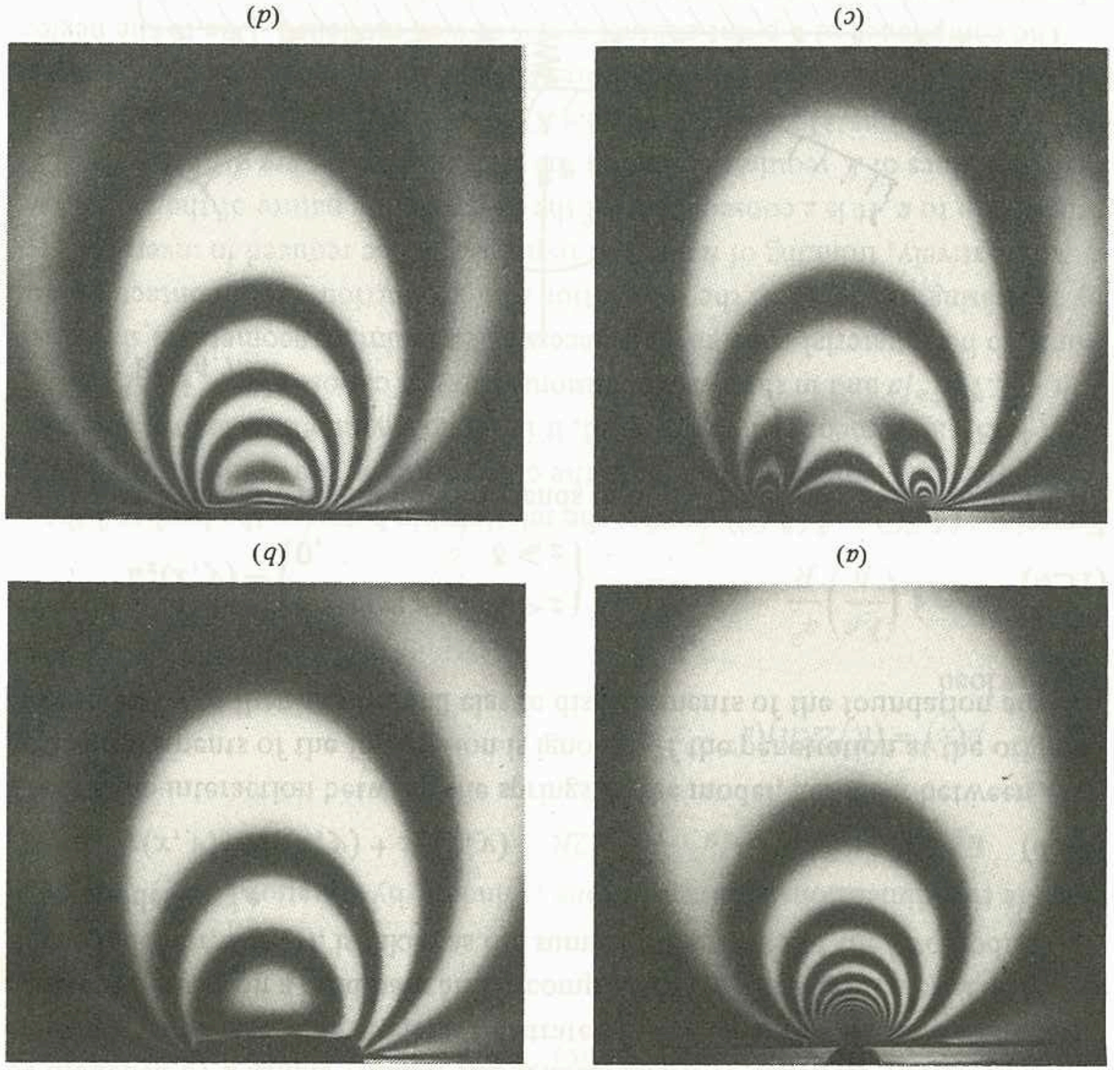
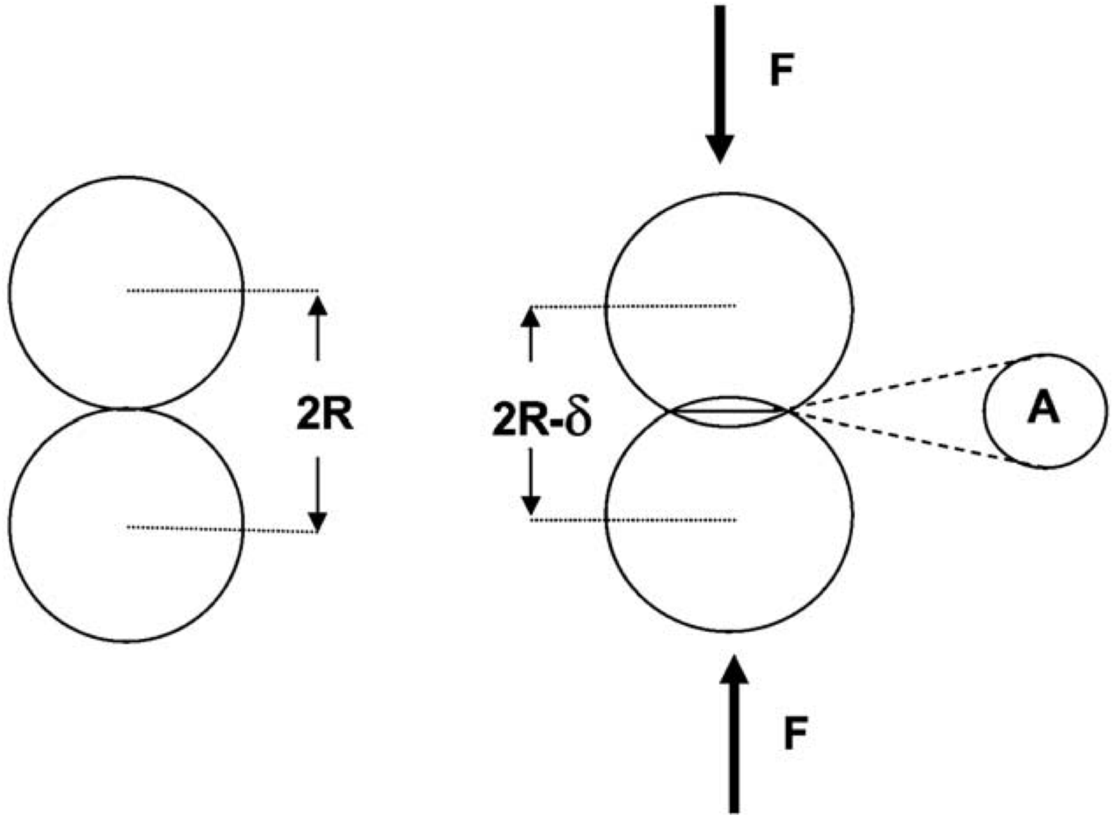


Fig. 4.6. Two-dimensional photo-elastic fringe patterns (contours of principal shear stress): (a) point load (§2.2); (b) uniform pressure (§2.5(a)); (c) rigid flat punch (§2.8); (d) contact of cylinders (§4.2(c)).

Hertzian contact law



Contact stiffness

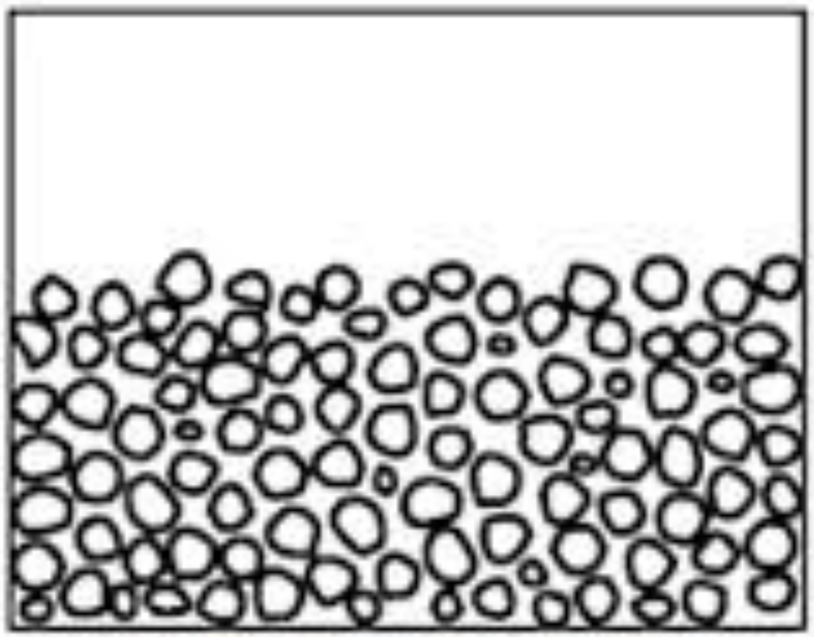
$$k = \mathbf{6}^{\mathbf{1/3}} R^{\mathbf{1/3}} \left(\frac{E}{\mathbf{1} - \nu^{\mathbf{2}}} \right)^{\mathbf{2/3}} F_n^{\mathbf{1/3}}$$

Solid state

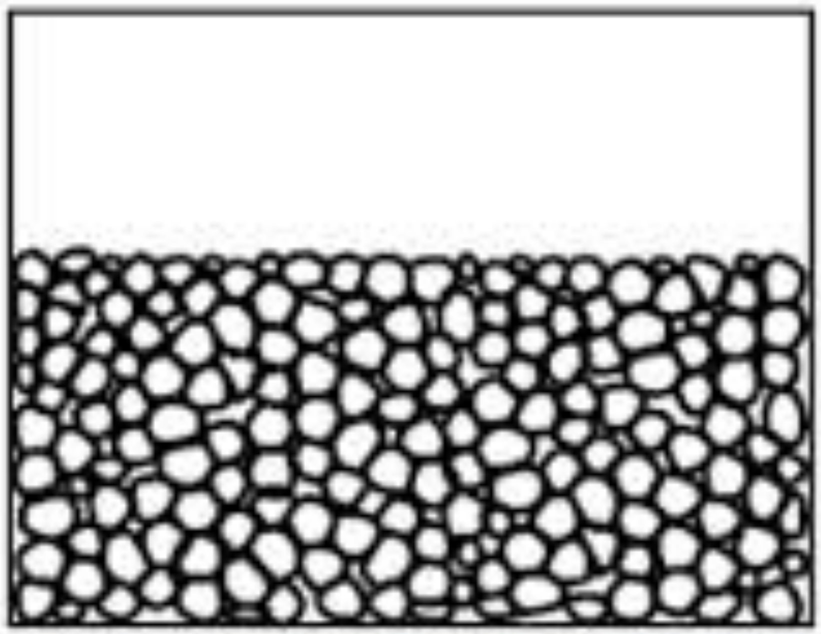
Small strain (elastic) stiffness

Classical solids: elastic stiffness is a material constant

Granular materials: elastic stiffness depends on **pressure** and **volume fraction**



Loose soil(Poor load support)



Compacted soil(Good load support)

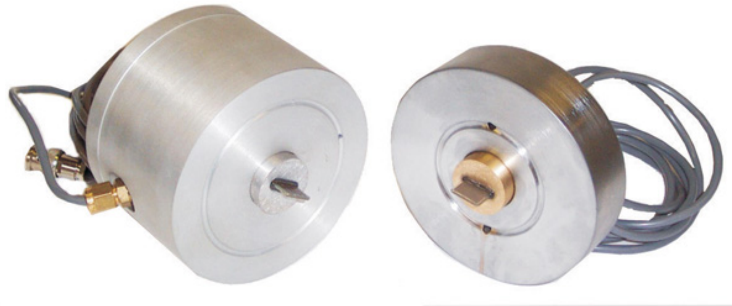
Experimental measurements

Static probing → *G*, *E*

Dynamic method → v_s, v_p

$$v_p = \sqrt{\frac{E}{\rho}} \qquad v_s = \sqrt{\frac{G}{\rho}}$$

Experimental measurements



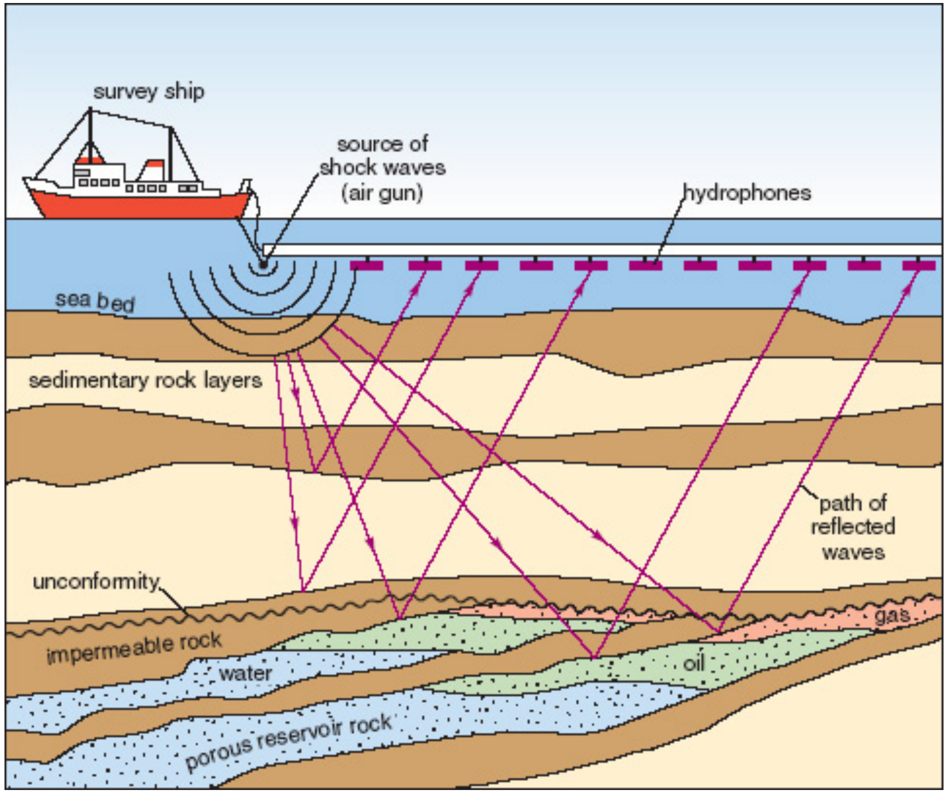
Dynamic method → v_s, v_p

Static probing → *G*, *E*

$$G = Af(\phi)\left(\frac{p}{p_{atm}}\right)^n$$

$$E = Ag(\phi)\left(\frac{p}{p_{atm}}\right)^m$$

Earthquakes



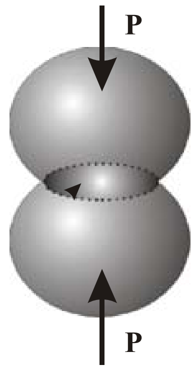
Oil/Gas exploration

Soil characterization

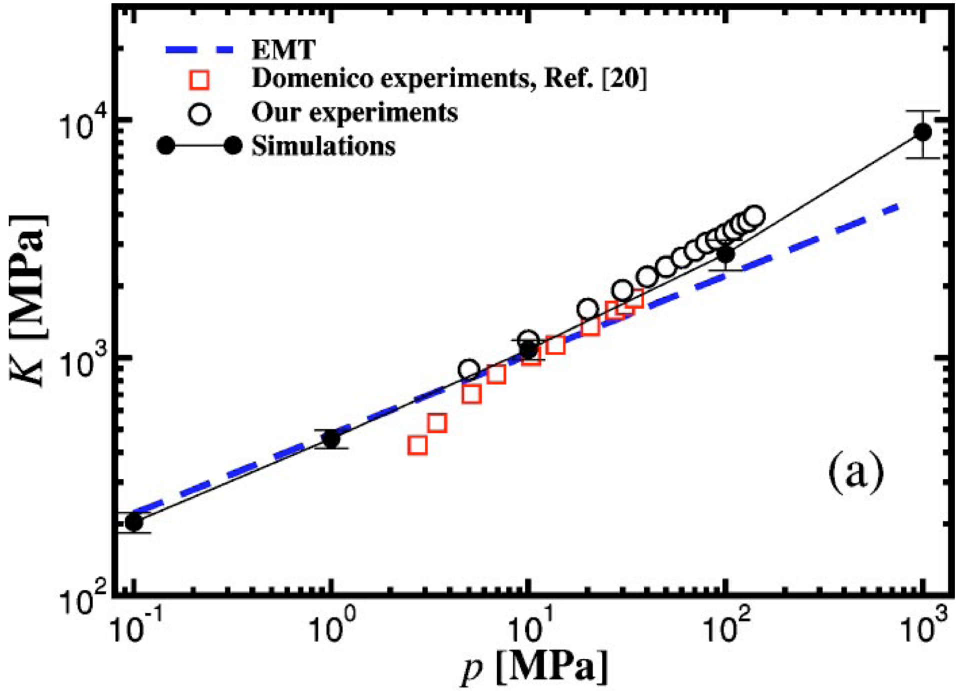
Small strain (elastic) stiffness

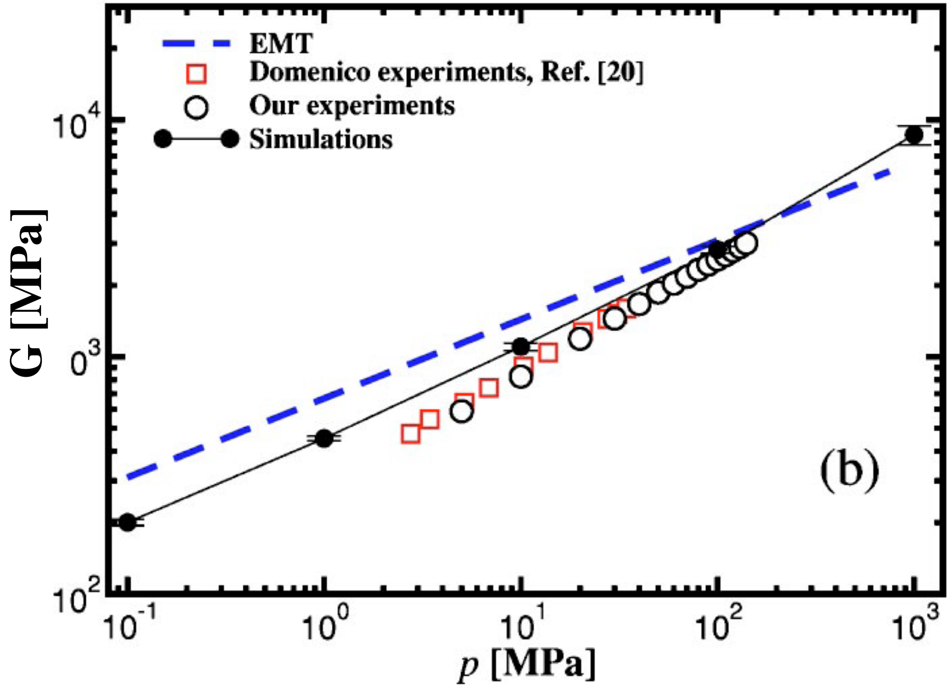
$$G_{bulk} \propto \frac{k}{R}$$

[Bathurst and Rothenburg, J. Appl. Mech. (1988)]



