Particle Technology

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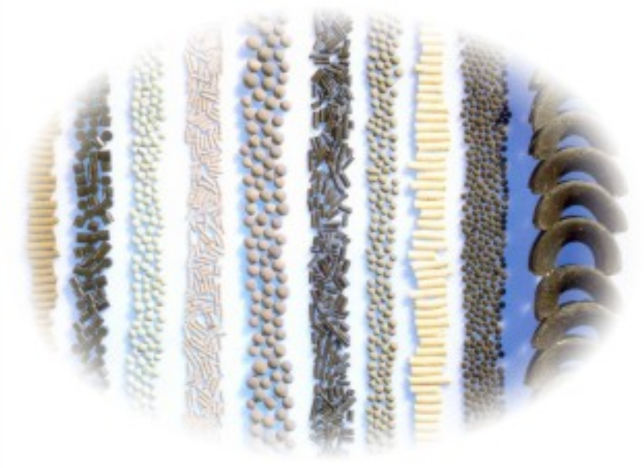
Applications in Industry

Iron ore pellets

Milk powder



Catalyst pellets

Fertilizer grains

Applications in everyday products

Pharmaceuticals

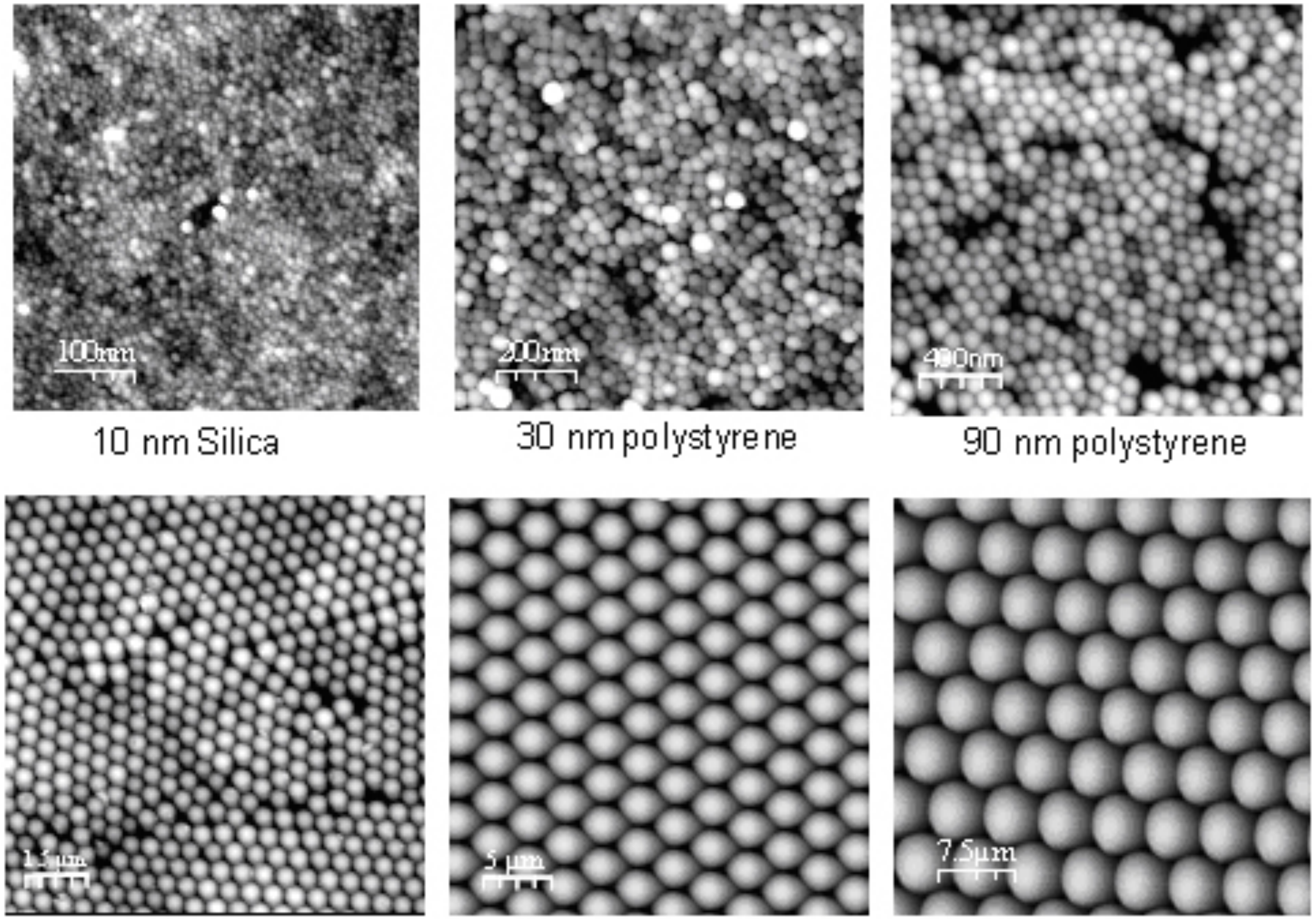
Detergent powders

Instant meals

And many more…

Particle Types

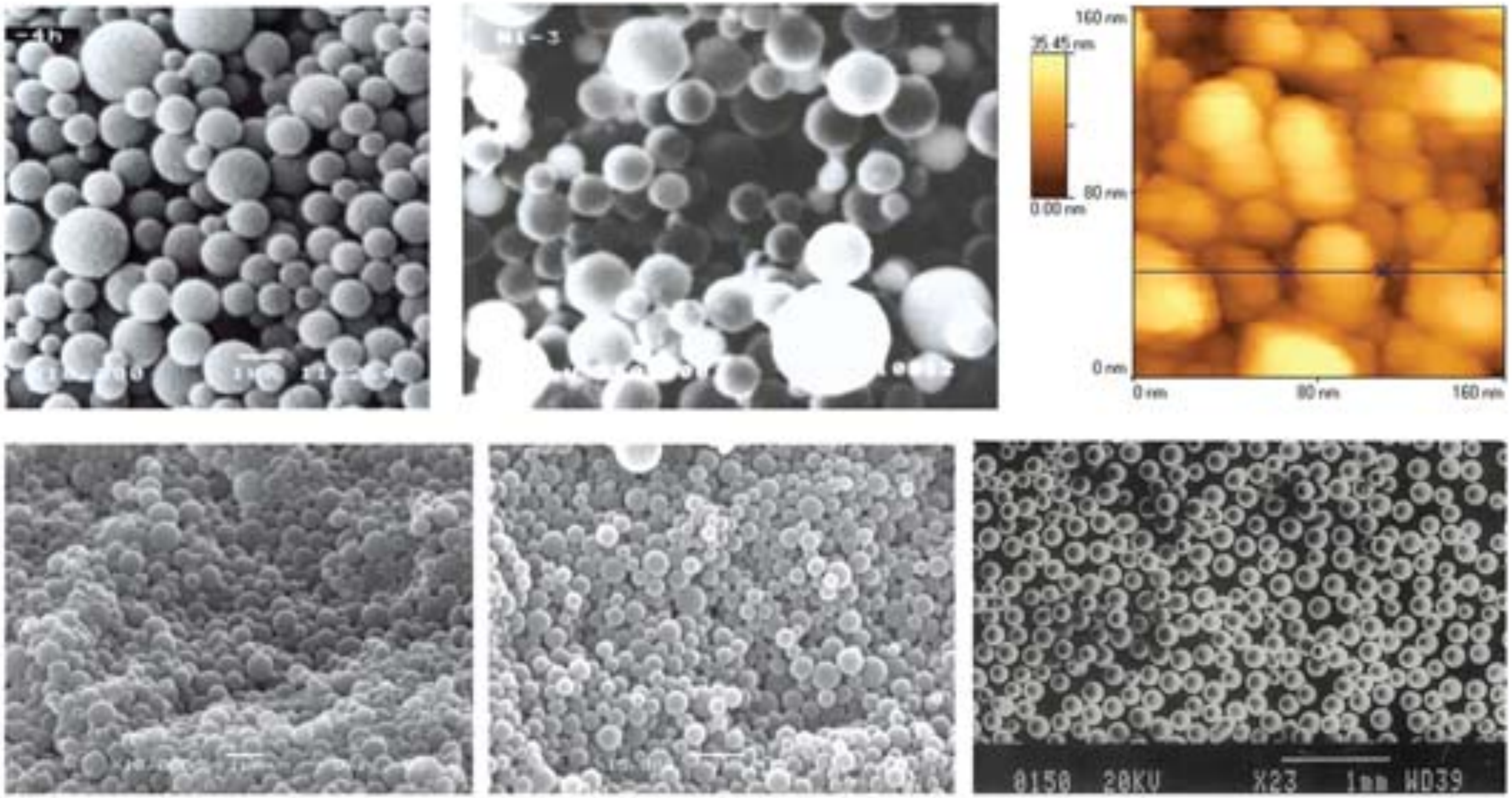
Size characterization of close packed nanoparticles



10 nm Silica | 30 nm polystyrene | 90 nm polystyrene

500 nm polystyrene | 3000 nm Silica | 5000 nm polystyrene

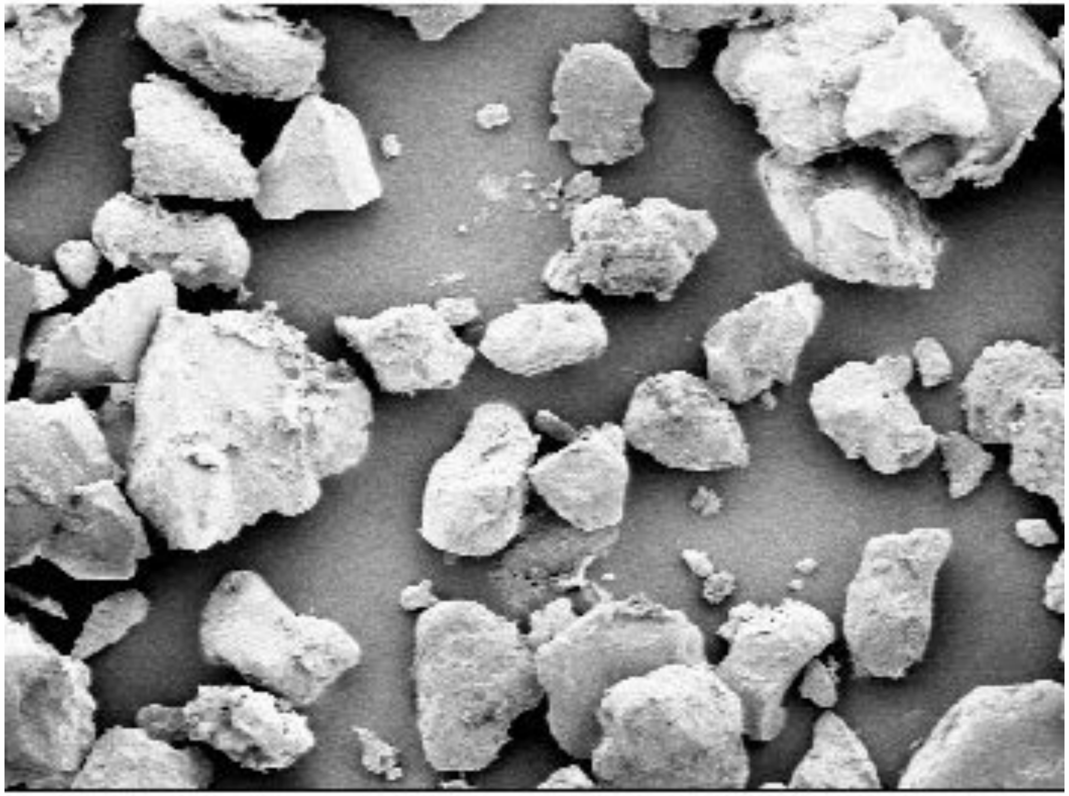
Particle Types



fine spherical particles of oxide, non-oxide, metallic, polymer and composite material: from nano to macro level