Because of Hertzian interaction we expect: $K(p) \propto G(p) \propto p^{1/3}$

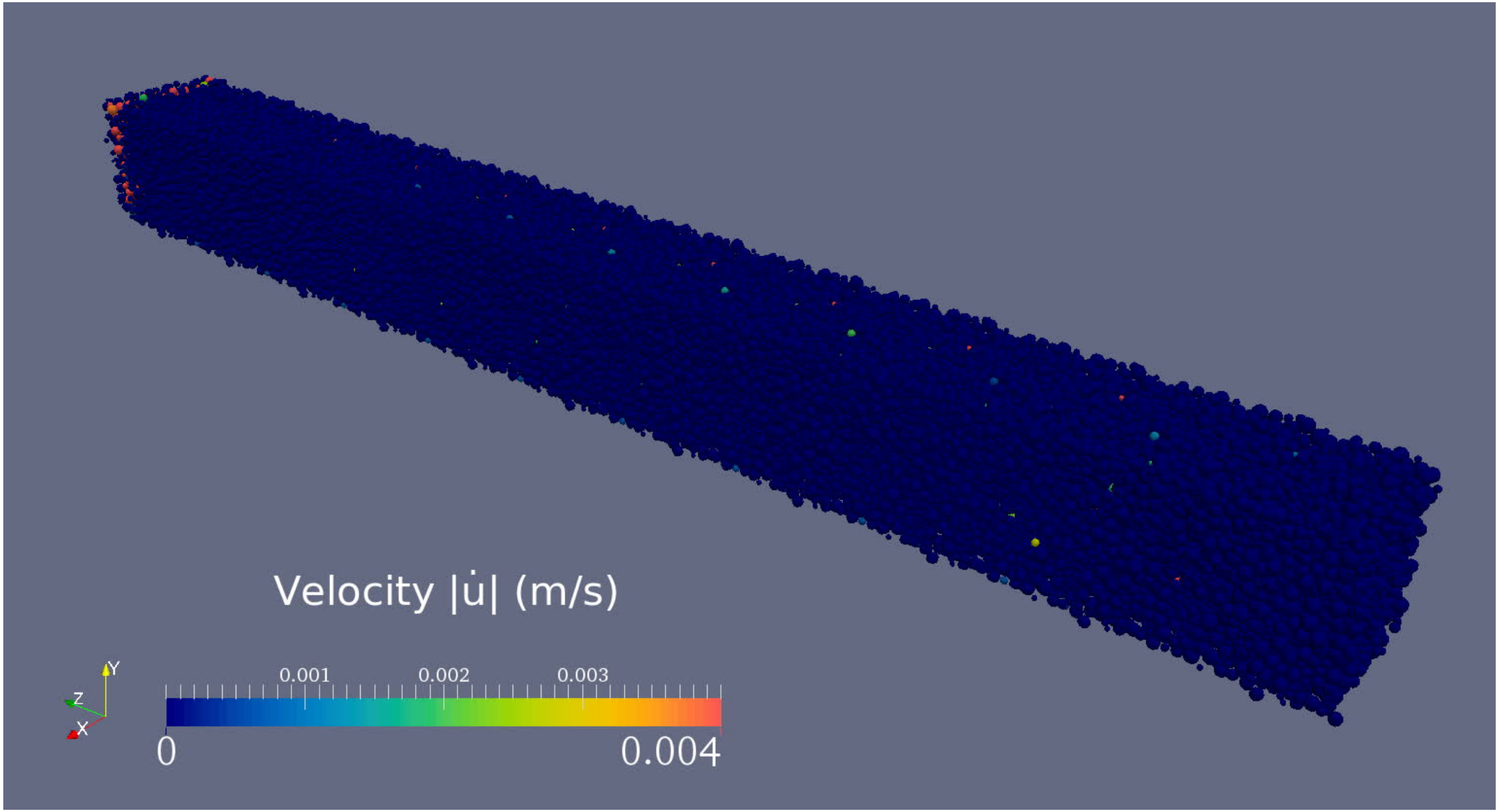




[Domenico (1977), Jia& Mills (2001), Wildenberg et al (2013), Gland et al., PRE (2005),…]

Wave propagation
(dynamic method)

Piezoelectric transducers



Wave propagation
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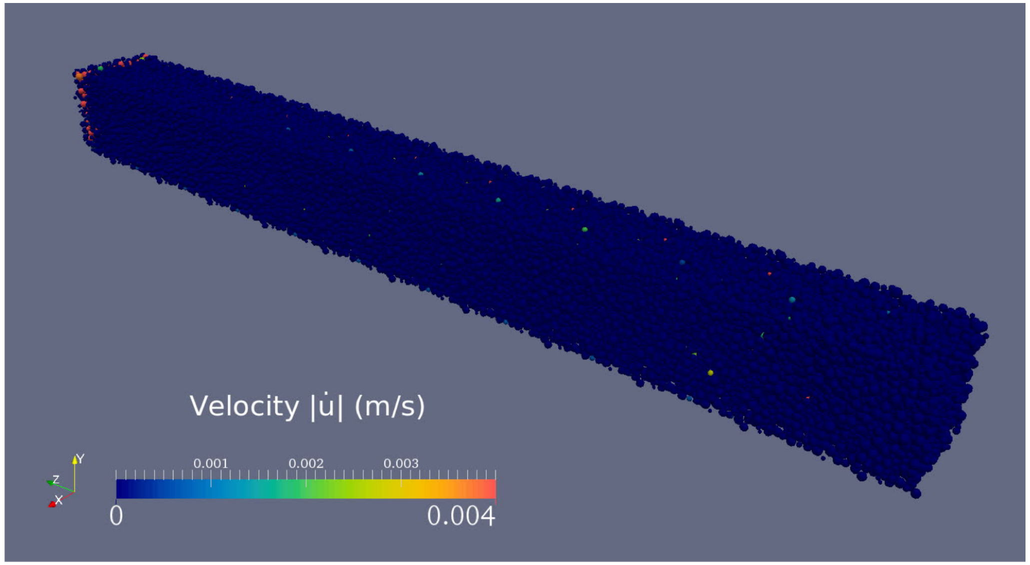
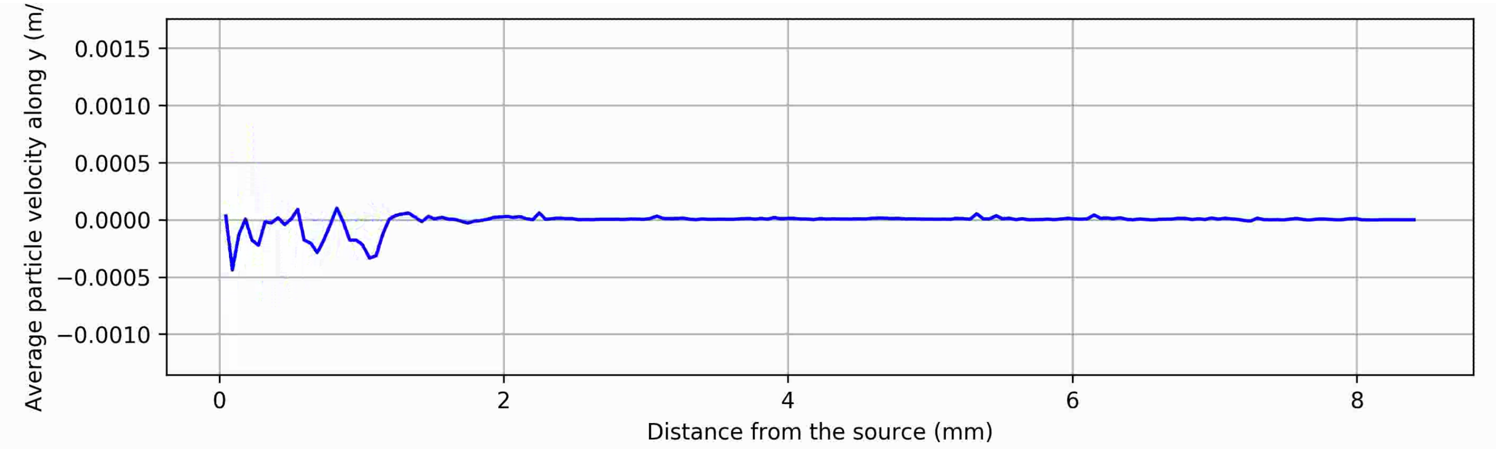
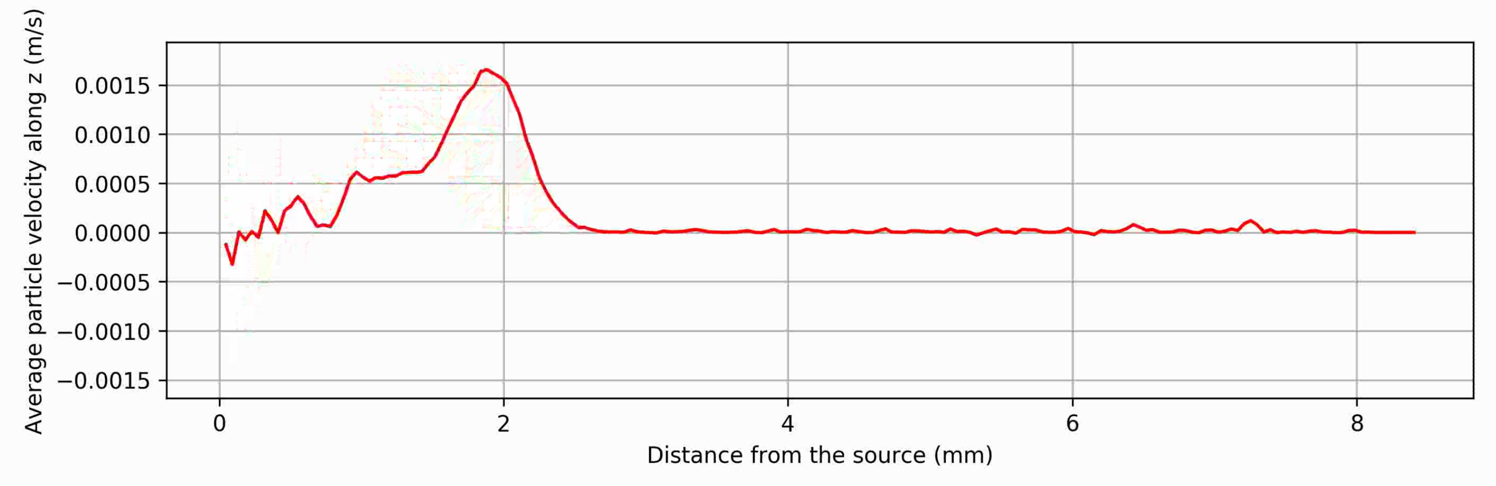


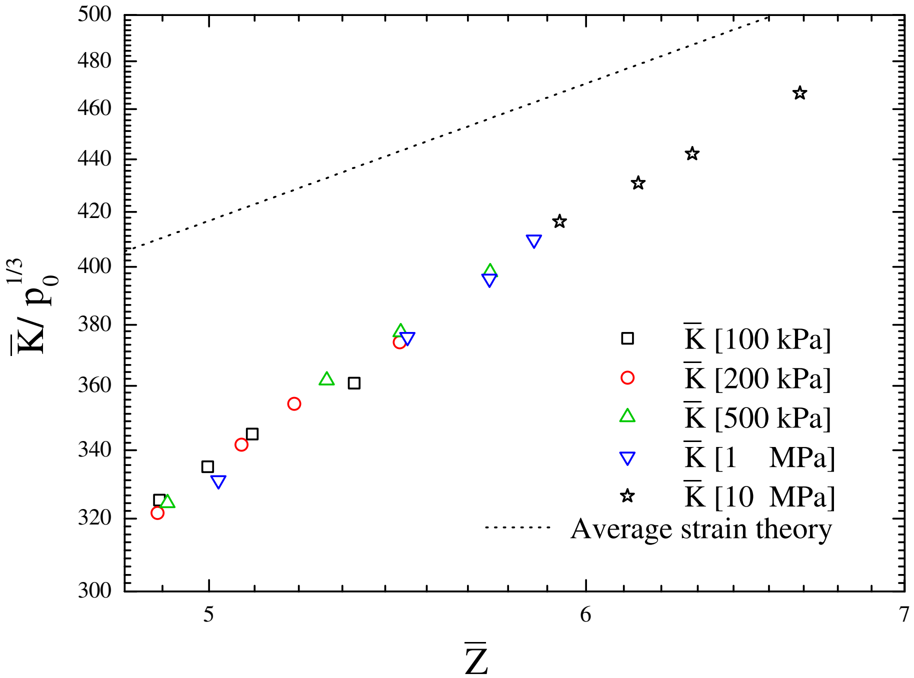


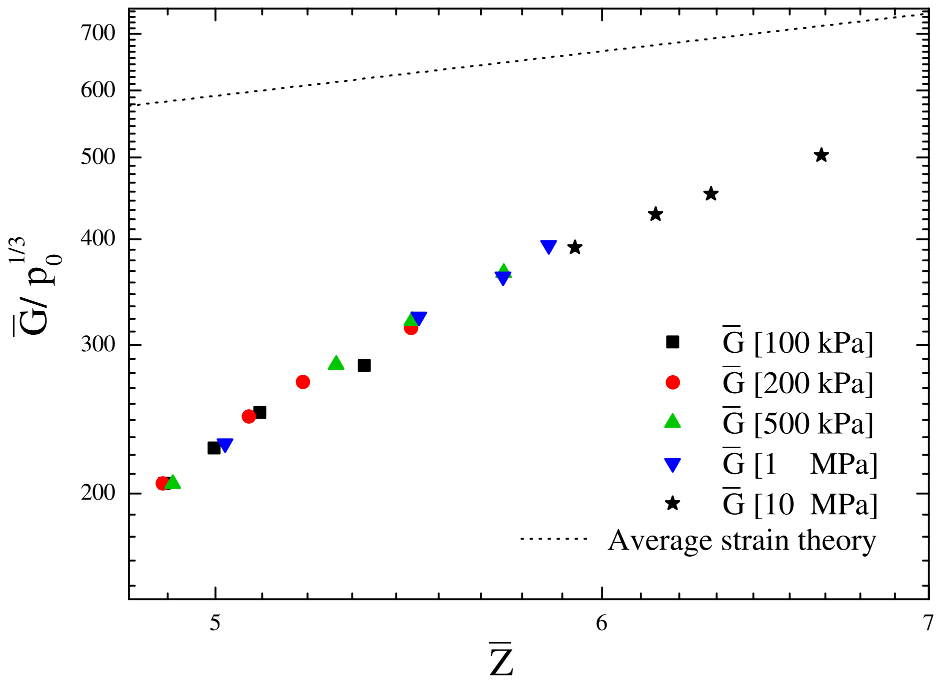


$$v_s = \sqrt{\frac{G}{\rho}}$$

$$v_p = \sqrt{\frac{E}{\rho}}$$

Dependence on coordination number





Coordination number

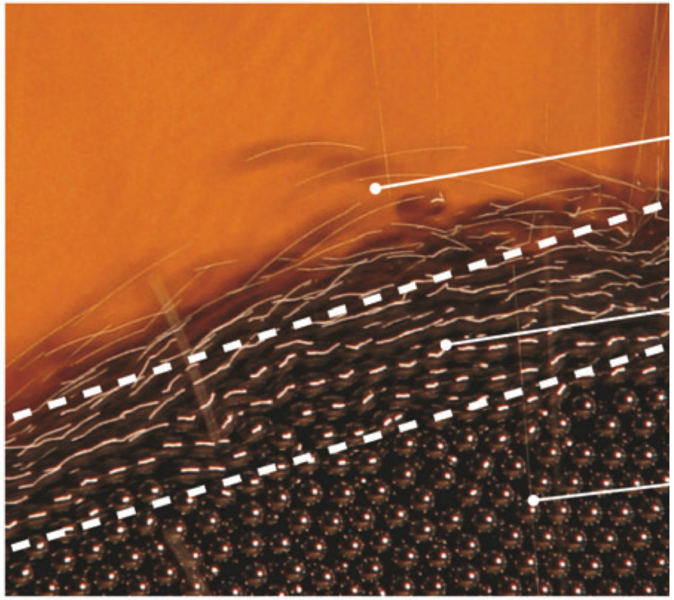
Avarage number of contacts in the system

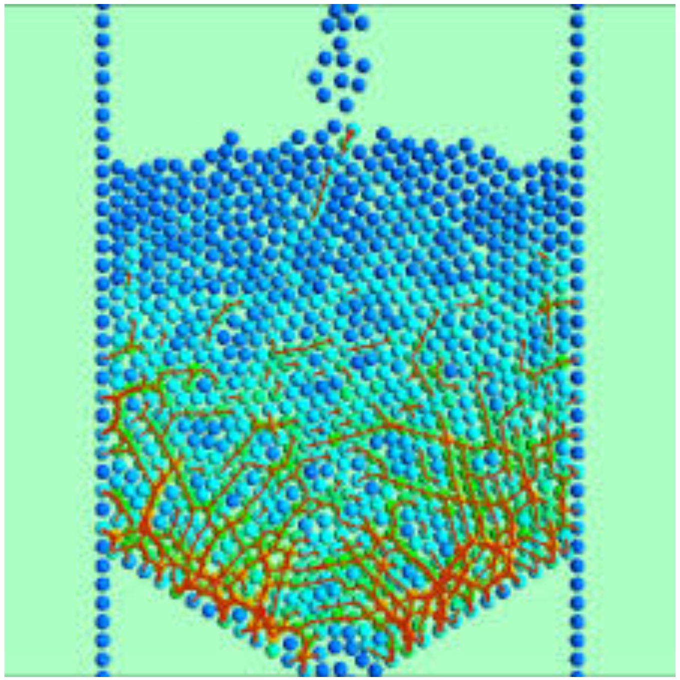
$$\overline{Z} = \frac{2Nc}{Np}$$

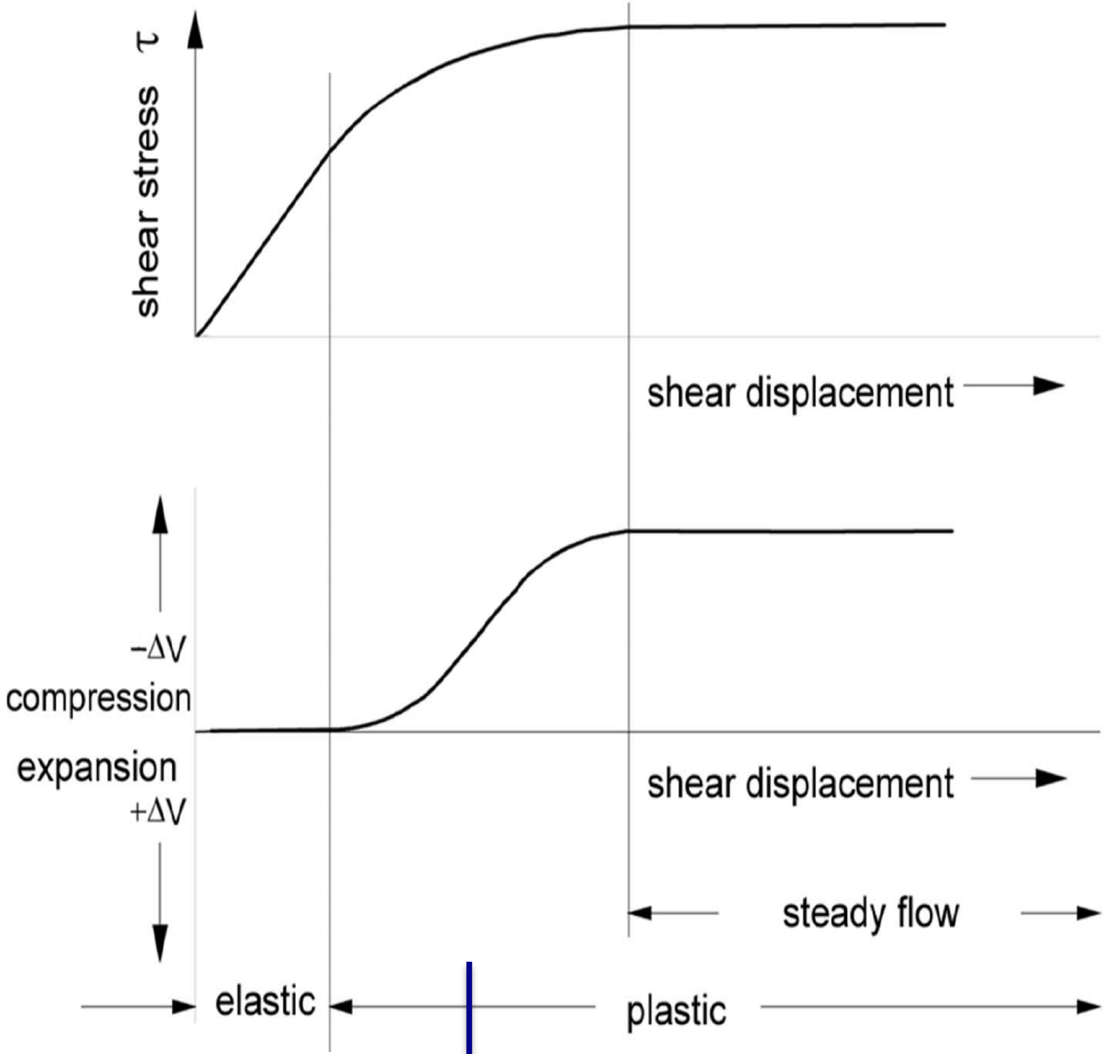
[Magnanimo et al., EPL (2008)]

Quasitatic behavior and flow threshold

Shearing







elasto-plasticity

…

Quasistatic behavior

Coulomb (1773)

Yielding of granular material as frictional process

Interested in prediction of soil failures for Civil Engineering

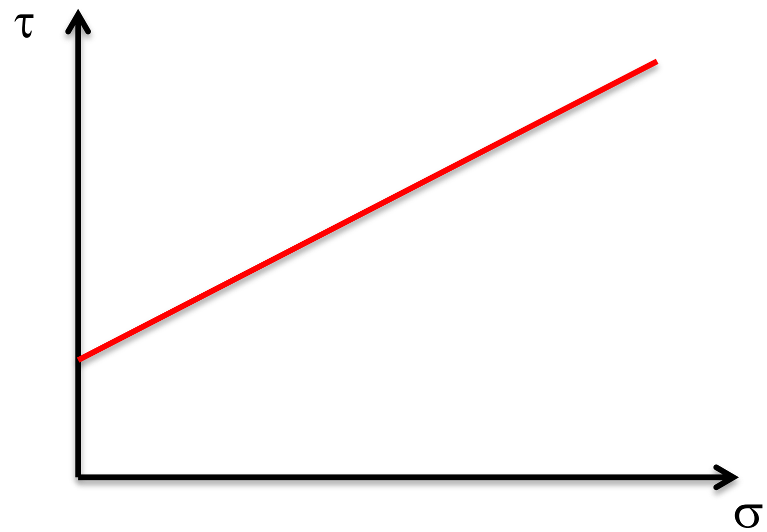
$$\tau < c + \sigma \tan\phi$$

When $\tau = c + \sigma \tan\phi$ the material yields and starts to flow

c = cohesion

ϕ = friction angle

ϕ and c are material **constant**



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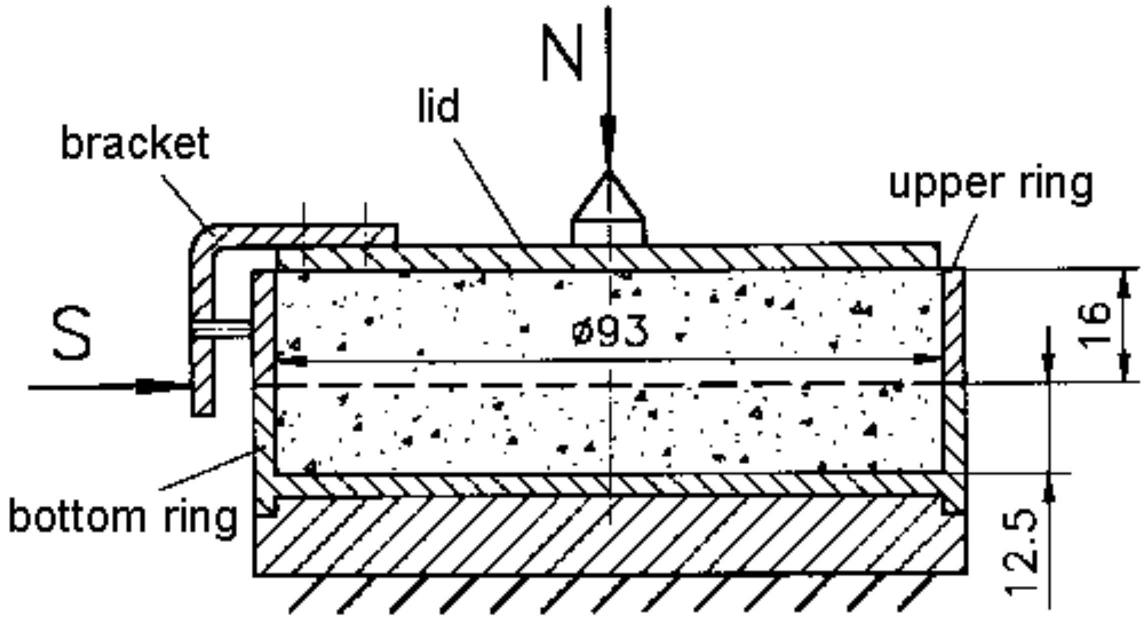
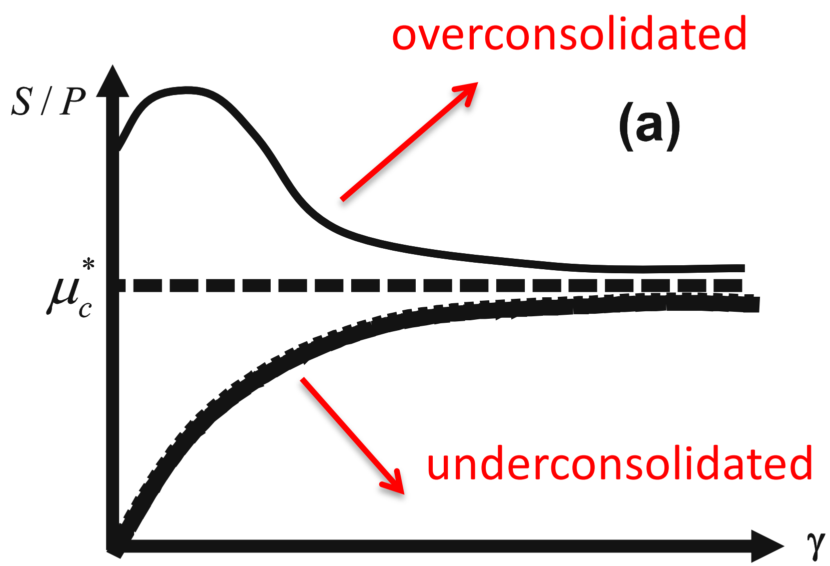
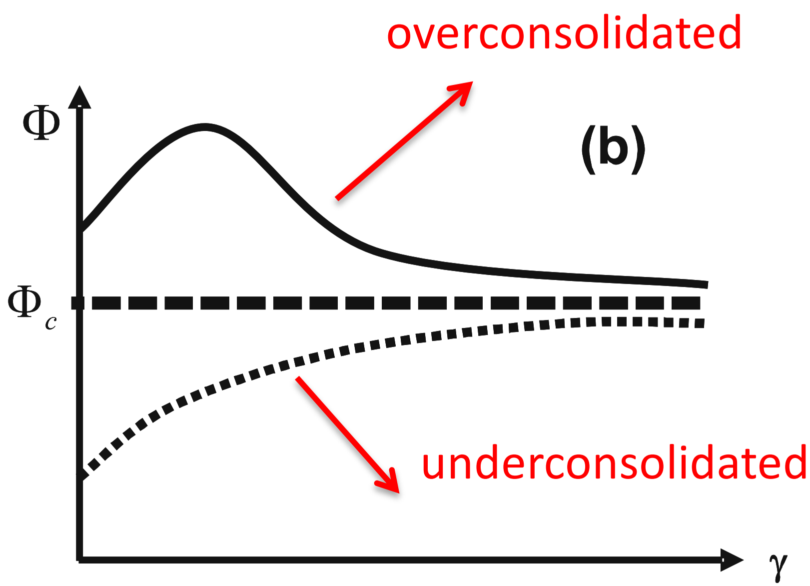


Critical state

A shearing granular material will ALWAYS approach a **critical** concentration
This is the **ONSET OF FLOW**





ϕ_c is again a material **constant**

The granular material **DILATES**

Critical state

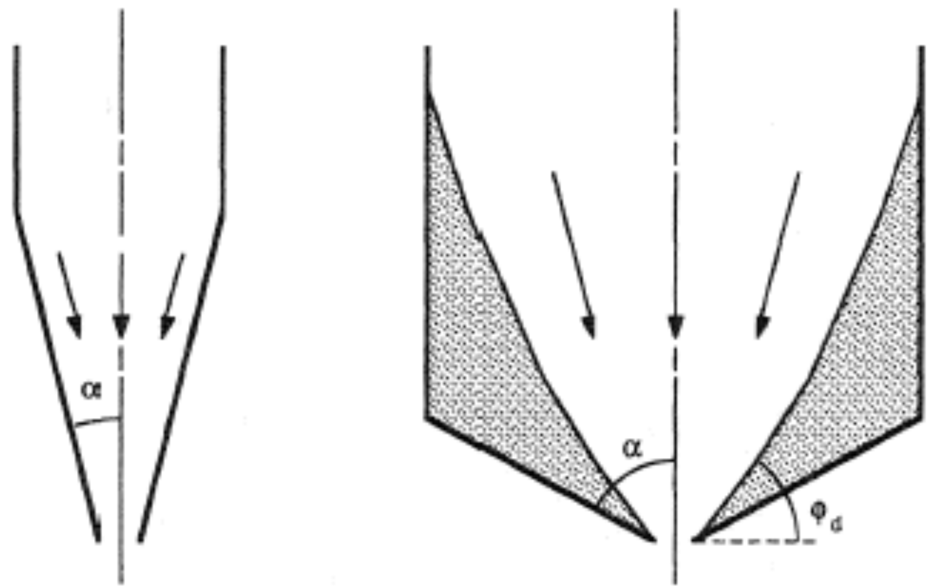
Soil mechanics: widely used

Particle Technology: flow behavior from silo (→ A. Kwade)

- when the material starts flowing is **always** yielding **everywhere** in the hopper (mass flow) or in a region (core flow)

$$\tau = c + \sigma \tan \phi$$

- the material is **always** at the critical concentration and it is **incompressible**.



Critical state

Soil mechanics: widely used

Particle Technology: flow behavior from silo (→ TUBS)

- when the material starts flowing is **always** yielding **everywhere** in the hopper (mass flow) or in a region (core flow)

$$\tau = c + \sigma \tan\phi$$

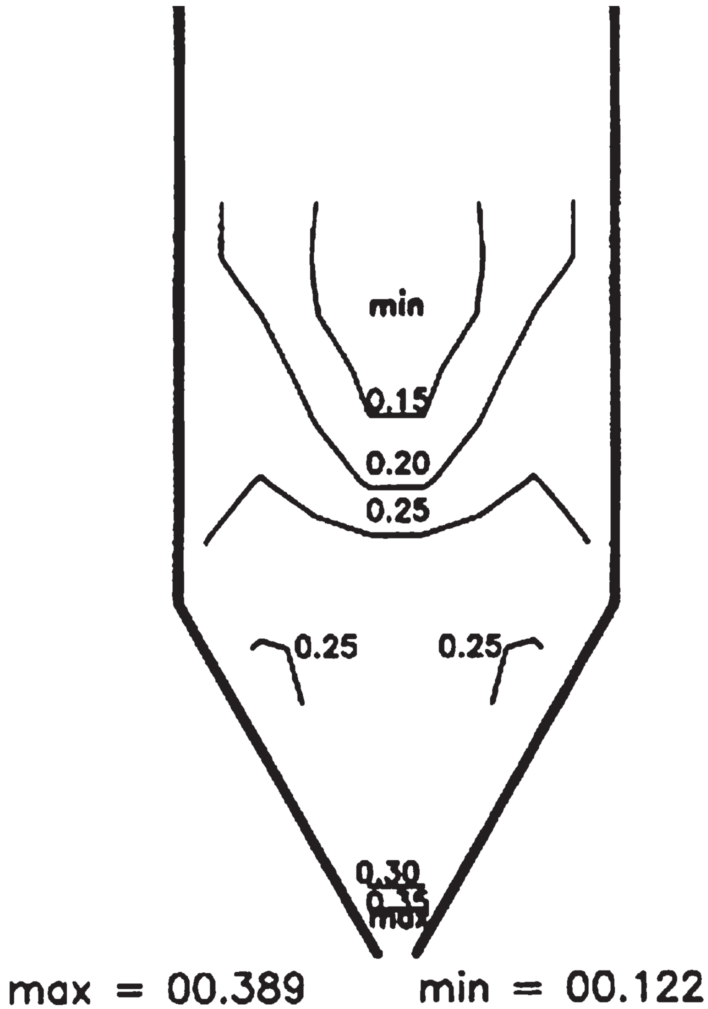
- the material is **always** at the critical concentration and it is **incompressible**.

N.B.!!

Application of Critical State theory in silos is based **on Janssen theory:**
the pressure at bottom of the silo is independent of bed height
→ the whole bulk material is in the critical state.

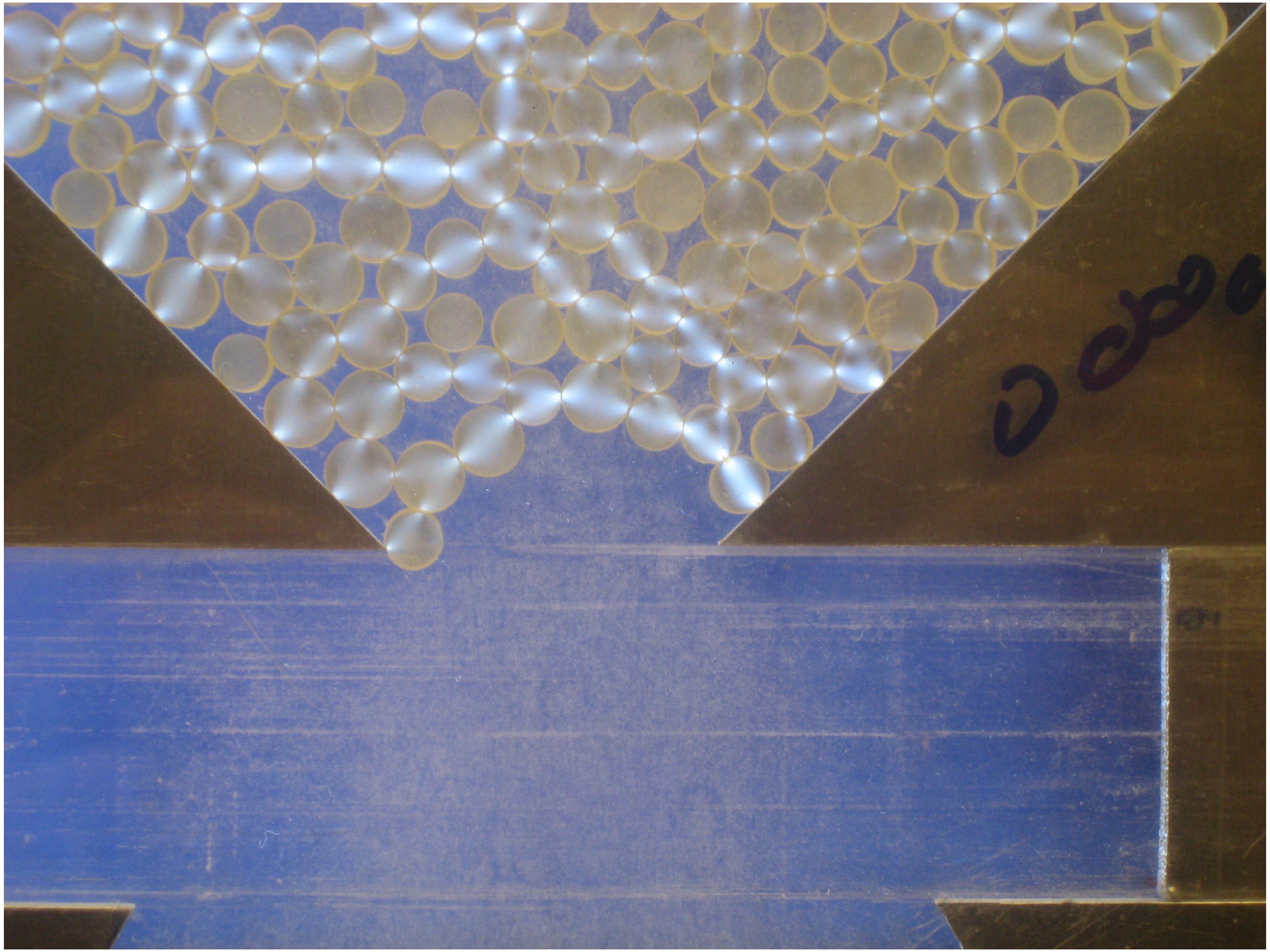
Critical state for silos - problems

ϕ is not constant in the silo



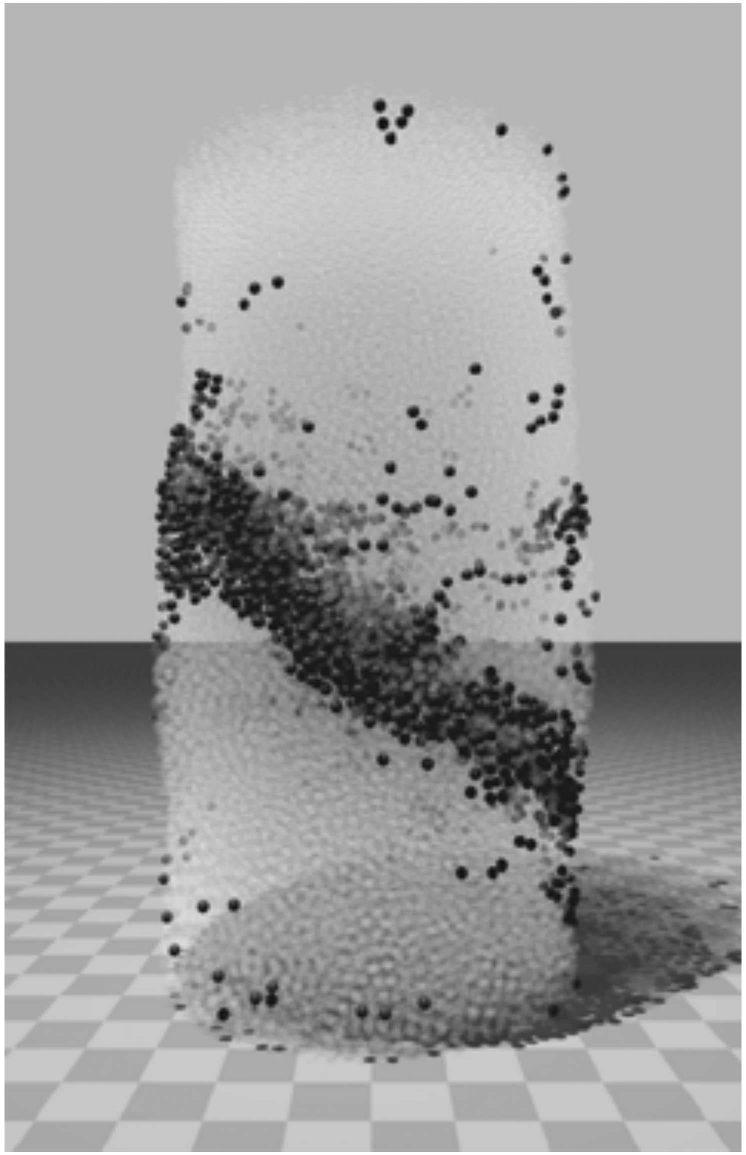
Friction and dilatancy laws

In solid and quasistatic flow, forces are transmitted through **force chains**



Shear bands and dilatant zones

Frictional Behavior = Mohr-Coulomb failure
Granular materials fail along narrow but **finite** zones: **SHEAR BANDS**