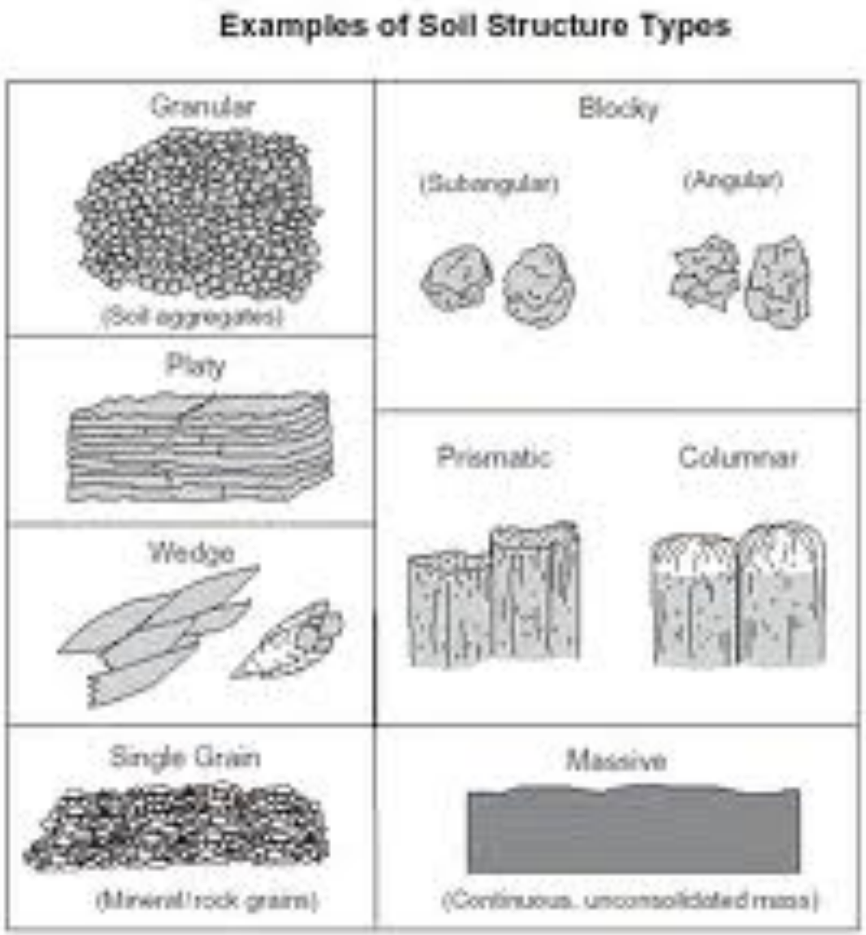
Particle Types

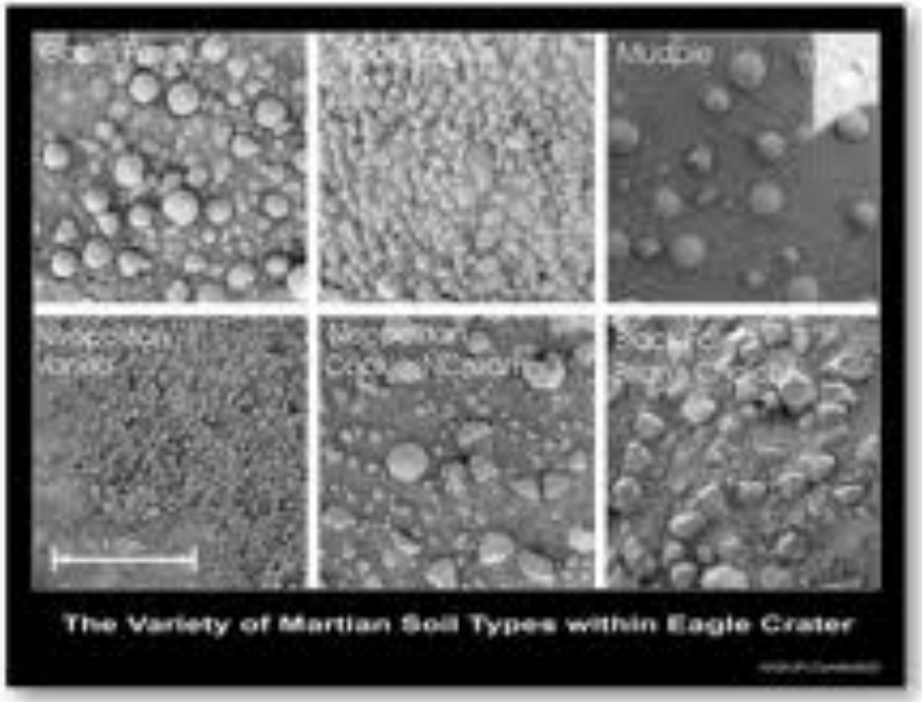
Soil



Carbon

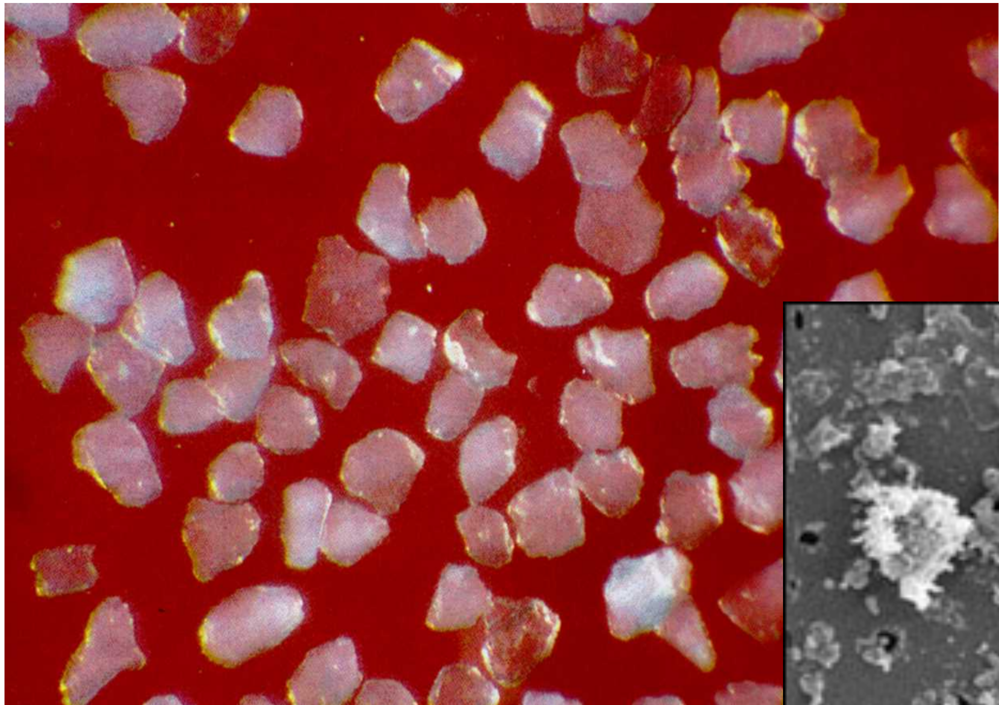
Particle Types



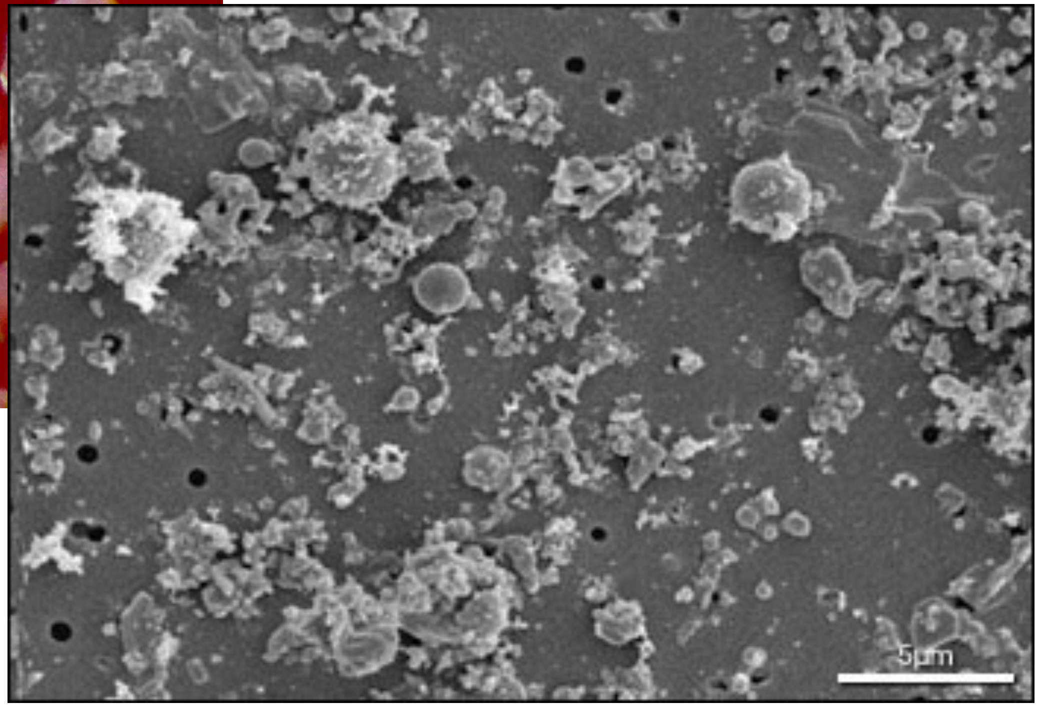


Particle Types

aerosol



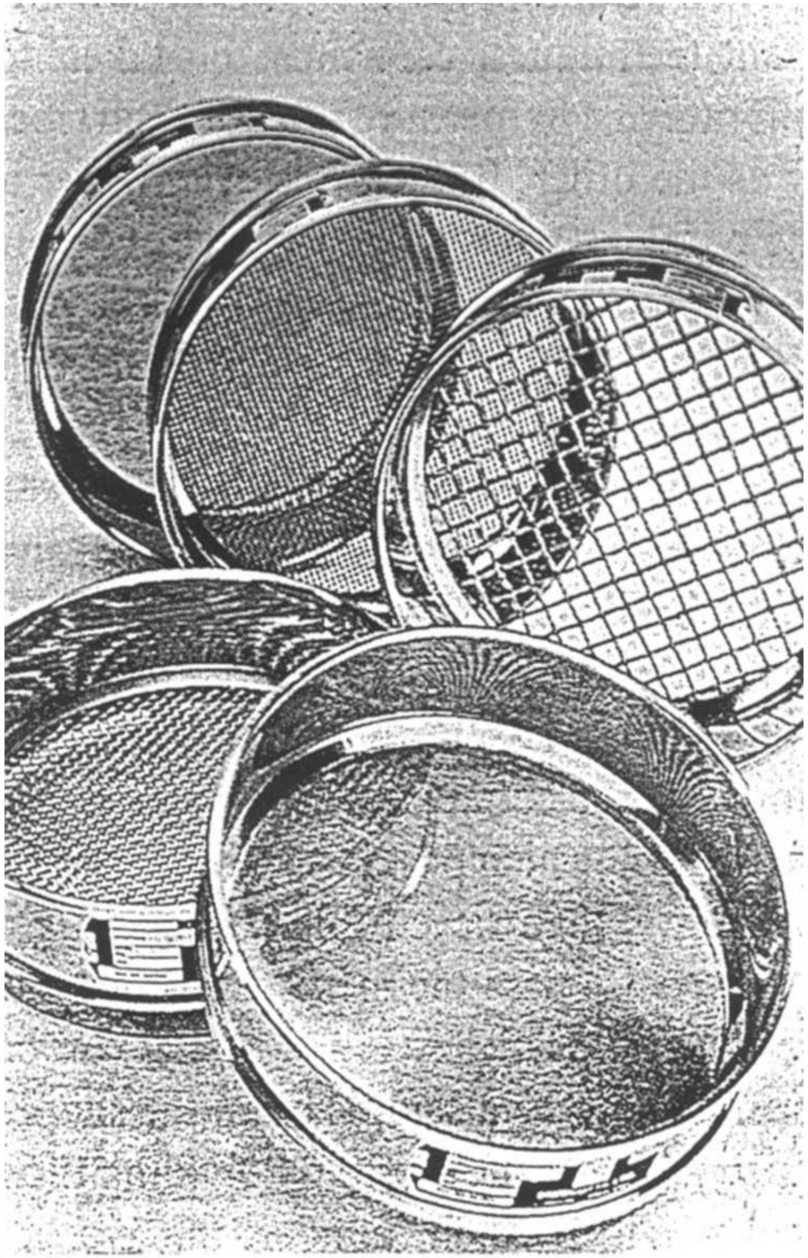
salt



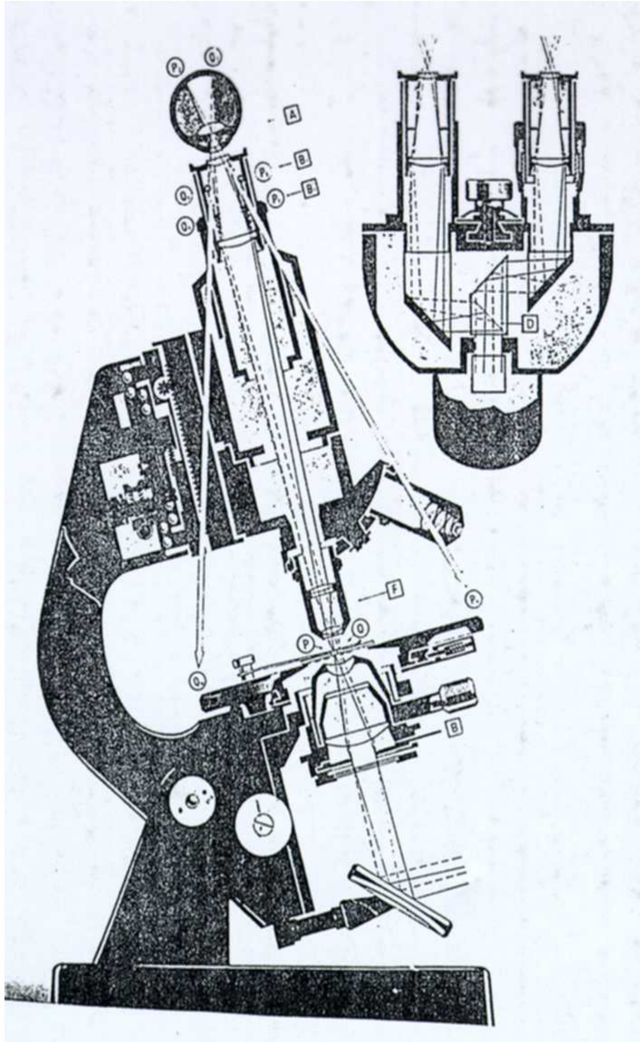
Particle description

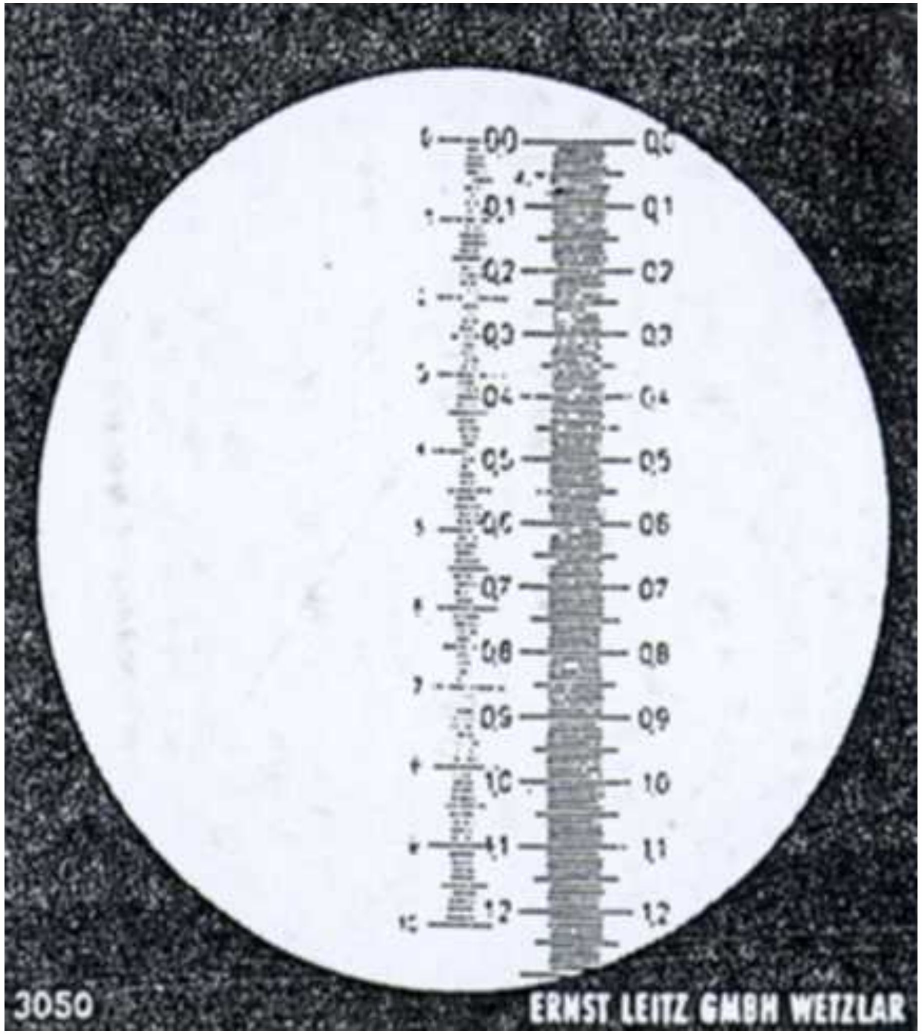
- GOAL: Describe the particle size
  (but use only ONE number)
- IMPOSSIBLE !?
- What will be done with the particle?