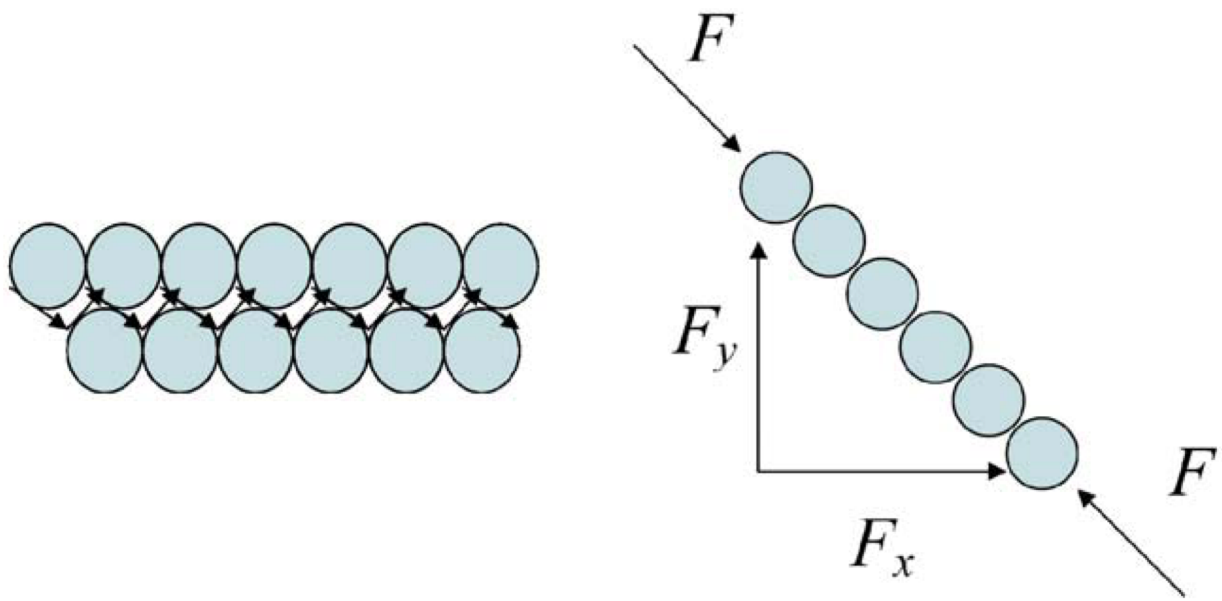


Shear bands and dilatant zones

Frictional Behavior = Mohr-Coulomb failure
Granular materials fail along narrow but **finite** zones: **SHEAR BANDS**

GLOBALLY → frictional behavior
LOCALLY → force chains

$$\frac{\tau_{xy}}{\tau_{yy}} = \frac{\langle F_x l_y \rangle}{\langle F_y l_y \rangle} = const$$



(a) (b)

Collisional or rapid granular flows

Dimensionless analysis (Buckingham Pi theorem)

Bagnold theory

$$\sigma_{ij} = f(\phi, \rho_p, d, \dot{\gamma})$$

$$\sigma_{ij} = f(\phi)\rho_p d^2 \dot{\gamma}^2$$

The shear stress varies as the square of the shear rate

Granular Temperature

1. granules moving in a flow = molecules in the kinetic theory of gases
2. random velocities = thermal motion of molecules.

Granular temperature = magnitude of fluctuating velocities

$$T_g = \frac{1}{3}\left|\left\langle u'^2_i \right\rangle\right| = \frac{1}{3}\left(\left\langle u'^2 \right\rangle + \left\langle v'^2 \right\rangle + \left\langle w'^2 \right\rangle\right)$$

Trace of the streaming stress tensor

$$T_g = \frac{1}{3\rho\phi} tr(\sigma^s_{ij})$$

deriving a set of equations for **Rapid Granular Flows**

Granular Hydrodynamic

Conservation of mass

$$\frac{D\rho\phi}{Dt} = \rho\phi\nabla\cdot\underline{u} = \mathbf{0}$$

Conservation of momentum

$$\rho\phi\frac{D\underline{u}}{Dt} = \nabla p(p,\phi,T_g,e) + \nabla\cdot\left(\eta(\rho,\phi,T_g,e)\nabla\underline{u}\right)$$

Conservation of granular energy (granular temperature)

$$\rho\phi\frac{DT_g}{Dt} = \nabla\cdot\left(\alpha(\rho,\phi,T_g,e)\nabla T_g\right) + \underline{\sigma}\cdot\nabla\underline{u} - \Gamma(\rho,\phi,T_g,e)$$

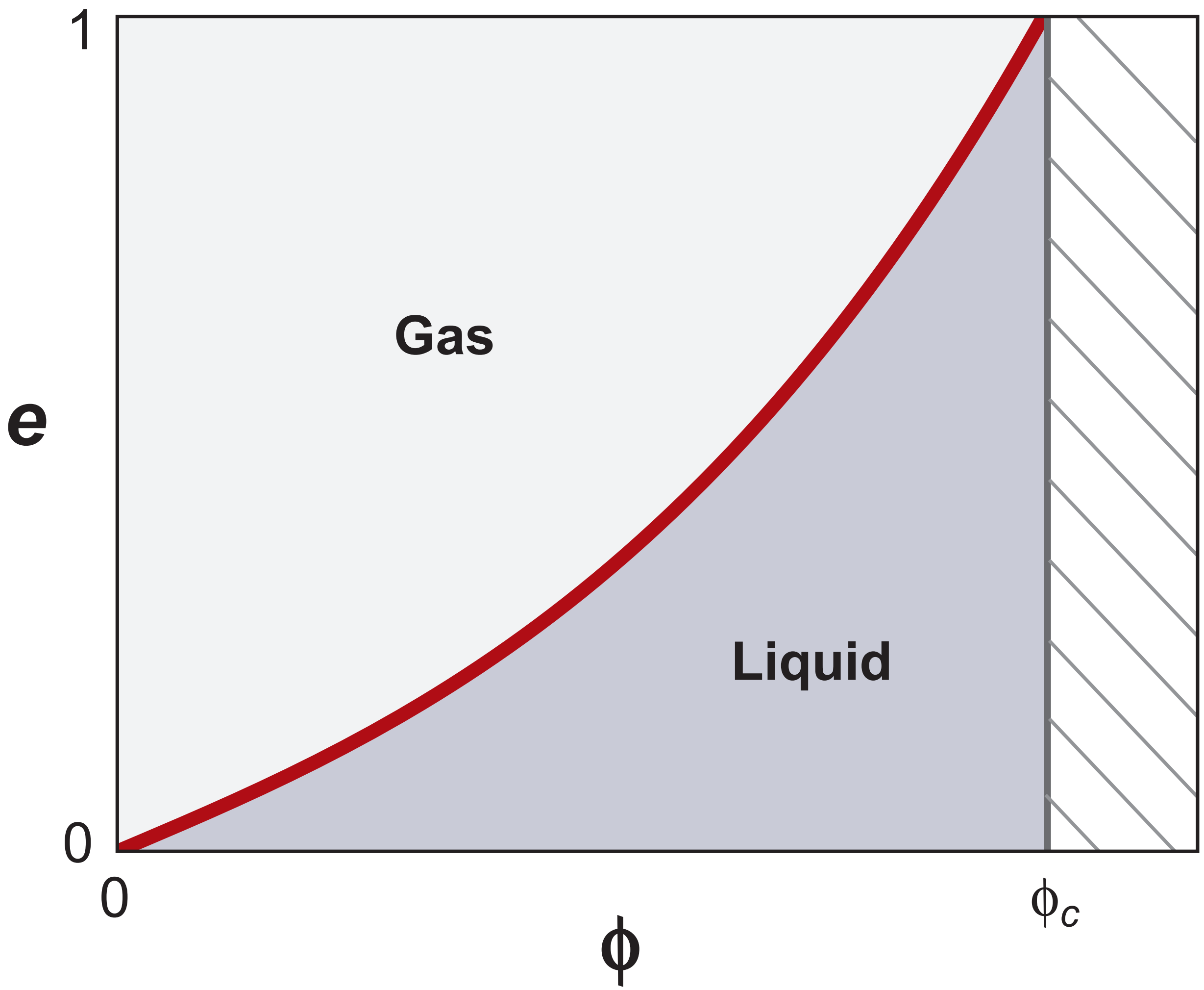
Kinetic Theory – Range of applicability

- **Nearly elastic particles** (e=0.9)
- **Extremely small concentration:**
  magnitude of thermal velocities is much larger than the relative velocities induced by shear

} **Binary collisions**

- **Isotropy in the angular distribution of collisions**
- **Molecular chaos:**
  no correlations in the velocities or positions of colliding particles
- **Absence of friction between particles and walls:**
  silos can not be modeled with kinetic theory

Kinetic Theory – Range of applicability



[Forterre and Pouliquen, Ann. Rev. Fluid Mech. (2008)]

Kinetic Theory – Extended theories

Jenkins, *Dense shearing flows of inelastic disks.* Phys.Fluids (2006)

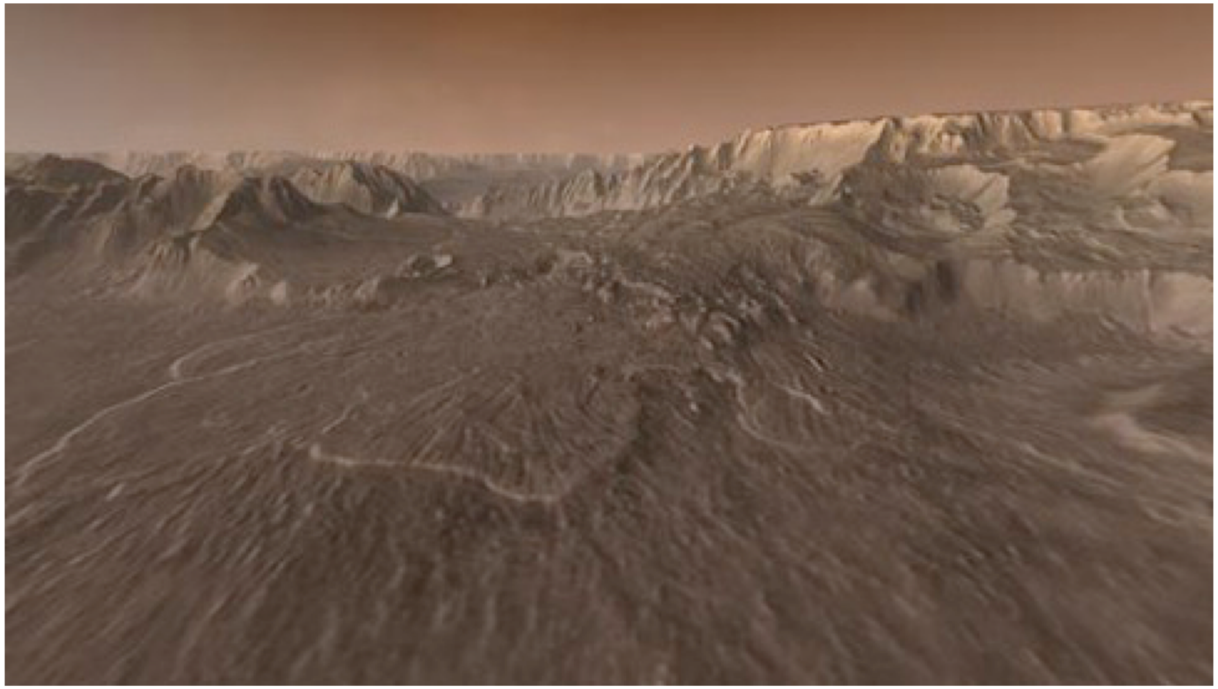
Vescovi, Di Prisco & Berzi, *From solid to granular gases: the steady state for granular materials* (2013)

[…]

Granular material: continuum approach

Solid: soil mechanics

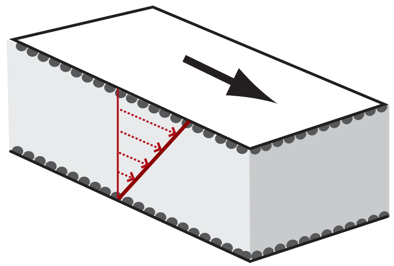
Gas: kinetic theory



Liquid ??

Dense (slow) flows
and inertial regime

Inertia number



For **large** systems – and **rigid** grains

Only based on dimensional analysis
The transition can be described trough a **single dimensionless number**

$$I = \frac{\dot{\gamma} d}{\sqrt{p / \rho_p}}$$

Inertia number

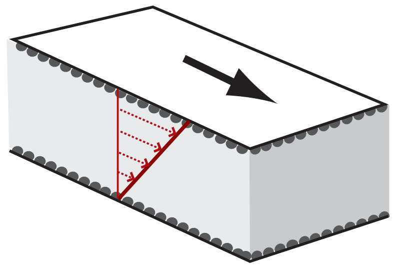
quasi-static (solid) dense (liquid) rapid (gas)

$\longrightarrow I$

10^{-3} 10^{-1}

[Da Cruz et al. (2005); Forterre and Pouliquen, Ann. Rev. Fluid Mech. (2008)]

Inertia number



$$I = \frac{\dot{\gamma} d}{\sqrt{P/\rho_p}}$$

I = micro time scale / macro time scale

$$\frac{d}{\sqrt{P/\rho_p}}$$

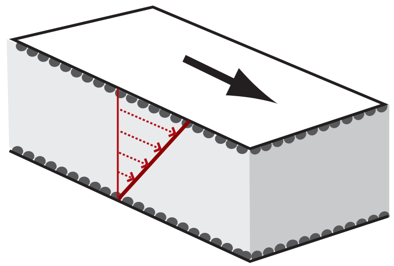
microscopic time scale
time needed for a particle to fall in a hole of size *d* under the pressure *P*
- typical time scale of rearrangements -

$$\frac{1}{\dot{\gamma}}$$

macroscopic time scale
linked to the mean deformation

[Da Cruz et al. (2005); Forterre and Pouliquen, Ann. Rev. Fluid Mech. (2008)]

Inertia number

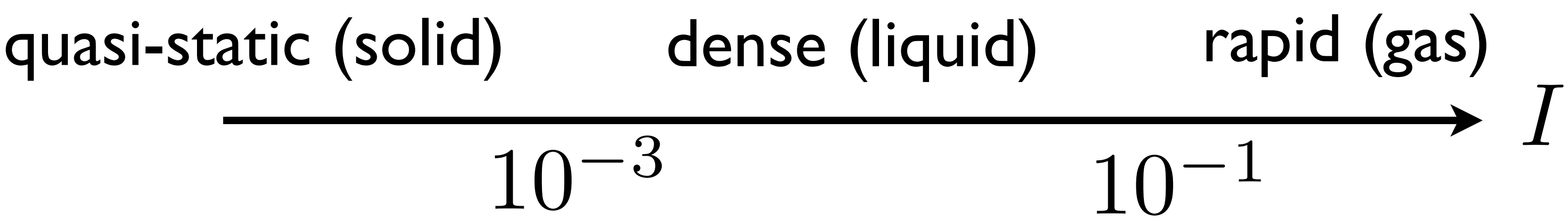


$$I = \frac{\dot{\gamma} d}{\sqrt{p / \rho_p}}$$

I = micro time scale / macro time scale

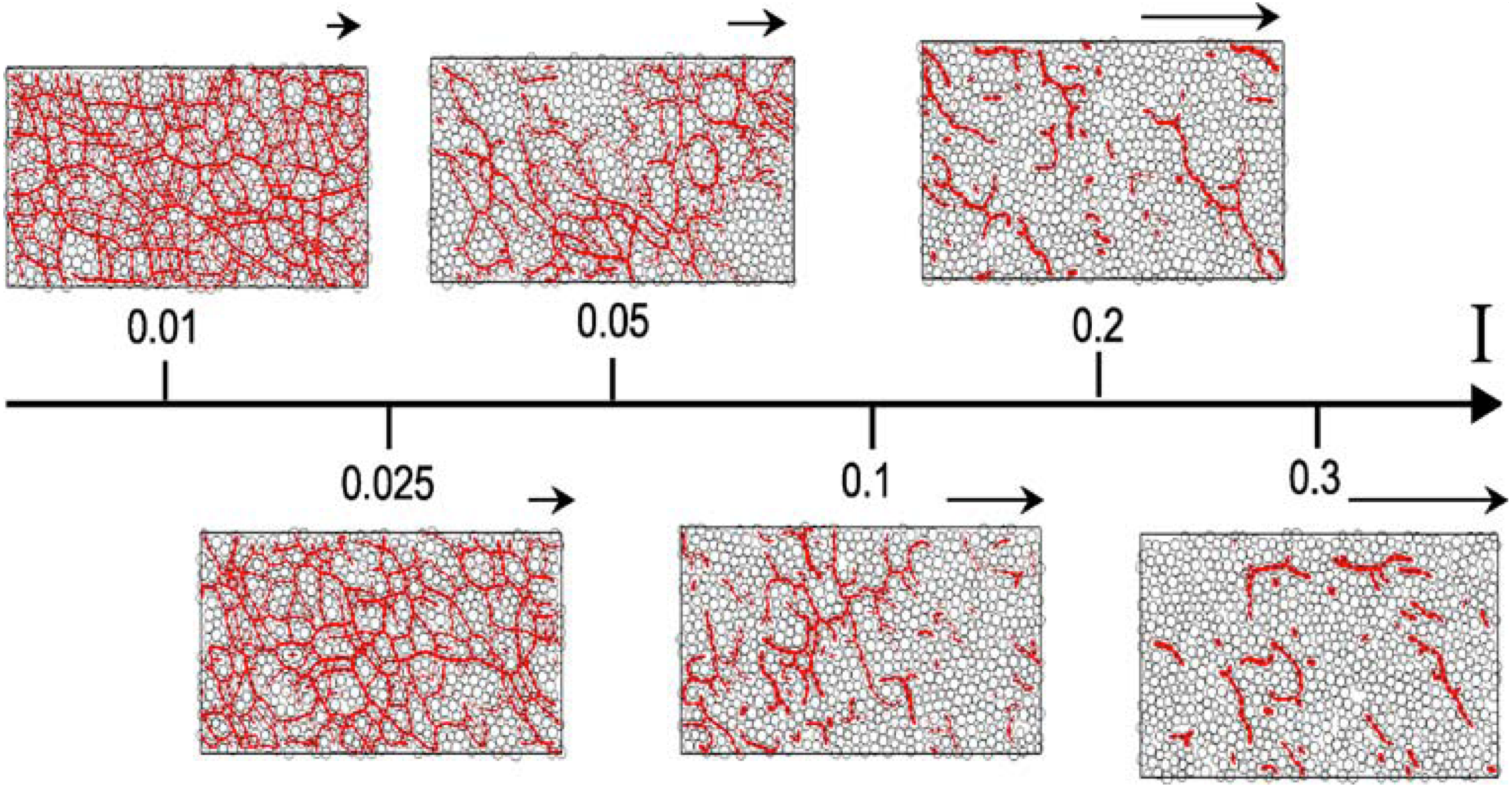
I **small –** quasi-static
macroscopic deformation is slow compared to microscopic rearrangement

I **large –** rapid flows



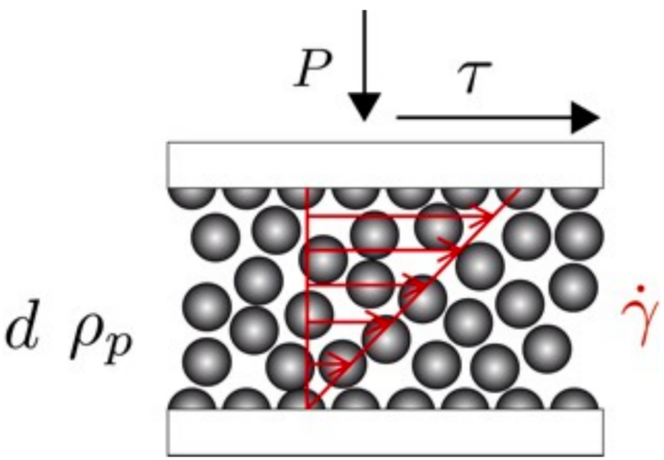
[Da Cruz et al. (2005); Forterre and Pouliquen, Ann. Rev. Fluid Mech. (2008)]

Quasistatic → Dense → Rapid



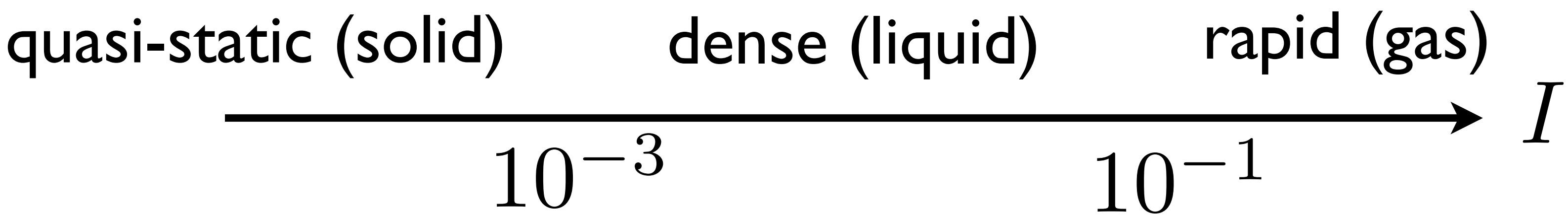
Granular flow model

Rheology of dense flows



Bulk friction
$$\mu(I) = \mu_0 + \frac{(\mu_2 - \mu_0)}{1 + I_0/I}$$

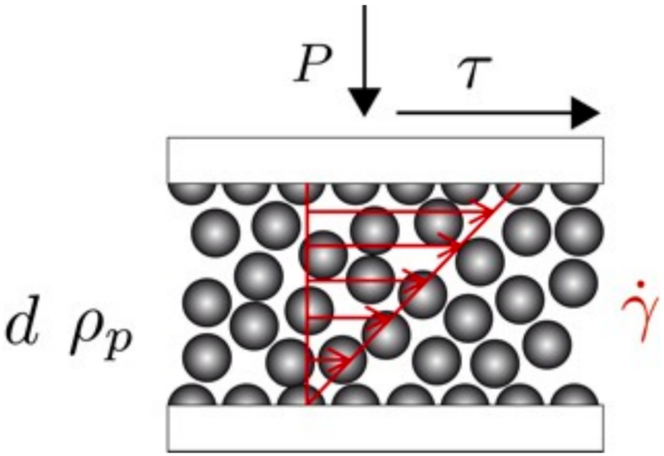
Dilatancy
$$\phi(I) = \phi_0\left(1 - \frac{I}{I_\phi}\right)$$



[MiDi, (2004). On dense granular flows. *The European Physical Journal E*, *14*(4), 341-365]
[Forterre Y, Pouliquen O. Flows of dense granular media. *Annu. Rev. Fluid Mech*., 2008, 40: 1-24]

Granular flow model

Rheology of dense flows



Bulk friction

$$\mu(I) = \mu_0 + \frac{(\mu_2 - \mu_0)}{1 + I_0/I}$$

Dilatancy

$$\phi(I) = \phi_0\left(1 - \frac{I}{I_\phi}\right)$$

Inertial number

$$I = \frac{\dot{\gamma} d}{\sqrt{p/\rho}}$$

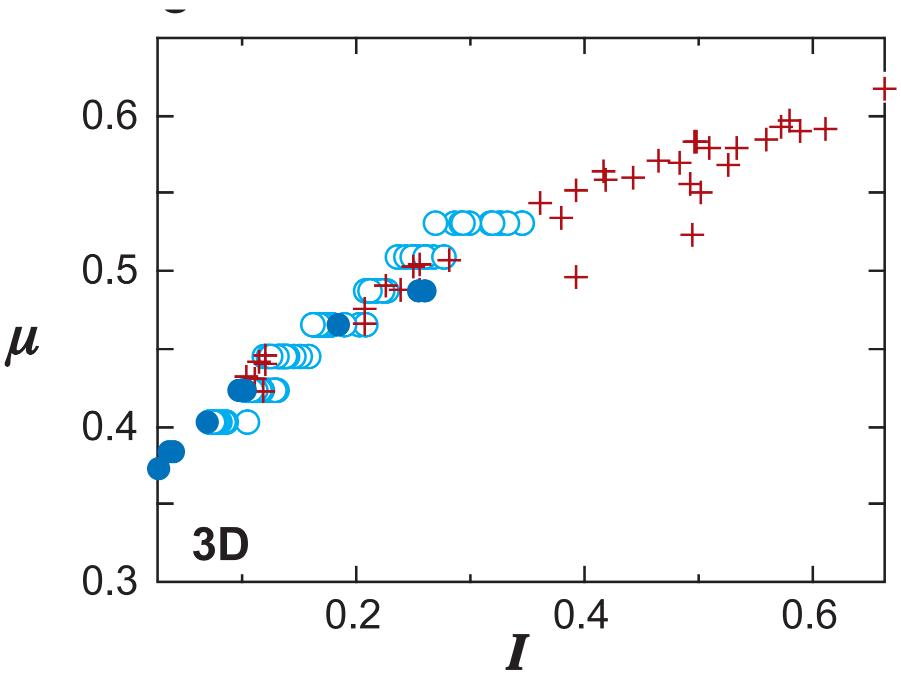
$\mu(I)$ Rheology:

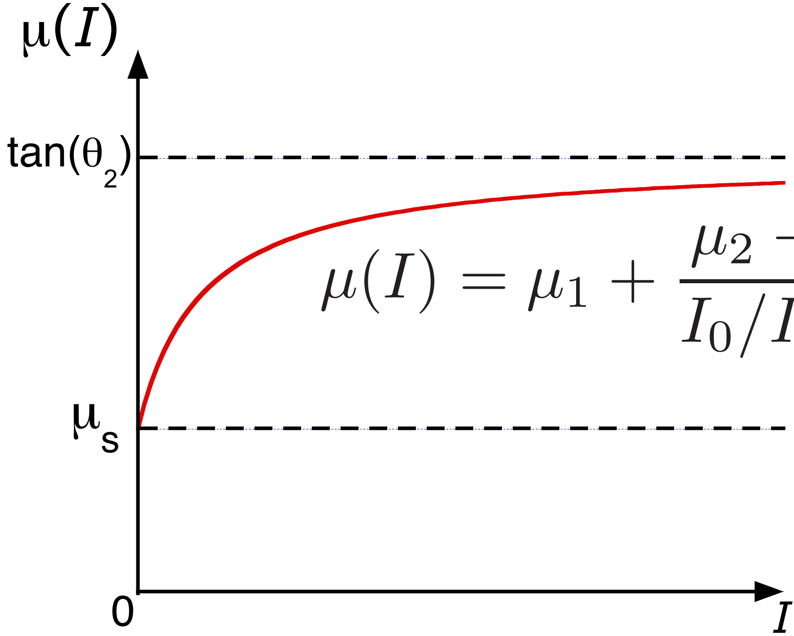
rigid
frictionless
cohesionless

[MiDi, (2004). On dense granular flows. *The European Physical Journal E*, *14*(4), 341-365]
[Forterre Y, Pouliquen O. Flows of dense granular media. *Annu. Rev. Fluid Mech*., 2008, 40: 1-24]

Pouliquen μ-I rheology – local constitutive relation

$\tau = \sigma\mu(I)$ $\phi = \phi(I)$ **(rigid** grains)





$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$

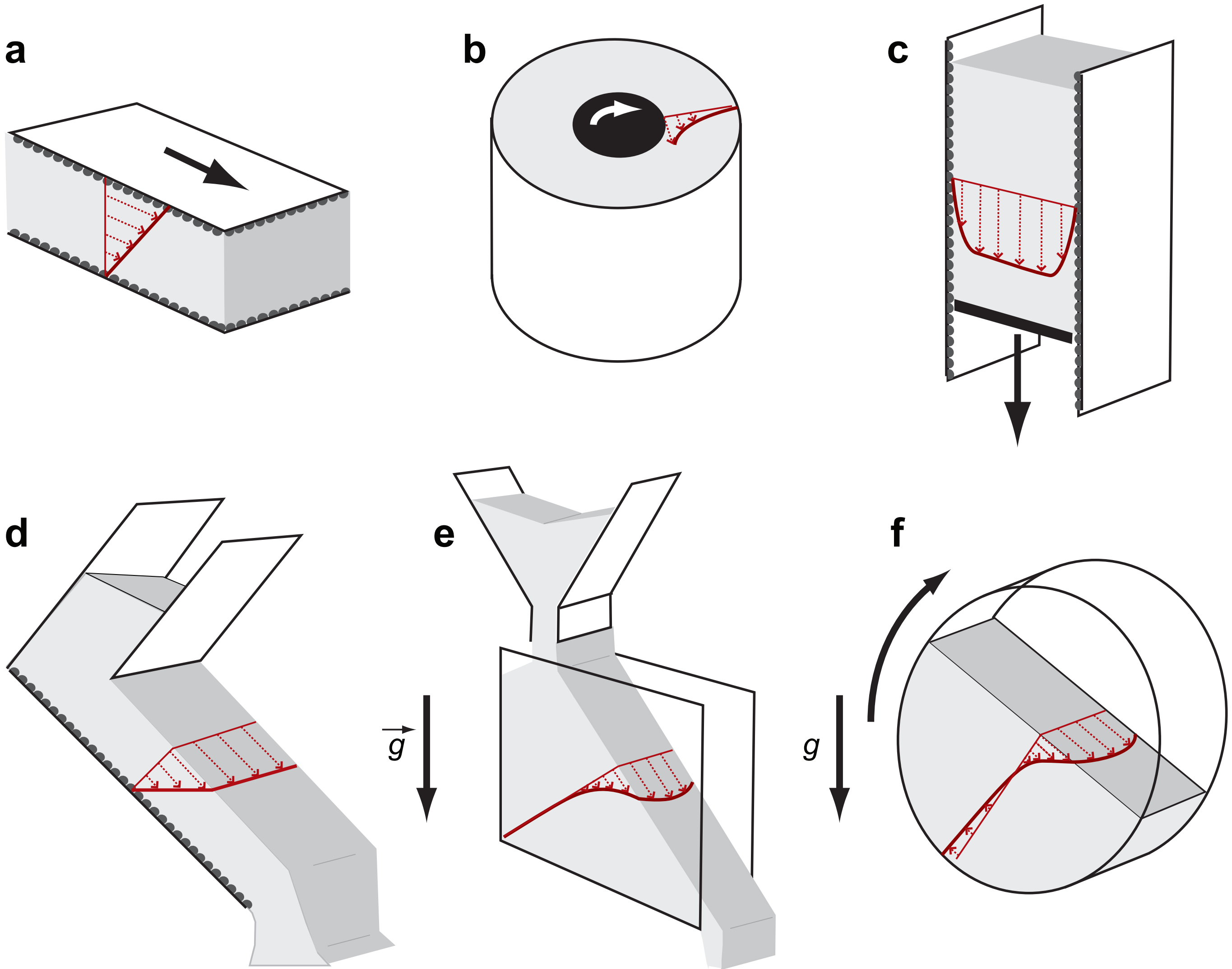
- inclined plane (exp, num)
- annular shear (exp)

glass beads : $\mu_1 = \tan 21^\circ$

$\mu_2 = \tan 33^\circ$ $I_0 = 0.3$

[Forterre and Pouliquen, Ann. Rev. Fluid Mech. (2008)]

Different geometries



[Forterre and Pouliquen, Ann. Rev. Fluid Mech. (2008), Weinhart et al. Phy. Fluids (2013)]

Additional characteristic length-scales

Micro-mechanical time scales

$$\tau_s = 1/\dot{\gamma} \qquad \tau_c = \sqrt{\bar{m}/k_n}$$

$$\tau_g = \sqrt{\bar{d}/g} \qquad \tau_P = \sqrt{\bar{m}/P\bar{d}}$$

$$\tau_{Bo} = \sqrt{\frac{\bar{m}\bar{d}}{f_{a,max}}}$$

Dimensionless numbers

Inertial number $I = \tau_P/\tau_s$

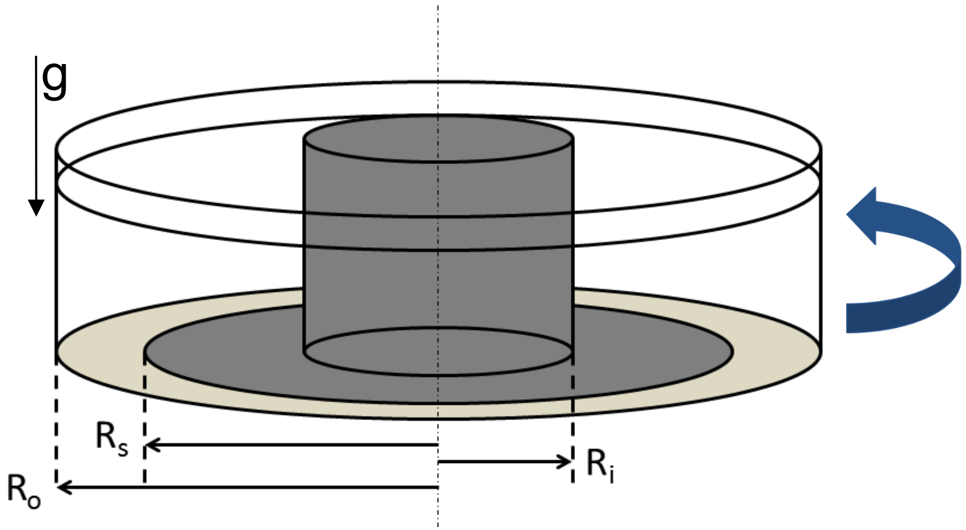
"Softness" $P^* = \left(\tau_c/\tau_P\right)^2$

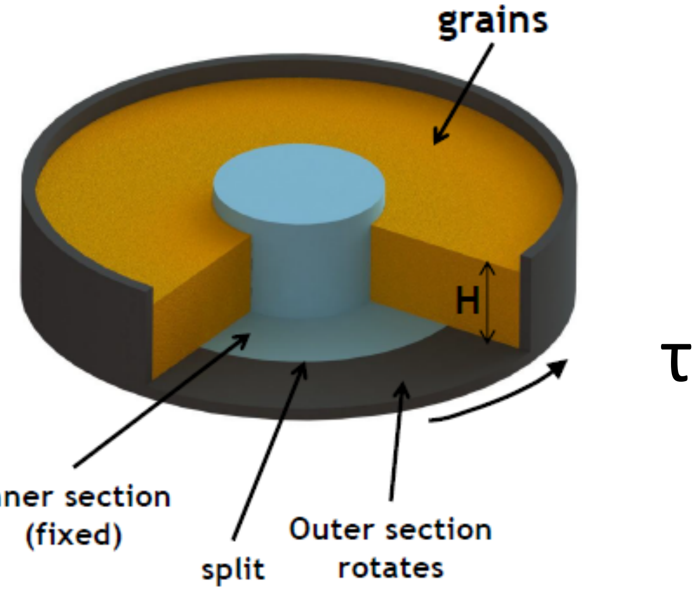
Bond number $Bo = \dfrac{f_{a,max}}{pd_p^2} = (\tau_P/\tau_{Bo})^2$

[MiDi, (2004). On dense granular flows. *The European Physical Journal E*, *14*(4), 341-365]
[A. Singh et al., (2015) NJP]

Influence of Softness

Split bottom shear cell





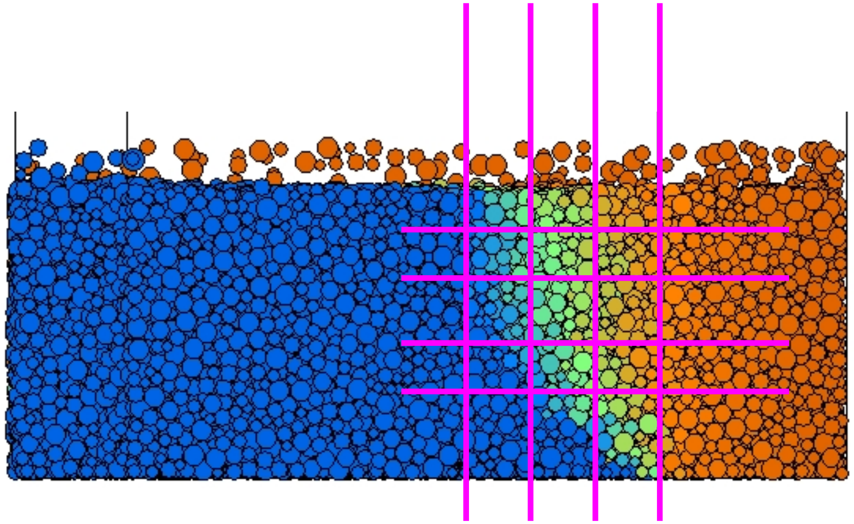
Wide and stable shear band
No side wall effect
Pressure induced by gravity