Traditional sieves



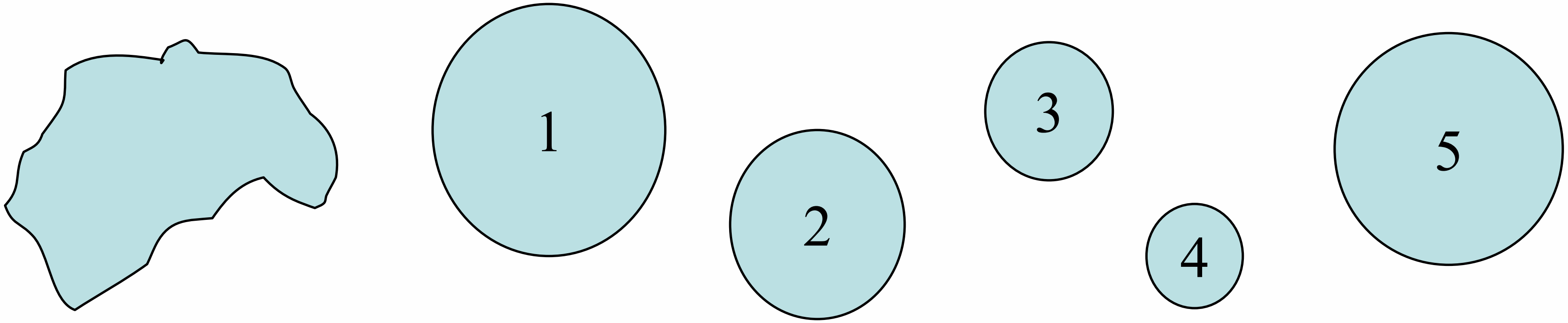
Microscope





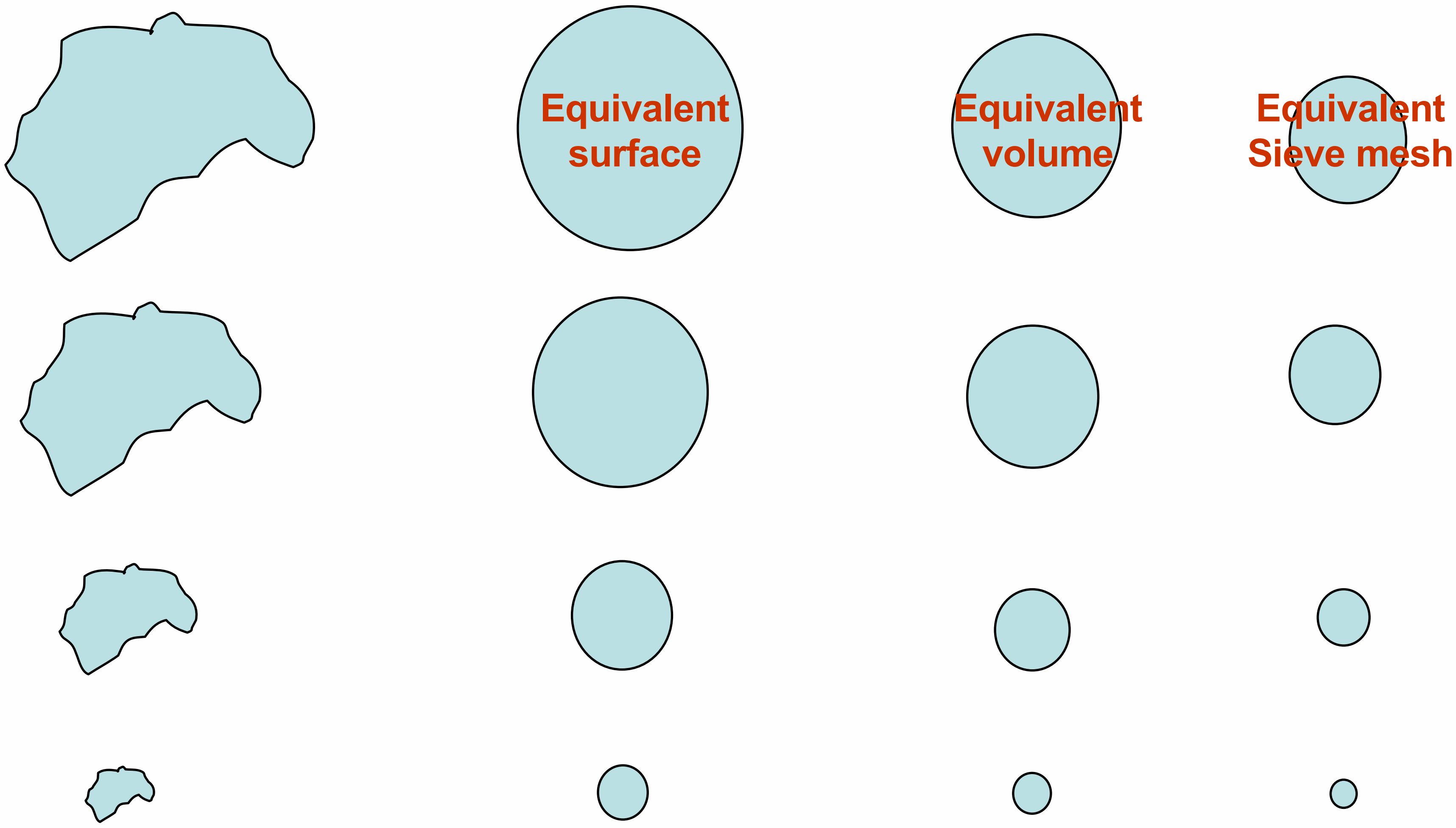
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1,0
1,1
1,2
3050
ERNST LEITZ GMBH WETZLAR

Equivalent diameters

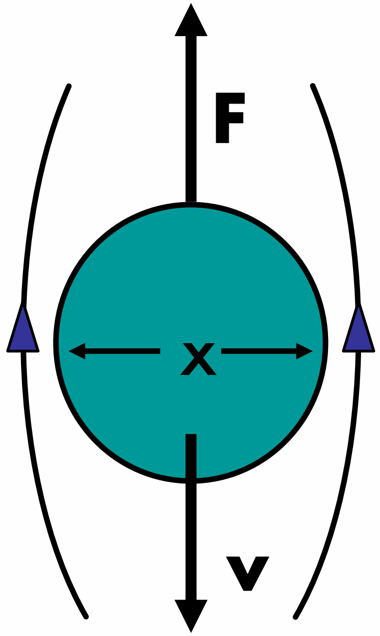


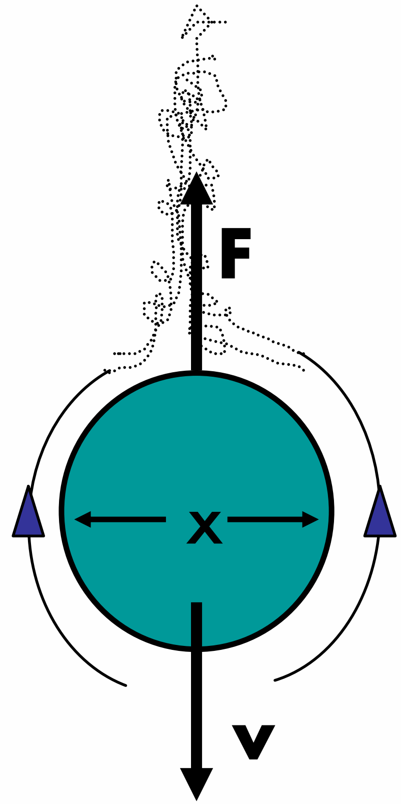
1. Sphere of equivalent surface
2. Sphere of equivalent volume
3. Sphere of equivalent settling velocity, low Re
4. Sphere of equivalent settling velocity, high Re
5. Sphere of equivalent sieve mesh