[Fenistein D, van Hecke M, *Nature*, 2003, 425(6955): 256-256]

Split bottom shear cell

Shear Band (steady/critical state) local mic-mac averaging =>

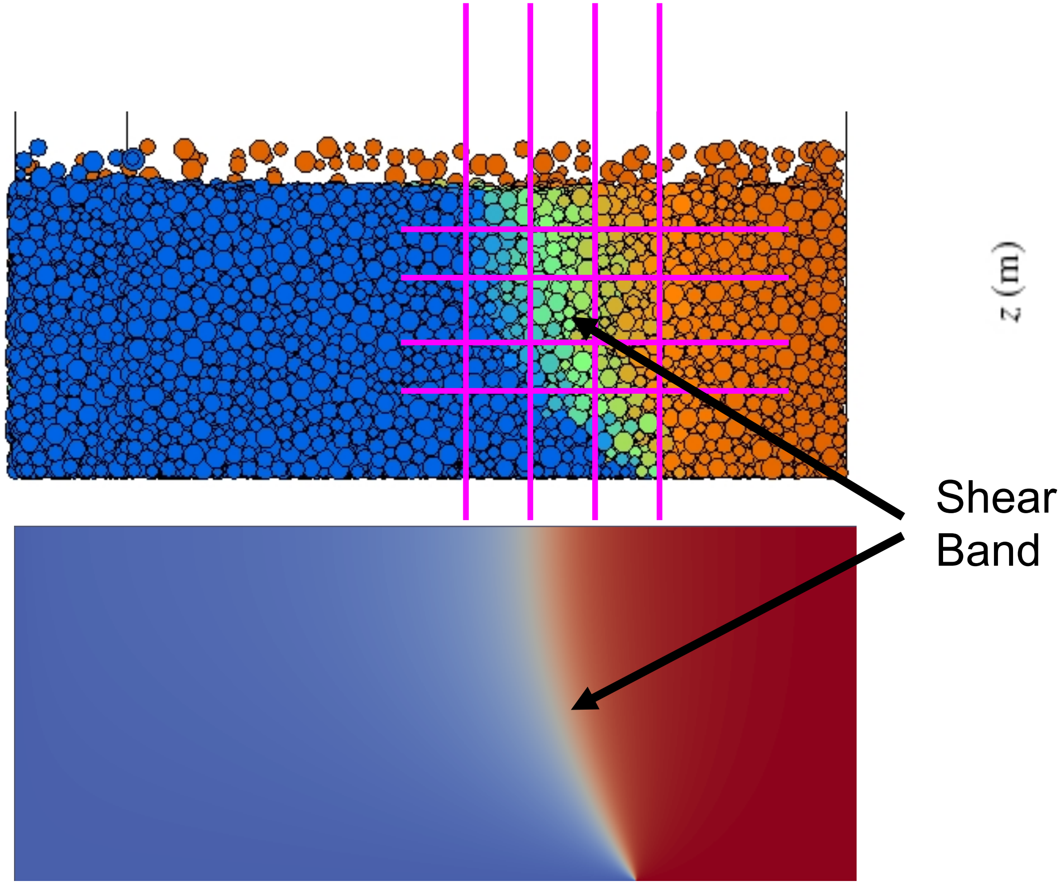
Constitutive relations

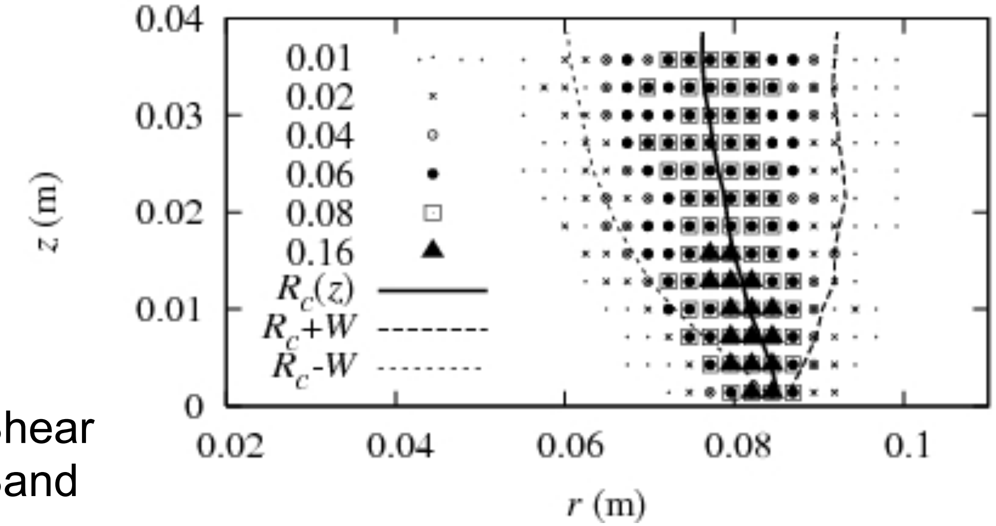


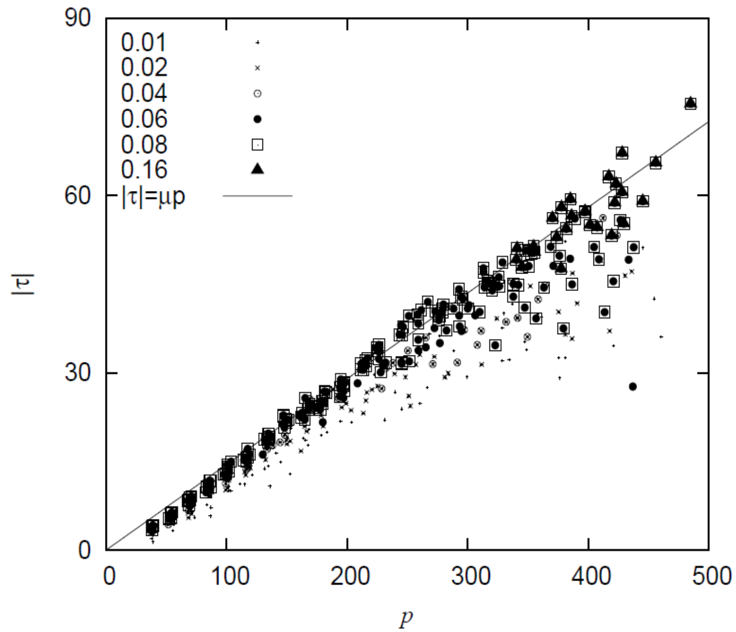
Split bottom shear cell

Shear Band (steady/critical state) local mic-mac averaging =>

Constitutive relations



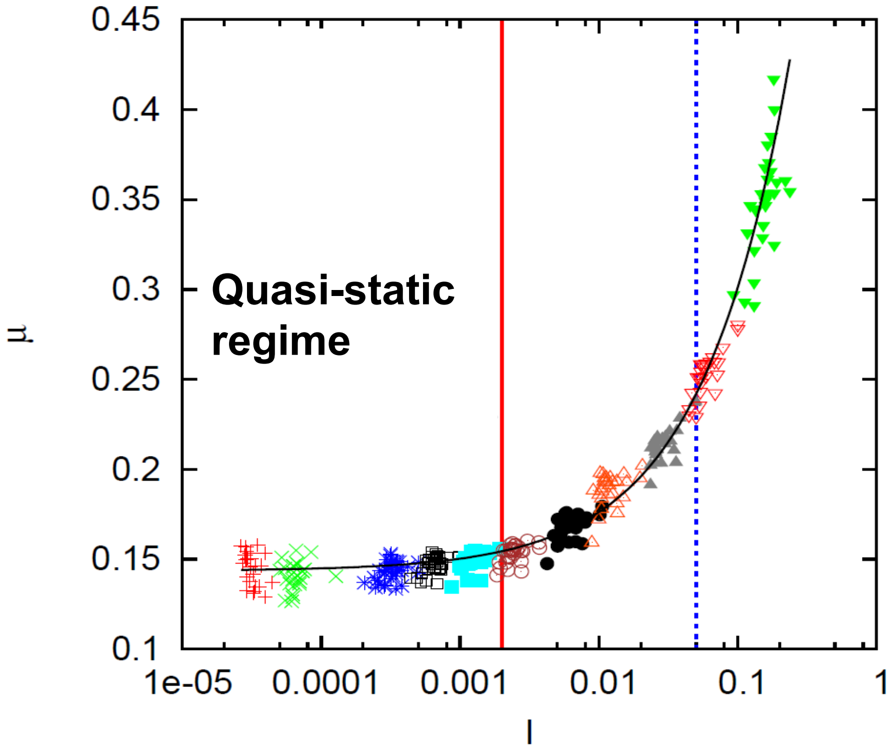




Local quantities

shear rate	$\dot{\gamma}$
shear stress	τ
pressure	P

Rigid particles – effect of strain-rate



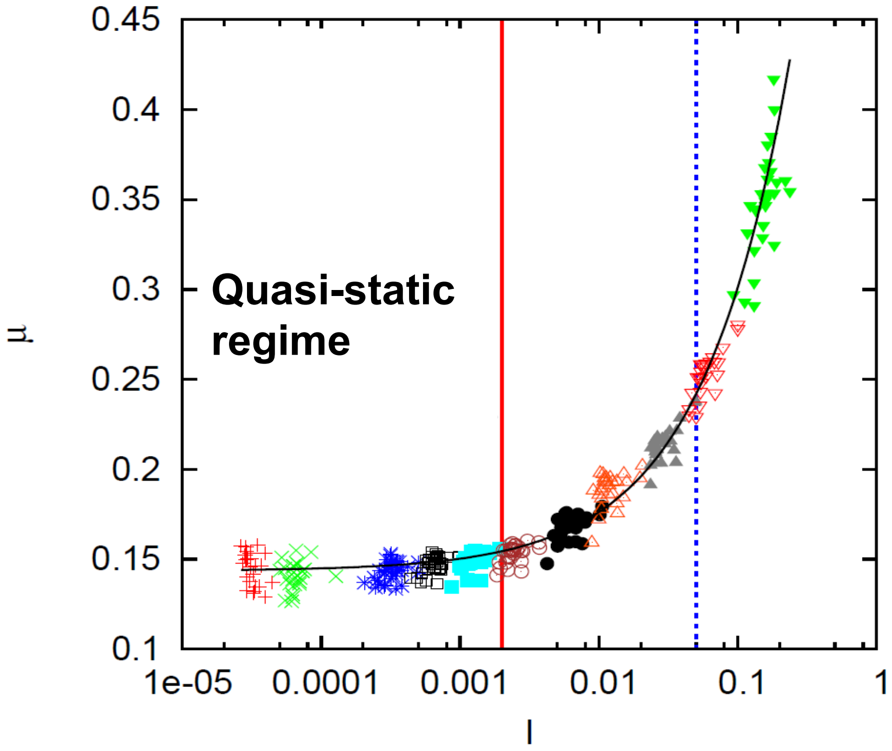
Macro-Friction coefficient

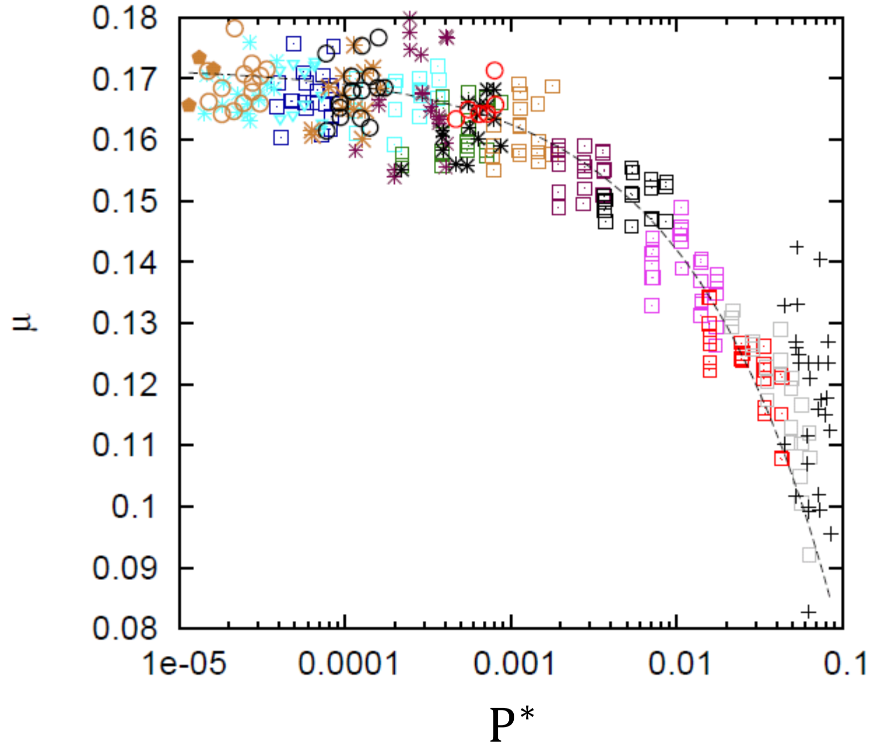
$$\mu(I) = \mu_0 + aI^{\alpha} \qquad \alpha \approx 1$$

In rigid quasi-static limit $\mu(I) \approx \mu_0$

Dependence on stiffness and gravity

$$P^* \equiv \frac{P\bar{d}}{k_n}$$





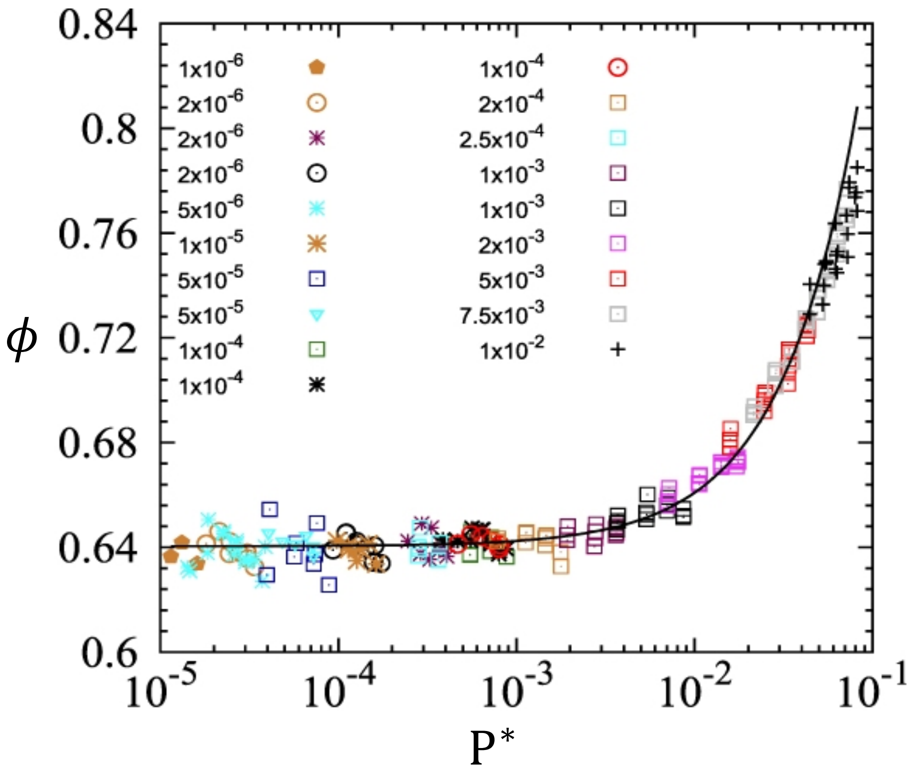
Macro-Friction coefficient

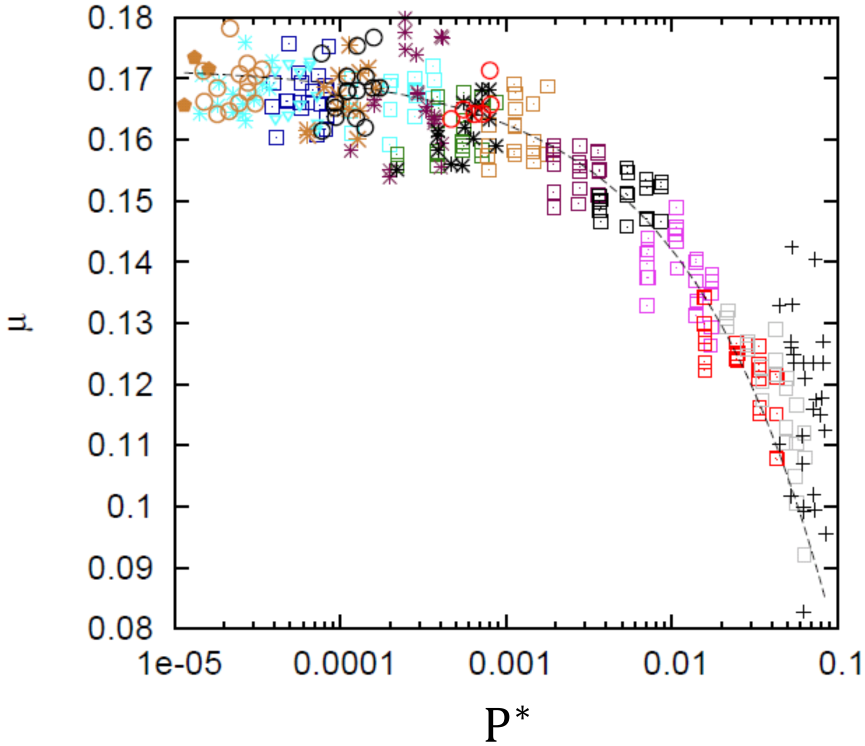
$$\mu(I) = \mu_0 + aI^{\alpha} \qquad \alpha \approx 1$$

In **soft** quasi-static limit $\mu(I) \approx \mu_0 \longrightarrow \mu(P^*) = \mu_0 - bP^{*\beta}$ $\beta \approx 0.50$

Dependence on stiffness and gravity

$$P^* \equiv \frac{P\bar{d}}{k_n}$$





Macro-Friction coefficient

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Inertial and soft flow-rheology

Dependence on **softness** in **inertial** flow states

$$\mu(I, P^*) = \mu_0 + aI^{\alpha} - bP^{*0.5}$$

$$\phi(I, P^*) = \phi_0 - a_\phi I + b_\phi P^*$$

Can't we do better? Yes: we make it multiplicative!

$$\mu(I, P^*) = \mu_0 f_I f_p = \mu_0 \left(1 + \frac{\mu(I)}{\mu_0}\right)\left(1 - \left(\frac{P^*}{P^*_\sigma}\right)^{0.5}\right)$$

$$\phi(I, P^*) = \phi_0 g_I g_p = \phi_0 \left(1 - \frac{I}{I_\phi}\right)\left(1 + \frac{P^*}{P^*_\phi}\right)$$

[A. Singh et al., NJP (2015) => S. Roy et al. NJP (2017)]

Let's add Cohesion (and Friction)

Contact model: liquid bridge

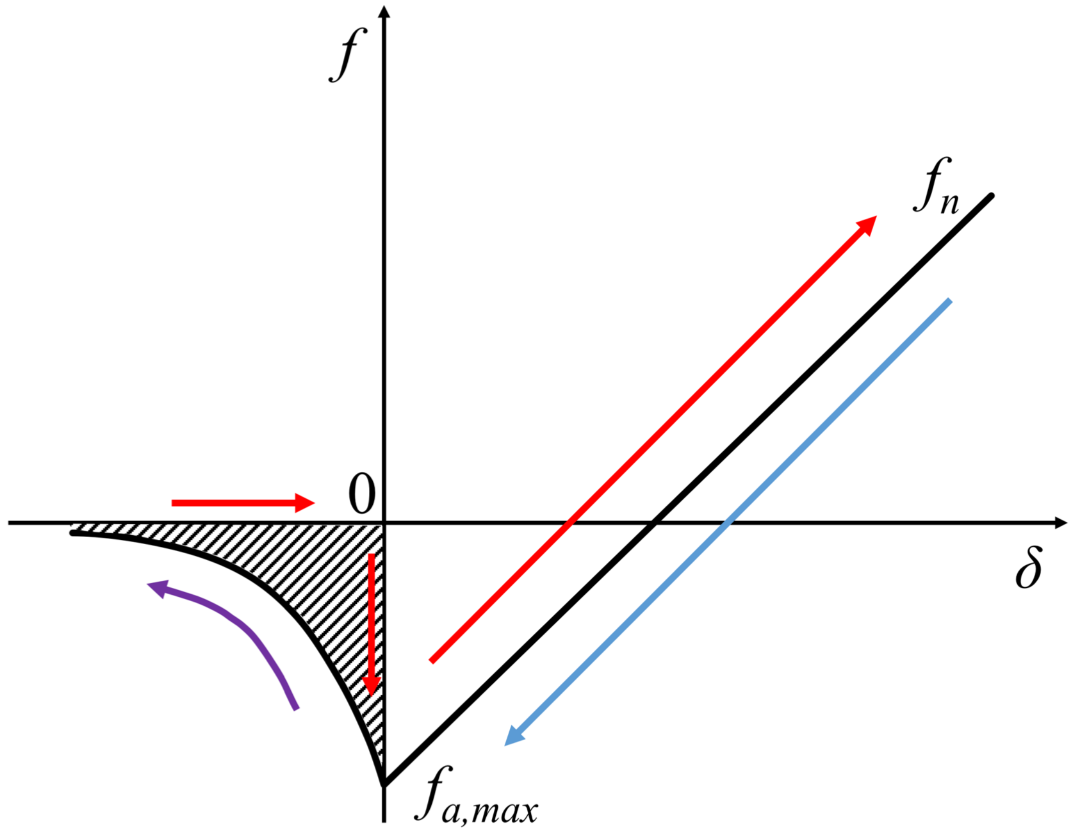
$$Bo = \frac{f_{a,max}}{p d_p^2}$$

Linear visco-elastic frictional with jump-in adhesive contact model

Loading - Particles are approaching each other.

Unloading – Particles are detaching.

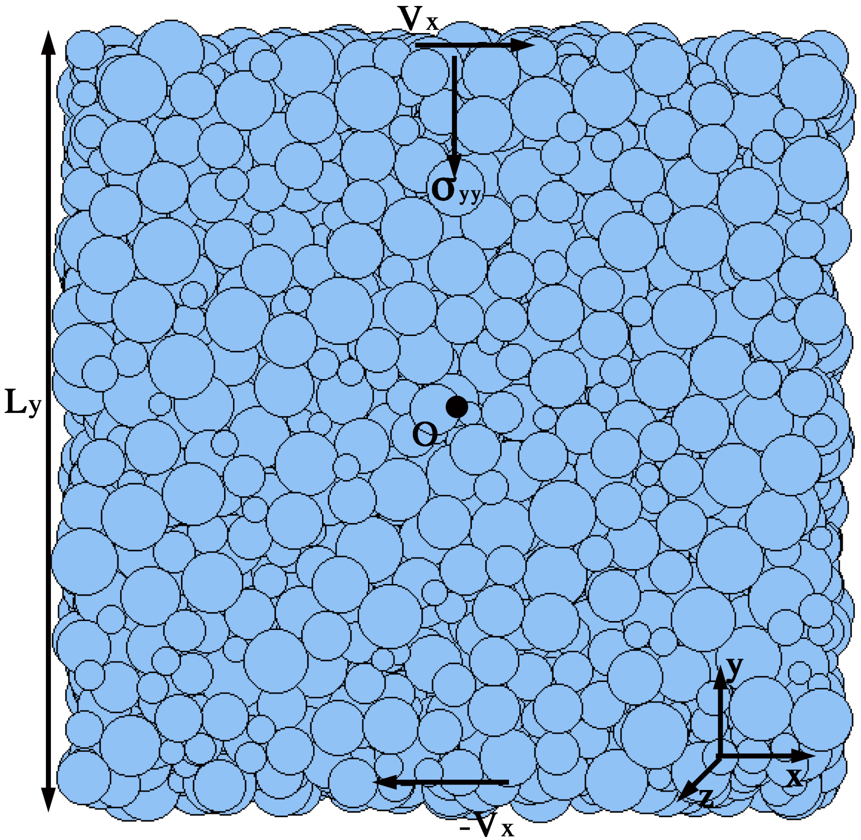
Tensile branch – Attractive forces are influencing detachment.



[Willett et al., Langmuir 16, 9396-9405 (2000)]
[S. Luding, Gran. Matter, 10(4), 235 (2008)]

Simple Shear REV

N = 4096 ν = 0.5-0.82
e = 0.802 $w = r_{max}/r_{min} = 2$,
μ_p = 0 to 1 and Bo: 0 to 5
No gravity



Symbols	Value	Scaled units	SI-unit units
t_u	1	µs	s
x_u	1	mm	m
m_u	1	µg	kg
ρ	2000	2000 (µg·mm^{-3})	2000 (kg·m^{-3})
d_{mean}	2.2	2.2 (mm)	0.0022 (m)
k_1	10^5	10^5 (µg·µs^{-2})	10^8 (kg·s^{-2})
k_2	k_1		
k_c	0		
P	1	1 (µg·mm^{-1}·µs^2)	10^8 (kg·m^{-1}·s^{-2})

Contact model: cohesive particles

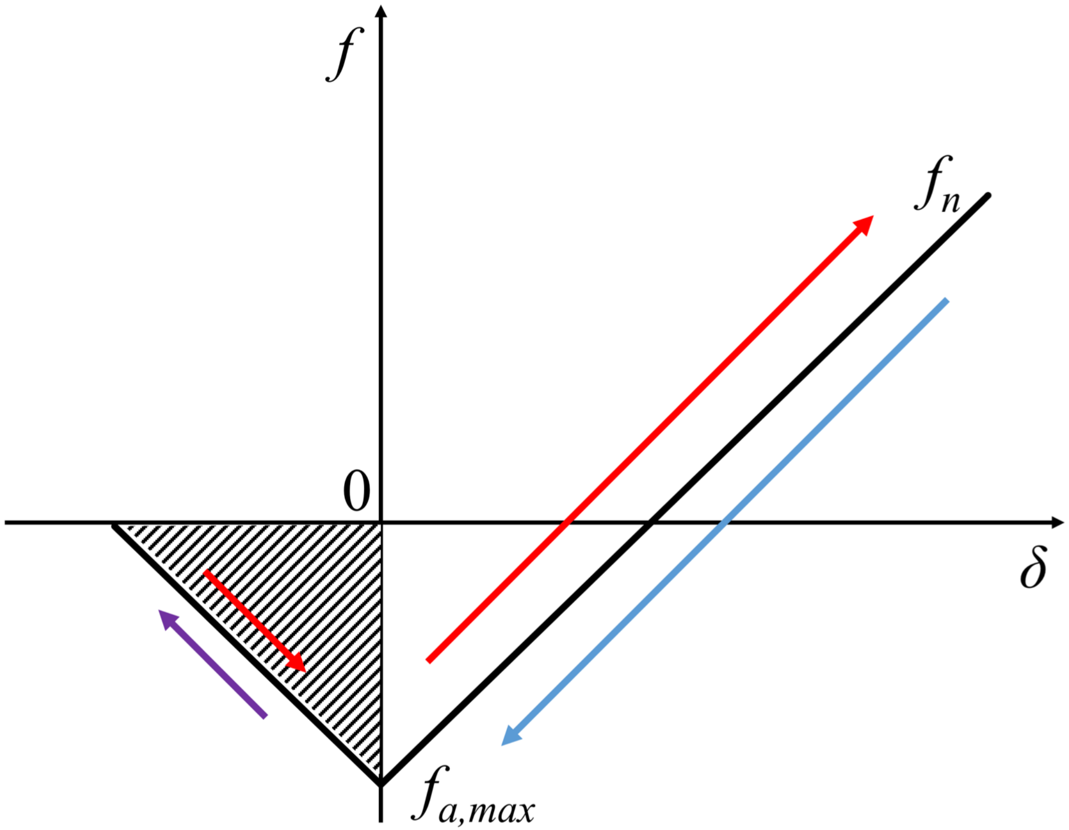
$$Bo = \frac{f_{a,max}}{p d_p^2}$$

Reversible linear visco-elastic frictional contact model

Loading - Particles are approaching each other.

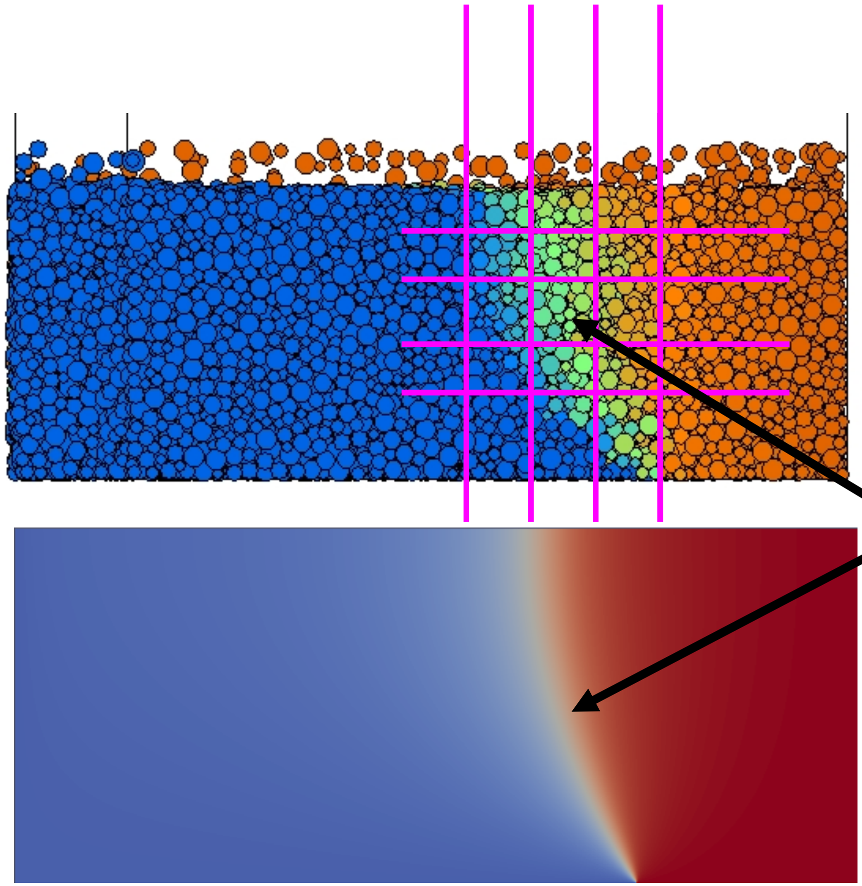
Unloading – Particles are detaching.

Tensile branch – Attractive forces are influencing detachment.

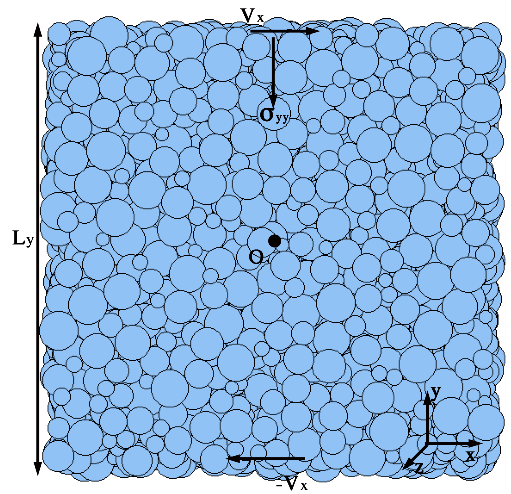


[S. Luding, Gran. Matter, 10(4), 235 (2008)]

Split bottom shear cell – Simple Shear REV

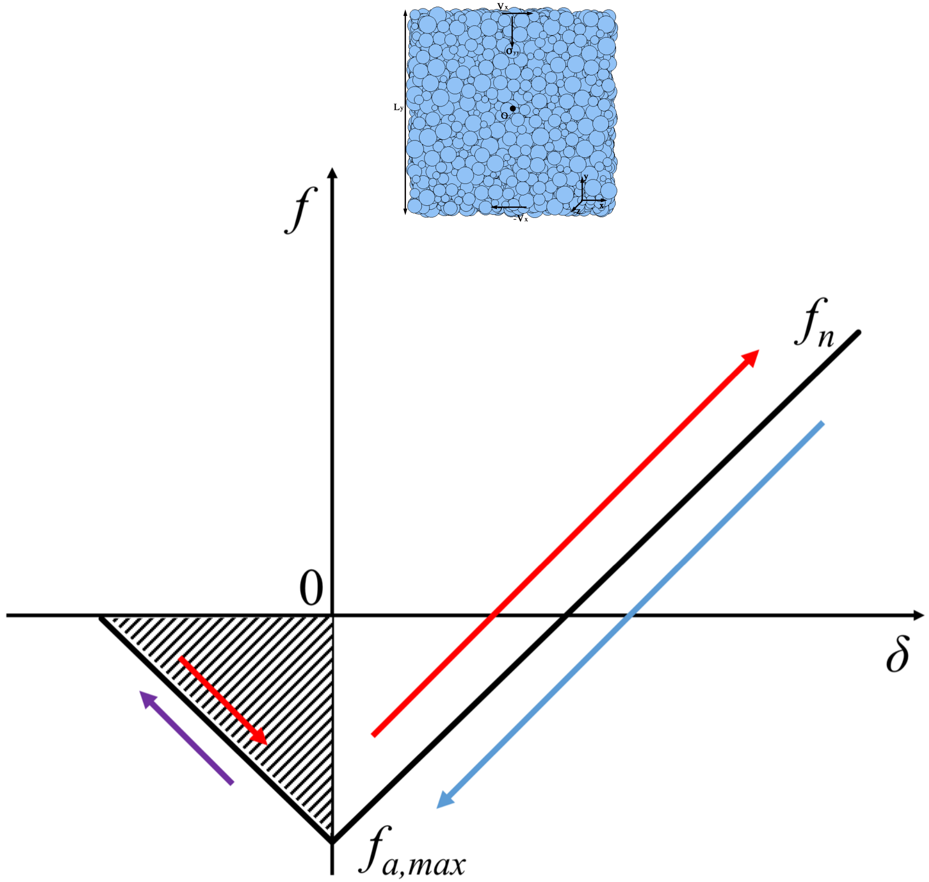


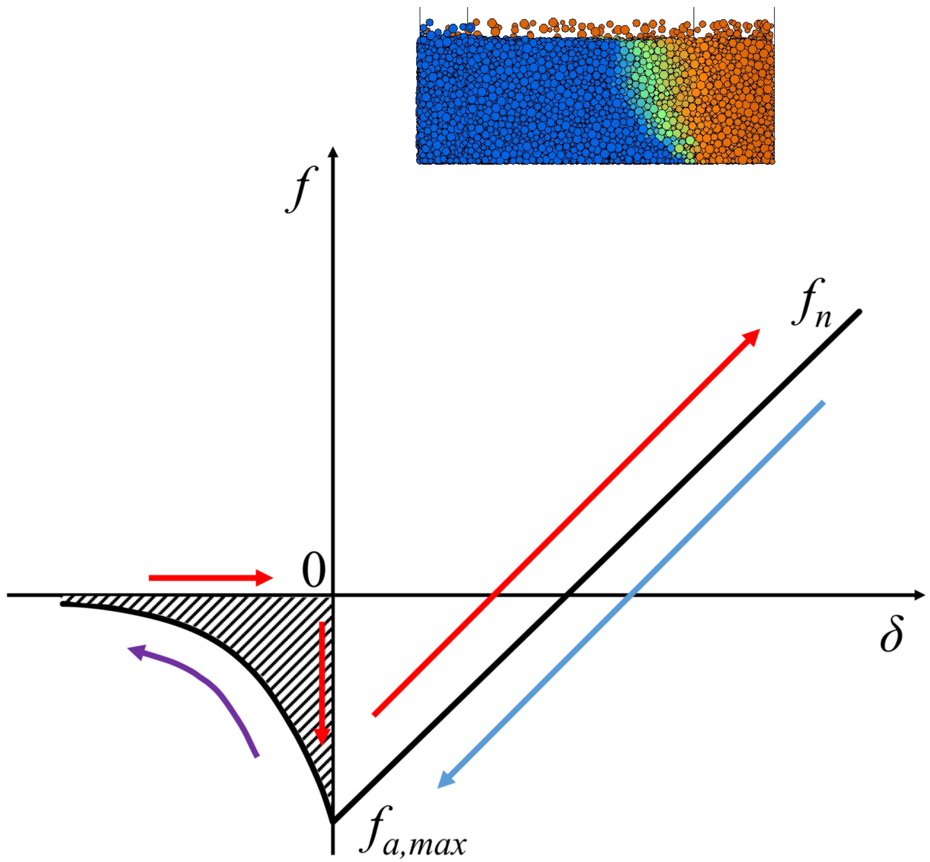
Shear Band = REVs ?



Can we represent our shear band zone with representative element volume (REV)?