Equivalent diameters



Particle Fluid Interactions





$$F = 3\pi\eta x v$$

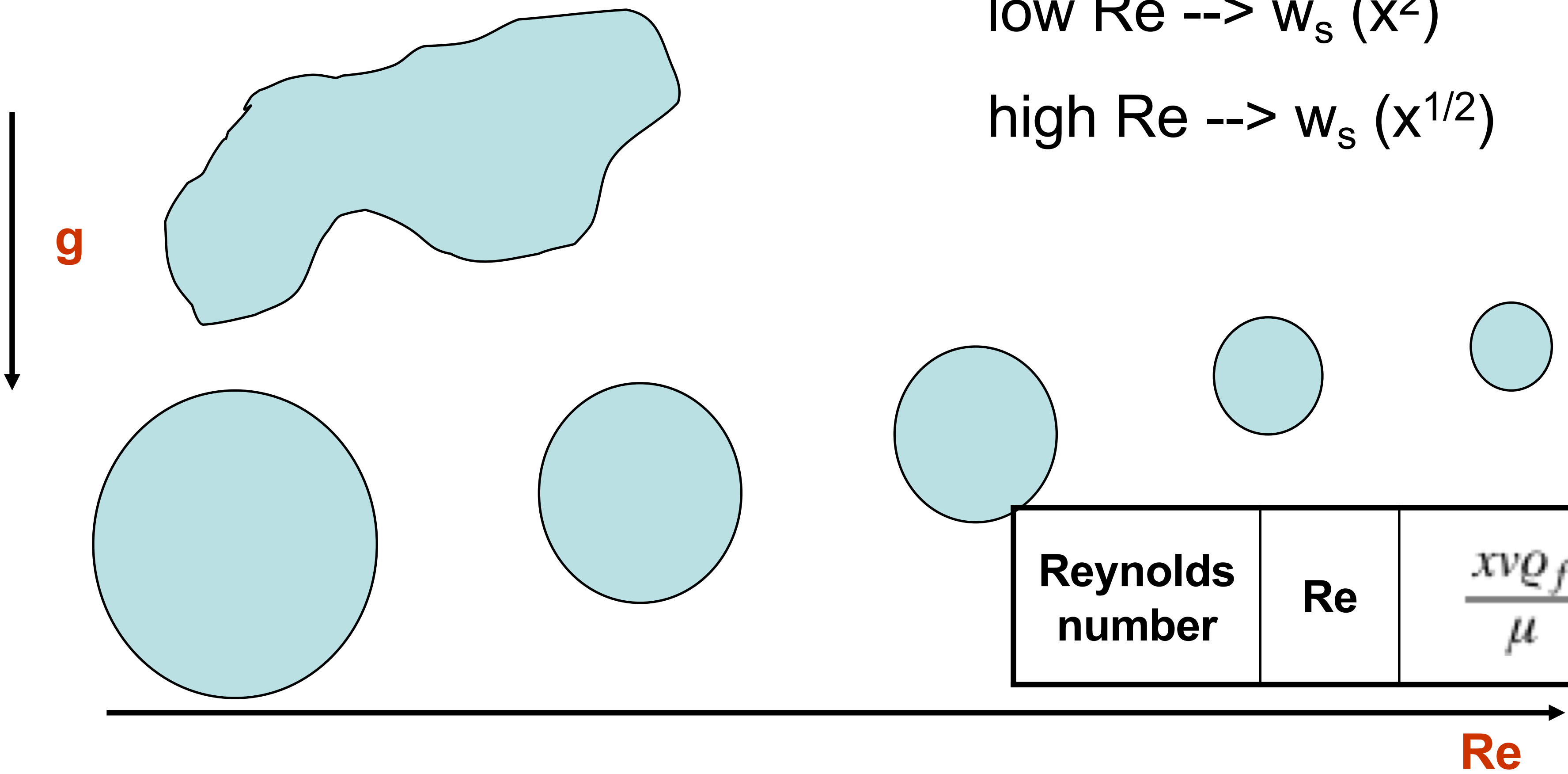
$$F = C_D \left(\frac{\pi x^2}{4} \right) \left(\frac{1}{2} \rho_f v^2 \right)$$

Equivalent settling diameter

The *Settling Velocity* (or *fall velocity* or *terminal velocity*) W_S is defined as the rate at which the sediment settles in still fluid

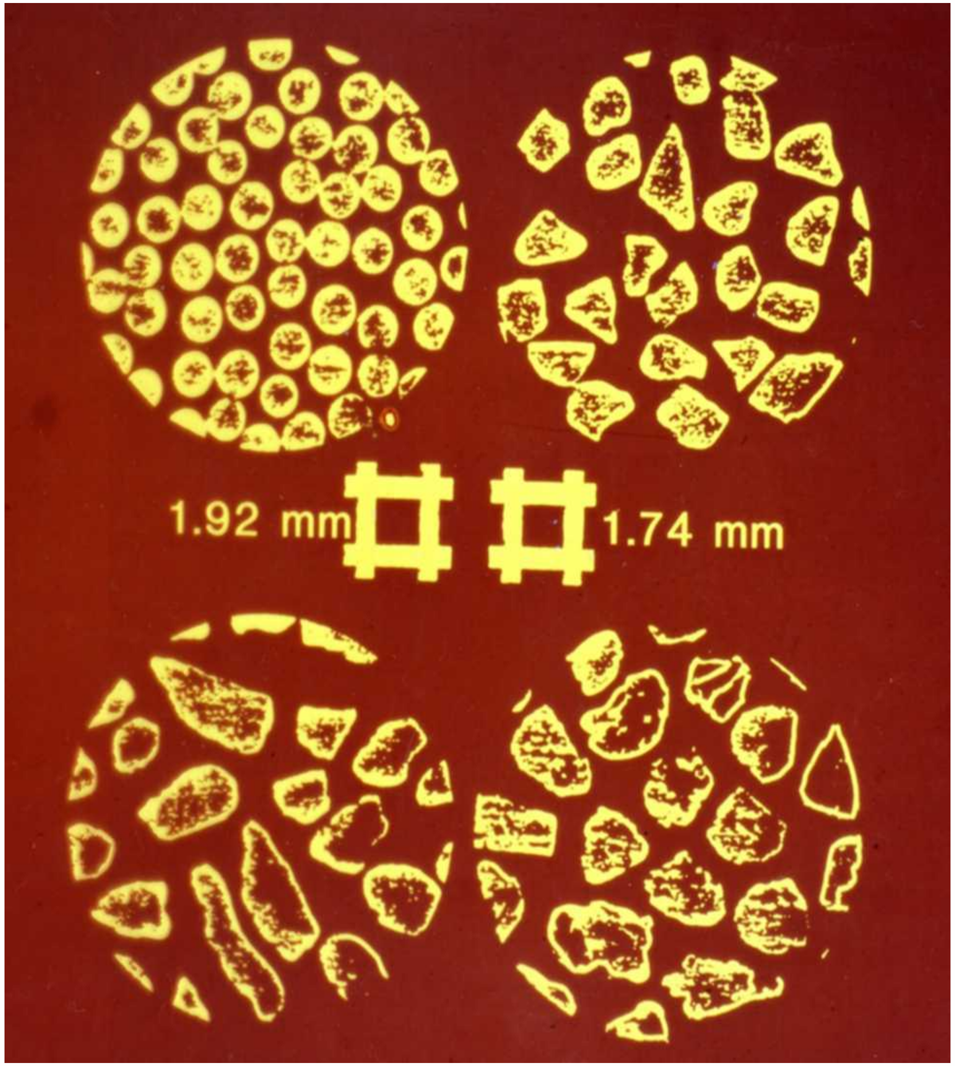
low Re --> w_s (x^2)

high Re --> w_s ($x^{1/2}$)



Reynolds number	Re	$\frac{x v \rho_f}{\mu}$

Near mesh particles



Exercises

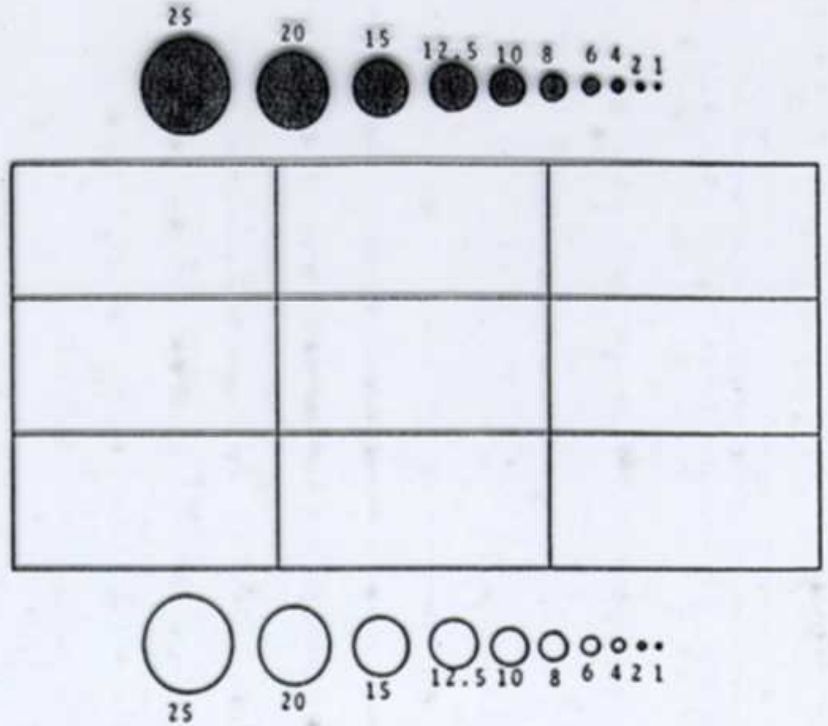
- Assume you have to design a process where the non-spherical particles are transported via a rapid pneumatic transport line. Which equivalent spherical diameter would you use for them? Why?

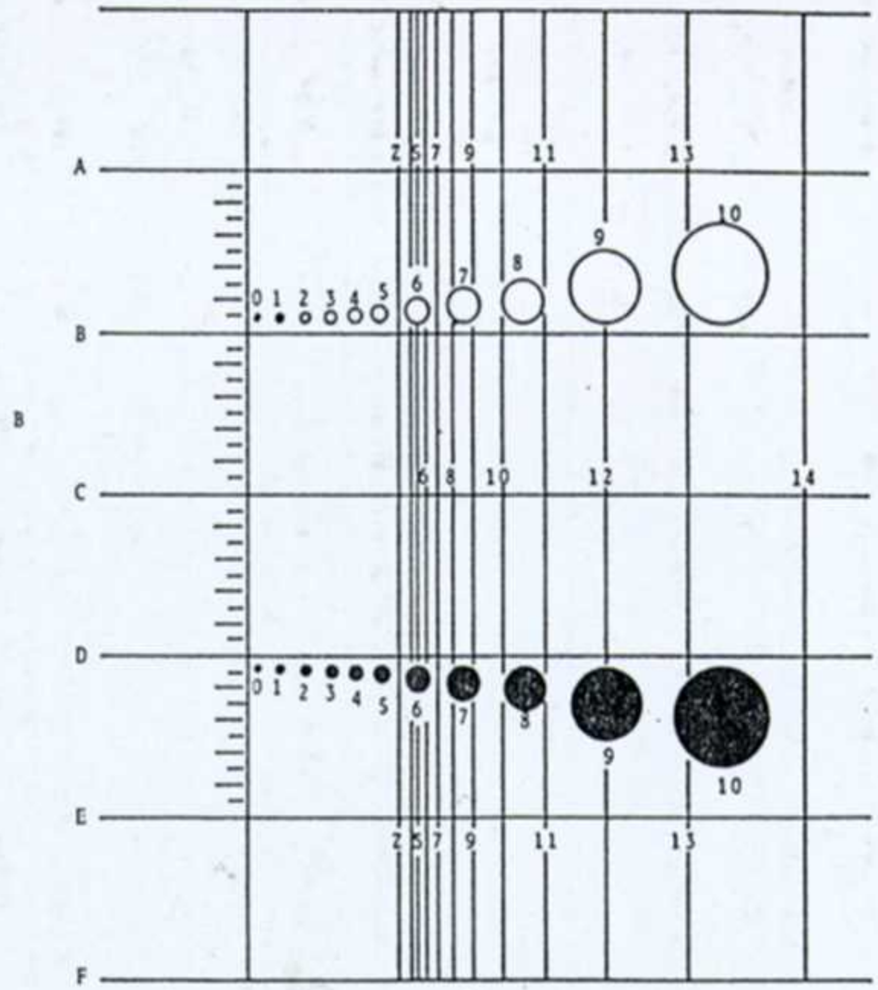
- Imagine a cylindrical particle with diameter 3 mm and height 1.0 mm. Calculate the equivalent volume diameter, the equivalent surface diameter (and the equivalent sieve diameter).

Summary

- Particle property of interest …
  Process dependent
  Measurement dependent
- Many particles are non-spherical
- Particles are NOT equal !