Different geometries and contact models





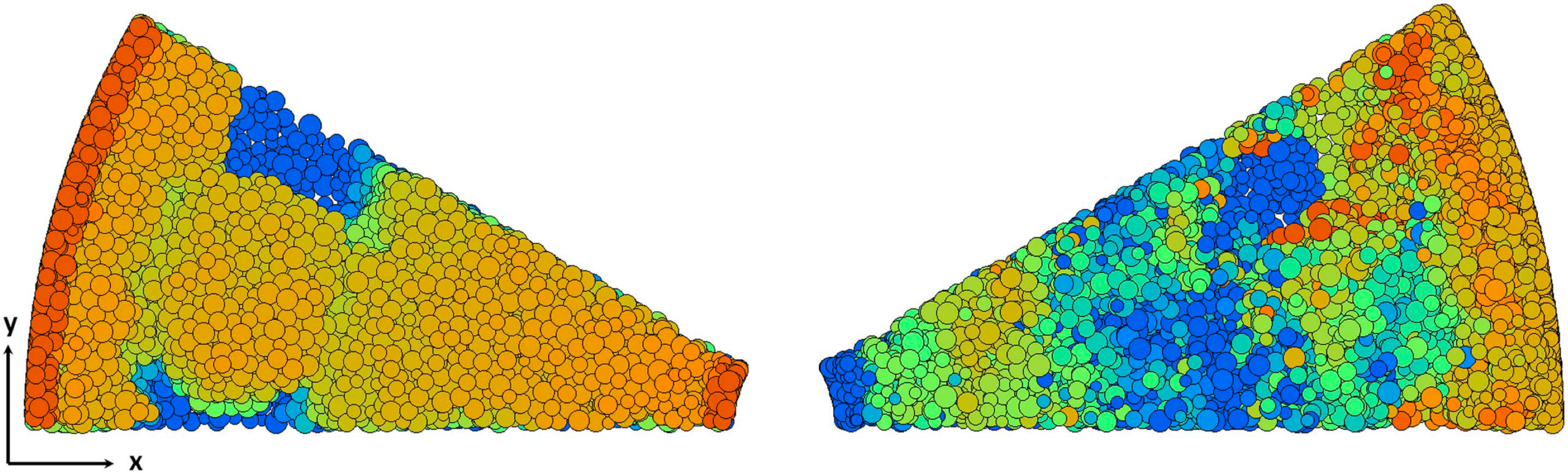
Stress controlled simple shear (SS)

Split bottom ring shear cell (SB)

Reversible linear visco-elastic frictional adhesive contact model

Irreversible linear visco-elastic frictional jump-in adhesive contact model

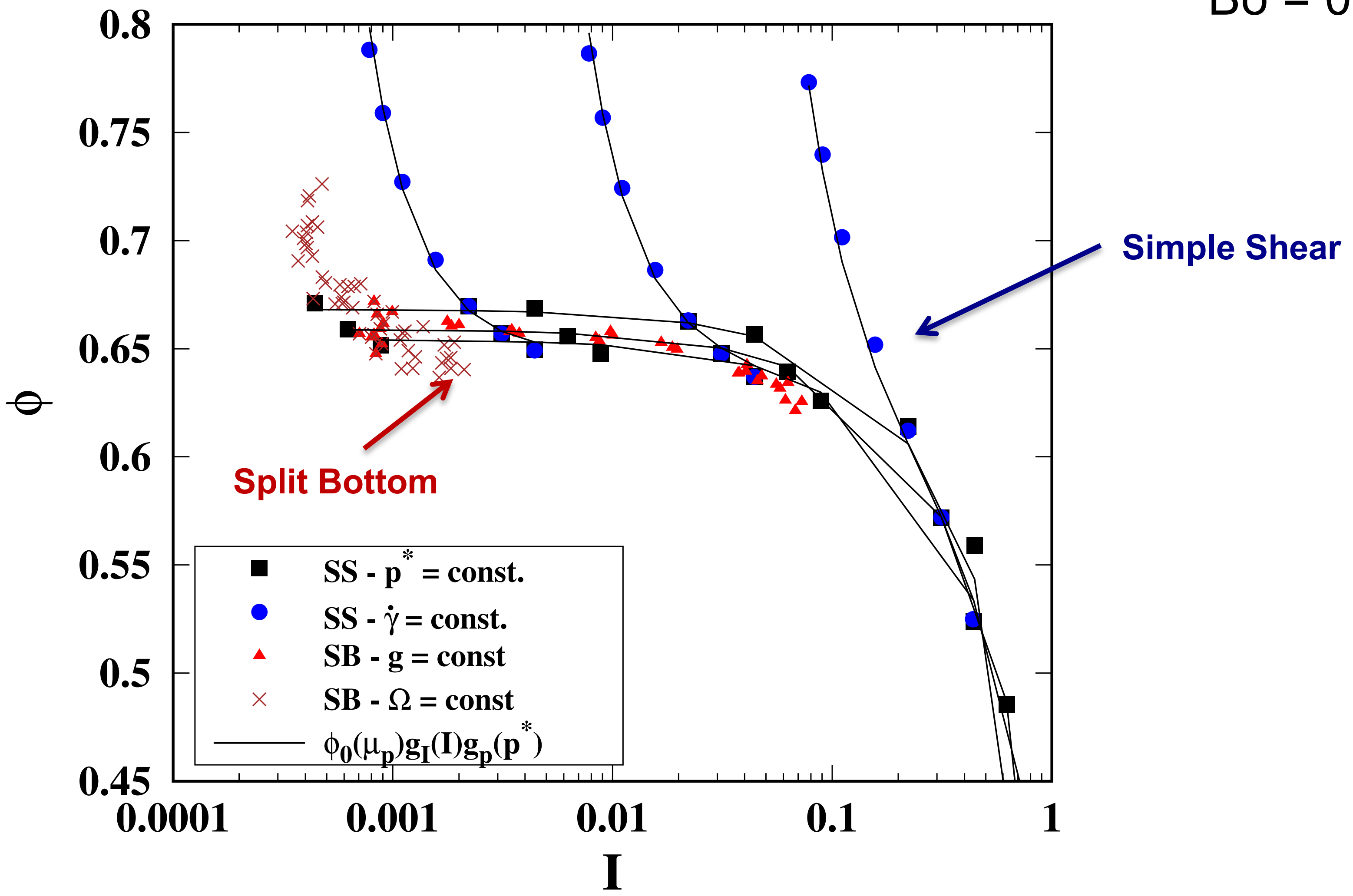
Cohesive material in the split bottom shear cell



[S. Roy et al. NJP (2017)]

Non-cohesive slightly frictional soft particles

$\mu_p = 0.01$
Bo = 0



$$\phi(I, p^*) = \phi_0 g_I g_p = \phi_0 \left(1 - \frac{I}{I_\phi}\right)\left(1 + \frac{p^*}{p^*_\phi}\right)$$

Cohesive frictional soft particles

Volume fraction ϕ:

$$\phi(I,p^*) = \phi_0 g_I g_p = \phi_0 \left(1 - \frac{I}{I_\phi}\right)\left(1 + \frac{p^*}{p^*_\phi}\right)$$

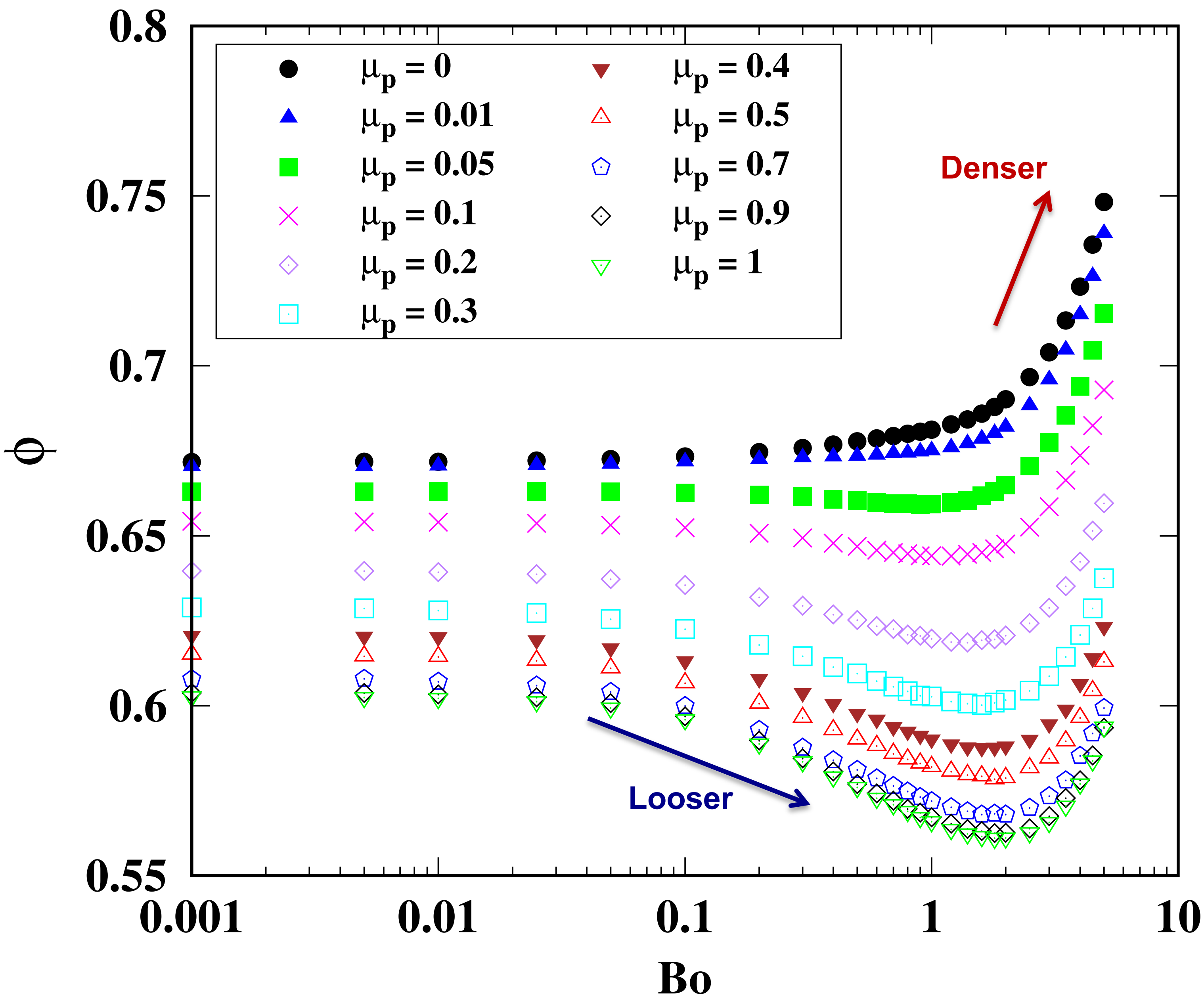
Valid for both local, inhomogeneous and global homogeneous systems

What about *Bo*?

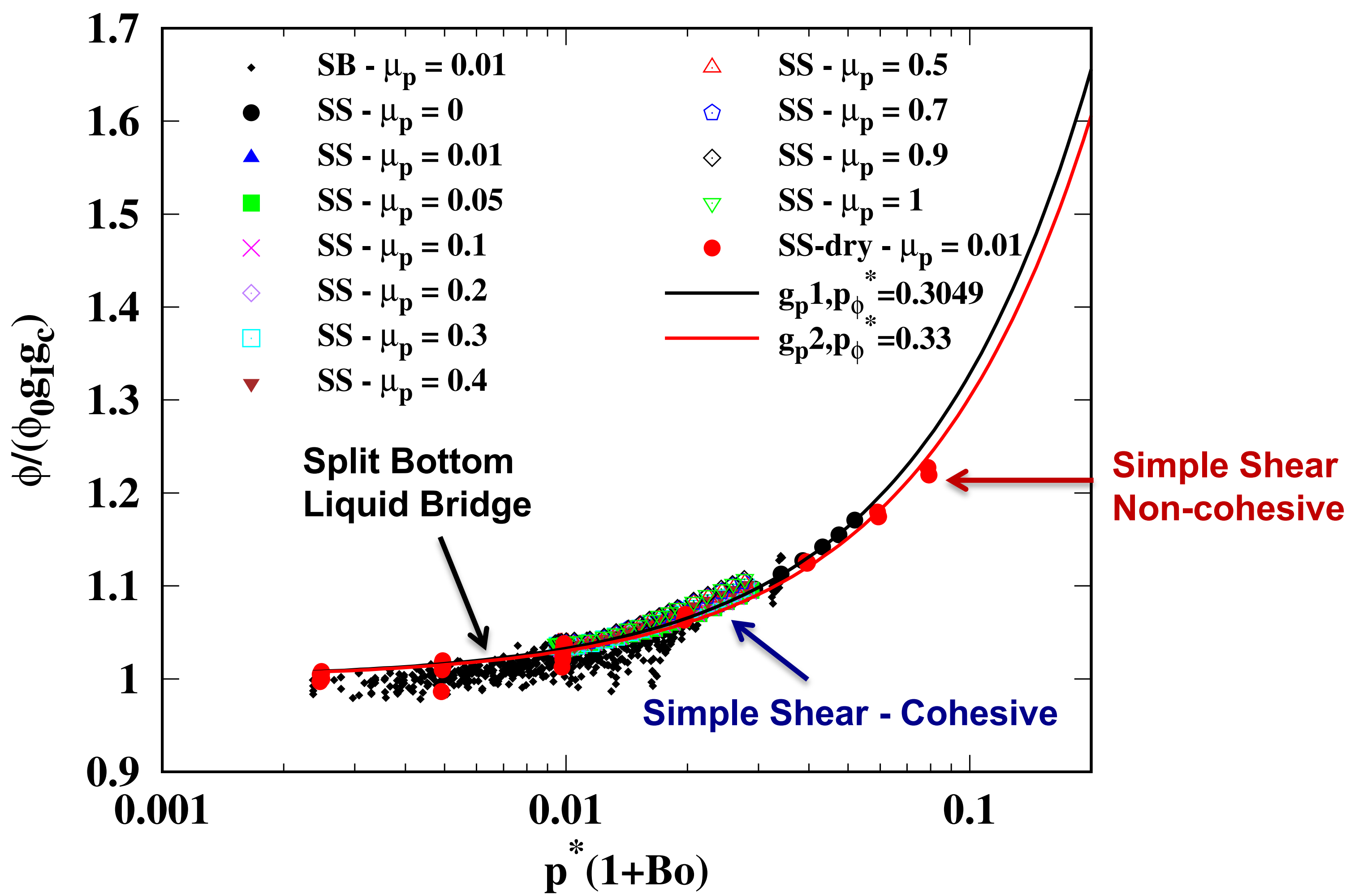
$$\phi(I,p^*,Bo) = \phi_0 g_I g_p g_c = \phi_0 \left(1 - \frac{I}{I_\phi}\right)\left(1 + \frac{p^*}{p^*_\phi}\right)?$$

Missing

Cohesive frictional soft particles (SS)



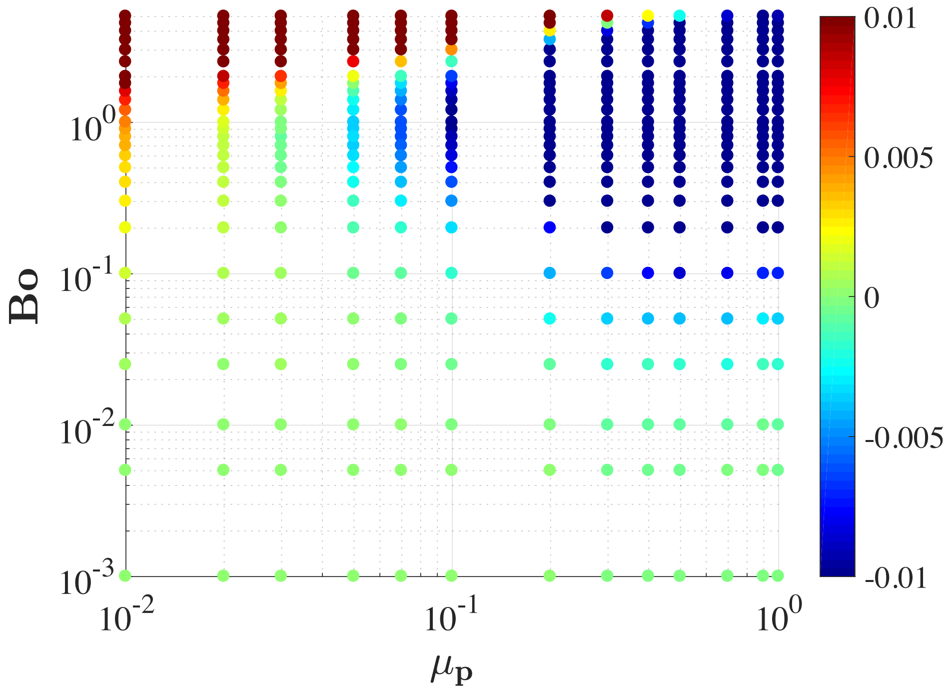
Cohesive frictional soft particles (SS)

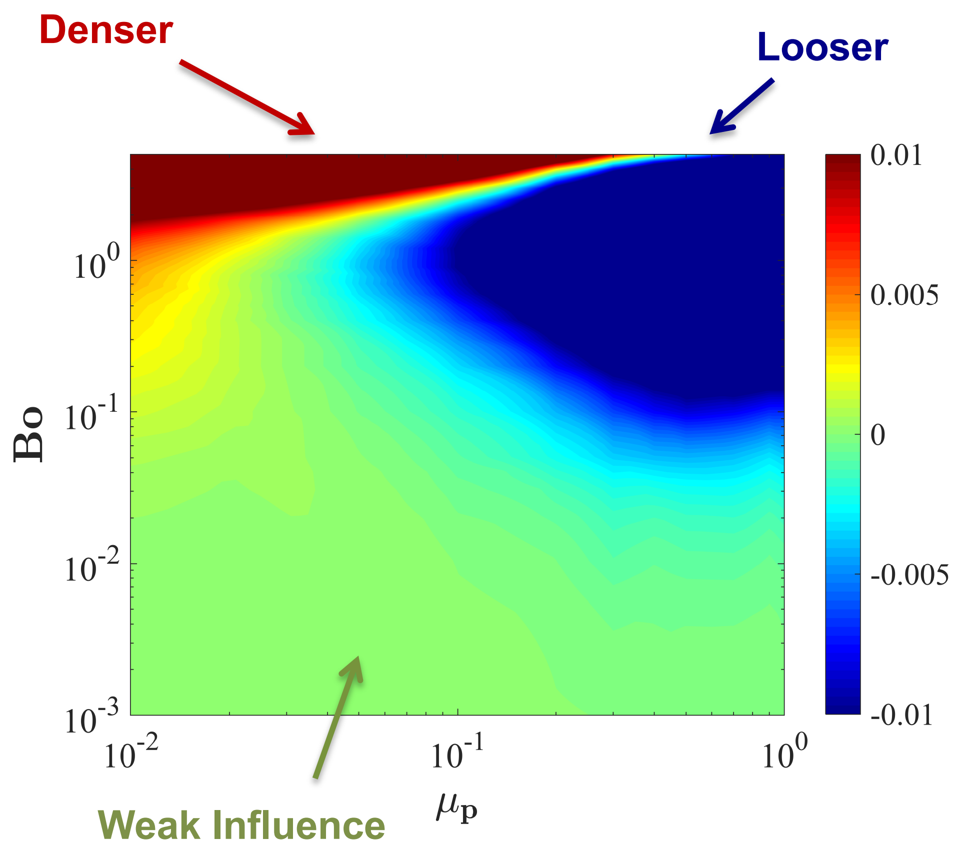


$$\phi(I, p^*, Bo) = \phi_0(\mu_p) g_I(I) g_p^{coh}(p^*, Bo) g_c(Bo)$$

Phase Diagram:
Coupled effect of friction and cohesion

$$\Delta\phi = \phi(Bo, \mu_p) - \phi(Bo = 0, \mu_p)$$





[H. Shi et al., under review, Gran. Matter (2019)]

Thank you