Size Distribution





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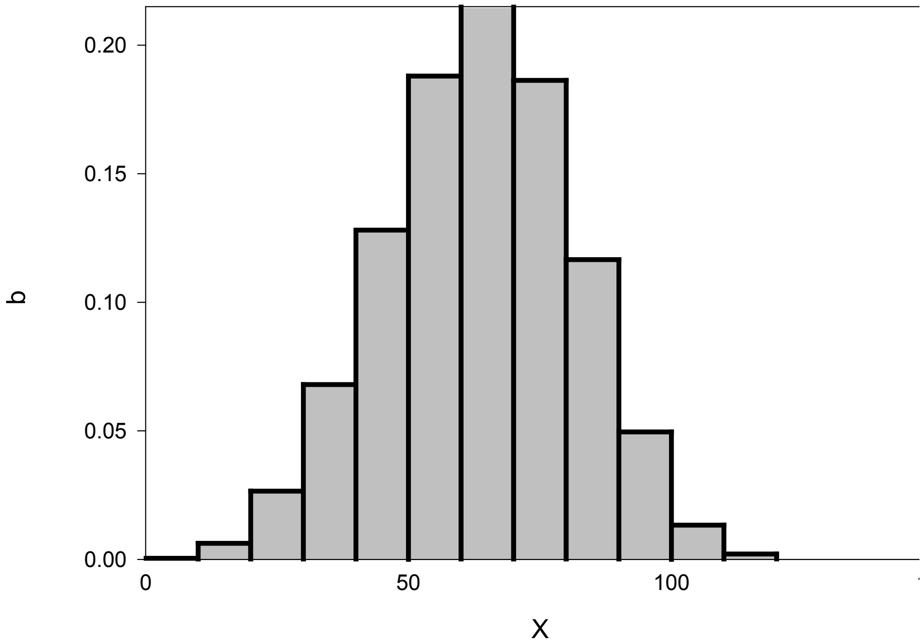
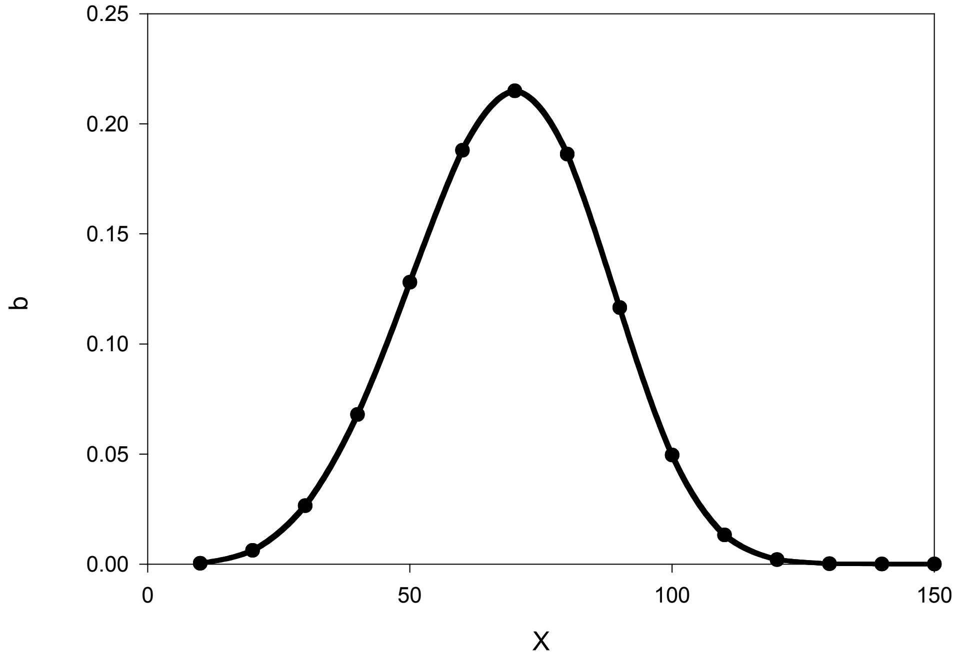
Distribution by histogram

Quantity **= q_N (numbers)**
= q_0 (normalized)

Particle Size = x





Normalization

Any distribution (e.g. by numbers/counts)

$$q_N(x) \qquad \int q_N(x)dx = N$$

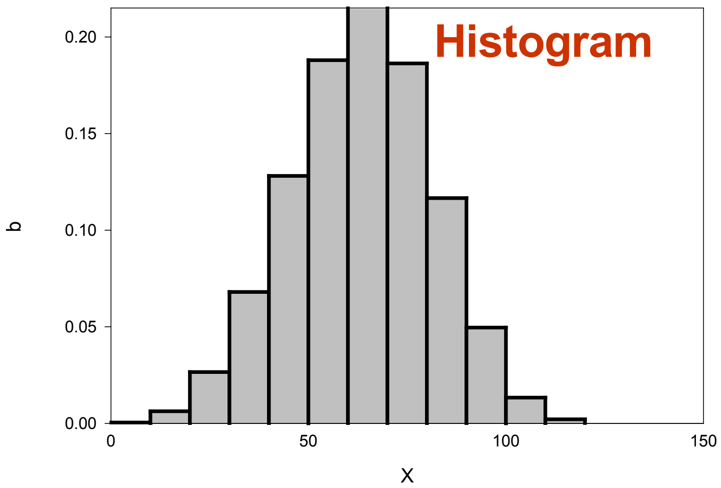
can be normalized

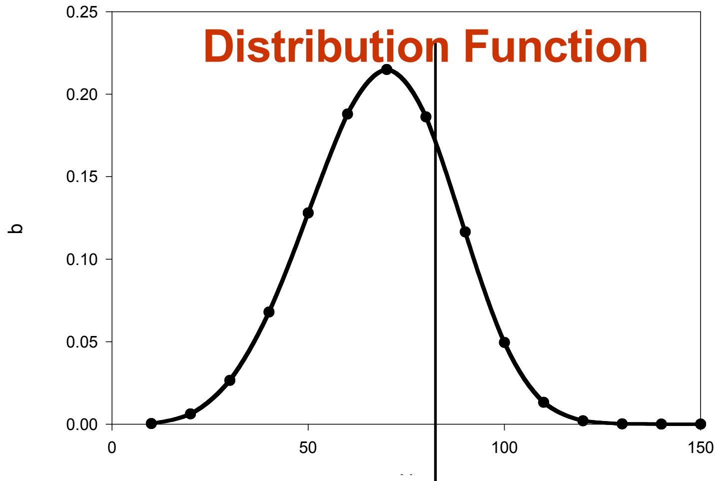
$$q_0(x) = \frac{q_N(x)}{\int_0^\infty q_N(x)\,dx} \qquad \int q_0(x)dx = 1$$

$$0 <= q_0(x) <= 1$$

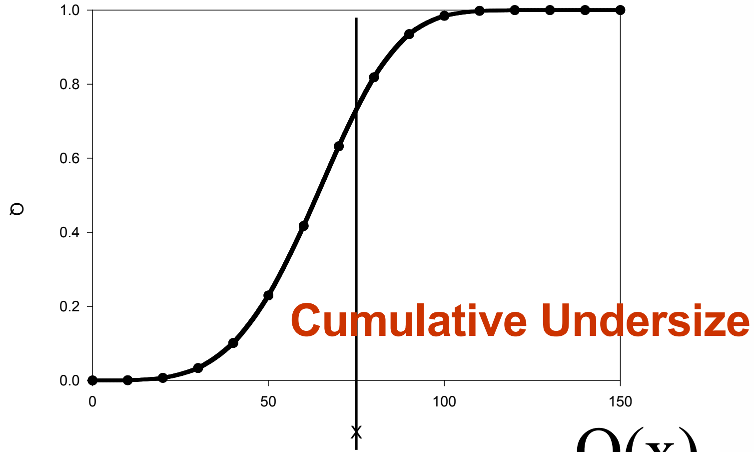
(integration boundaries are dropped if 0…∞)

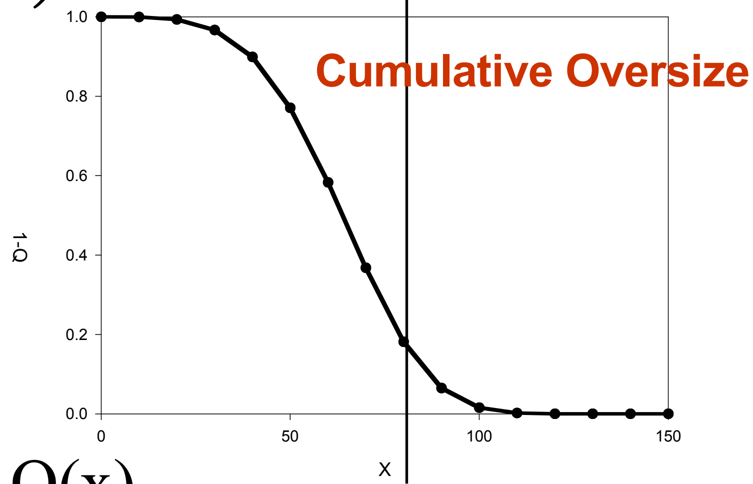
Graphical rapresentation





$$Q(x) = \int_{0}^{x} q_0(x')dx'$$





$$Q(x)_{undersaize} = 1 - Q(x)_{oversize}$$

Particle size distribution

Modal Size

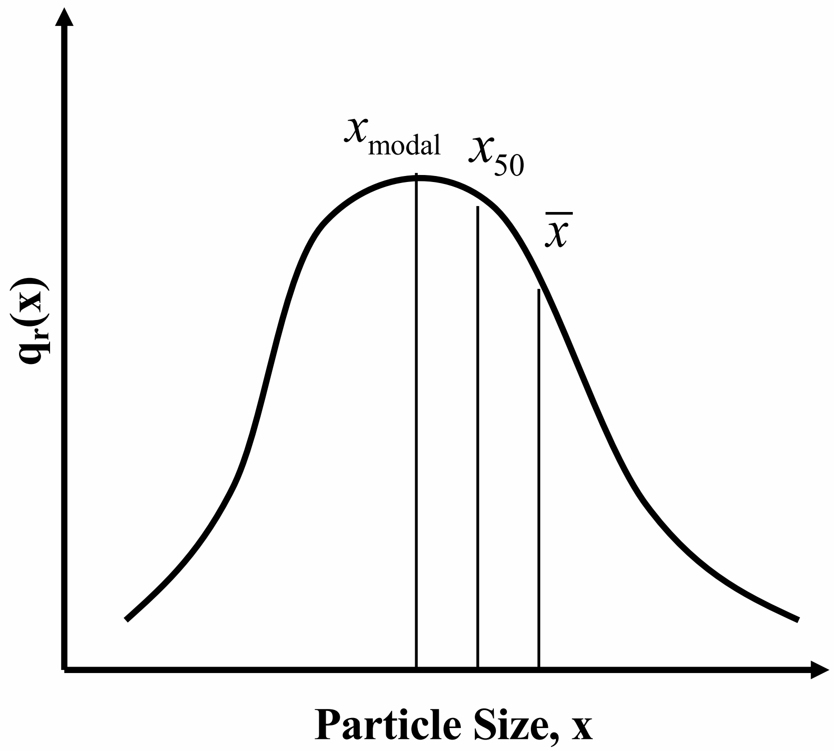
$$x_{\text{modal}} = \text{value at which } q_0(x) = \max$$

Median Size

$$x_{50} = 50\% \text{ value i.e. } \begin{matrix} \tfrac{1}{2} \text{ greater in size} \\ \tfrac{1}{2} \text{ less in size} \end{matrix}$$

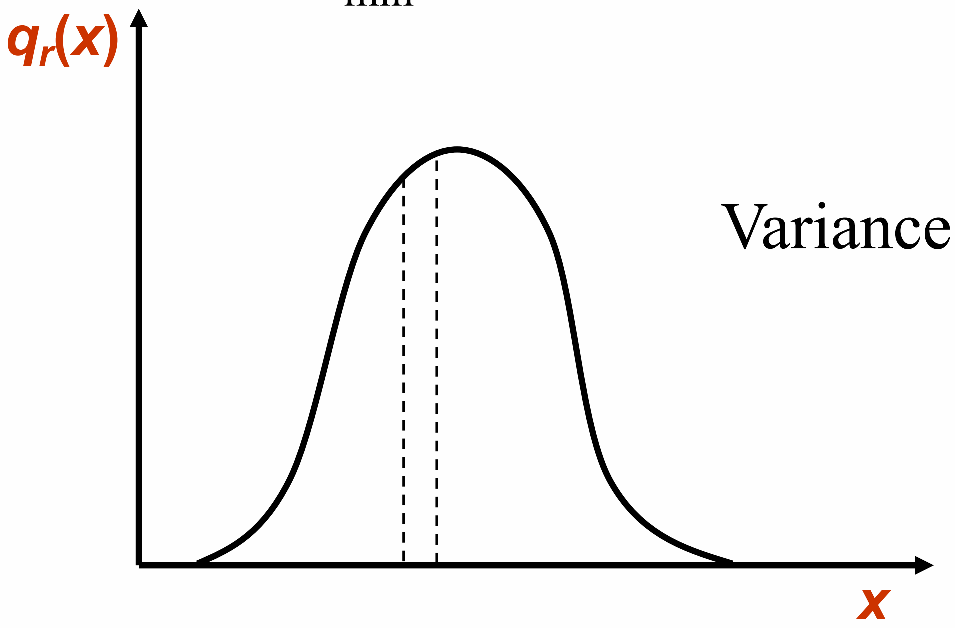
Mean Size

$$\overline{x} = \frac{\int q(x) x \, dx}{\int q(x) \, dx} = \int q_0(x) x \, dx$$



Moments of a distribution

$$M_k = \int_{x_{\min}}^{x_{\max}} q_0(x) x^k dx$$



Expectation

$$M_1 = \int_{x_{\min}}^{x_{\max}} q_0(x)\ x\ dx = \bar{x}$$

Variance

$$M_2 = \int_{x_{\min}}^{x_{\max}} q_0(x)\ x^2\ dx$$

$$M_3 = \int_{x_{\min}}^{x_{\max}} q_0(x)\ x^3\ dx$$

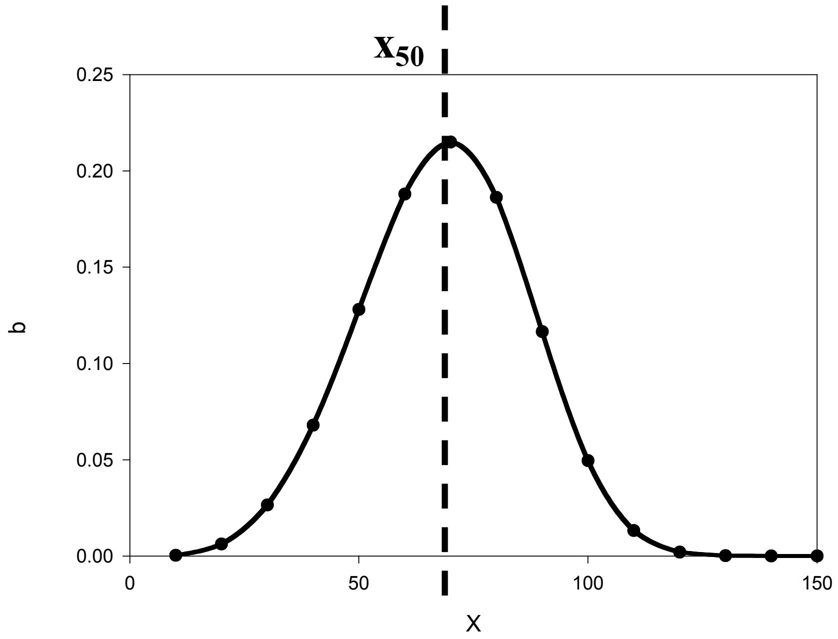
Skewness

Gaussian Normal distribution

Put $z = \dfrac{x - x_{50}}{s}$

where $s = \sqrt{\overline{\left(x - x_{50}\right)^2}}$

then $s\ dz = dx$



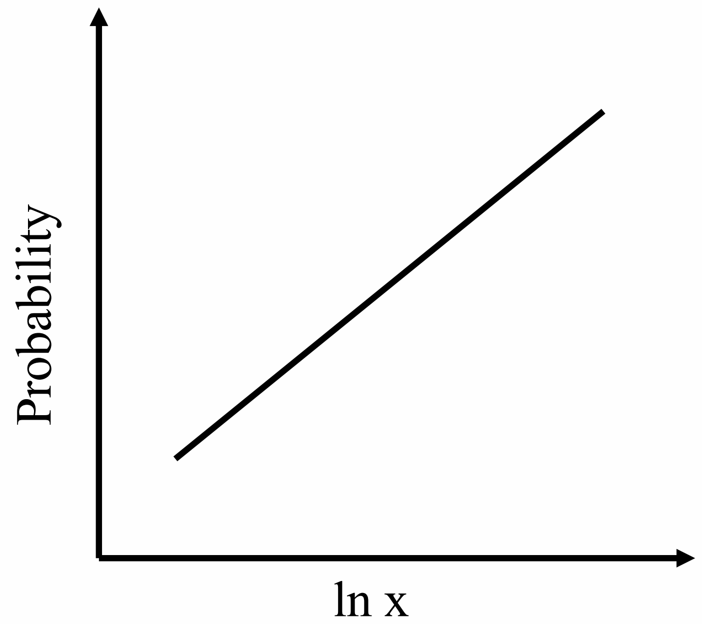
Thus $q\left(x\right) = \dfrac{1}{s\sqrt{2\pi}} \exp\left[-\dfrac{\left(x - x_{50}\right)^2}{2s^2}\right]$

Logarithmic Normal Distribution

Put:

$$z = \frac{\ln x - \ln x_{50}}{s_z}$$

$$s_z = \sqrt{\overline{(\ln x - \ln x_{50})^2}}$$



Then

$$q_{LN}(x) = \frac{1}{s_z\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(x/x_{50})}{s_z}\right)^2\right]$$

Exercises

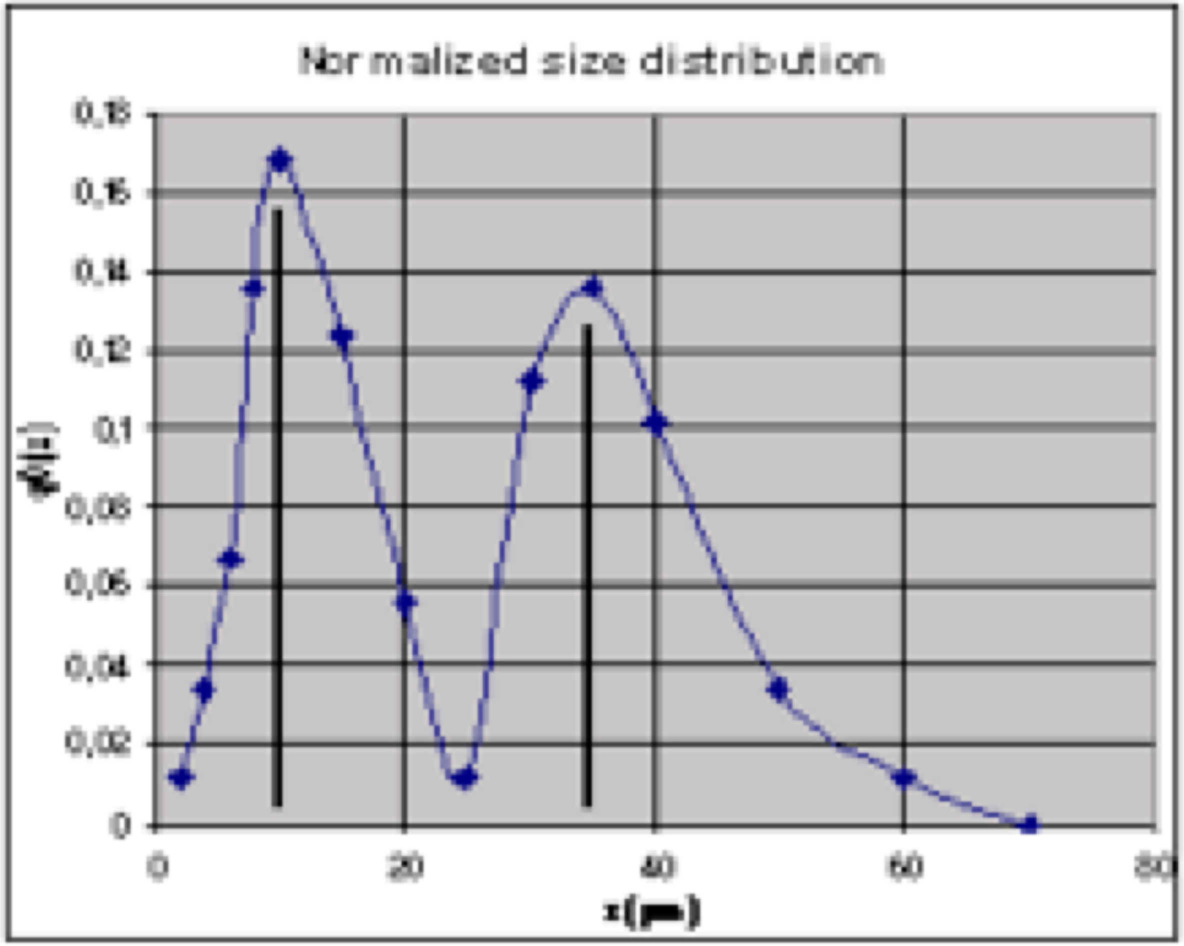
Given below is a size distribution of a sample of powder with a density of 2600 kg/m^3. The numbers is that what you get from your measurement. How do you check if the distribution is normalized? If not normalize.

x (μm)	2	4	6	8	10	15	20	25	30	35	40	50	60	70
$q(x)$	0.01	0.03	0.06	0.12	0.15	0.11	0.05	0.01	0.10	0.12	0.09	0.03	0.01	0
$q_0(x)$														
Q_u														

Exercises

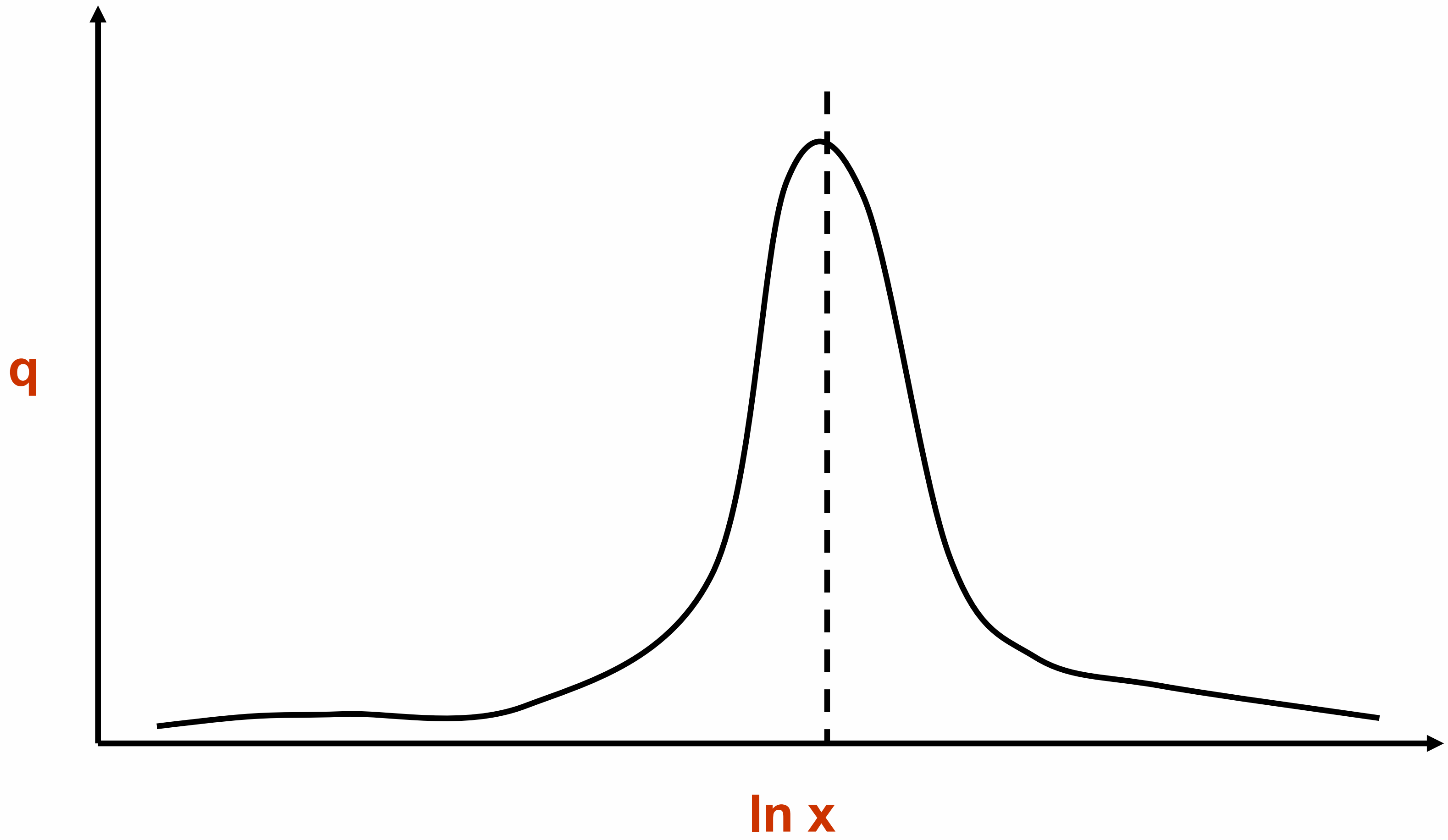
x(μm)	x(m)	q(x)		$q_0(x)$	Q_u	$q_2(x)$	$q_3(x)$
2	2.00E-06	0.01		0.01123596	0.01123596	1.41124E-13	4.70412E-20
4	4.00E-06	0.03		0.03370787	0.04494382	1.69348E-12	1.12899E-18
6	6.00E-06	0.06		0.06741573	0.11235955	7.62067E-12	7.62067E-18
8	8.00E-06	0.12		0.13483146	0.24719101	2.70957E-11	3.61276E-17
10	1.00E-05	0.15		0.16853933	0.41573034	5.29213E-11	8.82022E-17
15	1.50E-05	0.11		0.12359551	0.53932584	8.73202E-11	2.18301E-16
20	2.00E-05	0.05		0.05617978	0.59550562	7.05618E-11	2.35206E-16
25	2.50E-05	0.01		0.01123596	0.60674157	2.20506E-11	9.18773E-17
30	3.00E-05	0.10		0.11235955	0.71910112	3.17528E-10	1.58764E-15
35	3.50E-05	0.12		0.13483146	0.85393258	5.18629E-10	3.02534E-15
40	4.00E-05	0.09		0.1011236	0.95505618	5.08045E-10	3.38697E-15
50	5.00E-05	0.03		0.03370787	0.98876404	2.64607E-10	2.20506E-15
60	6.00E-05	0.01		0.01123596	1	1.27011E-10	1.27011E-15
70	7.00E-05	0.00		0	1	0	0
	Σq(x) =	0.89		1		2.00523E-09	1.21536E-14

Exercises

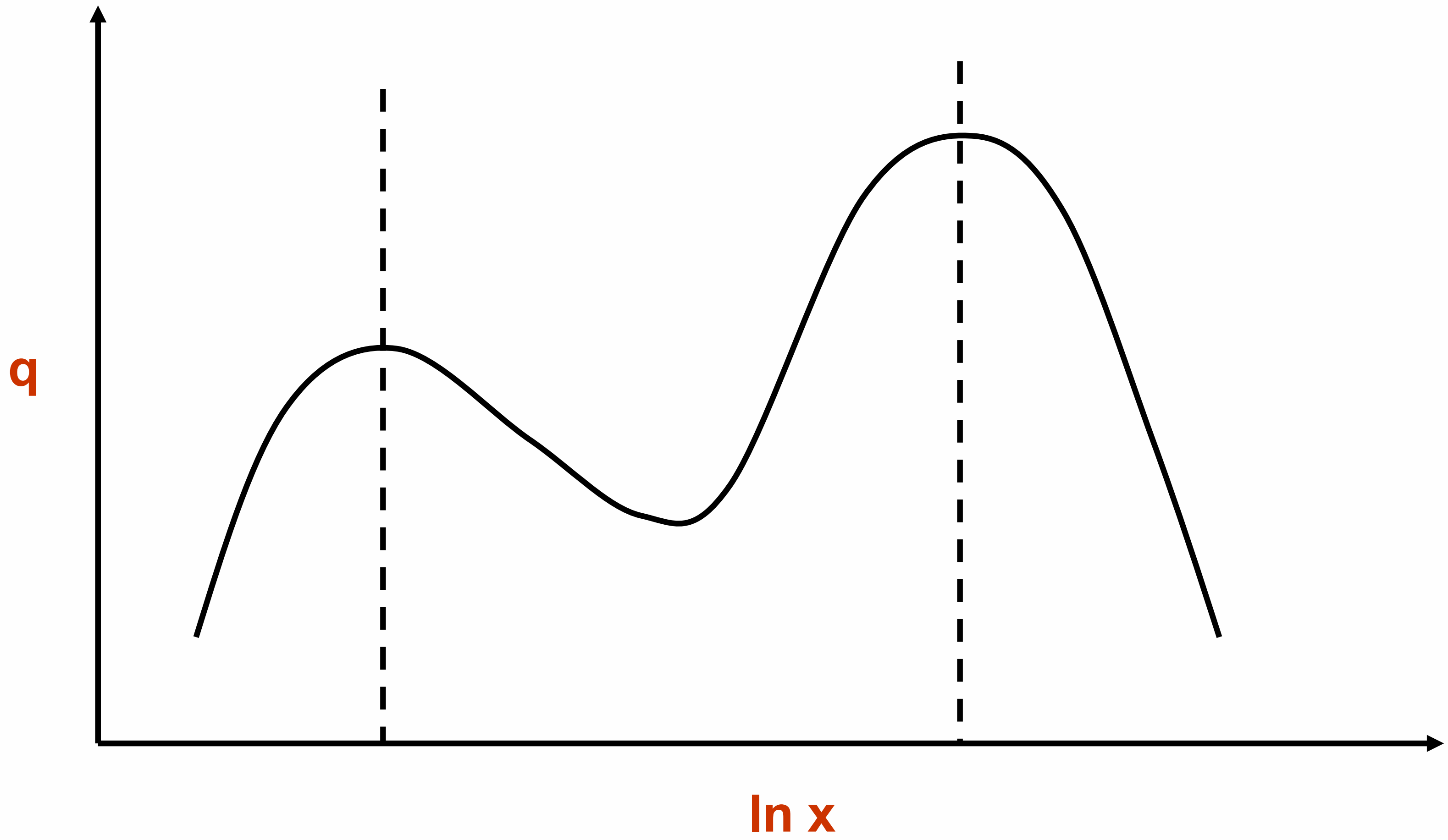


Normalized size distribution
0,18
0,16
0,14
0,12
0,1
0,08
0,06
0,04
0,02
0
0
20
40
60
80
x(μm)

Monodisperse distribution



Bi-Modal distribution

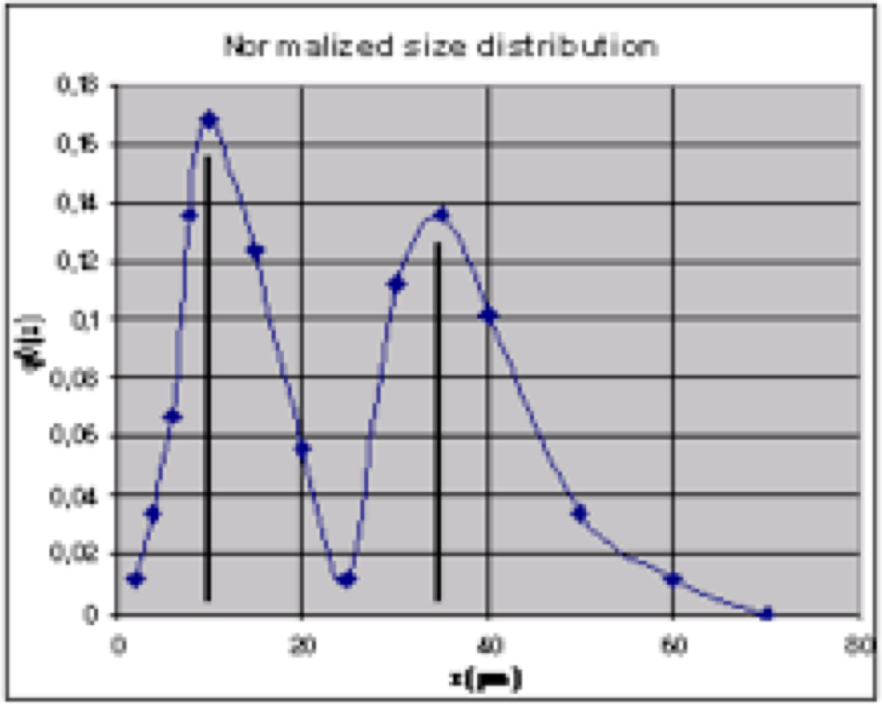


Exercises

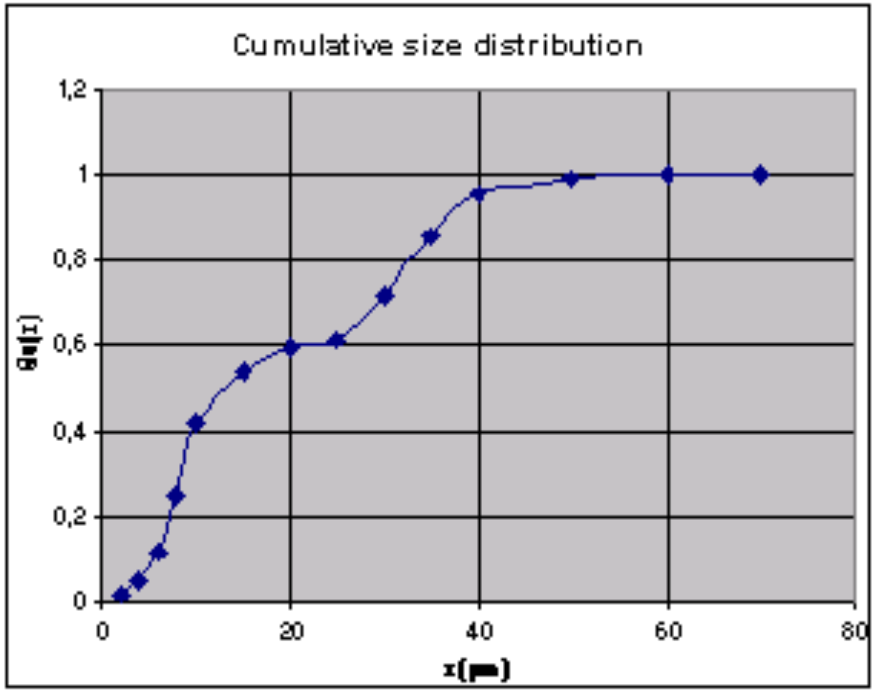
Determine and write down the cumulative undersize distribution Q_u. What is the size with 50 per-cent of the particles smaller/larger, x_{50} ?

x(µm)	x(m)	q(x)		$q_0(x)$	Q_u	$q_2(x)$	$q_3(x)$
2	2.00E-06	0.01		0.01123596	0.01123596	1.41124E-13	4.70412E-20
4	4.00E-06	0.03		0.03370787	0.04494382	1.69348E-12	1.12899E-18
6	6.00E-06	0.06		0.06741573	0.11235955	7.62067E-12	7.62067E-18
8	8.00E-06	0.12		0.13483146	0.24719101	2.70957E-11	3.61276E-17
10	1.00E-05	0.15		0.16853933	0.41573034	5.29213E-11	8.82022E-17
15	1.50E-05	0.11		0.12359551	0.53932584	8.73202E-11	2.18301E-16
20	2.00E-05	0.05		0.05617978	0.59550562	7.05618E-11	2.35206E-16
25	2.50E-05	0.01		0.01123596	0.60674157	2.20506E-11	9.18773E-17
30	3.00E-05	0.10		0.11235955	0.71910112	3.17528E-10	1.58764E-15
35	3.50E-05	0.12		0.13483146	0.85393258	5.18629E-10	3.02534E-15
40	4.00E-05	0.09		0.1011236	0.95505618	5.08045E-10	3.38697E-15
50	5.00E-05	0.03		0.03370787	0.98876404	2.64607E-10	2.20506E-15
60	6.00E-05	0.01		0.01123596	1	1.27011E-10	1.27011E-15
70	7.00E-05	0.00		0	1	0	0
	Σq(x) =	0.89		1		2.00523E-09	1.21536E-14

Exercises

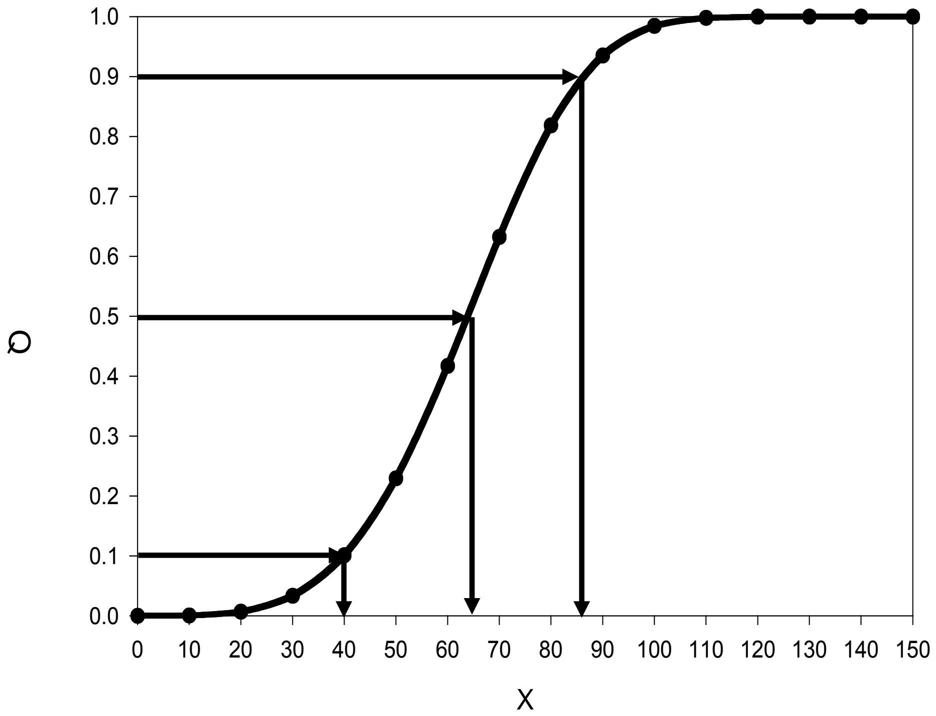


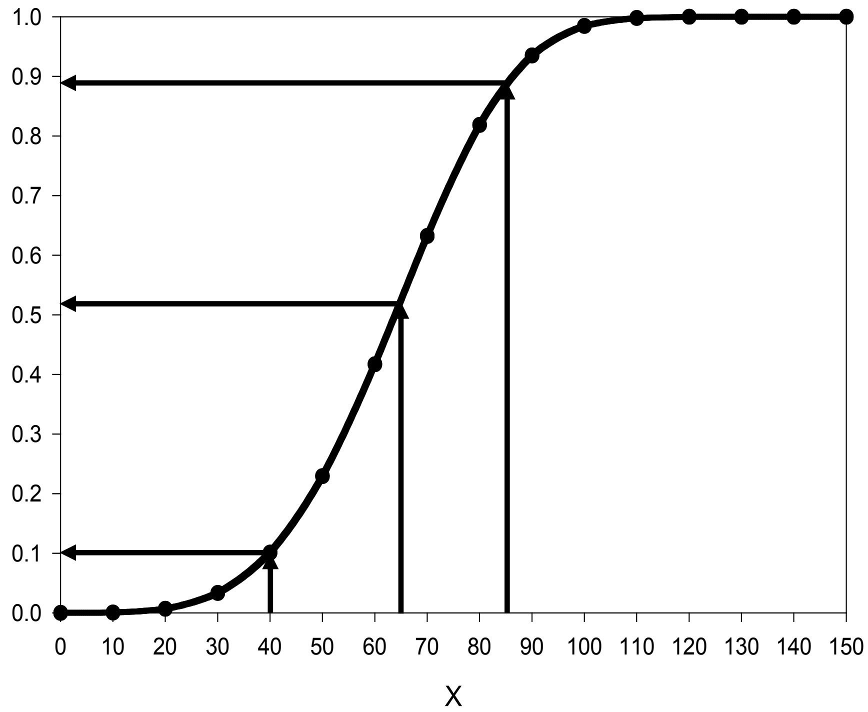
Normalized size distribution
0,18
0,16
0,14
0,12
0,1
0,08
0,06
0,04
0,02
0
0
20
40
60
80
x(µm)



Cumulative size distribution
1,2
1
0,8
0,6
0,4
0,2
0
0
20
40
60
80
x(µm)

Special Sizes





$$x_{10}, x_{50}, x_{90}$$

Index % are smaller than: x_{Index}

Transformations

Any normalized size-distribution

$$q_0(x)$$

$$\int q_0(x)\,dx = 1$$

can be transformed to:

$$q_S(x) = \int_0^x q_0(x')\,\pi(x')^2\,dx'$$

S — **Surface area distribution**

$$q_V(x) = \int_0^x q_0(x')\,\tfrac{\pi}{6}(x')^3\,dx'$$

V — **Volume distribution**

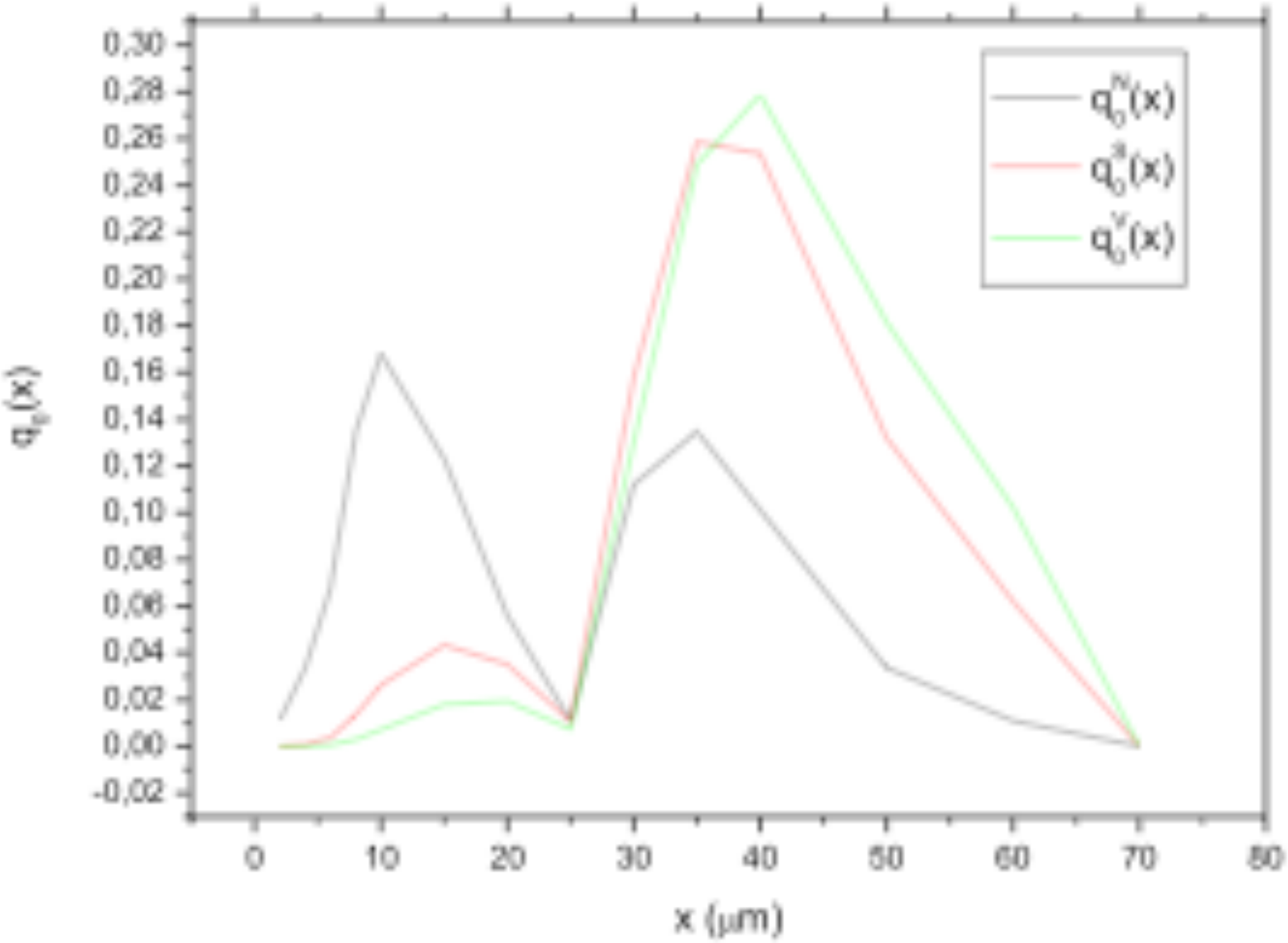
(These transformations are valid for spheres only)

Exercises

Assuming that all particles in the sample are spherical, estimate the specific surface area of the sample in m^2/kg.

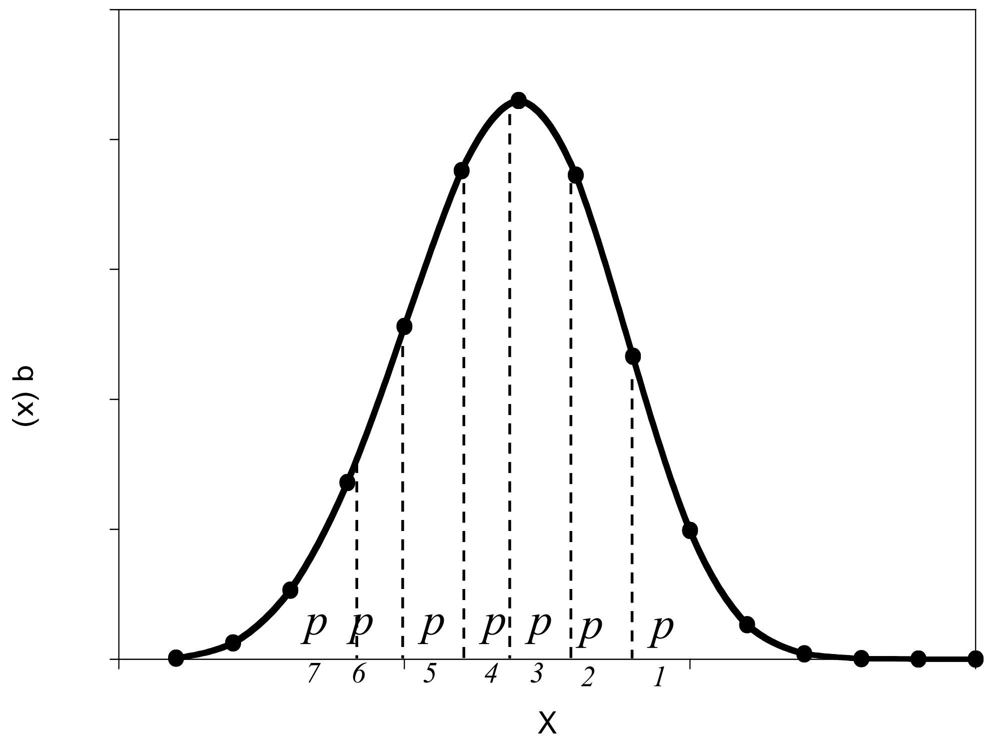
x(μm)	x(m)	q(x)		$q_0(x)$	Q_u	$q_2(x)$	$q_3(x)$
2	2.00E-06	0.01		0.01123596	0.01123596	1.41124E-13	4.70412E-20
4	4.00E-06	0.03		0.03370787	0.04494382	1.69348E-12	1.12899E-18
6	6.00E-06	0.06		0.06741573	0.11235955	7.62067E-12	7.62067E-18
8	8.00E-06	0.12		0.13483146	0.24719101	2.70957E-11	3.61276E-17
10	1.00E-05	0.15		0.16853933	0.41573034	5.29213E-11	8.82022E-17
15	1.50E-05	0.11		0.12359551	0.53932584	8.73202E-11	2.18301E-16
20	2.00E-05	0.05		0.05617978	0.59550562	7.05618E-11	2.35206E-16
25	2.50E-05	0.01		0.01123596	0.60674157	2.20506E-11	9.18773E-17
30	3.00E-05	0.10		0.11235955	0.71910112	3.17528E-10	1.58764E-15
35	3.50E-05	0.12		0.13483146	0.85393258	5.18629E-10	3.02534E-15
40	4.00E-05	0.09		0.1011236	0.95505618	5.08045E-10	3.38697E-15
50	5.00E-05	0.03		0.03370787	0.98876404	2.64607E-10	2.20506E-15
60	6.00E-05	0.01		0.01123596	1	1.27011E-10	1.27011E-15
70	7.00E-05	0.00		0	1	0	0
	Σq(x) =	0.89		1		2.00523E-09	1.21536E-14

Exercises



0,30
0,28
0,26
0,24
0,22
0,20
0,18
0,16
0,14
0,12
0,10
0,08
0,06
0,04
0,02
0,00
-0,02
$q_0(x)$
0
10
20
30
40
50
60
70
80
x (μm)
$q_0^N(x)$
$q_0^a(x)$
$q_0^\gamma(x)$

Matrix Representation



$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \end{bmatrix}$$

Summary

- Statistics - mathematics
- Distribution functions contain:

  Mean value

  Width

  & more
- Hard work ☹